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Efficient Computation of the Pareto Boundary for the Two-User MISO Interference Channel with Multi-User Decoding Capable Receivers

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Abstract—We study the two-user multiple-input single-output (MISO) interference channel for the scenario where the transmitters have perfect channel state information and employ singlestream beamforming. We assume that the receivers are able of decoding the data from both transmitters. Hence, the signal from the interfering transmitter might be decoded, treating the desired signal as noise, and subtracted from the received signal. We propose an efficient method for finding the Pareto boundary of the corresponding achievable rate region. This method has a complexity which is constant in the number of transmit antennas.

I. INTRODUCTION

We study a wireless system where two transmitter (TX) – receiver (RX) pairs, or links, operate simultaneously in the same frequency band. Hence, the links interfere with each other. This situation can be modeled via the so-called interference channel (IC) [1]. The TXs employ $n_T \ge 2$ antennas each, whereas the RXs are equipped with a single antenna each. Hence, the system is a multiple-input single-output (MISO) IC [2]. When the RXs treat the interference as noise, see e.g. [2]–[4], the interference can substantially degrade the performance of the two links.

The capacity region of the IC is unknown. However, we know that for strong interference, it is optimal to first decode the interference treating the desired signal as noise, subtract the interference, and then decode the desired message [5]. For weak interference, it is optimal to treat the interference as additive noise [6]. Motivated by these facts, we assume that the RXs are capable to decode the interference and subtract it from the received signal before decoding the intended data. Because of practical reasons, the RXs do the decoding independently. Moreover, we assume that both TXs have perfect channel state information and use Gaussian coding with single-stream beamforming. Given these assumptions, we obtain an achievable rate region. Herein, the focus is to efficiently find the so-called Pareto boundary of this region. The Pareto boundary consists of the points where we cannot increase the rate of one link without decreasing the rate of the other.

The MISO IC with multi-user decoding (MUD) capable RXs was first investigated in [2], where the authors illustrated

the potential gain of MUD compared to single-use decoding. In [7], an achievable rate region for the described scenario was defined. The authors of [7] proposed a parameterization of the beamforming vectors that achieve Pareto-optimal (PO) rate points. This parameterization does only yield necessary conditions that the beamforming vectors have to separately fulfill. That is, we only get pairs of beamforming vectors which *potentially* give PO operating points. In order to find the Pareto boundary, we have to perform a brute-force search over all rate pairs. However, the parameterization gives us some insight. When the RXs treat interference as noise, the PO beamforming vectors are obtained by trading off between maximizing the own rate and avoid creating interference. On the other hand, when the RXs decode the interference, we have a trade-off between maximizing the own rate and causing extra interference in order to aid the decoding of the interference.

Contributions: We propose a method that *jointly* finds a pair of beamforming vectors that yield an arbitrary PO point. We find the Pareto boundary in two steps. First, we compute the boundaries corresponding to the four scenarios of 1) both RXs decode the interference, 2) both RXs treat the interference as additive noise, 3) RX_1 decodes the interference while RX_2 treats it as noise, and 4) RX_1 treats the interference as noise while RX_2 decodes it. Second, the rate region for the MISO IC with MUD is obtained as the union of these four regions.

Notation: $\Pi_{\boldsymbol{x}} \triangleq \boldsymbol{x}\boldsymbol{x}^H / \|\boldsymbol{x}\|^2$ is the orthogonal projection onto the vector \boldsymbol{x} , whereas $\Pi_{\boldsymbol{x}}^{\perp} \triangleq \boldsymbol{I} - \Pi_{\boldsymbol{x}}$ is the orthogonal projection onto the orthogonal complement of \boldsymbol{x} . By $\boldsymbol{x} \sim \mathcal{CN}(0, \sigma^2)$ we say that \boldsymbol{x} is a zero-mean complex circularlysymmetric Gaussian random variable with variance σ^2 .

II. SYSTEM MODEL

We assume that the transmissions consist of scalar coding followed by beamforming and that all propagation channels are frequency-flat. The matched-filtered symbol-sampled complex baseband data received by RX_1 is modeled as¹

$$y_1 = \boldsymbol{h}_{11}^H \boldsymbol{w}_1 s_1 + \boldsymbol{h}_{21}^H \boldsymbol{w}_2 s_2 + e_1.$$
(1)

In (1), $h_{11}, h_{21} \in \mathbb{C}^{n_T}$, are the (conjugated) channel vectors for the links $TX_1 \rightarrow RX_1$ and $TX_2 \rightarrow RX_1$, respectively. We assume that the channels are perfectly known at the TXs. Also, $w_1, w_2 \in \mathbb{C}^{n_T}$ are the beamforming vectors employed by TX_1

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¹Whenever an expression is introduced only with respect to link 1, the expression for link 2 is obtained by interchanging the indices.

and TX₂, respectively, $s_1, s_2 \sim C\mathcal{N}(0, 1)$ are the transmitted symbols of TX₁ and TX₂, respectively, and $e_1 \sim C\mathcal{N}(0, \sigma^2)$ models the receiver noise at RX₁.

The achievable rates depend on the received power. Specifically, for RX₁ we define $p_1(w_1) \triangleq |h_{11}^H w_1|$ to be the power received from TX₁ over the direct channel and $q_1(w_2) \triangleq |h_{21}^H w_2|$ to be the power received from TX₂ over the cross-talk channel. There is a power constraint that we, without loss of generality, set to 1 and define the set of feasible beamforming vectors as $\mathcal{W} \triangleq \{w \in \mathbb{C}^{n_T} |||w||^2 \leq 1\}$.

III. AN ACHIEVABLE RATE REGION

In this section, we construct an achievable rate region for the described scenario. Each pair of beamforming vectors (w_1, w_2) and combination of decoding strategies (decode the interference (d) or treat it as noise (n)) is associated with maximum achievable rates. We denote these rates, in bits per channel use (bpcu), $R_i^{xy}(w_1, w_2)$, i = 1, 2, where x and y stand for the decoding strategies n or d. For each decoding strategy, we obtain a rate region by taking the union over all feasible beamforming vectors, i.e.

$$\mathcal{R}^{xy} \triangleq \bigcup_{(\boldsymbol{w}_1, \boldsymbol{w}_2) \in \mathcal{W}^2} (R_1^{xy}(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2^{xy}(\boldsymbol{w}_1, \boldsymbol{w}_2)), \quad (2)$$

where x and y stand for the decoding strategies n or d. The maximum achievable rate for each pair of beamforming vectors (w_1, w_2) is as follows [7]:

Both RXs treat the interference as noise: When both RXs treat the interference as noise, link 1 achieves the rate [2]

$$R_1^{nn}(\boldsymbol{w}_1, \boldsymbol{w}_2) = \log_2 \left(1 + p_1(\boldsymbol{w}_1) / (q_1(\boldsymbol{w}_2) + \sigma^2) \right).$$
 (3)

 \mathbf{RX}_1 decodes the interference, \mathbf{RX}_2 treats it as additive noise: Since \mathbf{RX}_1 decodes and subtracts the interference caused by \mathbf{TX}_2 , it experiences an interference-free signal and achieves the rate

$$R_1^{dn}(\boldsymbol{w}_1) = \log_2\left(1 + p_1(\boldsymbol{w}_1)/\sigma^2\right).$$
 (4)

 RX_1 will be able to decode interference from TX_2 if the rate of link 2 satisfies

$$R_2 \le \log_2 \left(1 + q_1(\boldsymbol{w}_2) / (p_1(\boldsymbol{w}_1) + \sigma^2) \right).$$
 (5)

Since RX_2 does not decode the interference, the rate of link 2 must also satisfy

$$R_2 \le \log_2 \left(1 + p_2(\boldsymbol{w}_2) / (q_2(\boldsymbol{w}_1) + \sigma^2) \right).$$
 (6)

The maximum achievable rate $R_2^{dn}(w_1, w_2)$ is the minimum of the right-hand sides of (5) and (6). For link 2, we note that the maximum achievable rate does not necessarily exploit the signal-to-interference-plus-noise (SINR) ratio at RX₂ to the full extent. Actually, it might hold back on rate to facilitate RX₁ to decode the signal of link 2. This fact was not exploited in [2], where the description leads to too restrictive conditions.

RX₂ decodes the interference, **RX**₁ treats it as additive noise: This case is identical to \mathcal{R}^{nd} , but with interchanged indices.

Both RXs decode the interference: Both RXs decode the interference before decoding their desired signals. When RX_1 has decoded the interference from TX_2 , the desired signal can be decoded if the rate of link 1 satisfies

$$R_1 \le \log_2(1 + p_1(\boldsymbol{w}_1)/\sigma^2).$$
 (7)

 RX_2 can decode the interference caused by TX_1 if the rate of link 1 satisfies

$$R_1 \le \log_2 \left(1 + q_2(\boldsymbol{w}_1) / (p_2(\boldsymbol{w}_2) + \sigma^2) \right)$$
 (8)

So, the maximum achievable rate of link 1, $R_1^{dd}(\boldsymbol{w}_1, \boldsymbol{w}_2)$, is the minimum of the right-hand sides of (7) and (8).

The achievable rate region: The rate region for the MISO IC with MUD capability is obtained as

$$\mathcal{R} = \mathcal{R}^{nn} \cup \mathcal{R}^{dn} \cup \mathcal{R}^{nd} \cup \mathcal{R}^{dd}.$$
(9)

We are interested in finding the so-called Pareto boundary of the region \mathcal{R} . The Pareto boundary consists of PO points, where Pareto-optimality is defined as follows.

Definition 1. A rate pair $(R_1^{\star}, R_2^{\star}) \in \mathcal{R}$ is Pareto-optimal if there is no other rate pair $(R_1, R_2) \in \mathcal{R}$ with $(R_1, R_2) > (R_1^{\star}, R_2^{\star})$. (The inequality is component-wise.)

Note that Def. 1 also includes the horizontal and vertical sections of the Pareto boundary. Hence the definition defines weak Pareto-optimality. Graphically, the Pareto boundary is the north-east boundary of the region. To find the Pareto boundary of \mathcal{R} , we first find the Pareto boundaries of \mathcal{R}^{nn} , \mathcal{R}^{dn} , \mathcal{R}^{nd} , and \mathcal{R}^{dd} . Second, we consider as boundary of \mathcal{R} the boundary of the union of \mathcal{R}^{nn} , \mathcal{R}^{dn} , \mathcal{R}^{nd} , and \mathcal{R}^{dd} . Second, we consider as boundary of \mathcal{R} the boundary of the union of \mathcal{R}^{nn} , \mathcal{R}^{dn} , \mathcal{R}^{nd} , and \mathcal{R}^{dd} . For each of these regions, we do as follows. Let (R_1^*, R_2^*) be an arbitrary point on the Pareto boundary. In order to find this point, we fix the rate of link 1 at R_1^* and maximize R_2 in order to get R_2^* . Due to the monotonicity of the logarithmic function, we formulate a SINR optimization problem. We define $\gamma_i^{xy}(\boldsymbol{w}_1, \boldsymbol{w}_2) \triangleq 2^{R_i^{xy}(\boldsymbol{w}_1, \boldsymbol{w}_2)} - 1$ to be the SINR needed to achieve $R_i^{xy}(\boldsymbol{w}_1, \boldsymbol{w}_2)$. The problem of finding the optimal γ_2^* for a given γ_1^* can be formulated as [3]

$$\underset{(\boldsymbol{w}_1,\boldsymbol{w}_2)\in\mathcal{W}^2}{\text{maximize}}\gamma_2^{xy}(\boldsymbol{w}_1,\boldsymbol{w}_2),\tag{10}$$

subject to
$$\gamma_1^{xy}(\boldsymbol{w}_1, \boldsymbol{w}_2) = \gamma_1^{\star}.$$
 (11)

The optimal solution of (10)–(11) is the pair of beamforming vectors $(\boldsymbol{w}_1^{\star}, \boldsymbol{w}_2^{\star})$ enabling $(R_1^{\star}, R_2^{\star})$.

IV. EFFICIENT COMPUTATION OF THE PARETO BOUNDARY

In this section, we propose efficient methods for finding the boundaries of \mathcal{R}^{nn} , \mathcal{R}^{dn} , \mathcal{R}^{nd} , and \mathcal{R}^{dd} . The focus is on \mathcal{R}^{dn} and \mathcal{R}^{dd} . Due to symmetry, the problem of computing the boundary of \mathcal{R}^{nd} is identical to that of finding the boundary of \mathcal{R}^{dn} . For \mathcal{R}^{nn} , we have previously proposed two methods. In [3] we computed an arbitrary point on the boundary via a sequence of second-order cone (SOC) programs. In [4], we gave a closed-form parameterization of the beamforming vectors that yield PO rate points. The methods for finding \mathcal{R}^{dn} , \mathcal{R}^{nd} , and \mathcal{R}^{dd} devised in the sequel are novel.

A. Only One RX Decodes Interference

Here, we consider the boundary of the region \mathcal{R}^{dn} , i.e. the region consisting of the points where RX_1 is able to decode the interference while RX_2 treats it as noise. We insert (4)–(6) in (10)–(11) and obtain the resulting problem

$$\begin{array}{ll} \underset{\gamma_{2} \in \mathbb{R}_{+}, (\boldsymbol{w}_{1}, \boldsymbol{w}_{2}) \in \mathcal{W}^{2}}{\text{maximize}} & \gamma_{2} \\ \text{subject to} & p_{1}(\boldsymbol{w}_{1})/\sigma^{2} = \gamma_{1}^{\star}, \end{array}$$
(12)

subject to

$$q_1(\boldsymbol{w}_2)/(p_1(\boldsymbol{w}_1) + \sigma^2) \ge \gamma_2, \quad (14)$$

$$p_2(\boldsymbol{w}_2)/(q_2(\boldsymbol{w}_1) + \sigma^2) \ge \gamma_2.$$
 (15)

This is nonconvex, because (13) is a quadratic equality and (14), (15) are nonconvex quadratic inequalities parameterized by γ_2 . However, in [7] it was shown that the beamforming vectors that solve (12)–(15) can be parameterized as

$$\boldsymbol{w}_{1} = x_{1} \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}}{\|\boldsymbol{\Pi}_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|} + y_{1} \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11}}{\left\|\boldsymbol{\Pi}_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11}\right\|},$$
(16)

$$\boldsymbol{w}_{2} = x_{2} \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{22}} \boldsymbol{h}_{21}}{\|\boldsymbol{\Pi}_{\boldsymbol{h}_{22}} \boldsymbol{h}_{21}\|} + y_{2} \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{22}}^{\perp} \boldsymbol{h}_{21}}{\left\|\boldsymbol{\Pi}_{\boldsymbol{h}_{22}}^{\perp} \boldsymbol{h}_{21}\right\|}, \qquad (17)$$

where $(x_i, y_i) \in \mathcal{Q} \triangleq \{(x, y) | x, y \ge 0, x^2 + y^2 \le 1\}$. We see that Q is a quarter disk, which is a convex set. Using the parameterization (16)–(17), we propose a closed-form solution of (12)-(15). By using the parameterization (16)-(17), we note that y_1 does not affect $q_2(w_1)$ and y_2 does not affect $p_2(w_2)$. Hence, we can ignore the complex phases of $h_{12}^H w_1$ and $h_{22}^H w_2$. Also, we see that the inner products $h_{11}^H w_1$ and $h_{21}^H w_2$ are real and positive. Then, we define $t \triangleq \sqrt{\gamma_2}$, insert (13) in (14) and equivalently write (12)–(17) as

$$\max_{t \in \mathbb{R}_+, (x_i, y_i) \in \mathcal{Q}, i=1,2} t$$
(18)

 $\alpha x_1 + \tilde{\alpha} y_1 = \sqrt{\gamma_1^* \sigma^2},$ (19)subject to

$$\beta_2 x_2 + \tilde{\beta}_2 y_2 / \sqrt{(\gamma_1^* + 1)\sigma^2} \ge t$$
, (20)

$$g_{22}x_2/\sqrt{g_{12}^2x_1^2+\sigma^2} \ge t.$$
 (21)

The coefficients in (18)-(21) are defined in Tab. I. We solve (18)–(21) in two steps. First, we solve for (x_1, y_1) and we call the optimal solution $(x_1^{\star}, y_1^{\star})$. We note that x_1 and y_1 only appear in constraints (19) and (21). We make the left-hand side of (21) as large as possible by minimizing x_1 subject to the constraint (19):

$$\min_{\substack{(x_1,y_1) \in \mathcal{Q}}} x_1 \tag{22}$$

ubject to
$$\alpha x_1 + \tilde{\alpha} y_1 = \sqrt{\gamma_1^* \sigma^2}.$$
 (23)

Second, we insert the optimal solution x_1^{\star}, y_1^{\star} of (22)–(23) into (18)–(21) and obtain

S

t

$$\underset{\in \mathbb{R}_+, (x_2, y_2) \in \mathcal{Q}}{\text{maximize}} \quad t \tag{24}$$

subject to
$$\beta_2 x_2 + \tilde{\beta}_2 y_2 / \sqrt{(\gamma_1^{\star} + 1)\sigma^2} \ge t$$
, (25)

$$g_{22}x_2/\sqrt{g_{12}^2(x_1^\star)^2 + \sigma^2} \ge t.$$
 (26)

$\alpha \triangleq \ \mathbf{\Pi}_{\boldsymbol{h}_{12}}\boldsymbol{h}_{11}\ ,$	$\tilde{\alpha} \triangleq \left\ \boldsymbol{\Pi}_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11} \right\ $
$\beta_1 \triangleq \ \boldsymbol{\Pi}_{\boldsymbol{h}_{11}} \boldsymbol{h}_{12}\ ,$	$ ilde{eta}_1 riangleq \left\ oldsymbol{\Pi}_{oldsymbol{h}_{11}}^{ot} oldsymbol{h}_{12} ight\ $
$\beta_2 \triangleq \ \mathbf{\Pi}_{\boldsymbol{h}_{22}}\boldsymbol{h}_{21}\ ,$	$ ilde{eta}_2 riangleq \left\ oldsymbol{\Pi}_{oldsymbol{h}_{22}}^{ot} oldsymbol{h}_{21} ight\ $
$g_{ij} riangleq \ oldsymbol{h}_{ij}\ ,$	i,j=1,2
$g_{ij} = \ \boldsymbol{\kappa}_{ij} \ ,$	i, j = 1, 2

TABLE I DEFINITION OF CONSTANTS.

The solutions of (22)–(23) and (24)–(26) are summarized in the following proposition.

Proposition 1. The optimal solution $(x_1^{\star}, y_1^{\star})$ of (22)–(23) is

$$x_{1}^{\star} = \max\left\{0, \frac{1}{g_{11}^{2}} \left(\alpha \sqrt{\gamma_{1}^{\star} \sigma^{2}} - \tilde{\alpha} \sqrt{g_{11}^{2} - \gamma_{1}^{\star} \sigma^{2}}\right)\right\}, (27)$$
$$y_{1}^{\star} = \left\{\begin{array}{l}\sqrt{\gamma_{1}^{\star} \sigma^{2}} / \tilde{\alpha}, & x_{1}^{\star} = 0, \\\sqrt{1 - (x_{1}^{\star})^{2}}, & otherwise.\end{array}\right.$$
(28)

Then, the optimal value of (24)–(26) is given as

$$\gamma_{2}^{\star} = \begin{cases} g_{22}^{2}/((x_{1}^{\star}g_{12})^{2} + \sigma^{2}), & a \leq b, \\ g_{21}^{2}/(\sigma^{2}(\gamma_{1}^{\star} + 1)), & ab > b^{2} + c^{2} \\ \frac{g_{22}^{2}c^{2}}{((x_{1}^{\star}g_{12})^{2} + \sigma^{2})((a - b)^{2} + c^{2})}, & otherwise, \end{cases}$$

$$(29)$$

for

(13)

$$x_{2}^{\star} = \begin{cases} 1, & a \leq b, \\ b/\sqrt{b^{2} + c^{2}}, & ab > b^{2} + c^{2} \\ c/\sqrt{c^{2} + (a - b)^{2}}, & otherwise, \end{cases}$$
(30)

$$y_2^{\star} = \sqrt{1 - (x_2^{\star})^2} \tag{31}$$

where

$$\begin{cases} a \triangleq g_{22}/\sqrt{(x_1^{\star})^2 g_{12}^2 + \sigma^2}, \\ b \triangleq \beta_2/\sqrt{\sigma^2(\gamma_1^{\star} + 1)}, \\ c \triangleq \tilde{\beta}_2/\sqrt{\sigma^2(\gamma_1^{\star} + 1)}. \end{cases}$$
(32)

The optimal $(w_1^{\star}, w_2^{\star})$ is obtained by inserting (27)–(28) and (30)-(31) into (16)-(17).

Prop. 1 provides a scheme for evaluating the Pareto boundary quickly and exactly, by providing γ_2^{\star} as an explicit function of γ_1^{\star} , in closed-form. In order to find the entire boundary, we vary γ_1^{\star} over the interval $[0, g_{11}^2/\sigma^2]$. Note that the upper bound, g_{11}^2/σ^2 , is the largest value that $p_1(w_1)/\sigma^2$ can assume when $w_1 \in \mathcal{W}$ and corresponds to the rightmost segment of the Pareto boundary. We note that once the constants in Tab. I are computed, the complexity is constant with respect to the number of transmit antennas. From Prop. 1 we note that TX₂ will always use full power at the Pareto boundary, whereas TX_1 might use less than full power. This was proven in [7].

B. Both RXs Decode Interference

Here we consider the boundary of the regions \mathcal{R}^{dd} , i.e. the region consisting of the points where both RX_1 and RX_2 decode the interference. We insert (7)-(8) in (10)-(11) and obtain the resulting problem

$$\underset{\gamma_2 \in \mathbb{R}_+, (\boldsymbol{w}_1, \boldsymbol{w}_2) \in \mathcal{W}^2}{\text{maximize}} \quad \gamma_2 \tag{33}$$

$$p_1(\boldsymbol{w}_1)/\sigma^2 \ge \gamma_1^\star,\tag{34}$$

$$q_2(w_1)/(p_2(w_2) + \sigma^2) \ge \gamma_1^{\star},$$
 (35)

$$p_2(\boldsymbol{w}_2)/\sigma^2 \ge \gamma_2,$$
 (36)

$$q_1(\boldsymbol{w}_2)/(p_1(\boldsymbol{w}_1) + \sigma^2) \ge \gamma_2.$$
 (37)

This is a nonconvex problem, but the beamforming vectors that solve it can be parameterized as [7]

$$\boldsymbol{w}_{i} = x_{i} \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{ii}} \boldsymbol{h}_{ij}}{\|\boldsymbol{\Pi}_{\boldsymbol{h}_{ii}} \boldsymbol{h}_{ij}\|} + y_{i} \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{ii}}^{\perp} \boldsymbol{h}_{ij}}{\|\boldsymbol{\Pi}_{\boldsymbol{h}_{ii}}^{\perp} \boldsymbol{h}_{ij}\|}$$
(38)

for i, j = 1, 2 and $j \neq i$, where $(x_i, y_i) \in Q$. Inserting (38) in (33)–(37) yields the equivalent optimization problem

$$\underset{t \in \mathbb{R}_+, (x_i, y_i) \in \mathcal{Q}, i=1,2}{\text{maximize}} t \tag{39}$$

subject to
$$g_{11}x_1 \ge \sqrt{\gamma_1^\star \sigma^2},$$
 (40)

$$\beta_1 x_1 + \tilde{\beta}_1 y_1 \ge \sqrt{\gamma_1^{\star} (g_{22}^2 x_2^2 + \sigma^2)},$$
 (41)

$$g_{22}x_2 \ge \sigma t,\tag{42}$$

$$\beta_2 x_2 + \tilde{\beta}_2 y_2 \ge t \sqrt{g_{11}^2 x_1^2 + \sigma^2}.$$
 (43)

The coefficients in (39)–(43) are defined in Tab. I. The objective function (39) and the constraints (40) and (42) are linear in the optimization variables. The set Q is convex. Constraint (41) defines a SOC. However, (43) is a SOC constraint parameterized by t. So, we solve (39)–(43) by bisection over t [8, Ch. 4]. An upper bound, U, on t is obtained by setting $\gamma_1^* = 0$ and solving (39)–(43). Then, we have $x_1^* = y_1^* = 0$ and (43) is a linear constraint. We set the lower bound to L = 0 and set t := (U + L)/2. For t, we solve the following convex SOC feasibility problem.

$$\inf_{(x_i, y_i) \in \mathcal{Q}, i=1,2} (x_1, y_1, x_2, y_2)$$
(44)

 $g_{11}x_1 \ge \sqrt{\gamma_1^\star \sigma^2},$

subject to

subject to

$$\beta_1 x_1 + \tilde{\beta}_1 y_1 > \sqrt{\gamma_1^{\star}(q_{22}^2 x_2^2 + \sigma^2)},$$
 (46)

(45)

$$g_{22}x_2 \ge \sigma t,\tag{47}$$

$$\beta_2 x_2 + \tilde{\beta}_2 y_2 \ge t \sqrt{g_{11}^2 x_1^2 + \sigma^2}.$$
 (48)

If (44)–(48) is feasible we set L := t, otherwise, we set U := t. We iterate this procedure until convergence. Typically, a handful of iterations is needed. We get the optimal beamforming vectors $(\boldsymbol{w}_1^*, \boldsymbol{w}_2^*)$, by inserting the optimal solution $(x_1^*, y_1^*, x_2^*, y_2^*)$ into (38). In order to find the entire boundary, we let γ_1^* go from 0 to the maximum of $\min\{p_1(\boldsymbol{w}_1), q_2(\boldsymbol{w}_1)\}/\sigma^2$ for $\boldsymbol{w}_1 \in \mathcal{W}$. Once the constants in Tab. I are computed, the complexity of (44)–(48) is constant with respect to the number of transmit antennas. In [7] it was proven that the TXs might not necessarily use the maximum available power at the Pareto boundary. However, it is easy to verify that at least one TX uses maximum power.



Fig. 1. Example of regions for $n_T = 3$ and $\sigma^2 = 0.5$

V. NUMERICAL EXAMPLE AND CONCLUSION

In Fig. 1, we illustrate the rate regions \mathcal{R}^{nn} , \mathcal{R}^{dn} , \mathcal{R}^{nd} , and \mathcal{R}^{dd} for one realization of the channels, where $h_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, i, j = 1, 2. In this specific example, we see that \mathcal{R}^{dn} constitutes almost the entire rate region \mathcal{R} . It is a typical result that the union of the two regions obtained when one RX decodes interference (\mathcal{R}^{dn} and \mathcal{R}^{nd}) is larger than the region \mathcal{R}^{dd} obtained when both RXs decode interference. The reason for why \mathcal{R}^{dd} is not the largest region is that in order to decode the interference we need extra power over the cross-talk channel. This comes at the cost of decreased power over the direct channel. So, when both RXs decode interference low power received from the direct channel. This implies low achievable rates.

In this paper we proposed an efficient method for finding the Pareto boundary of the rate region for the MISO IC with MUD capable receivers. The method is efficient in the sense that it has a complexity that is constant with respect to the number of transmit antennas. Also, the boundary can partly be found in closed form. The merit of the proposed method, compared to the previously known methods, is that it avoids the brute-force search over all feasible beamforming vectors.

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