Improved Techniques for Retransmission and Relaying in Wireless Systems

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“Anyone who fights for the future, lives in it today.”

– Ayn Rand
Abstract

The last three decades have seen significant advances in the wireless communication field. As the data rates of wireless systems are increasing, the demand for mobile services also is increasing rapidly. Like other natural resources, radio spectrum suitable for mobile wireless communications is also limited. In order to keep up with this increasing demand, there is a requirement of new signal processing algorithms.

Diversity is a technique used in wireless systems to combat the effects of fading and thereby improve reliability of data transfer. There are many ways in which algorithms can exploit diversity in wireless channels. Hybrid-automatic repeat request (H-ARQ) schemes and relaying mechanisms are two such diversity extracting techniques. Even though these diversity achieving techniques have been well understood in theory, there are many ways in which one can optimize these techniques for specific application scenarios. In this thesis, we focus on improving the performance of retransmission schemes and relaying systems.

In the first part of the thesis, we improve the performance of H-ARQ schemes in the 3GPP- long term evolution (LTE) system by improving the performance of feedback signaling. We employ complex-field coding to extract the inherent frequency diversity available in the resources. Next, we provide a sub-optimal solution to the outage-optimal power allocation problem in incremental redundancy based H-ARQ system, whose performance is practically the same as that of the optimal solution.

In the later part of the thesis, we propose a retransmission scheme based on superposition coding (SPC) for the symmetric relaying scenario. We provide packet error probability (PEP) expressions and solutions for the optimal fraction of power allocated for the partner’s data. Finally, we study the optimal bits-to-symbol mappings for SPC and its effect on an H-ARQ scheme and the symmetric relaying scenario using SPC.
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T. V. K. Chaitanya
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Part I

Introduction
Introduction

From the first generation analog Nordic mobile telephony (NMT) system in the early 1980’s to the current wireless broadband systems like 3GPP-long term evolution (LTE) and LTE-advanced, the peak data rates for wireless communication have increased many orders of magnitude. Significant advances have been made in many aspects during this evolution in the last three decades. Some of these advances are in terms of channel access methods, modulation methods and efficient signal processing algorithms. The main contributions to this evolution involve the usage of multi-carrier modulation techniques and the invention of multi-antenna systems.

Reliable and efficient communication over wireless networks has been the focus of research in the telecommunications field for a long time. Often, reliability over wireless networks is effected by fading, which causes the signal strength to vary at the receiver and this results in a loss of data packets. Even though fading is not good for reliable communication, the idea of independent fading channels lead to the concept of diversity. Diversity is an approach used in wireless systems to combat the effects of fading and thereby to provide reliable data transfer [1]. Retransmissions (time diversity), multiple-input-multiple-output (MIMO) methods (antenna or space diversity) are two commonly used diversity achieving techniques in many practical systems like WLAN [2], WiMAX [3], 3GPP-LTE [4]. Other forms of diversity such as multiuser diversity [5] and cooperative diversity [6] are also finding their presence in current wireless standards in terms of opportunistic scheduling and relay communications, respectively.
Even though many of these diversity achieving techniques have been well understood in theory, there are many ways in which one can optimize these techniques for specific application scenarios. In this thesis, we focus on improving the performance of retransmission schemes and relaying systems.

In the following, we give a brief introduction to different hybrid-automatic repeat request (H-ARQ) schemes along with their information theoretic analysis in Section 1. In Section 2, we briefly describe various relaying schemes. A brief introduction to superposition coding and its applications to various communication systems is presented in Section 3. Finally, we list the contributions of the thesis along with short descriptions of the included papers in Section 4.

1 Hybrid ARQ Schemes

Early versions of retransmission schemes were called automatic repeat request (ARQ) schemes. These schemes are used when a feedback channel from the receiver to the transmitter is available. One such ARQ mechanism is to append error detection bits to the data packet before transmitting it. After receiving the data packet, the receiver checks for errors in the received data. If there are errors, the receiver sends a negative-acknowledgment (NACK) signal on the feedback channel asking for a retransmission. If the packet is received without any error, it sends an acknowledgment (ACK) signal to the transmitter. Many types of ARQ schemes have been proposed for efficient usage of resources [7]. However, ARQ schemes suffer from a reduction in throughput. In H-ARQ schemes, throughput performance is improved by combining conventional ARQ mechanisms with forward error correction (FEC) schemes. There are different types of H-ARQ schemes proposed in literature, and they are mainly classified into type I and type II H-ARQ schemes [8]:

- **Type I H-ARQ Schemes**: In these schemes, the receiver does not combine the information across different (re)transmissions. If the packet is in error, the receiver discards it.

- **Type II H-ARQ Schemes**: In these schemes, the receiver combines the information from different (re)transmissions to decode the packet. These are further classified into:
1. Hybrid ARQ Schemes

– **Diversity Combining Schemes:** These are the type-II H-ARQ schemes in which all the retransmissions carry the same information, i.e., the message is encoded using the same channel code during each transmission and the transmitted signal contains the same coded bits. The receiver uses maximum-ratio-combining (MRC) to decode the data packet. These schemes are also known as H-ARQ schemes with chase combining (CC).

– **Code Combining Schemes:** In these type-II H-ARQ schemes, successive retransmissions carry ‘new’ information to the receiver. During retransmissions, additional parity bits are sent to the destination, thereby effectively forming a longer codeword with a smaller rate. The receiver uses code combining to decode the data packet. These schemes are also known as H-ARQ schemes with incremental redundancy (IR).

One disadvantage of retransmission schemes in practice is that additional delays are introduced in the system. However in many practical systems, the number of retransmissions allowed is limited to avoid an unacceptable time delay before the successful transmission of a packet. This limit on the number of allowed retransmissions however introduces residual packet errors.

1.1 Importance of the Feedback Channel

Reliability of the ACK/NACK feedback signaling is important for the system performance. Errors in decoding these feedback signals result in wastage of resources (if an ACK signal is decoded as NACK) and loss of data at the receiver (if NACK signal is decoded as ACK). Many practical systems use one bit for H-ARQ acknowledgments. Now with the introduction of MIMO systems, the transmitter can use layered transmissions to increase the throughput. In such cases, the receivers can send more than one bit feedback (one for each layer). For example, in 3GPP-LTE systems, in the downlink, the base station sends a one bit H-ARQ acknowledgment. In the uplink, the mobile station can send either one or two bits feedback depending upon the number layers in the downlink transmission. Since there is only one bit for feedback in LTE downlink, this bit is repetition coded (with rate $\frac{1}{3}$) to improve reliability. However in the uplink, in case of extended cyclic prefix (CP) configuration, the H-ARQ acknowledgment bits are encoded together with other control signaling bits using the $(N_I, 20)$ Reed-Muller code [9], where $N_I \leq 13$ denotes the total number of bits for control signaling.
1.2 Information Theoretic Analysis of H-ARQ Schemes

In this section, we give an information theoretic understanding of H-ARQ schemes involving CC and IR. A block-fading communication system model using a H-ARQ scheme is shown in Fig. 1. A block of $U$ information bits are encoded (both error detection and error correction capabilities) to obtain $V$ coded bits for transmission on the channel. The coded bits are then modulated using a $Q$-ary constellation $S$ with equal probability for all the constellation points. The modulation symbols $s_1, s_2, \ldots, s_W$ are then divided into $B$ code blocks $x_1, x_2, \ldots, x_B$ of $N_1, N_2, \ldots, N_B$ symbols each. $W$ denotes the total number of modulation symbols. Depending on the H-ARQ scheme, a code block $x_b$ can be transmitted $T_b$ times. The propagation channel is assumed to be block-fading, which means that the channel gain remains constant during one code block transmission and changes independently between different code blocks. The total number of fading blocks involved in the transmission is $\sum_{b=1}^{B} T_b$. The total number of channel symbols is $N_s = \sum_{b=1}^{B} T_b N_b$. The $l$th reception of the $b$th code block can be written as:

$$ y_{b,l} = h_{b,l} x_b + w_{b,l}, \quad 1 \leq b \leq B, 1 \leq l \leq T_b $$

(1)

where $h_{b,l}$ and $w_{b,l}$ are the complex fading coefficient and the additive white Gaussian noise (AWGN), respectively. The noise samples are assumed to be i.i.d. and independent across the code blocks with distribution $CN(0,1)$. 

Figure 1: System model for H-ARQ schemes.
1. Hybrid ARQ Schemes

Let SNR\(_{b,l}\) denote the average symbol signal-to-noise ratio (SNR) experienced by the \(l\)th reception of the code block \(x_b\), then \(\text{SNR}_{b,l} \triangleq |h_{b,l}|^2\). Let \(\text{SNR}\) denote the collection of \(\text{SNR}_{b,l}\) values. In this thesis, we use information outage probability as the performance metric for comparison of different H-ARQ schemes. Which can be defined as:

\[
P_{\text{out}} \triangleq \Pr(I(\text{SNR}) \leq U)
\]

where \(I(\text{SNR})\) is the accumulated mutual information at the receiver conditioned on \(\text{SNR}\). The accumulated mutual information can be written as:

\[
I(\text{SNR}) = \sum_{b=1}^{B} N_b C \left( \sum_{l=1}^{T_b} \text{SNR}_{b,l} \right)
\]

where \(C(\text{SNR})\) is the symmetric AWGN capacity\(^1\) of \(S\) at signal-to-noise-ratio SNR, given by [10]:

\[
C(\text{SNR}) \triangleq \log_2 Q - \int_{\mathbb{C}} e^{-|v|^2} \pi Q \sum_{s \in \mathbb{S}} \log_2 \left( \sum_{\hat{s} \in \mathbb{S}} e^{\left| |v|^2 - |v + \sqrt{\text{SNR}(s-\hat{s})}|^2 \right|} \right) dv
\]

and \(\mathbb{C}\) is the complex-field. The relative performance of different H-ARQ schemes can be compared based on (2).

In our work, we assume that there is a limit on the number of retransmissions, say \(L\). For CC based H-ARQ schemes, we have \(B = 1, N_1 = N\). The single code block is repeated \(T_1 = L\) times. For CC case, we have:

\[
I_{\text{CC}}(\text{SNR}) = N C \left( \sum_{l=1}^{L} \text{SNR}_l \right)
\]

where SNR\(_l\) is the SNR for the \(l\)th transmission. In case of CC, if the accumulated mutual information of the initial transmission \(N C(\text{SNR}_1)\) is smaller than the number of information bits, then the transmission is in outage. The transmitter sends the same code block and hence there by improving \(N C(\sum_l \text{SNR}_l)\) so that the combined received packet has a small probability of being in outage.

For IR, assuming that all the code blocks have the same size and each code block is transmitted only once, we have \(B = L, N_b = N\) and \(T_b = 1\), for \(1 \leq\)

---

\(^1\)Throughout the thesis, we use logarithm with base 2 and hence the unit for capacity is bits per channel use (bpcu).
Introduction

The three-terminal communication channel [11, 12] has attracted much research interest recently in the wireless communications field [6, 13, 14]. A basic model of relay communication channel is shown in Fig. 2. A source node $S$ wants to communicate with a destination node $D$. A node $R$ overhears this communication and assists $S$ in such a way that the reliability of its information at the destination node is increased. Node $R$ is generally called a relay node. The main idea of relay networks is to increase the reliability of the data transmission when the source-to-destination link is in outage. The basic functionality of a relay communication in itself is similar to the retransmission of data using H-ARQ in point-to-point communication. With adaptive relaying, the relay only transmits the additional information needed to decode the source packet, which is similar to the IR based H-ARQ schemes.
2. Relaying Schemes

We are interested only in half-duplex relays with non-overlapping transmissions from the source node and the relay node. Assuming that $S$ and $R$ transmit in orthogonal time slots, the relay communication in Fig. 2 can be explained as follows. Assuming coded transmissions, during the first time slot, $S$ encodes an information block $X$ to obtain a coded information block $X_s = \alpha(X)$ and transmits to $D$, and $R$ also listens to this transmission. $\alpha$ denotes the encoding function. Assuming a block-fading channel, the received signal at nodes $D$ and $R$ during the first time slot can be expressed as:

\[
Y_{D_1} = h_{SD}X_s + N_{D_1} \\
Y_R = h_{SR}X_s + N_R
\]

where $N_{D_1}$ and $N_R$ denote the AWGN at the destination and the relay during the first time slot. $h_{SD}$ and $h_{SR}$ denote the fading channel gains for the links $S \rightarrow D$ and $S \rightarrow R$. During the second time slot, $R$ forwards the code block $X_R = f(Y_R)$ to $D$. The received signal at the destination node during the second time slot can be written as:

\[
Y_{D_2} = h_{RD}X_R + N_{D_2}
\]

where $N_{D_2}$ is the AWGN at the destination during the second time slot. $h_{RD}$ is the channel gain for the link $R \rightarrow D$. Assuming perfect channel knowledge at node $D$, it uses $Y_{D_1}$ and $Y_{D_2}$ to decode $X$.

Depending on the resources available, $R$ can perform different operations on $Y_R$ and these operations $f(.)$ form the basis for different relaying schemes\textsuperscript{2}. We list some of them here:

- Amplify-and-forward (AF): In AF scheme, the relay node $R$ simply amplifies $Y_R$ and forwards it to the destination, i.e.,

\[
X_R = f(Y_R) = gY_R, \; g > 0
\]

where $g$ is a scaling factor whose value depends on the channel gain $h_{SR}$ and the average power constraint at $R$. The outage analysis of the AF scheme can be found in [13].

\textsuperscript{2}Note that there are schemes which can be used when relay does not have any memory. Such relaying methods are known as instantaneous relaying schemes [15, 16, 17].
• Decode-and-forward (DF): In DF based schemes, the relay decodes the information received from $S$ and then re-encodes it to obtain $X_R$ and forwards it to the destination. In this case we can write the relaying operation as:

$$X_R = f(Y_R) = \beta \left( \arg \max_X p(X_S|Y_R, h_{SR}) \right)$$

where $\beta$ denotes the encoding function at the relay node. If $\beta = \alpha$ and $R$ forwards the complete $X_R$, the relaying scheme is called DF with repetition coding [13]. If $R$ repeats only a part of the codeword $X_R$, the relaying scheme is known as DF with partial repetition coding [18]. However if $R$ uses a different channel code ($\beta \neq \alpha$), the relaying scheme is known as DF with parallel coding [19]. Outage analysis of various DF schemes is treated in depth in [18].

• Compress-and-forward (CF): In CF based schemes, the relay node $R$ forwards a quantized and compressed version of $Y_R$ to the destination. During the compress operation, $R$ exploits the fact that $Y_R$ and $Y_{D1}$ are correlated [12].

When multiple relays (which are geographically separated) assist the source-to-destination link, their transmit antennas can form a distributed antenna array. In such cases, one can apply the techniques of space-time coding to extract the transmit diversity as well [20].

3 Superposition Coding

A large part of our work in this thesis uses the concept of superposition coding (SPC) [1]. In a superposition coded system, two or more users data is transmitted simultaneously. Note that there are many ways in which one can superpose users data, for example in the modulation domain as in [21, 22] or in the GF (2) domain as in [23, 24]. In the following, we consider only the superposition in the modulation domain for simplicity, and since that is used in the rest of the thesis.
3. Superposition Coding

Consider a $K$-user downlink broadcast channel model as shown in Fig. 3. The received signal at each user can be written as:

$$y_k[m] = h_k x[m] + w_k[m], \quad k = 1, 2, \ldots, K \text{ and } m = 1, 2, \ldots$$  \hspace{1cm} (12)

where $w_k[m] \sim \mathcal{CN}(0, N_0)$ is i.i.d. complex Gaussian noise and $y_k[m]$ is the received signal for user $k$ at time $m$. $h_k$ denotes the fixed channel gain from the base station to user $k$ and $x[m]$ is the transmitted signal from the base station. If $h_k$ is known to both the base station and the user $k$, the optimal transmission strategy is to encode the information of each user using an i.i.d. Gaussian code spread over the entire bandwidth. The total available transmit power $P$ at the base station is split between the users’ streams such that

$$x[m] = \sum_{k=1}^{K} x_k[m], \quad m = 1, 2, \ldots$$  \hspace{1cm} (13)

with $\mathbb{E}|x_k[m]|^2 = P_k$ and $\sum_{k=1}^{K} P_k = P$. Assuming that each users data stream is formed using a capacity achieving code, the boundary of the capacity region is characterized by the parametrized rate tuples [1]:

$$R_k = \log_2 \left( 1 + \frac{P_k |h_k|^2}{N_0 + \left( \sum_{j=k+1}^{K} P_j \right) |h_k|^2} \right), \quad k = 1, 2, \ldots, K$$

\(^3\text{Note that } X_n \text{ should satisfy the average power constraint at } R.\)
Introduction

\[ -\sqrt{\gamma} - \sqrt{1 - \gamma} \quad \sqrt{\gamma} - \sqrt{1 - \gamma} \quad \sqrt{\gamma} + \sqrt{1 - \gamma} \]

(a) Constellation for \( x \) when \( x_1, x_2 \in \{\pm 1\} \).

\[ 2\sqrt{\gamma} \quad 2(\sqrt{1 - \gamma} - \sqrt{\gamma}) \]

(b) Constellation for \( x \) when \( x_1, x_2 \in \{\pm 1 \pm j\} \).

Figure 4: Constellation for the super-symbol \( x \) for a two user case when \( x_1, x_2 \) belong to BPSK and QPSK constellations with \( \rho_1 = \sqrt{\gamma}, \rho_2 = \sqrt{1 - \gamma} \) and \( \theta_1 = \theta_2 = 0 \) deg. \( \gamma \) denotes the fraction of the total power allocated to one of the users.

for all possible power splits \( \sum_{k=1}^{K} P_k = P \). The boundary is achieved by the superposition coding operation in (13) at the transmitter and successive interference cancellation (SIC) at the receivers. In SIC, the users with stronger channel gains first decode the weaker users data and subtract it before decoding their own data.

3.2 SPC for Finite Input Symbol Alphabets

If practical systems, the modulation symbols \( x_k [m] \) in (13) come from a finite input alphabet. For example if \( x_k \in \mathcal{A}, \forall k \), where \( \mathcal{A} \) corresponds to a finite dimensional constellation such as PAM or QAM with unit average energy,
3. Superposition Coding

the superposed modulation symbols in general can be written as \[25\]^4:

\[ x = \sum_{k=1}^{K} \rho_k e^{j\theta_k} x_k. \] (14)

where \(\rho_k\) denotes the amplitude scaling factor and \(\theta_k\) denotes the phase rotation parameter (in case of complex modulation symbols) for user \(k\)'s data. Note that when \(\rho_k = \frac{1}{\sqrt{K}}\) and \(\theta_k = 0, \forall k,\) then the signal points corresponding to the super-symbol \(x \in \mathcal{B}\) exhibit a non-uniform nature. Example constellations of \(x\) for a two user superposition case when \(x_1, x_2\) belong to BPSK and QPSK constellations are shown in Fig. 4. In Rayleigh fading channels, one can optimize \(\theta_k\) to maximize the product distance between the constellation points for the super-symbol \(x\) to obtain performance improvements as in [26]. In this thesis, we work with the superposition of BPSK constellations with \(\theta_k = 0, \forall k.\)

3.3 Applications of SPC

SPC has found its application in relaying systems, where a single relay or multiple relays assist a set of users to convey their information to a destination using SPC. Recently, SPC has also been proposed for H-ARQ systems for efficient utilization of resources. In H-ARQ schemes using SPC, during the retransmissions, the source sends superposed information of a new data packet as well as additional parity bits of the erroneous packet. These two applications along with other practical aspects of SPC are discussed in [25].

The signal-space diversity (SSD) concept introduced in [27] can also be interpreted as a variation of SPC, in which the modulation symbols are precoded with an orthonormal (unitary for complex case) matrix and transmitted over the channel to extract the diversity available in the channel. For example, suppose that two real uncoded modulation symbols \(x_1\) and \(x_2\) are sent to the destination in two channel uses with fading gains \(h_1\) and \(h_2\) respectively. If any of these channel fades are deep, then the receiver cannot detect the symbols correctly. In SSD, instead of sending \(x_1\) and \(x_2\) on the channel, we send the precoded modulation symbols \(s_1\) and \(s_2\) obtained as follows:

\[
\begin{bmatrix}
  s_1 \\
  s_2 \\
\end{bmatrix}
= \begin{bmatrix}
  \psi_{11} & \psi_{12} \\
  \psi_{21} & \psi_{22} \\
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
\end{bmatrix}
\]

(15)

\[\equiv \Phi\]

\[^4\text{We drop the index } m \text{ for notational convenience.}\]
where $\Psi$ is an orthonormal matrix. As we can see from (15), the precoded modulation symbols are also linear combinations of $x_1$ and $x_2$, similar to the operation shown in (14). The SSD concept was later generalized to complex modulation symbols with elements of $\Psi \in \mathbb{C}$ and evolved as complex-field coding (CFC) [28]. CFC has also found its application in relay communications [29].

4 Contributions of the Thesis

In the major part of the thesis, we consider different aspects of H-ARQ schemes in point-to-point communication as well as relay communications. In the last part, we consider bits-to-symbol-mapping for SPC, with applications in H-ARQ and relay systems.

Brief summaries of the papers included in this thesis are as follows:

Paper A: Improved Error Protection for Uplink Control Signaling in 3GPP-LTE via Complex-Field Coding


Published in the proceedings of 71st Vehicular Technology Conference, 2010.

We study the uplink control signaling in 3GPP-Long Term Evolution (LTE) systems. Specifically, we propose a precoding method that uses complex-field coding (CFC) to improve the performance of the PUCCH format 2 control signaling. In the case of perfect channel state information (CSI) at the receiver and with a single receive antenna, the proposed method offers significant gains compared to the coding currently used in 3GPP-LTE. However the gains are marginal with two receive antennas. In order to examine the impact of channel estimation errors, we also derive the optimal detector for the case of imperfect receiver CSI, both for conventional coding and for the proposed CFC method.

Paper B: Outage-Optimal Power Allocation for Hybrid ARQ with Incremental Redundancy


4. Contributions of the Thesis

We consider the optimization of power in incremental redundancy (IR) based hybrid automatic repeat request (HARQ) schemes when the maximum number of (re)transmissions is fixed. We formulate two optimization problems: (i) minimizing the packet drop probability (PDP) under a total average transmit power constraint, and (ii) minimizing the average transmit power under a fixed PDP constraint. We consider in detail the special case of only two allowed transmissions, and we prove that the two optimization problems are equivalent. For this special case, we also provide a sub-optimal root-finding solution and compare its performance with the optimal solution obtained through an exhaustive search. The results show that the optimal power allocation can provide significant gains over the equal power solution in terms of average transmit power spent. The performance of the proposed root-finding solution is practically the same as that of the optimal solution.

**Paper C: Superposition Modulation Based Symmetric Relaying with Hybrid ARQ: Analysis and Optimization**


This paper is an evolved version of [31], presented at IEEE VTC, 2010.

We present a retransmission scheme based on superposition modulation for the symmetric relaying scenario when the number of retransmissions for a data packet is limited. We consider both diversity combining based as well as code combining based retransmission schemes. Under the assumption that the receiver implements a mechanism that use all accumulated received mutual information when decoding the message, we derive packet error probability (PEP) expressions for the proposed retransmission scheme for the case when only one retransmission is allowed. Based on the PEP expressions derived, we provide a closed-form solution for the optimal superposition ratio (the fraction of power used for the relaying operation). Simulation results show that the proposed retransmission scheme offers significant gains compared to a retransmission scheme based on classical decode-and-forward (DF) relaying.

**Paper D: Bits-to-Symbol Mappings for Superposition Coding**


Submitted to the IEEE Global Communications Conference, 2011.
Superposition coding can be used in scenarios where orthogonal sharing of resources is not efficient. In this paper, we consider two such scenarios in which superposition coding is used to send two or more users data (or packets of data) to the destination with efficient utilization of resources. Conventional way of superposing users data in Euclidean space leads to a natural mapping of bits-to-superposed-symbols. We propose the use of Gray mapping of bits-to-superposed-symbols and study the effect in terms of mutual information at the destination for the two scenarios considered in this paper. We also present some numerical results showing that using a Gray mapping of bits-to-superposed-symbols has better performance compared to the conventional natural mapping. From the results, we also conclude that the choice of optimal superposition ratio values for the AWGN channel using Gray mapping of bits for two and three user scenarios results in 4-PAM and 8-PAM constellation points for the superposed modulation symbols respectively.
Bibliography


