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Eleftherios Karipidis, Nicholas Sidiropoulos, Amir Leshem and Li Youming, Capacity Statistics for Short DSL Loops from Measured 30 MHz Channel Data, 2005, Proceedings of the 6th IEEE International Workshop on Signal Processing Advances for Wireless Communications (SPAWC), 174-178.

<http://dx.doi.org/10.1109/SPAWC.2005.1505891>

Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-67044>

Capacity Statistics for Short DSL Loops from Measured 30 MHz Channel Data

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Abstract — In recent years, there is growing interest in hybrid fiber-copper access solutions, as in fiber to the basement (FTTB) and fiber to the curb/cabinet (FTTC), combined with advanced vectored transmission modalities. The twisted pair segment in these architectures is in the range of a few hundred meters, thus supporting transmission over up to 30 MHz. In this paper, we assess the capacity potential of these very short loops using full-binder channel measurements collected by France Telecom R&D. Key statistics are provided for both uncoordinated and vectored transmission, and the vectoring benefit is evaluated. The results provide useful bounds for developers and providers alike.

I. INTRODUCTION

Fiber to the basement (FTTB) and fiber to the curb/cabinet (FTTC) architectures have attracted considerable interest in recent years as promising low-overhead solutions for broadband network access to businesses and residential premises. Unlike, e.g., ADSL, wherein the twisted-pair copper loop length is in the order of 2–3 kilometers, FTTB/FTTC architectures entail much shorter copper segments, typically a few hundred meters. Insertion loss (IL) decays more gracefully with frequency at these lengths, potentially supporting transmission over up to 30 MHz for the shorter loops, which is 50% more than in VDSL 998. At the same time, far-end crosstalk (FEXT) becomes more prominent at these shorter loop lengths - e.g., at 75m, FEXT behaves quite similarly to near-end crosstalk (NEXT), as is intuitive [7]. For this reason, coordinated (also known as *vector*) multiple - input multiple - output (MIMO) transmission [5] becomes even more important in this context.

While there have been considerable advances in vectoring techniques, and several companies (both start-up and major players) have been developing MIMO prototypes for use in both ADSL and VDSL-like FTTB/FTTC architectures, there has been no publicly available experimental evaluation of the capacity potential of these schemes, using measured channel data. This is important for developers and providers alike, for enhanced capacity is the key selling point.

One reason for this lack of capacity blueprints is the associated lack of broadband short-length copper channel measurements. For the NEXT-limited case, [11] provides capacity estimates based on measurements by Telcordia [8], but the data is limited to ADSL frequencies (see also [2]). As part of the EU-FP6 U-BROAD project # 506790, France Telecom R&D conducted a comprehensive measurement campaign for short VDSL loops up to 30 MHz, during which IL, NEXT, and FEXT channels were measured for S88 cable comprising 14 quads, i.e., 28 loops. The measured lengths were 75m, 150m, 300m, and 590m. For each length, all 378 (28 picks 2) crosstalk channels of each type

(NEXT, FEXT) were measured, for a total of over 3000 crosstalk channels. For each channel, a log-frequency sweeping scheme was used to measure the inphase/quadrature (I/Q)¹ components of the frequency response from 10 KHz – 30 MHz, yielding 801 complex samples per channel. Piecewise cubic Hermite interpolation was used to convert these complex samples to a linear frequency scale. This paper describes the results of the associated capacity analysis.

The capacity of copper transmission channels depends strongly on the realization of crosstalk - that is, the selection and type of active loops in the binder. Thus, capacity is a random variable, characterized by a density parameterized by the type and number of crosstalk interferers. For brevity, we focus herein on reporting key capacity statistics (minimum, mean, maximum per-loop capacity); but we also provide a few representative plots of capacity distribution functions. We consider both “light” and “full”, FEXT-only, and NEXT plus FEXT crosstalk models, as well as the effect of coordination.

In the downstream direction, one can distinguish two MIMO communication scenarios of interest: point-to-point (P2P), wherein not only the transmitters but also the receivers of the MIMO subsystem are physically co-located, and thus joint receive processing is possible; and point-to-multi-point (P2M), wherein joint receive processing is not possible. Whereas the capacity of P2P MIMO can be readily calculated under standard assumptions, computing the capacity for P2M scenarios is far more difficult. Still, the capacity of a P2M MIMO system is bounded above by the capacity of the associated P2P MIMO system. For these reasons, we focus on computing P2P capacity statistics. Similar comments hold for the upstream direction, except that it is relatively easier to compute the (so-called *medium access control*-) capacity when joint receive processing is possible, but the transmitters cannot be coordinated.

We begin with a short introduction to capacity calculations for single-loop and coordinated DSL transmission subject to crosstalk. For coordinated MIMO transmission, we start from the general MIMO capacity formula, and work our way through the various application-specific assumptions and simplifications. In particular, we briefly discuss self-NEXT echo-cancellation, and self-FEXT transmit pre-compensation, and their effect on the crosstalk covariance.

Since capacity depends on the specific configuration of interferers (not only on the number thereof), it is important to calculate the capacity distribution and associated summary metrics as a function of the number of coordinated loops and the type and number of interferers. For this purpose, one can either opt for full enumeration-based computation of the capacity distributions, or computationally simpler Monte-Carlo sampling-based estimates. We will use both options in the sequel.

¹Contact author. Work supported by the EU-FP6 under U-BROAD STREP contract # 506790

¹Note that magnitude information alone is not sufficient for assessing capacity in the case of coordinated transmission - phase is also important.

II. CAPACITY CALCULATIONS AND RELATED ASSUMPTIONS

For general background on capacity, we refer the reader to Cover & Thomas [3]. On the capacity of MIMO systems, see [4, 10]. References [6, 1] are good background on DSL system capacity. On coordinated transmission over a pair of DSL lines see [9], and also Ginis & Cioffi [5] for an overview of the current state-of-art regarding coordinated transmission for multi-pair DSL. Crosstalk cancelation techniques for DSL systems are discussed in [12].

Let there be N loops in the binder ($N = 28$ for France Telecom's data), out of which L loops are employed for coordinated (vectored) transmission. Let L_N (L_F) denote the number of NEXT (respectively, FEXT) loops that interfere with the L coordinated loops. Let σ^2 denote the additive white Gaussian noise (AWGN) power spectral density (PSD) - typically at -140 dBm/Hz (or 10^{-17} W/Hz in absolute scale) for DSL systems.

For a single direct loop ($L = 1$), and a certain configuration of interfering loops, the Shannon capacity is given by [3, 6]

$$C = \int_{\text{BW}} \log_2(1 + \text{SINR}(f)) df \quad (1)$$

where BW denotes the available bandwidth, and the Signal to Interference plus Noise Ratio

$$\text{SINR}(f) = \frac{|H_{\text{IL}}(f)|^2 p(f)}{\sigma^2 + \sum_{i=1}^{L_N} |H_{N,i}(f)|^2 p_N(f) + \sum_{j=1}^{L_F} |H_{F,j}(f)|^2 p_F(f)} \quad (2)$$

Here, $H_{\text{IL}}(f)$ is the insertion loss (frequency response) of the direct channel, $H_{N,i}(f)$ ($H_{F,j}(f)$) is the frequency response of the i -th NEXT (resp. j -th FEXT) channel, $p(f)$ is the PSD (or, *spectral mask*) employed for the direct channel, while $p_N(f)$ ($p_F(f)$) is the PSD of NEXT (resp. FEXT) interferers. Assuming that all spectral masks are equal, equation (2) simplifies to

$$\text{SINR}(f) = \frac{|H_{\text{IL}}(f)|^2}{\frac{\sigma^2}{p(f)} + \sum_{i=1}^{L_N} |H_{N,i}(f)|^2 + \sum_{j=1}^{L_F} |H_{F,j}(f)|^2} \quad (3)$$

In the vectored case ($L > 1$) the capacity of the coordinated system is given by

$$C = \int_{\text{BW}} \log_2 \det(\mathbf{I} + p(f) \mathbf{H}(f) \mathbf{R}_{nn}^{-1}(f) \mathbf{H}^H(f)) df \quad (4)$$

where $\mathbf{H}(f)$ is the $L \times L$ input-output MIMO channel transfer matrix at frequency f , H denotes Hermitian (conjugate) transpose, and $\mathbf{R}_{nn}(f)$ is the $L \times L$ interference plus noise covariance matrix at the output of the MIMO subsystem at frequency f :

$$\mathbf{R}_{nn} = p(f) \mathbf{G}_N(f) \mathbf{G}_N^H(f) + p(f) \mathbf{G}_F(f) \mathbf{G}_F^H(f) + \sigma^2 \mathbf{I} \quad (5)$$

where $\mathbf{G}_N(f)$ is a $L \times L_N$ crosstalk transfer matrix, whose (m, ℓ) -element is the complex coupling coefficient from the ℓ -th NEXT disturber to the m -th loop in the vectored subsystem at frequency f ; and similarly for the $L \times L_F$ FEXT coupling matrix, $\mathbf{G}_F(f)$.

The matrix $\mathbf{H}$ is diagonally-dominated in DSL systems, wherein insertion loss is usually 20 or more dB higher than interference. For this reason, it is possible to pre-equalize $\mathbf{H}$ at the transmitter's side, without a significant penalty in terms of transmission power. Thus, upon pre-multiplication by

$$\mathbf{H}^{-1} \text{diag}([H_{\text{IL},1}(f), \dots, H_{\text{IL},L}(f)]),$$

the effective MIMO channel transfer matrix becomes

$$\text{diag}([H_{\text{IL},1}(f), \dots, H_{\text{IL},L}(f)]).$$

If we further assume that all insertion loss channels of the vectored subsystem are approximately equal [11] (this is well-justified, for insertion loss primarily depends on length, termination and bridge taps), then the capacity expression further simplifies to

$$C = \int_{\text{BW}} \log_2 \det(\mathbf{I} + |H_{\text{IL}}(f)|^2 p(f) \mathbf{R}_{nn}^{-1}(f)) df \quad (6)$$

A few remarks are in order:

- Equation (6) effectively assumes that self-FEXT (from within the vectored subsystem) has been pre-compensated at the transmitter. External (often called *alien*) FEXT, from the remaining loops in the binder, is accounted for in $\mathbf{R}_{nn}(f)$ (the $p_F(f) \mathbf{G}_F(f) \mathbf{G}_F^H(f)$ term).
- Self-NEXT at the receiver can be mitigated by employing *echo cancellation* techniques, which essentially amount to subtracting self-NEXT interference to a given loop from other loops in the coordinated subsystem, taking into account the associated frequency-dependent coupling factor. If echo cancellation is employed at the receiver, then there is only alien NEXT, if any. If upstream-downstream Frequency Division Duplex (FDD) is further employed, then alien NEXT is effectively suppressed as well. In this case, only alien FEXT remains. In non-FDD systems, however, alien NEXT is the performance-limiting factor, for FEXT is usually much lower than NEXT, even for relatively short loops (one significant exception is very short loops, under 100 meters, wherein FEXT looks much like NEXT, for obvious reasons).
- Because interference is generally correlated ($\mathbf{R}_{nn}(f)$ in Equation (6) is not diagonal), the capacity in Equation (6) implicitly assumes that the coordinated system's receiver employs multi-user detection.
- Capacity in Equation (4) depends on the particular configuration of coordinated loops and interferers in the binder. Even under the simplifying assumptions leading to Equation (6), capacity depends not only on the number, but also on the particular configuration of interferers, through the coupling coefficients in $\mathbf{G}_N(f)$ and $\mathbf{G}_F(f)$. For this reason, we are interested in assessing the *capacity distribution* for a given number of FEXT interferers, as is appropriate for echo-cancelled, FDD, transmit pre-compensated coordinated subsystems of order L . We are interested in measuring the per-loop capacity as a function of L , and L_F . The per-loop capacity is the MIMO capacity of the coordinated subsystem divided by L . Furthermore, in the case of non-FDD systems, we are interested in the distribution of C/L as a function of L , L_N , and L_F . This will allow us to assess the capacity benefit afforded by vectoring, i.e., the impact of coordinated transmission on the per-loop capacity as a function of the size of the coordinated subsystem and the number, type, and configuration of disturbers.
- For each L , L_N , and L_F , we will thus obtain a capacity distribution. This distribution can be summarized by means of a few key statistics: minimum, mean, maximum per-loop capacity. We compute those for a range of configurations of interest, and plot as a function of the parameters L , L_N , and L_F .
- Capacity also depends on the spectral mask, $p(f)$. Depending on whether or not frequency planning is used (e.g., FDD and coarse frequency-dependent power-loading), $p(f)$ may vary with f . We will thus consider two cases: one in which $p(f)$ is set to -60 dBm/Hz (10^{-9} W/Hz in absolute scale) for all f (this is a typical PSD level for DSL modems, and maintaining it across

the 30 MHz bandwidth will yield slightly optimistic capacity results; but also use a standard FDD frequency plan (VDSL 998) and its extension to frequencies up to 30 MHz (extended VDSL 998). In the first case (constant $p(f)$ at -60 dBm/Hz), we compute the total link capacity, which can be split between upstream and downstream using, e.g., TDMA or suitable band plan. Furthermore, since we are interested in assessing theoretical bounds on performance, we will also ignore implementation issues like modulation loss, noise margin, and coding gain. In the second case (VDSL 998 or extended VDSL 998), we will incorporate these practical considerations. Pursuing both scenarios will allow us to gauge how far practical implementations will be from theoretical capacity.

III. CAPACITY STATISTICS VIA ENUMERATION

Assuming, as we did, that the insertion loss for all coordinated loops is identical, the clear-cut way of calculating the capacity distribution is to go over all possible configurations of (L_N, L_F) alien interferers, calculate the capacity for each configuration using Equation (6), and then generate a histogram of the results. The difficulty is that certain combinations of L , L_N , and L_F generate an immense number of possible configurations; e.g., for a total of $N = 28$ loops in the binder, $L = 4$, and $L_N = L_F = 12$, generates about 246 million possibilities, for each of which the integral in Equation (6) must be approximated. To reduce complexity, we assume that whenever L is even, *quads* (i.e., pairs of loops in the same quad) are chosen for vectoring, as is likely to be the case in practice. However, it is still easy to get into situations wherein generating the capacity distribution for a given L , L_N , and L_F takes several weeks in a state-of-art PC. For this reason, we let L take all possible values, but restrict the choice of the number of alien interferers to two extreme cases: the case of “light” crosstalk (only one interfering loop); and “full” crosstalk (all but one of the remaining loops in the binder are interfering). In Section IV we will complement our results with Monte-Carlo sampling-based estimates of the capacity distribution, wherein loops are drawn at random from the binder. Monte-Carlo techniques yield an *estimate* of the capacity distribution, but allow us to consider cases that are essentially intractable using the enumeration-based approach. The results of the enumeration approach for loop lengths of 75, 150, 300, and 590 meters are summarized in Figures 1–4, and 5–8, for the non-FDD and the FDD case, respectively.

IV. CAPACITY CCDFs VIA MONTE-CARLO SAMPLING

In this section, we present estimated Complementary Cumulative Distribution Functions (CCDFs) for capacity, using Monte-Carlo sampling to randomly select loops from the binder. This is a computationally simpler alternative to enumeration-based full-binder capacity calculations, presented in the previous section. We assume that a coordinated system with L channels is employed, along with L_F alien FEXT interferers drawn from the same binder. Unlike the results presented in the previous section (which assumed a flat transmission power spectrum at -60 dBm/Hz), in this section we employ two frequency band plans, and associated spectral masks. The first is VDSL 998 with downstream (DS) frequency bands as 0.1–3.75 MHz and 5.2–8.5 MHz; while the second plan is called extended VDSL 998 with DS frequency bands as 0.1–3.75 MHz, 5.2–8.5 MHz, and 12–20 MHz. In all frequency bands, the transmitter has PSD of -60 dBm/Hz. The noise is assumed to be AWGN with PSD of -140 dBm/Hz. Due to space constraints, we only present CCDF plots for 300m and $L = 8$ coordinated pairs.

Attaining Shannon capacity requires Gaussian signaling and long codes. In reality, there is an effective SINR loss due to the use of practical modulation schemes and the need to guarantee a certain noise margin. This SINR penalty is alleviated to a certain extent by the use of

advanced coding techniques. The combined effect of modulation loss, noise margin, and coding gain can be captured by replacing SINR by SINR/Γ , where the gap Γ in dB scale is $\Gamma_{\text{dB}} \triangleq \ell_{\text{mod}} + m_{\text{noise}} - g_{\text{coding}}$. Here, ℓ_{mod} is the modulation loss (9.8 dB for QAM at BER of 10^{-7}), m_{noise} is the noise margin (set to 6 dB for our M-C simulations), and g_{coding} is the coding gain. For coding gain we use $g_{\text{coding}} = 3.8$ or 7.8 dB (Reed Solomon or advanced coding), which correspond to a gap $\Gamma_{\text{dB}} = 12$ or 8 dB, respectively, at BER 10^{-7} . Figures 9 and 10 depict the average achieved rate per loop CCDFs, assuming $L_F = 12$ alien FEXT disturbers and using 400 independent Monte-Carlo runs, for the VDSL 998 band plan and the extended one, respectively. We also provide capacity results ($\Gamma_{\text{dB}} = 0$ dB) for comparison. Figure 11 depicts capacity CCDFs for the extended VDSL 998 band plan and for different choices of the number L_F of alien FEXT disturbers.

V. CONCLUSIONS

Mean per-loop capacity ranges from about 0.72 Gbps down to 34 Mbps, depending on length and scenario considered. The benefit afforded by coordination in terms of mean capacity manifests itself at relatively heavy system loads, as expected. Note, however, that coordination can significantly reduce the capacity spread even in lightly loaded systems, which is important from the operators’ perspective. The typical per-loop mean capacity with coordination is under $2\times$ the capacity without coordination, but in certain cases it can be as high as $4.4\times$. For light crosstalk, $L = 6$ is a breakpoint, beyond which virtually all of the coordination benefits are reaped. For full crosstalk, one has to coordinate at least half the binder ($L = 14$) in order to see any improvement. In this case, the improvements are rapidly amplified with increasing L .

The statistics provided herein are relatively optimistic, in the sense that they do not account for, e.g., shaping loss due to modulation, noise margin, coding gain, etc. Nevertheless, they are useful bounds on what is attainable in practice without spectral optimization. E.g., for high SINR, every 3 dB in gap yield capacity loss $\sim BW$ Mbps.

Acknowledgment: We would like to thank Dr. Meryem Ouzzif and Dr. Rabah Tarafi of France Telecom R&D, who conducted the VDSL channel measurements under the auspices of the U-BROAD project.

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Figure 1: Minimum/Average/Maximum capacity per loop; Non-FDD, 75m

Figure 2: Minimum/Average/Maximum capacity per loop; Non-FDD, 150m

Figure 3: Minimum/Average/Maximum capacity per loop; Non-FDD, 300m

Figure 4: Minimum/Average/Maximum capacity per loop; Non-FDD, 590m

Figure 5: Minimum/Average/Maximum capacity per loop; FDD, 75m

Figure 6: Minimum/Average/Maximum capacity per loop; FDD, 150m

Figure 9: DS capacity and achievable rates CCDF; VDSL 998, $L = 8$, $L_F = 12$, 300m

Figure 7: Minimum/Average/Maximum capacity per loop; FDD, 300m

Figure 10: DS capacity and achievable rates CCDF; extended VDSL 998, $L = 8$, $L_F = 12$, 300m

Figure 8: Minimum/Average/Maximum capacity per loop; FDD, 590m

Figure 11: DS capacity CCDF; extended VDSL 998, $L = 8$, $L_F = \{0, 1, 4, 8, 12\}$, 300m