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Ebrahim Avazkonandeh Gharavol and Erik G. Larsson, Robust Joint Optimization of MIMO Two-Way Relay Channels with Imperfect CSI, 2011, Proceedings of Allerton Conference on Communication, Control, and Computing.

Postprint available at: Linköping University Electronic Press
http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-71365

Robust Joint Optimization of MIMO Two-Way Relay Channels with Imperfect CSI

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***Abstract*—In this paper we deal with the problem of the joint optimization of the precoders, equalizers and relay beamformer of a multiple-input multiple-output two-way relay channel. Unlike the conventional design procedures, we assume that the Channel State Information (CSI) is not known perfectly. The imperfect CSI is described using the norm bounded error framework. We use a system-wide Sum Mean Square Error (SMSE) based problem formulation which is constrained using the transmit power of the terminals and the relay node. The problem at hand, from a worst-case design perspective, is a multilinear, and hence, a nonconvex problem which is also semi-infinite in its constraints. We use a generalized version of the Peterson's lemma to handle the semi-infiniteness and reduce the original problem to a single Linear Matrix Inequality (LMI). However, this LMI is not convex, and to resolve this issue we propose an iterative algorithm based on the alternating convex search methodology to solve the aforementioned problem. Finally simulation results are included to asses the performance of the proposed algorithm.**

I. Introduction

A two-way relay channel is used to provide a full-duplex communication link between two independent terminals via a linear amplify-and-forward (AF) relay node. Two time slots are required to complete a communication process: in the first time slot, both terminals transmit their signals toward the relay channel and in the second time slot, the relay node, transmits a linear combination of the received signals toward both terminals. To best exploit the spatial diversity of the multiple antennas used in the system, precoding, equalizing and beamforming are essential in the terminals and the relay node.

The training-based channel estimation under the AF relay scheme is considered in [1]. A maximum likelihood and a conventional least square estimator are used in this process. A similar process is undergone for an orthogonal-frequency-division multiplexing modulated system in [2]. A similar problem with power allocation is considered in [3]. The distributed space-time coding for the two-way wireless relay networks, where communication between two terminals is assisted by the relay nodes and the relaying occurred using two, three, and four time slots is proposed in [4]. Optimal distributed beamforming for two-way relay networks, for three different two-way relaying schemes is the center of [5]. To solve this problem, the authors exploit two different approaches, a signal to noise ratio balancing approach, as well as a total power minimization approach. In each approach different properties of the beamforming weights are provided. Multiuser two-way AF relay processing and power control methods for the beamforming systems are studied in [6]. The relay is optimized based on both zero-forcing and minimum mean-square-error criteria under relay power constraints, and various transmit and receive beamforming methods, for example, eigen beamforming, antenna selection, random beamforming, and modified equal gain beamforming are examined.

The cited works above, all assume that the Channel State Information (CSI) is available at all the nodes of the network, however, this is not a realistic assumption, and should possibly be avoided. Walking along the line of this comment, the two-way relay channel with imperfect CSI is studied in [7], [8], and [9] focusing on the superposition coding, linear minimum mean squared error estimation and sum-rate of the system, respectively.

In this paper we study the joint optimization of the linear precoders and equalizers of the terminals, as well as the linear beamformer of the relay node. We also use a linear minimum sum mean squared error estimator and a norm-bounded error model is used to to describe the CSI imperfectness in our model. We shed a light on the problem from the worst-case design approach.

In this paper the following notations are used: Boldfaced letters are used to show vectors or the matrices while scalar variables or constants are shown with no specific formatting. The field of the complex numbers, the field of the n-dimensional complex-valued vector spaces and the field of the $m \times n$ complex-valued matrices are denoted using $\mathbb{C}$, $\mathbb{C}^n$ and $\mathbb{C}^{m\times n}$, respectively. For any vector $\boldsymbol{x}$ or for any matrix $\boldsymbol{X}$, $\|\boldsymbol{x}\|$ and $\|\boldsymbol{X}\|_F$ denote the Euclidean and the Frobenius norms of that vector or matrix, respectively. The positive semidefinite matrix $\boldsymbol{X}$ is denoted using $\boldsymbol{X} \succeq 0$. To show a vertically concatenated vectorized version of a matrix, $\mathsf{vec}\,[]$, to show a vertical concatenation of a set of matrices to build a taller block matrix, $\mathsf{MAT}\,[]$, and to show the block diagonal concatenation of a set of matrices $\mathsf{blkdiag}\,[\ldots]$

This paper describes work undertaken in the context of the LOLA project - Achieving LOw-LAtency in Wireless Communications (www.ict-lola.eu). The research leading to these results has received funding from the European Communitys Seventh Framework Program under grant agreement No. 248993. E. Larsson is a Royal Swedish Academy of Sciences (KVA) Research Fellow supported by a grant from the Knut and Alice Wallenberg Foundation.

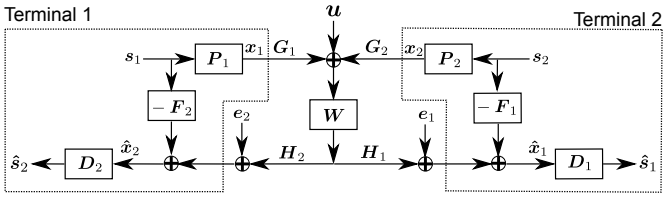

Fig. 1. TWRC Signal Flow Graph

are used, respectively. Finally, $\mathsf{E}_{\boldsymbol{x}}\left[f(\boldsymbol{x})\right]$ is the mathematical expectation of $f(\boldsymbol{x})$ with respect to the stochastic variable $\boldsymbol{x}$.

II. System Model and Problem Formulation

A wireless two-way relay system is depicted in Fig. 1. In this system two distinct terminals simultaneously communicate with each other in two consecutive time slots. Each terminal has its own transmit and receive sides. Unlike the one-way relay channel in which, to initiate a half-duplex communication service, two distinct time slots are required, in this system using two time lots, a full-duplex communication is provided. In the first time slot, both terminals transmit their data toward the relay node, and in the second time slot, the relay node retransmits the combination of the received signals, after linearly beamforming, towards the other end of the communication link. By means of this relaying process, the data is exchanged between these terminals. To transmit $S_i, i=1,2$ independent streams of zero-mean, unit-variance data ($\boldsymbol{s}_i \in \mathbb{C}^{S_i}, i=1,2$) with independent elements toward the either ends of the communication link, each terminal is equipped with $T_i, i=1,2$ transmit antennas. The destinations are equipped with $R_i, i=1,2$ receive antennas, while the relay has r and t receive and transmit antennas, respectively. To better compensate the fading channels, the source nodes precode the data using the precoding matrices $\boldsymbol{P}_i \in \mathbb{C}^{T_i \times S_i}$. The precoded data is sent over two wireless fading channels, namely $\boldsymbol{G}_i \in \mathbb{C}^{r \times T_i}, i=1,2$. At the relay node, the received signal is amplified using $\boldsymbol{W} \in \mathbb{C}^{t \times r}$ and the resultant signal is transmitted to the destinations in the next time slot over the wireless channels $\boldsymbol{H}_i \in \mathbb{C}^{R_i \times t}, i=1,2$.

At the destination, since each terminal knows its own transmit signal, first a self cancellation filter, $\boldsymbol{F}_i \in \mathbb{C}^{R_i \times S_{-i}}$, is applied to the received signal. where $\boldsymbol{X}_{-i}$ refers to a quantity belonging to the other index set, in a bi-index notation we used here[1]. The aim of this filter is to minimize the effect of "self-talk" signals. After appropriately combining the received signals and decoding using linear decoders $\boldsymbol{D}_i \in \mathbb{C}^{S_i \times R_i}, i=1,2$, the received signals are as follows:

$$\begin{aligned}\hat{\boldsymbol{s}}_i = \boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{G}_i\boldsymbol{P}_i\boldsymbol{s}_i + (\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{G}_{-i}\boldsymbol{P}_{-i} - \boldsymbol{D}_i\boldsymbol{F}_i)\boldsymbol{s}_{-i} \\ \boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{u} + \boldsymbol{D}_i\boldsymbol{e}_i, \quad i=1,2 \end{aligned} \tag{1}$$

In this equation $\boldsymbol{u} \in \mathbb{C}^r$ and $\boldsymbol{e}_i \in \mathbb{C}^{R_i}$ are the additive zero-mean noise signals with independent elements and σ_u^2 and $\sigma_{e_i}^2$ variances, respectively. Due to the limited feedback between the nodes, it is assumed that only the nominal value of the CSI is known to the system. In other words, the CSI follows the norm bounded error model:

$$\boldsymbol{G}_i \in \mathcal{G}_i = \{\tilde{\boldsymbol{G}}_i + \boldsymbol{\Delta}_{G_i} \mid \|\boldsymbol{\Delta}_{G_i}\|_F \le \delta_{G_i}\},\ i=1,2, \tag{2a}$$

$$\boldsymbol{H}_i \in \mathcal{H}_i = \{\tilde{\boldsymbol{H}}_i + \boldsymbol{\Delta}_{H_i} \mid \|\boldsymbol{\Delta}_{H_i}\|_F \le \delta_{H_i}\},\ i=1,2. \tag{2b}$$

where $\tilde{\boldsymbol{G}}_i$, and $\tilde{\boldsymbol{H}}_i$ are the fixed nominal values of the CSI for each of the channels and $\boldsymbol{\Delta}_{G_i}$, $\boldsymbol{\Delta}_{H_i}$, and are the random norm-bounded variations (uncertainties) around these nominal values.

In this paper, our goal is to jointly optimize the source precoders, the relay beamformer, and the destination equalizers. To do so, we can either use the system-wide Sum Mean Square Error (SMSE) as the performance measure of the system, while restricting the optimization problem with the power budgets of both terminals' sources and the relay node, or we can use the transmit power of the relay node as the performance measure and restrict the optimization problem with target MSEs for each link. Regardless of the problem formulation, by employing a worst-case design approach, the optimal solutions will be valid for all the realizations of the CSI that satisfy (2). To facilitate the computation of the MSE of each link, and the transmit powers of the sources and the relay nodes, we use the following lemma.

Lemma 1: For any set of zero-mean, independent and identically distributed random vectors with independent elements and individual variances of $\mathsf{E}_{\boldsymbol{x}_i}\left[\boldsymbol{x}_i^*\boldsymbol{x}_i\right] = \sigma_i^2$ we have

$$\mathsf{E}_{\{\boldsymbol{x}_i\}_i}\left[\left\|\sum_i \boldsymbol{A}_i\boldsymbol{x}_i\right\|^2\right] = \sum_i \sigma_i^2\|\boldsymbol{A}_i\|_F^2 \tag{3}$$

Proof: The proof is omitted due to length constraints. ■

Based on this Lemma 1, the MSE of the links ($\mathtt{MSE}_i, i=1,2$), and the transmit powers of the source ($\mathtt{TxP}_{s_i}, i=1,2$) and the relay ($\mathtt{TxP}_r$) nodes are defined as follows:

$$\mathtt{MSE}_i \triangleq \mathsf{E}_{\boldsymbol{s}_{1,2},\boldsymbol{u},\boldsymbol{e}_{1,2}}\left[\|\hat{\boldsymbol{s}}_i - \boldsymbol{s}_i\|^2\right], \quad i=1,2 \tag{4a}$$

$$\begin{aligned} &= \|\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{G}_i\boldsymbol{P}_i - \boldsymbol{I}\|_F^2 + \sigma_u^2\|\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\|_F^2 \\ &\quad + \|\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{G}_{-i}\boldsymbol{P}_{-i} - \boldsymbol{D}_i\boldsymbol{F}_i\|_F^2 + \sigma_{e_i}^2\|\boldsymbol{D}_i\|_F^2 \end{aligned} \tag{4b}$$

$$\mathtt{TxP}_{s_i} \triangleq \mathsf{E}_{\boldsymbol{s}_i}\left[\|\boldsymbol{P}_i\boldsymbol{s}_i\|_2^2\right], \quad i=1,2 \tag{4c}$$

$$= \|\boldsymbol{P}_i\|_F^2, \tag{4d}$$

$$\mathtt{TxP}_r \triangleq \mathsf{E}_{\boldsymbol{s}_{1,2},\boldsymbol{u}}\left[\|\boldsymbol{W}(\boldsymbol{G}_1\boldsymbol{P}_1\boldsymbol{s}_1 + \boldsymbol{G}_2\boldsymbol{P}_2\boldsymbol{s}_2) + \boldsymbol{W}\boldsymbol{u}\|_2^2\right] \tag{4e}$$

$$= \|\boldsymbol{W}\boldsymbol{G}_1\boldsymbol{P}_1\|_F^2 + \|\boldsymbol{W}\boldsymbol{G}_2\boldsymbol{P}_2\|_F^2 + \sigma_u^2\|\boldsymbol{W}\|_F^2. \tag{4f}$$

Remark 1: It is clear that $\mathtt{TxP}_{s_i}, i=1,2$, $\mathtt{TxP}_r$, and $\mathtt{MSE}_i, i=1,2$, are linear, bilinear, and trilinear functions of the design variables, respectively. Hence, $\mathtt{TxP}_r$, and $\mathtt{MSE}_i, i=1,2$, are not convex in the design variables. ♦

[1]That is $\boldsymbol{X}_{-i} = \begin{cases}\boldsymbol{X}_2, & i=1,\\ \boldsymbol{X}_1, & i=2.\end{cases}$

Using these quantities, the problem formulation in its epigraph form will be either to minimize the SMSE:

$$\underset{\boldsymbol{P}_i,\boldsymbol{W},\boldsymbol{D}_i,\boldsymbol{F}_i,\tau_i\geq 0}{\text{minimize}} \quad \tau_1+\tau_2 \tag{5a}$$
$$\text{subject to} \quad \mathtt{TxP}_{s_i}\leq P_{s_i},\ i=1,2, \tag{5b}$$
$$\mathtt{TxP}_r\leq P_r,\ \forall \boldsymbol{G}_i\in\mathcal{G}_i,\ i=1,2, \tag{5c}$$
$$\mathtt{MSE}_i\leq \tau_i,\ \forall \boldsymbol{G}_i\in\mathcal{G}_i,\boldsymbol{H}_i\in\mathcal{H}_i,\ i=1,2, \tag{5d}$$

or to minimize the transmit power of the relay station:

$$\underset{\boldsymbol{P}_i,\boldsymbol{W},\boldsymbol{D}_i,\boldsymbol{F}_i,\tau\geq 0}{\text{minimize}} \quad \tau \tag{6a}$$
$$\text{subject to} \quad \mathtt{TxP}_{s_i}\leq P_{s_i},\ i=1,2, \tag{6b}$$
$$\mathtt{TxP}_r\leq \tau,\ \forall \boldsymbol{G}_i\in\mathcal{G}_i,\ i=1,2, \tag{6c}$$
$$\mathtt{MSE}_i\leq \gamma_i,\ \forall \boldsymbol{G}_i\in\mathcal{G}_i,\boldsymbol{H}_i\in\mathcal{H}_i,\ i=1,2, \tag{6d}$$

where P_{s_i} and P_r are the power limits of the source and the relay nodes, and $\gamma_i, i=1,2$ are the MSE targets for each link.

Remark 2: In these robust problem formulations, the last constraints are semi-infinite constraints, i.e., they have infinitely many realizations, while the constraint functions are not simultaneously convex in the design variables. Both these features make the proposed problem very hard to solve (a NP-hard problem). Additionally, it should be mentioned that the last constraint is threefold semi-infinite (in three different and independent variables). ♦

Remark 3: Clearly if we set $\delta_{G_i}=\delta_{H_i}=0$, $\mathcal{G}_i$ and $\mathcal{H}_i$ would become singleton sets, and these sets are reduced to the nominal values of the CSI corresponding to each channel. This case is the perfect CSI scenario which is studied deeply in the literature [1]-[4]. We will use this scenario as a benchmark to compare the performance of the proposed algorithm here. ♦

In the following section we choose a two stage mechanism to simplify the problem (5) to an equivalent problem, and then we provide an iterative algorithm which solves the equivalent problem optimally.

III. Solution

In this section, we deal with the nonconvex problem (5) and convert it into a convex equivalent problem, for which computationally efficient interior point methods exist. A similar procedure can be performed to simplify (6). To do so, we employ a two stage process: first, we deal with the semi-infiniteness of the last two constraints of (5) using the generalized version of Petersen's lemma for complex valued matrices, and then we propose an iterative algorithm based on the Alternating Convex Search (ACS), to suboptimally solve the multilinear (nonconvex) problem. We start with the last constraint which, based on Remark 2, is multilinear in the design variables and threefold semi-infinite. The $\mathtt{TxP}_r$ constraint needs a similar procedure which is not repeated here. To deal with the MSE constraints, first using $\|\boldsymbol{A}_i\|_F=\|\mathsf{vec}\left[\boldsymbol{A}_i\right]\|_2$, the $\mathtt{MSE}_i, i=1,2$ is recast as follows:

$$\mathtt{MSE}_i=\left\|\begin{bmatrix}\mathsf{vec}\left[\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{G}_i\boldsymbol{P}_i\right]-\mathsf{vec}\left[\boldsymbol{I}\right]\\ \mathsf{vec}\left[\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\boldsymbol{G}_{-i}\boldsymbol{P}_{-i}-\boldsymbol{D}_i\boldsymbol{F}_i\right]\\ \sigma_u\mathsf{vec}\left[\boldsymbol{D}_i\boldsymbol{H}_i\boldsymbol{W}\right]\\ \sigma_{e_i}\mathsf{vec}\left[\boldsymbol{D}_i\right]\end{bmatrix}\right\|^2. \tag{7}$$

After inserting the form of (2) into the above equation, and neglecting the higher order uncertainty terms[2], it is possible to recast the $\mathtt{MSE}_i$ as $\mathtt{MSE}_i\triangleq\|\boldsymbol{\mu}_i\|^2$ where

$$\boldsymbol{\mu}_i=\tilde{\boldsymbol{\mu}}_i+\boldsymbol{M}_{H_i}\ \mathsf{vec}\left[\boldsymbol{\Delta}_{H_i}\right]+\sum_{j=i,-i}\boldsymbol{M}_{G_j}\ \mathsf{vec}\left[\boldsymbol{\Delta}_{G_j}\right], \tag{8}$$

where

$$\tilde{\boldsymbol{\mu}}_i=\begin{bmatrix}\mathsf{vec}\left[\boldsymbol{D}_i\tilde{\boldsymbol{H}}_i\boldsymbol{W}\tilde{\boldsymbol{G}}_i\boldsymbol{P}_i\right]-\mathsf{vec}\left[\boldsymbol{I}\right]\\ \mathsf{vec}\left[\boldsymbol{D}_i\tilde{\boldsymbol{H}}_i\boldsymbol{W}\tilde{\boldsymbol{G}}_{-i}\boldsymbol{P}_{-i}-\boldsymbol{D}_i\boldsymbol{F}_i\right]\\ \sigma_u\mathsf{vec}\left[\boldsymbol{D}_i\tilde{\boldsymbol{H}}_i\boldsymbol{W}\right]\\ \sigma_{e_i}\mathsf{vec}\left[\boldsymbol{D}_i\right]\end{bmatrix}\in\mathbb{C}^{S_i'}, \tag{9a}$$

and subsequently

$$\boldsymbol{M}_{G_i}=\begin{bmatrix}\boldsymbol{P}_i^T\otimes\boldsymbol{D}_i\tilde{\boldsymbol{H}}_i\boldsymbol{W}\\ \boldsymbol{0}\end{bmatrix}\in\mathbb{C}^{S_i'\times rT_i}, \tag{9b}$$

$$\boldsymbol{M}_{G_{-i}}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{P}_{-i}^T\otimes\boldsymbol{D}_i\tilde{\boldsymbol{H}}_i\boldsymbol{W}\\ \boldsymbol{0}\end{bmatrix}\in\mathbb{C}^{S_i'\times rT_i}, \tag{9c}$$

$$\boldsymbol{M}_{H_i}=\begin{bmatrix}(\boldsymbol{W}\tilde{\boldsymbol{G}}_i\boldsymbol{P}_i)^T\otimes\boldsymbol{D}_i\\ (\boldsymbol{W}\tilde{\boldsymbol{G}}_{-i}\boldsymbol{P}_{-i})^T\otimes\boldsymbol{D}_i\\ \sigma_u\boldsymbol{W}^T\otimes\boldsymbol{D}_i\\ \boldsymbol{0}\end{bmatrix}\in\mathbb{C}^{S_i'\times R_it}, \tag{9d}$$

where $S_i'=S_i(S_i+S_{-i}+r+R_i)$. Using the Schur complement lemma [10], the $\mathtt{MSE}_i$ constraint can be recast as the following LMI:

$$\begin{bmatrix}\tau_i & \tilde{\boldsymbol{\mu}}_i^*\\ \tilde{\boldsymbol{\mu}}_i & \boldsymbol{I}\end{bmatrix}\succeq-\sum_{\zeta\in\{G_{i,-i},H_i\}}\begin{bmatrix}0 & (\boldsymbol{M}_\zeta\mathsf{vec}\left[\boldsymbol{\Delta}_\zeta\right])^*\\ \boldsymbol{M}_\zeta\mathsf{vec}\left[\boldsymbol{\Delta}_\zeta\right] & \boldsymbol{0}\end{bmatrix} \tag{10}$$

To proceed with this constraint, we formally generalize the Petersen's Lemma to multiple complex valued uncertainties [11]-[12]. It is noteworthy that the complex valued version of this lemma for a single uncertainty is proved in [13].

Lemma 2: Given matrices $\boldsymbol{A}$ and $\{\boldsymbol{P}_i,\boldsymbol{Q}_i\}_{i=1}^N$ with $\boldsymbol{A}=\boldsymbol{A}^*$, the semi-infinite LMI of the form of

$$\boldsymbol{A}\succeq\sum_{i=1}^N\left(\boldsymbol{P}_i^*\boldsymbol{X}_i\boldsymbol{Q}_i+\boldsymbol{Q}_i^*\boldsymbol{X}_i^*\boldsymbol{P}_i\right),\quad\forall i,\boldsymbol{X}_i:\|\boldsymbol{X}_i\|\leq\varkappa_i;$$

holds if and only if there exist nonnegative real numbers $\epsilon_1,\cdots,\epsilon_N$ such that

$$\begin{bmatrix}\boldsymbol{A}-\sum_{i=1}^N\epsilon_i\boldsymbol{Q}_i^*\boldsymbol{Q}_i & \mathsf{MAT}\left[\{-\varkappa_i\boldsymbol{P}_i\}_{i=1}^N\right]^*\\ \mathsf{MAT}\left[\{-\varkappa_i\boldsymbol{P}_i\}_{i=1}^N\right] & \mathsf{blkdiag}\left[\{\epsilon_i\boldsymbol{I}\}_{i=1}^N\right]\end{bmatrix}\succeq 0. \tag{11}$$

[2] $\boldsymbol{D}\boldsymbol{\Delta}_{H_i}\boldsymbol{W}\boldsymbol{\Delta}_{G_i}\boldsymbol{P}$ and $\boldsymbol{D}\boldsymbol{\Delta}_{H_i}\boldsymbol{W}\boldsymbol{\Delta}_{G_{-i}}\boldsymbol{P}$ have a very small norm relative to the other terms and introduce a nonlinearity to the system which makes it mathematically intractable.

Proof: The proof is omitted due to length constraints. ■

Using Lemma 2, and by appropriately choosing its parameters as follows (we have included i to distinguish between two different constraints),

$$\begin{aligned}
\boldsymbol{A}_i &= \begin{bmatrix} \tau_i & \tilde{\boldsymbol{\mu}}_i^* \\ \tilde{\boldsymbol{\mu}}_i & \boldsymbol{I} \end{bmatrix} \in \mathbb{C}^{(1+S_i')\times(1+S_i')} \\
\boldsymbol{Q}_{i1} &= \boldsymbol{Q}_{i2} = \boldsymbol{Q}_{i3} = \begin{bmatrix} -1 & \boldsymbol{0}^T \end{bmatrix} \in \mathbb{C}^{1\times(1+S_i')} \\
\boldsymbol{P}_{i1} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{M}_{H_i}^* \end{bmatrix} \in \mathbb{C}^{R_i T\times(1+S_i')}, \\
\boldsymbol{P}_{i2} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{M}_{G_i}^* \end{bmatrix} \in \mathbb{C}^{rT_i\times(1+S_i')}, \\
\boldsymbol{P}_{i3} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{M}_{G_{-i}}^* \end{bmatrix} \in \mathbb{C}^{rT_{-i}\times(1+S_i')}, \\
\boldsymbol{X}_{i1} &= \mathsf{vec}\left[\boldsymbol{\Delta}_{H_i}\right], \boldsymbol{X}_{i2} = \mathsf{vec}\left[\boldsymbol{\Delta}_{G_i}\right], \boldsymbol{X}_{i3} = \mathsf{vec}\left[\boldsymbol{\Delta}_{G_{-i}}\right],
\end{aligned}$$

It is possible to rewrite the $\mathtt{MSE}_i$ constraint as the following finite(single) LMIs:

$$\begin{bmatrix} \begin{bmatrix} \tau_i - \sum_{j=1}^3 \epsilon_{ij} & \tilde{\boldsymbol{\mu}}_i^* \\ \tilde{\boldsymbol{\mu}}_i & \boldsymbol{I} \end{bmatrix} & \boldsymbol{\Upsilon}_i^* \\ \boldsymbol{\Upsilon}_i & \mathsf{blkdiag}\left[\{\epsilon_{ij}\boldsymbol{I}\}_{j=1}^3\right] \end{bmatrix} \succeq 0, \quad (13a)$$

$$\mathsf{diag}\left[\{\tau_i, \epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3}\}_i\right] \succeq 0, \quad (13b)$$

where

$$\boldsymbol{\Upsilon}_i = \begin{bmatrix} -\delta_{H_i}\boldsymbol{P}_{i1}^T & -\delta_{G_i}\boldsymbol{P}_{i2}^T & -\delta_{G_{-i}}\boldsymbol{P}_{i3}^T \end{bmatrix}^T. \quad (13c)$$

Using a similar procedure for the other semi-infinite constraint, i.e., the $\mathtt{TxP}_r$ constraint, it is possible to show that $\mathtt{TxP}_r = \|\boldsymbol{\pi}\|^2$, where

$$\boldsymbol{\pi} = \tilde{\boldsymbol{\pi}} + \sum_{i=1}^{2} \boldsymbol{P}_{G_i}\ \mathsf{vec}\left[\boldsymbol{\Delta}_{G_i}\right], \quad (14a)$$

$$\tilde{\boldsymbol{\pi}} = \begin{bmatrix} \mathsf{vec}\begin{bmatrix} \boldsymbol{W}\tilde{\boldsymbol{G}}_1\boldsymbol{P}_1 \end{bmatrix} \\ \mathsf{vec}\begin{bmatrix} \boldsymbol{W}\tilde{\boldsymbol{G}}_2\boldsymbol{P}_2 \end{bmatrix} \\ \sigma_u \mathsf{vec}\left[\boldsymbol{W}\right] \end{bmatrix} \in \mathbb{C}^{S''}, \quad (14b)$$

$$\boldsymbol{P}_{G_1} = \begin{bmatrix} \boldsymbol{P}_1^T \otimes \boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix} \in \mathbb{C}^{S''\times rT_1}, \quad (14c)$$

$$\boldsymbol{P}_{G_2} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{P}_2^T \otimes \boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix} \in \mathbb{C}^{S''\times rT_2}, \quad (14d)$$

where $S'' = t(S_1 + S_2 + r)$. Similarly, it is possible to replace this constraint with the following single LMI:

$$\begin{bmatrix} \begin{bmatrix} P_r - \epsilon_4 - \epsilon_5 & \tilde{\boldsymbol{\pi}}^* \\ \tilde{\boldsymbol{\pi}} & \boldsymbol{I} \end{bmatrix} & \boldsymbol{\Pi}^* \\ \boldsymbol{\Pi} & \mathsf{blkdiag}\left[\epsilon_4\boldsymbol{I}, \epsilon_5\boldsymbol{I}\right] \end{bmatrix} \succeq 0, \quad (15a)$$

$$\mathsf{diag}\left[\epsilon_4, \epsilon_5\right] \succeq 0, \quad (15b)$$

where

$$\boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{0} & -\delta_{G_1}\ \boldsymbol{P}_{G_1}^* \\ \boldsymbol{0} & -\delta_{G_2}\ \boldsymbol{P}_{G_2}^* \end{bmatrix} \in \mathbb{C}^{r(T_1+T2)\times(1+S'')}. \quad (15c)$$

Putting all these equivalent constraints together will result in the following LMI which replaces (5):

$$\begin{aligned}
\underset{\boldsymbol{P}_i, \boldsymbol{W}, \boldsymbol{D}_i, \tau_i}{\text{minimize}} \quad & \tau_1 + \tau_2 \quad \text{subject to } (i = 1, 2) \qquad (16) \\
& \|\boldsymbol{P}_i\|_F^2 \le P_{s_i} \\
& \mathsf{diag}\left[\{\tau_i, \epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3}\}_i, \epsilon_4, \epsilon_5\right] \succeq 0 \\
& \begin{bmatrix} \begin{bmatrix} P_r - \epsilon_4 - \epsilon_5 & \tilde{\boldsymbol{\pi}}^* \\ \tilde{\boldsymbol{\pi}} & \boldsymbol{I} \end{bmatrix} & \boldsymbol{\Pi}^* \\ \boldsymbol{\Pi} & \mathsf{blkdiag}\left[\epsilon_4\boldsymbol{I}, \epsilon_5\boldsymbol{I}\right] \end{bmatrix} \succeq 0, \\
& \begin{bmatrix} \begin{bmatrix} \tau_i - \sum_{j=1}^3 \epsilon_{ij} & \tilde{\boldsymbol{\mu}}_i^* \\ \tilde{\boldsymbol{\mu}}_i & \boldsymbol{I} \end{bmatrix} & \boldsymbol{\Upsilon}_i^* \\ \boldsymbol{\Upsilon}_i & \mathsf{blkdiag}\left[\{\epsilon_{ij}\boldsymbol{I}\}_{j=1}^3\right] \end{bmatrix} \succeq 0.
\end{aligned}$$

This problem is not a semi-infinite problem, but it is still nonconvex. Due to biconvex and multilinear structure of the elements of $\boldsymbol{\Pi}$ and $\boldsymbol{\Upsilon}_i$, we resort to an iterative algorithms derived based on the ACS method, i.e., Algorithm 1.

Algorithm 1

Require: ε (the desired accuracy) and K_{max} (the maximum number of iterations)

1: *Initialization step:* set $k \leftarrow 0$, set the beamformer matrices randomly: $\boldsymbol{W} \leftarrow \boldsymbol{W}^{[0]}, \boldsymbol{D}_i \leftarrow \boldsymbol{D}_i^{[0]}, i = 1, 2$, and choose arbitrary $\tau_{i,new} \gg 0, i = 1, 2$.
2: **repeat**
3: $k \leftarrow k + 1$ and $\tau_{i,old} \leftarrow \tau_{i,new}, i = 1, 2$.
4: Solve (16) to find $\boldsymbol{P}_i, \boldsymbol{F}_i, i = 1, 2,$.
5: Solve (16) to update $\boldsymbol{W}$ for fixed $\boldsymbol{P}_i, \boldsymbol{F}_i, i = 1, 2$, found in the previous step.
6: Solve (16) to update $\boldsymbol{D}_i$ and $\tau_{i,new}, i = 1, 2$ for fixed $\boldsymbol{P}_i, \boldsymbol{F}_i$ and $\boldsymbol{W}$ found in the previous steps.
7: **until** $k \le K_{max}$ **or** $\sum_{i=1}^2 \tau_{i,new} - \tau_{i,old} \ge \varepsilon$

The convergence of the above algorithm is inherent from the ACS method. Since the original problem is not a convex problem, we may have different solutions due to different initialization processes. It is easily possible to show that if $\delta_{G_i} = \delta_{H_i} = 0$ the above problem reduces to problem with full (perfect) CSI. In that case, (16) becomes a simple SOCP as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_i, \boldsymbol{W}, \boldsymbol{D}_i, \boldsymbol{F}_i, \tau_i}{\text{minimize}} \quad & \tau_1 + \tau_2 \qquad (17) \\
\text{subject to} \quad & \|\boldsymbol{P}_i\|_F^2 \le P_{s_i},\ i = 1, 2, \\
& \|\tilde{\boldsymbol{\pi}}\|^2 \le P_r, \\
& \|\tilde{\boldsymbol{\mu}}_i\|^2 \le \tau_i,\ i = 1, 2.
\end{aligned}$$

IV. Simulation Results

To assess the performance of the proposed algorithm, the following simulation is done, and the results are summarized here in this section. The simulation setup is as follows: the system is used to transfer $S_1 = S_2 = 2$ streams of independent data between the source and the destination. The number of transmit and receive antennas in the source, relay and destination are equal to $T_i = R_i = t = r = 4, i = 1, 2$.

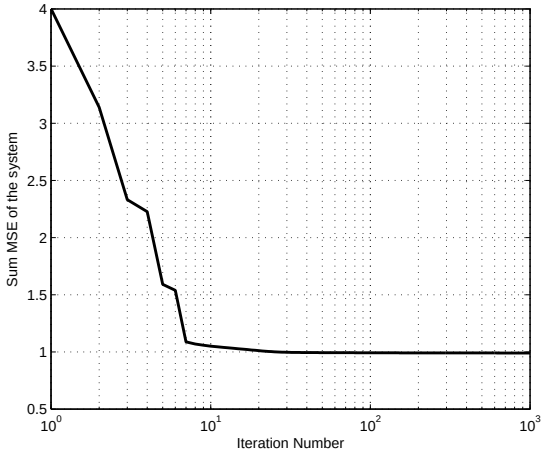

Fig. 2. Convergence of the SMSE minimization

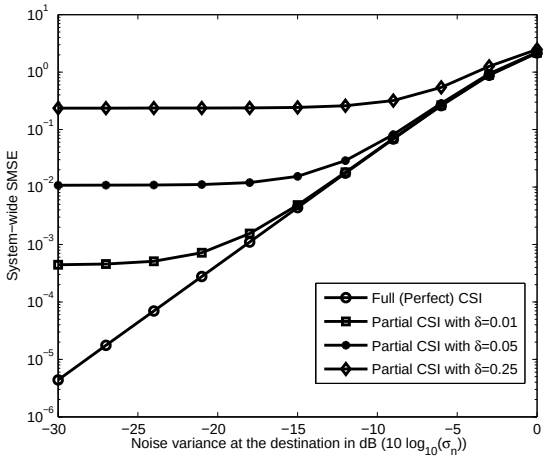

Fig. 3. System-wide SMSE

Both source and relay power budgets are set to be equal to $P_r = P_s = 1$. The convergence parameters of the algorithm are set to $K_{max} = 1000$, and $\varepsilon = 10^{-4}$. The initial value of the relay precoder and the destination equalizer matrices are set to be some random matrices. The set of channels are generated randomly to model Rayleigh fading channels.

To numerically demonstrate the convergence of the proposed algorithm, the SMSE of the system is depicted in terms of the iteration number in Fig. 2. As it is clear at each iteration the SMSE of the system decreases monotonically and after a couple of iterations the rate of the change of the SMSE is small enough to terminate the algorithm. It is clear that the algorithm converges a to local minimum with no guarantee about the global optimality. This figure shows the convergence of the algorithm for $\delta_{G_i} = \delta_{H_i} = 0.01$ and $\sigma_{e_i}^2 = \sigma_u^2 = -3\ dB$.

In Fig. 3 the SMSE of the relay system is depicted. As can be seen, the SMSE increases proportionally with the increase of the uncertainty size. It is expected since $\boldsymbol{\mu}_i$ is a linear combination of the uncertainty matrices, and the uncertainty size is the norm of these matrices. For smaller noise powers, the SMSE is mostly dominated by the uncertainty terms rather than the noise terms, and because of that the SMSE is more or less constant with respect to the noise power. However, for higher noise powers, the SMSE is dominated by the noise terms and on the rightmost part, the SMSE is only a function of the noise power but not the uncertainty size. Since the SMSE is proportional to the uncertainty size, the perfect (full) CSI case outperforms the other cases with uncertainty, because the full CSI case is a special case of the partial CSI case with $\delta_{G_i} = \delta_{H_i} = 0$.

V. Conclusion

In this paper the problem of the robust joint optimization of a two-way relay channel is studied. The system-wide SMSE-based problem formulation which is nonconvex in nature, is considered. An approximate convex solution for this problem is proposed here. Simulation results of the solution show that the SMSE of the system increases with the increase of the uncertainty size of the CSI. For the smaller noise powers, the SMSE is dominated by the uncertainty terms while for the larger noise powers SMSE is dominated by the noise terms.

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