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Dark and grey compressional dispersive Alfvén solitons in plasmas

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The amplitude modulation of compressional dispersive Alfvén (CDA) waves in a low-β plasma is considered. It is shown that the dynamics of modulated CDA waves is governed by a cubic nonlinear Schrödinger equation, which depicts the formation of a dark/grey envelope CDA soliton. © *2011 American Institute of Physics*. [doi:10.1063/1.3602463]

The nonlinear propagation of hydromagnetic waves is of significant interest in understanding the properties of intrinsic localized excitations of electromagnetic disturbances in magnetized space and laboratory plasmas.[1] Hydromagnetic waves include the Alfvén waves propagating along and across the external magnetic field direction, as well as the coupled fast and slow Alfvén waves and the kinetic and inertial Alfvén waves[2] that propagate obliquely to the magnetic field. Dispersive hydromagnetic waves, which can be parametrically excited,[3] have been used for heating ions and electrons in plasmas.

Our low-frequency compressional dispersive Alfvén (CDA) wave propagates here across a homogeneous magnetic field ($\hat{\mathbf{z}}B_0$), where $\hat{\mathbf{z}}$ is the unit vector along the z-axis in a Cartesian coordinate system and B_0 the strength of the magnetic field in a uniform magnetoplasma. The restoring force on the CDA wave comes from the wave magnetic pressure, and the ion mass provides inertia to sustain the CDA wave. The CDA wave dispersion arises due to the electron polarization drift in the wave electric field $\mathbf{E}_\perp = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y axes, respectively. In a quasi-neutral plasma, the x components of the electron and ion fluid velocities are identical, but the x and y components of the electron fluid velocities differ owing to the electron polarization. The electrons carry current only along the y-direction. The CDA wave compresses the magnetic field-lines without bending them.

The nonlinear propagation of one-dimensional CDA waves along the x-axis is governed by the ion continuity equation

$$\frac{dn}{dt} + n\frac{\partial u}{\partial x} = 0, \tag{1}$$

the ion momentum equation

$$\frac{du}{dt} + \frac{V_A^2}{2n}\frac{\partial}{\partial x}\left(B^2 + 2\beta \ln n\right) = 0, \tag{2}$$

and Faraday's law of electromagnetic induction

$$\frac{d}{dt}\left(1 - \lambda_e^2\frac{\partial^2}{\partial x^2}\right)B + B\frac{\partial u}{\partial x} = 0, \tag{3}$$

where $d/dt = (\partial/\partial t) + u\partial/\partial x$, n is the normalized (by the equilibrium density n_0) ion number density, u the $x-$component of the ion fluid velocity, B the normalized (by B_0) compressional (along $\hat{\mathbf{z}}$) magnetic field, $V_A = B_0/(4\pi n_0 m_i)^{1/2}$ the Alfvén speed, m_i the ion mass, $\beta = C_s^2/V_A^2$ the plasma beta, C_s the ion sound speed, $\lambda_e = c/\omega_{pe}$ the electron skin depth, c the speed of light in vacuum, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ the electron plasma frequency, e the magnitude of the electron charge, and m_e the electron mass. It should be noted that in deriving (2) and (3), we used the electric field from the electron momentum equation, while (2) also employed Ampère's law, which is valid for $\partial/\partial t \ll c\partial/\partial x$. The λ_e -term in (3) comes from the electron inertia.

Letting $n = 1 + N_1$ and $B = 1 + b_1$, where $N_1 = n_1/n_0 \ll 1$ and $b_1 = B_1/B_0 \ll 1$, we linearize (1)–(3) and combine the resultant equations to obtain the wave equation

$$\left(1 - \lambda_e^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2}{\partial t^2} - C_s^2\frac{\partial^2}{\partial x^2}\right)b_1 - V_A^2\frac{\partial^2 b_1}{\partial x^2} = 0, \tag{4}$$

which can be Fourier transformed by supposing that b_1 is proportional to $\exp(-i\omega t + ikx)$, where the frequency ω and the wavenumber are related by[4]

$$\omega^2 = k^2C_s^2 + \frac{k^2V_A^2}{(1 + k^2\lambda_e^2)}. \tag{5}$$

We note that for $k^2\lambda_e^2 \gg 1$, Eq. (5) gives $\omega^2 = k^2C_s^2 + \omega_{LH}^2$, where $\omega_{LH} = \sqrt{\omega_{ce}\omega_{ci}}$ is the lower-hybrid resonance frequency. Here, we have denoted $\omega_{ce} = eB_0/m_e c$ and $\omega_{ci} = eB_0/m_i c$ as the electron and ion gyrofrequencies, respectively.

Several comments are in order. First, in the past, many authors[5–7] discussed non-envelope soliton[5,6] and cnoidal wave[7] solutions of the CDA waves based on Eqs. (1)–(3) without the β -term in (2). Second, without the electron inertia induced dispersion and finite β effects, Eqs. (2) and (3)

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admit self-similar solutions in the form of simple waves and the formation of shocks.[8] Third, ignoring the electron inertial effect, Shukla *et al*.[9] examined the effect of finite β on the shock structures. Fourth, the combined influence of dispersion and dissipation on non-stationary CDA waves in a zero-β plasma has been examined by Chakrabarti *et al*.[10] Finally, when $\partial(B^2 + 2\beta \ln n)/\partial x = 0$ in Eq. (2), we end up obtaining the solution of Eqs. (1) and (2) in the form[11] $n(t,x) = n_0(\xi)/[1 + tu_0'(\xi)]$ and $u(t,x) = u_0(\xi)$, where n_0 is the initial density profile, u_0 the initial velocity $\xi = x - ut$, and the prime denotes the differentiation with respect to ξ. It turns out that for this particular case, the ion density blows up and becomes infinite if $u_0'(\xi)$ is negative at time $t^* = -1/u_0'(\xi^*)$, where ξ^* is the value of ξ where $u_0'(\xi)$ has its largest negative value.

In the following, we consider the amplitude modulation of a finite amplitude CDA pump with the magnetic field $B_1 \exp(-i\omega_0 t + ik_0 x) +$ complex conjugate, where B_1 is the wave magnetic field perturbation along the z-axis and $\omega_0 = k_0 V_A/(1 + k_0^2\lambda_e^2)^{1/2}$ for $\beta \ll 1$. The nonlinear interaction between the CDA pump and quasi-stationary compressional magnetic field perturbations give rise to a slowly varying envelope of CDA waves whose magnetic field $\mathcal{B}$ varies slowly on the temporal-spatioscales (T, X). The evolution of the CDA envelope is given by

$$i\left(\frac{\partial}{\partial T} + V_g \frac{\partial}{\partial X}\right)\mathcal{B} + P\frac{\partial^2 \mathcal{B}}{\partial X^2} - \omega_0 B_s \mathcal{B} = 0, \tag{6}$$

where $\partial\mathcal{B}/\partial T \ll \omega_0 \mathcal{B}$, $V_g = \partial\omega_0/\partial k_0 = V_A/(1 + k_0\lambda_e^2)^{3/2}$ is the group velocity of the pump, $P = (1/2)\partial V_g/\partial k_0 = -3\omega_0\lambda_e^2/2(1 + k_0^2\lambda_e^2)^2 \equiv -P_0$ represents the pump wave group dispersion $B_s = B_{1s}/B_0$, and $B_{1s}(\ll B_0)$ the compressional magnetic field perturbation associated with a quasi-stationary plasma slow motion. We note that (6) has been deduced from (1)-(3) in the WKB approximation.[12]

The expression for B_s is determined by averaging the inertialess ion momentum Equation (2) over $2\pi/\omega_0$. We have

$$\left\langle V_i \frac{\partial V_i}{\partial X}\right\rangle = -V_A^2 \frac{\partial B_s}{\partial X} - \frac{V_A^2}{2B_0^2}\frac{\partial \langle \mathcal{B}^2\rangle}{\partial X}, \tag{7}$$

where the angular bracket denotes an ensemble average over the CDA wave-period, and the x-component of the ion fluid velocity in the CDA wave magnetic field is $V_i = (1 + k_0^2\lambda_e^2)^{1/2} V_A \mathcal{B}/B_0$. Accordingly, Eq. (7) yields

$$B_s = -\frac{(2 + k_0^2\lambda_e^2)}{2B_0^2}|\mathcal{B}|^2. \tag{8}$$

Substituting for B_s from (8) into (6) we obtain the cubic nonlinear Schrödinger equation

$$i\left(\frac{\partial}{\partial T} + V_g \frac{\partial}{\partial X}\right)\mathcal{B} - P_0\frac{\partial^2 \mathcal{B}}{\partial X^2} + Q|\mathcal{B}|^2\mathcal{B} = 0, \tag{9}$$

where $Q = \omega_0(2 + k_0^2\lambda_e^2)/2B_0^2$. Since the product of the group dispersion $-P_0$ and the coefficient of nonlinearity Q is negative, a finite amplitude CDA pump is stable against quasi-stationary magnetic perturbations.

Possible stationary solutions of Eq. (9) are the dark and grey envelope solitons.[13,14] The profile of the dark envelope soliton is

$$\mathcal{B} = \sqrt{D(\xi, T)}\exp(i\Theta), \tag{10}$$

where $\xi = X - V_g T$, and

$$D(\xi, T) = D_0\left[1 - \mathrm{sech}^2(\xi/L_d)\right] = D_0 \tanh^2(\xi/L_d), \tag{11}$$

where $\Theta = (1/2P_0)\left[V_g X - \left(0.5V_g^2 - 2P_0 Q D_0^2\right)T\right]$ is a phasor, $L_d = \sqrt{2P_0/QD_0}$ the soliton pulse width, and D_0 is the value of D as $|\xi| \to \infty$. Furthermore, the profile of the grey envelope soliton is

$$\mathcal{B} = \sqrt{G(\xi, T)}\exp(i\Theta), \tag{12}$$

where

$$G(\xi, T) = G_0\left[1 - H^2 \mathrm{sech}^2(\xi/L_g)\right], \tag{13}$$

with the soliton pulse width $L_g = (1/H)\sqrt{2P_0/QG_0}$. Here H is a dimensionless parameter, representing the modulation depth $(0 < H \le 1)$. It is given by $H^2 = 1 + (V_g - V_0)^2/2P_0 Q G_0$, where V_0 is an independent real constant, satisfying the condition $V_0 - \sqrt{2P_0 Q G_0^2} \le V_g \le V_0 + \sqrt{2P_0 Q G_0^2}$. For $V_0 = V_g$, we have $H = 1$ and thus recover the dark soliton. Both dark and grey envelope solitons represent a localized region of negative wave magnetic density propagating with the group velocity V_g. Phases inside the envelope solitons oscillate rapidly.

In this Brief Communication, we have considered the amplitude modulation of the CDA waves against quasi-stationary magnetic field perturbations in a plasma with $\beta \ll 1$. It is found that the modulated CDA wave packet remains stable and it propagates in the form of a dark/grey envelope soliton. The latter is composed of a localized region of negative magnetic wave density that is trapped in a self-created magnetic field depression. The present result may help to interpret experimental data which have a signature of localized compressional magnetic field perturbations in very low-β magnetized space and laboratory plasmas.

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