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Eleftherios Karipidis, Nicholas Sidiropoulos and Leandros Tassiulas, Joint QoS Multicast Power / Admission Control and Base Station Assignment: A Geometric Programming Approach, 2008, Proceedings of the 5th IEEE Workshop on Sensor Array and Multi-Channel Signal Processing (SAM), 155-159.

<http://dx.doi.org/10.1109/SAM.2008.4606845>

Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-67052>

JOINT QOS MULTICAST POWER / ADMISSION CONTROL AND BASE STATION ASSIGNMENT: A GEOMETRIC PROGRAMMING APPROACH

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ABSTRACT

The joint power control and base station (BS) assignment problem is considered under Quality-of-Service (QoS) constraints. If a feasible solution exists, the problem can be efficiently solved using existing distributed algorithms. Infeasibility is often encountered in practice, however, which brings up the issue of optimal admission control. The joint problem is NP-hard, yet important for QoS provisioning and bandwidth-efficient operation of existing and emerging cellular and overlay/underlay networks. Recognizing this, there have been several attempts to develop reasonable heuristics for joint admission and power control. This contribution takes a more disciplined approach. The joint problem is first concisely formulated as a constrained optimization problem, whose objective combines the BS assignment, admission, and power control components. The formulation also allows for multicasting. A geometric programming approximation is then developed, which forms the core of a heuristic, yet well-motivated centralized algorithm that generates approximate solutions to the original NP-hard problem. Numerical results against an enumeration baseline illustrate the merits of the approach.

1. INTRODUCTION

Consider a wireless network comprising K single-antenna receivers. In a general multicast scenario, there are $G \in \{1, \dots, K\}$ distinct data streams to be carried over the network. Each receiver is interested in a single stream, but in multicast mode several receivers may request the same stream. The two extreme values of G are of particular interest: when $G = 1$ all receivers are interested in the same content (i.e., the broadcast scenario), and when $G = K$ each receiver is interested in its own stream (e.g., the multiuser downlink scenario). Let $\mathcal{G}_i$ denote the index set of the receivers forming the i th multicast group, i.e., listening to the i th multicast stream. Under this setup, the multicast groups are mutually exclusive and partition the entire set of receivers, i.e., $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset$, $j \neq i$ and $\cup_i \mathcal{G}_i = \mathcal{K} := \{1, \dots, K\}$.

Regarding the transmissions, it is assumed that there are N degrees of freedom in the network. Specifically, each multicast stream can be transmitted in one or more (up to N) different dimensions. As dimensions we refer to base stations (BS's) located spatially apart but transmitting in the same channel, but the setup can also accommodate orthogonal dimensions, such as disjoint time slots or frequency bands. In this work, we will focus on the BS assignment scenario.

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Let $p_i^{(n)}$ denote the power at which BS $n \in \mathcal{N} := \{1, \dots, N\}$ transmits multicast stream $i \in \mathcal{G} := \{1, \dots, G\}$. The allowed or available transmission power is upper bounded by $P_i^{(n)}$, due to regulatory or power amplifier limitations. For every receiver $k \in \mathcal{G}_i$, let $\alpha_{i,k}^{(n)}$ and $\alpha_{j,k}^{(n)}$ denote the corresponding power gain of the direct and coupling links, respectively, to the n th BS. The link gains include the effects of propagation loss, shadowing, and fading, as well as beamforming and coding gains if any. They are considered known to the design center.

Due to the broadcast nature of the wireless medium and the cochannel transmissions, there is interference which must be accounted for. The quality of communication that receiver $k \in \mathcal{G}_i$ experiences, when tuned to the n th BS, is measured by the received signal-to-interference-and-noise ratio (SINR)

$$\text{SINR}_k^{(n)} = \frac{\alpha_{i,k}^{(n)} p_i^{(n)}}{\sum_{j \neq i} \alpha_{j,k}^{(n)} p_j^{(n)} + \sum_{m \neq n} \sum_{i=1}^G \alpha_{i,k}^{(m)} p_i^{(m)} + \sigma_k^2} \quad (1)$$

where σ_k^2 denotes receiver's noise variance. Interference consists of two terms: the first accounts for the interfering multicasts coming from the n th BS, and the second for all the transmissions from the other BS's. Note that the stream of interest is included in the second term, due to incoherent combining. The multicast stream can be decoded successfully by the k th receiver when the received SINR is equal to or greater than a threshold γ_k , determined by application requirements, and the modulation and coding scheme. A receiver k is considered *assigned* to the n th BS when $\text{SINR}_k^{(n)} \geq \gamma_k$, and *served* or *admitted* when its Quality of Service (QoS) requirement can be guaranteed, i.e., when it is assigned to any BS.

The joint QoS power control and BS assignment problem can be efficiently solved, provided that a feasible solution exists, using existing distributed algorithms, e.g., [6], [11], [5]. However, infeasibility is often encountered in practice, which brings up the issue of optimal admission control. Joint admission and power control is NP-hard; reasonable heuristics have been developed to find approximate solutions, e.g., [3], [10]. This contribution takes a more disciplined approach to the joint QoS power control and BS assignment problem, proposing a general constrained optimization formulation that incorporates admission control and multicasting. Explicitly stated, the problem of interest is to find the optimum i) assignment of receivers to BS's, (including a "void" assignment that accounts for admission control); and ii) transmission powers, in order to maximize the number of admitted receivers and minimize the total transmission power required to serve them.

2. JOINT PROBLEM FORMULATION

Let us introduce the auxiliary binary variables $\{s_k^{(n)} \in \{0, 1\}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$, one for every possible receiver-BS pair. The role of the binary variable $s_k^{(n)}$ is to determine whether the k th receiver can be assigned to the n th BS. Using these auxiliary variables, the joint QoS multicast power and admission control, and BS assignment problem can be written as the constrained optimization

$$\min_{\{p_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}, \{s_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}} \epsilon \sum_{i=1}^G \sum_{n=1}^N p_i^{(n)} + (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + s_k^{(n)}} \quad (2)$$

$$\text{s. t. } 0 \leq p_i^{(n)} \leq P_i^{(n)} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}, \quad (3a)$$

$$\text{SINR}_k^{(n)} \geq \gamma_k s_k^{(n)} \quad \forall k \in \mathcal{K} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}, \quad (3b)$$

$$s_k^{(n)} \in \{0, 1\} \quad \forall k \in \mathcal{K} \quad \forall n \in \mathcal{N}, \quad (3c)$$

$$\sum_{n=1}^N s_k^{(n)} \leq 1 \quad \forall k \in \mathcal{K}, \quad (3d)$$

where the $\text{SINR}_k^{(n)}$ term in the left side of inequalities (3b) is defined in (1) to be the SINR of the k th receiver when tuned to the n th BS.

As per (3b), N SINR constraints (one per available BS) are *a priori* defined for every receiver. Focusing on the (k, n) th inequality of (3b), it is seen that the auxiliary variable $s_k^{(n)}$ is multiplicative to the SINR threshold γ_k . The value of the binary variable determines whether the respective SINR constraint is accounted (or not) in the power control part of the joint problem; thus, $s_k^{(n)}$ can be interpreted as a switch that activates (or not) the inequality. Specifically, for $s_k^{(n)} = 1$, the constraint is enforced on the transmission powers, so that $\text{SINR}_k^{(n)}$ is requested to be at least equal to γ_k . On the contrary, for $s_k^{(n)} = 0$, the respective inequality (3b) is reduced to the trivial nonnegativity constraint for the transmission power $p_i^{(n)}$, which is already included in the left part of (3a); hence, no further constraint is actually imposed. Note that when no receiver of the i th multicast group is assigned to the n th BS (i.e., $s_k^{(n)} = 0 \quad \forall k \in \mathcal{G}_i$), then, the optimization problem (2)–(3) zeroes the respective transmission power (i.e., $p_i^{(n)} = 0$). This is because $p_i^{(n)}$ counts solely as interference in this case, since it appears only in the denominator of “active” SINR constraints (3b) and the left term of the objective function (2) minimizes the total transmission power. Hence, there is no need to explicitly account for the non-transmitted multicast streams in the denominator of $\text{SINR}_k^{(n)}$ (cf. (1)) in (3b).

The inequalities in (3d) constrain each receiver to be assigned to at most one BS. Since QoS is already guaranteed to a receiver when just one out of N respective constraints (3b) is “active”, assigning him to many BS’s solely increases the interference in the wireless network. In case there are extra degrees of freedom (i.e., other feasible assignments), the system would rather utilize them to serve more receivers or decrease the total transmission power. Note that by letting the sum in (3d) take values smaller than 1 (actually 0), admission control is included in the joint optimization problem (2)–(3). When there are not enough resources, the system is allowed to deny service to any receiver, say k , (setting $s_k^{(n)} = 0 \quad \forall n \in \mathcal{N}$) in order to ensure service to the remaining ones. If the inequalities (3d) were replaced with equalities, the resulting joint problem would be a restriction of (2)–(3), since the set of feasible solutions would then be a subset of the original one. The restricted problem becomes infeasible when it

is not possible to admit all receivers. On the contrary, inequalities (3d) imply the following result.

Claim 1 *Optimization problem (2)–(3) is always feasible.*

Proof: A trivial feasible solution, i.e., satisfying the constraints (3), is $\{s_k^{(n)} = 0\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ and any $\{p_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}$ conforming to (3a). ■

However, this trivial solution is the worst possible from a QoS perspective, since none of the K receivers is served. As noted in Section 1, the primal design goal of the joint QoS problem of interest, formulated in (2)–(3), is to maximize the number of receivers served, which, by virtue of (3d), equals to the value of the auxiliary variables’ total sum. For this reason, apart from the total power minimization, a second term has been included in the objective function (2), which is pertinent to the BS assignment / admission part of the joint problem. This term is a function that favors the value 1 for the binary variables, by increasing the penalty when they take the value 0 instead. Specifically, focusing on any receiver, say k , the objective function term of the BS assignment / admission problem is $\prod_{n=1}^N 1/(1 + s_k^{(n)})$. The associated cost for assigning the receiver to a BS, say n , is 2^{-1} ($s_k^{(n)} = 1$ and $s_k^{(m)} = 0, \forall m \neq n$) and 1 for not admitting it ($s_k^{(n)} = 0, \forall n \in \mathcal{N}$). From (2), the total assignment cost is the product of the individual ones; hence, it is equal to $2^{-\hat{K}}$, where $\hat{K} \in \{0, 1, \dots, K\}$ is the number of receivers served. When all receivers are served, the assignment cost is equal to 2^{-K} and doubles for every receiver that is not admitted. Observe that the assignment cost is discrete-valued with minimum step size of 2^{-K} that doubles for every step.

The overall cost of the joint QoS problem (2)–(3) is the weighted sum of the power and the assignment cost. The power is continuous and upper bounded by

$$\sum_{i=1}^G \sum_{n=1}^N p_i^{(n)} \stackrel{(3a)}{\leq} \sum_{i=1}^G \sum_{n=1}^N P_i^{(n)} \leq GNP, \quad (4)$$

where $P \geq \max_{i \in \mathcal{G}, n \in \mathcal{N}} P_i^{(n)}$. The weighting factor $\epsilon \in [0, 1]$ is chosen so that service is denied to some receivers only if this event cannot be avoided. Adapting the ruler analogy argument in [8] to a logarithmic scale, ϵ is chosen so that the minimum penalty paid for denying service to a receiver is higher than the maximum potential gain from the total power minimization, i.e.,

$$(1 - \epsilon)2^{-K} > \epsilon GNP \Leftrightarrow \epsilon < \frac{2^{-K}}{GNP + 2^{-K}}. \quad (5)$$

Claim 2 *For $\epsilon < \frac{2^{-K}}{GNP + 2^{-K}}$, optimization problem (2)–(3) maximizes the number of admitted receivers and minimizes the total transmission power required to serve them.*

Proof: Let $\{\{\tilde{p}_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}, \{\tilde{s}_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}\}$ denote an optimal solution of problem (2)–(3). Assume the existence of another feasible solution $\{\{\hat{p}_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}, \{\hat{s}_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}\}$ which serves 1 receiver more than the optimal solution, i.e., $\sum_{k=1}^K \sum_{n=1}^N \hat{s}_k^{(n)} = \sum_{k=1}^K \sum_{n=1}^N \tilde{s}_k^{(n)} + 1$. Then, it holds that

$$\prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \hat{s}_k^{(n)}} = \frac{1}{2} \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \tilde{s}_k^{(n)}} \quad (6)$$

and

$$\prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \tilde{s}_k^{(n)}} \geq 2^{-K} \stackrel{(6)}{\Rightarrow} \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}} \geq 2^{-K+1}. \quad (7)$$

The cost of the feasible solution is

$$\begin{aligned} & \epsilon \sum_{i=1}^G \sum_{n=1}^N \tilde{p}_i^{(n)} + (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \tilde{s}_k^{(n)}} \stackrel{(4), (6)}{\leq} \\ & \epsilon GNP + (1 - \epsilon) \frac{1}{2} \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}} = \\ & \epsilon GNP + (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}} - (1 - \epsilon) \frac{1}{2} \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}} \stackrel{(7)}{\leq} \\ & (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}} + \underbrace{\epsilon GNP - (1 - \epsilon) 2^{-1} 2^{-K+1}}_{< 0 \text{ (5)}} < \\ & (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}} \leq \\ & \underbrace{\epsilon \sum_{i=1}^G \sum_{n=1}^N \check{p}_i^{(n)}}_{> 0 \text{ (3a)}} + (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{1 + \check{s}_k^{(n)}}. \end{aligned}$$

Hence, it is smaller than the minimum objective value, which contradicts optimality of $\{\check{p}_i^{(n)}\}_{i \in \mathcal{G}}, \{\check{s}_k^{(n)}\}_{k \in \mathcal{K}}\}$ for problem (2)–(3). Thus, solution of (2)–(3) serves the maximum number of receivers possible under the constraints in (3). Given the optimum assignment $\{\check{s}_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ of receivers to BS's, the joint QoS problem (2)–(3) reduces to a *feasible* multicast power control problem, with respect to the variables $\{p_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}$, which minimizes the total transmission power, while ensuring the QoS targets of the admitted receivers. ■

The joint QoS problem (2)–(3) is nonconvex, since the auxiliary variables are binary. The computationally intensive components of the problem are the BS assignment and admission control. Due to its combinatorial nature, the optimization with respect to the variables $\{s_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ has exponential complexity—it can be proven that problem (2)–(3) is NP-hard. Given an assignment, the joint problem (2)–(3) reduces to the multicast power control problem, which is linear programming (LP) optimization, with respect to the variables $\{p_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}$, see, e.g. [10], [7].

3. RELAXATION TO GEOMETRIC PROGRAMMING

The binary constraints (3c) on the auxiliary variables are nonconvex. They can be equivalently written as

$$s_k^{(n)}(s_k^{(n)} - 1) = 0 \quad \forall k \in \mathcal{K} \quad \forall n \in \mathcal{N} \quad \text{or} \quad (8)$$

$$\begin{cases} s_k^{(n)}(s_k^{(n)} - 1) \leq 0 \\ s_k^{(n)}(s_k^{(n)} - 1) \geq 0 \end{cases} \quad \forall k \in \mathcal{K} \quad \forall n \in \mathcal{N}. \quad (9)$$

The quadratic inequalities on the upper branch of (9) are convex, whereas the others on the lower are not. Discarding the nonconvex ones, the auxiliary variables $\{s_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ are merely allowed to take any value in the interval $[0, 1]$, which is obviously convex. An immediate consequence of this relaxation is that the auxiliary variables

lose their strict functionality of taking “yes” or “no” decisions for the admission control and BS assignment problems. Now, they merely measure the ratio of SINR target satisfaction in (3b). Hence, the motivation for introducing constraint (3d) no longer holds, so that it can be discarded as well.

The relaxation of the feasible set for the auxiliary variables has the side effect that the right term of the objective function (2) does not take discrete values anymore. Furthermore, due to the absence of (3d), it now tries to push *all* auxiliary variables as close as possible to 1, i.e., it tries to assign every receiver to all BS's. Note that the denominator of the right objective function term consists of all possible cross-products of the auxiliary variables. Since all variables are positive and smaller than or equal to 1, the smallest term in the denominator, dominating the cost, is the product of the highest order, i.e., the one involving all variables. Thus, the denominator may be well approximated by this term. The resulting objective function term has simpler form: it is a monomial. It upper bounds the original objective function term and has essentially the same functionality, i.e., it penalizes severely the values of the auxiliary variables close to 0.

After the aforementioned relaxations, the joint QoS problem (2)–(3) is written as

$$\min_{\{p_i^{(n)}\}_{i \in \mathcal{G}}^{n \in \mathcal{N}}, \{s_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}} \epsilon \sum_{i=1}^G \sum_{n=1}^N p_i^{(n)} + (1 - \epsilon) \prod_{k=1}^K \prod_{n=1}^N \frac{1}{s_k^{(n)}} \quad (10)$$

$$\text{s. t.} \quad 0 \leq p_i^{(n)} \leq P_i^{(n)} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}, \quad (11a)$$

$$\begin{aligned} & \sum_{j \neq i} \alpha_{j,k}^{(n)} p_j^{(n)} + \sum_{m \neq n} \sum_{i=1}^G \alpha_{i,k}^{(m)} p_i^{(m)} + \sigma_k^2 \leq \\ & \alpha_{i,k}^{(n)} p_i^{(n)} (\gamma_k s_k^{(n)})^{-1} \quad \forall k \in \mathcal{K} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}, \end{aligned} \quad (11b)$$

$$0 \leq s_k^{(n)} \leq 1 \quad \forall k \in \mathcal{K} \quad \forall n \in \mathcal{N}. \quad (11c)$$

The relaxed problem (10)–(11) is a posynomial minimization subject to posynomial upper bound inequality constraints. Hence, it is a Geometric Programming (GP) problem in *standard form* [1, 2]. It can be readily transformed into *convex form*, by a logarithmic change of variables, and efficiently solved by means of interior point methods. In fact, there exist freely available MATLAB toolboxes, e.g., GGPLAB [9] and CVX [4], that accept as input GP problems in standard form.

4. ALGORITHM AND RESULTS

The introduction, in Section 2, of the auxiliary variables $\{s_k^{(n)}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ has enabled us to approach the QoS multicast power control, the admission control, and the BS assignment problems *jointly* with the formulation of the constrained optimization (2)–(3). However, this optimization is intractable, due to its combinatorial nature. The relaxation, in Section 3, of the binary constraints (3c) to the intervals (11c) has lead, on one hand, to the convex approximation (10)–(11) of the problem of interest. On the other hand, convexity is obtained at a considerable cost; the strict functionality of the auxiliary variables has been sacrificed in favor of the relaxation. However, the GP problem (10)–(11) is still useful; it can be used as the core component of heuristic algorithms to find approximate solutions to the original joint QoS problem. In the following, an iterative algorithm is presented, which takes binary BS assignment / admission decisions by pruning degrees of freedom in each iteration.

Let $\mathcal{G}_{i,n} \subseteq \mathcal{G}_i$ denote the set of receivers, interested in the i th multicast, which are *tentatively* tuned to the n th BS. Using these sets, the GP problem (10)–(11) is equivalently rewritten as

$$\min_{\{p_i^{(n)}\}, \{s_k^{(n)}\}_{k \in \mathcal{G}_{i,n}}}_{n \in \mathcal{N}} \epsilon \sum_{i=1}^G \sum_{n=1}^N p_i^{(n)} + (1-\epsilon) \prod_{n=1}^N \prod_{i=1}^G \prod_{k \in \mathcal{G}_{i,n}} \frac{1}{s_k^{(n)}} \quad (12)$$

$$\text{s. t. } 0 \leq p_i^{(n)} \leq P_i^{(n)} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}, \quad (13a)$$

$$\sum_{j \neq i} \alpha_{j,k}^{(n)} p_j^{(n)} + \sum_{m \neq n} \sum_{i=1}^G \alpha_{i,k}^{(m)} p_i^{(m)} + \sigma_k^2 \leq \alpha_{i,k}^{(n)} p_i^{(n)} (\gamma_k s_k^{(n)})^{-1} \quad \forall k \in \mathcal{G}_{i,n} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}, \quad (13b)$$

$$0 \leq s_k^{(n)} \leq 1 \quad \forall k \in \mathcal{G}_{i,n} \quad \forall i \in \mathcal{G} \quad \forall n \in \mathcal{N}. \quad (13c)$$

As noted in Section 2 for problem (2)–(3), when no receiver of the i th multicast group is assigned to the n th BS, i.e., $\mathcal{G}_{i,n} = \emptyset$, the optimization (12)–(13) yields $p_i^{(n)} = 0$. Moreover, it is important to take into consideration a peculiarity of the QoS multicast power control problem. The power of each multicast stream is controlled by one variable, but many SINR constraints (one for each receiver of the respective multicast group) need to be fulfilled at once. Typically, for a feasible instance of the problem, optimization yields SINR values which are greater than the targeted thresholds, for all but one receivers per multicast group.

The following algorithm initially assumes that all receivers can be tuned to all BS's that transmit the multicast stream of interest. Then, it iteratively rejects possible BS assignments in order to decrease the interference level, until (if possible) a single assignment is preserved per receiver that satisfies the requested QoS target.

Algorithm

1. Initialization: $\mathcal{G}_{i,n} = \mathcal{G}_i$, $\forall n \in \mathcal{N}$, $\forall i \in \mathcal{G}$.
2. Solve the GP problem (12)–(13) and denote the optimum solution $\left\{ \{p_i^{(n)}\}, \{s_k^{(n)}\}_{k \in \mathcal{G}_{i,n}} \right\}_{n \in \mathcal{N}}_{i \in \mathcal{G}}$.
3. **If** there exist receiver(s), assigned to multiple BS's, whose SINR target is reached by more than one BS, **then** find, among them, the receiver-BS pair $\{\tilde{k}, \tilde{n}\}$ with the largest SINR over-satisfaction, preserve assignment of receiver $\tilde{k}$ only to BS $\tilde{n}$ (i.e., $\mathcal{G}_{i,n} = \mathcal{G}_{i,n} - \tilde{k}$, $\forall n \neq \tilde{n}$, $\forall i \neq \tilde{i}$, for $\tilde{k} \in \mathcal{G}_{\tilde{i}}$), and go back to step 2, **else** go to next step.
4. **If** there exist receiver(s), assigned to multiple BS's, whose SINR target is reached by a single BS, **then** find, among them, the pair $\{\tilde{k}, \tilde{n}\}$ with the largest SINR, preserve assignment of receiver $\tilde{k}$ only to BS $\tilde{n}$, and go back to step 2, **else** go to next step.
5. **If** there exist receiver(s), assigned to multiple BS's, whose SINR target is not reached by any BS, **then** find, among them, the pair $\{\tilde{k}, \tilde{n}\}$ with the largest SINR, preserve assignment of receiver $\tilde{k}$ only to BS $\tilde{n}$, and go back to step 2, **else** go to next step.
6. **If** there exist receiver(s) whose SINR target is not reached, **then** find, among them, the pair $\{\tilde{k}, \tilde{n}\}$ with the minimum SINR, drop assignment of receiver $\tilde{k}$ to BS $\tilde{n}$ (i.e., $\mathcal{G}_{i,\tilde{n}} =$

$\mathcal{G}_{i,\tilde{n}} - \tilde{k}$, for $\tilde{k} \in \mathcal{G}_{\tilde{i}}$), and go back to step 2,

else the sets $\mathcal{G}_{i,n}$, $\forall i \in \mathcal{G}$, $\forall n \in \mathcal{N}$, contain a feasible assignment, i.e., $\{\{s_k^{(n)} = 1\}_{k \in \mathcal{G}_{i,n}}\}_{n \in \mathcal{N}}_{i \in \mathcal{G}}$, and the solution $\{p_i^{(n)}\}_{i \in \mathcal{G}}_{n \in \mathcal{N}}$ determines the minimum power levels required for service.

Note that in each iteration, the number of variables and constraints (hence, the complexity) of the core GP problem decreases. The overall worst-case complexity of the algorithm is $O((KN)^{4.5})$.

Fig. 1. Wireless network instance with fixed transmitters' locations

The performance of the algorithm is evaluated on a $1\text{km} \times 1\text{km}$ network topology, an instance of which is depicted in Fig. 1. 2 multicast streams are transmitted, each by 2 dedicated transmitters located on the anti-diagonal vertices of the square. There are K receivers requesting service, randomly distributed within the network, forming 2 equally-sized multicast groups. Receivers are assumed stationary and the direct link gains account only the propagation loss with an attenuation exponent of 4, whereas the coupling gains are further reduced by a factor equal to 8. This setup can refer to a CDMA network of 4 different single-antenna BS's, each transmitting a single stream, where the coupling reduction is due to spreading. Alternatively, the antenna pairs on the left and right (or the upper and lower) side of the square may be viewed as separate BS's, each spatially multiplexing the 2 multicasts. Then, the difference in the values of the direct and coupling link gains of each receiver-BS pair is due to beamforming. All receivers ask for a SINR threshold of 10 dB and have a noise power level of 10^{-13} W. The transmission power per stream is upper bounded by 10 W.

Simulations were performed for $K = \{4, 6, 8, 10\}$ and 50 network instances. The network size has been kept small, to enable comparison with a brute-force approach, i.e., enumeration over all possible BS assignments, including the "void" one for admission control, and solution of QoS multicast power control. The toolboxes GGPLAB [9] and CVX [4] were used to solve GP and LP optimizations, respectively. The average number of receivers admitted, versus the number of receivers requesting service, is plotted in Fig. 2. Focusing on the case of $K = 10$, Fig. 3 depicts the number of admitted receivers versus the network instance. It is observed that on 60% of the occasions the algorithm serves the maximum possible number of receivers, and on nearly all others it serves just 1 receiver less. The total transmission power, averaged over the 30 network instances that the admission control problem is solved optimally, is reported in Fig. 4. For $K = 10$, the total power for these network instances is shown in Fig. 5. It can be seen that in 70% of those, the algorithm solves optimally the joint QoS problem.

Fig. 2. Average admitted receivers

Fig. 4. Average total transmission power

Fig. 3. Admitted receivers

Fig. 5. Total transmission power

5. CONCLUSIONS

A disciplined convex approximation approach was developed for a wide class of cross-layer network optimization problems. The formulation is general enough to account for interfering cochannel transmissions, unicast, broadcast, or multicast modalities, admission control, and base station selection. The nonconvex and NP-hard joint optimization problem is approximated by a suitable geometric program, which is relatively easy to solve. This approximation forms the core of an iterative algorithm that generates approximate solutions to the original problem. While heuristic, this algorithm has a solid footing, and this shows in numerical comparisons to the optimum solution.

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