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Robust Joint Optimization of MIMO Interfering Relay Channels with Imperfect CSI

Ebrahim A. Gharavol and Erik G. Larsson
Division of Communication Systems,
Electrical Engineering Department (ISY)
Linköping University, 581 83 Linköping, Sweden

Abstract—In this paper we deal with the problem of the joint optimization of the precoders, equalizers and relay beamformer of a multiple-input multiple-output interfering relay channel. This network can be regarded as a generalized model for both one-way and two-way relay channels with/without direct interfering links. Unlike the conventional design procedures, we assume that the Channel State Information (CSI) is not known perfectly. The imperfect CSI is described using the norm bounded error framework. We use a system-wide Sum Mean Square Error (SMSE) based problem formulation which is constrained using the transmit power of the terminals and the relay node. The problem at hand, from a worst-case design perspective, is a multilinear, and hence, a nonconvex problem which is also semi-infinite in its constraints. We use a generalized version of the Peterson's lemma to handle the semi-infiniteness and reduce the original problem to a single Linear Matrix Inequality (LMI). However, this LMI is not convex, and to resolve this issue we propose an iterative algorithm based on the alternating convex search methodology to solve the aforementioned problem. Finally simulation results, i.e., the convergence of the proposed algorithm and the SMSE properties, are included to assess the performance of the proposed algorithm.

I. INTRODUCTION

A Gaussian Interfering Relay Channel (GIFRC) is a generalized model for both interfering channels as well as relay channels. From the point of view of the relay channels, it can be considered as a general model for both one-way and two way relay channels. If only one link is presented in this network, or only one of them is actively transmitting while the other one is quite, GIFRC reduces to a one-way relay channel. If each of the destination nodes are cross co-located with the source nodes, and a sort of a self cancellation procedure is assumed, GIFRC can be considered as a two-way relay channel. In the literature, to the best of our knowledge, the achieving rates of such a network is studied only and the network optimization did not receive much attention.

In [1] the GIFRC with different relaying schemes like compress and forward, compute and forward, and finally hash-and forward is studied. The authors use a new approach to find an upper bound on the sum rate capacity of such a

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system. The capacity of a cognitive relay assisted Gaussian interference channel is studied in [2]. An achievable rate region for the system is derived by combining the Han-Kobayashi coding scheme for the general interference channel with dirty paper coding. A novel sum-rate outer bound for the Gaussian interference channel with a relay is presented in [3]. The power allocation problem for interference relay channels is considered in [4]. Due to the competitive nature of the multiuser environment, the problem is modeled as a strategic non-cooperative game. The existence of the Nash equilibrium is proved, and it is shown that this game always has a unique Nash equilibrium for any system profile. A hash-and-forward relay is studied in [5]. The improvements achieved through this method are proved and compared to compress- and amplify-and-forward methods.

In this paper we study the joint optimization of the linear precoders and equalizers of the terminals, as well as the linear beamformer of the relay node in a GIFRC. The performance measure which is central to our study is the minimum means square error error (M-MSE). To optimize the network, the precoders, equalizers and the relay beamformer, depend heavily on the Channel State Information (CSI). While it is not a practical assumption, conventionally it is assumed that the CSI is known a priori. A norm-bounded error model is used to describe the imperfectly known CSI in this paper. We shed a light on the problem from the worst-case design approach.

In this paper we use conventionally accepted notations except that of the following two notations introduced here: to show a vertical concatenation of a set of matrices to build a taller block matrix, $\text{MAT} \begin{bmatrix} \\ \\ \end{bmatrix}$ is used, and if an expression like $f(\mathbf{X}) + f_{\mathbf{X}}$ is valid for different variables, say for $\mathbf{A}, \mathbf{B}, \dots, \mathbf{Z}$, we write it in compact form as: $f(\mathbf{X}) + f_{\mathbf{X}}, \mathbf{X} \leftrightarrow \{\mathbf{A}, \mathbf{B}, \dots, \mathbf{Z}\}$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A wireless Interfering relay system is depicted in Fig. 1. In this system two distinct communication pairs simultaneously communicate with each other in two consecutive time slots. Each pair has its own transmit and receive sides. Unlike the one-way relay channel in which, to initiate a half-duplex communication service, two distinct time slots are required, in this system using two time slots, a full-duplex communication is provided. In the first time slot, both terminals transmit their

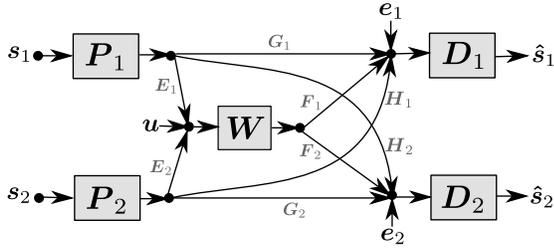


Fig. 1. Signal Flow Graph of Gaussian Interfering Relay Channel

data toward the destination and the relay nodes. This signal propagates to the destination nodes as well. The received signal at the intended receiver node contributes to increase the Signal to Interference plus Noise (SINR) while the received signal at the other node will decrease that SINR. In the second time slot, the relay node retransmits the combination of the received signals, after linearly beamforming, towards the other end of the communication link. By means of this relaying process, the exchange of the data is facilitated between these terminals by increasing the SINR. To transmit $S_i, i = 1, 2$ independent streams of zero-mean, unit-variance data ($s_i \in \mathbb{C}^{S_i}, i = 1, 2$) with independent elements toward the either end of the communication link, each transmitter is equipped with $T_i, i = 1, 2$ transmit antennas. The receivers are equipped with $R_i, i = 1, 2$ receive antennas, while the relay has r and t receive and transmit antennas, respectively. To better compensate the fading channels, the source nodes precode the data using the precoding matrices $P_i \in \mathbb{C}^{T_i \times S_i}$. The precoded data is sent over wireless fading channels, namely $G_i \in \mathbb{C}^{R_i \times T_i}, H_i \in \mathbb{C}^{R_i \times T_i}, E_i \in \mathbb{C}^{r \times T_i}, i = 1, 2$, where \mathbf{X}_{-i} refers to a quantity belonging to the other index set, in a bi-index notation we used here¹. At the relay node, the received signal is amplified using $W \in \mathbb{C}^{t \times r}$ and the resultant signal is transmitted to the destinations in the next time slot over the wireless channels $F_i \in \mathbb{C}^{R_i \times t}, i = 1, 2$.

After appropriately combining the received signals and decoding using linear decoders $D_i \in \mathbb{C}^{S_i \times R_i}, i = 1, 2$, the received signals are as follows:

$$\hat{s}_i = D_i(G_i + F_i W E_i)P_i s_i + D_i F_i W u + D_i e_i + D_i(H_i + F_i W E_{-i})P_{-i} s_{-i}, \quad i = 1, 2 \quad (1)$$

In this equation $u \in \mathbb{C}^r$ and $e_i \in \mathbb{C}^{R_i}$ are the additive zero-mean noise signals with independent elements and σ_u^2 and $\sigma_{e_i}^2$ variances, respectively. Due to the limited feedback between the nodes, it is assumed that only the nominal value of the CSI is known to the system. In other words, the CSI follows the norm bounded error model, i.e., for any complex-valued matrix quantity like $E_{i,-i}, F_{i,-i}, G_{i,-i}, H_{i,-i}$, say \mathbf{X} , we have:

$$\mathbf{X} \in \mathcal{X} = \{\tilde{\mathbf{X}} + \Delta_{\mathbf{X}} \mid \|\Delta_{\mathbf{X}}\|_F \leq \delta_{\mathbf{X}}\}, \quad (2)$$

¹That is $\mathbf{X}_{-i} = \begin{cases} \mathbf{X}_2, & i = 1, \\ \mathbf{X}_1, & i = 2. \end{cases}$

where $\tilde{\mathbf{X}}$, is the fixed nominal value of the CSI for each of the channels and $\Delta_{\mathbf{X}}$, is the random norm-bounded variation (uncertainty) around these nominal value.

In this paper, our goal is to jointly optimize the source precoders, the relay beamformer, and the destination equalizers. To do so, we can either use the system-wide Sum Mean Square Error (SMSE) as the performance measure of the system, while restricting the optimization problem with the power budgets of both sources and the relay node, or we can use the transmit power of the relay node as the performance measure and restrict the optimization problem with target MSEs for each link. Regardless of the problem formulation, by employing a worst-case design approach, the optimal solutions will be valid for all the realizations of the CSI that satisfy (2). To facilitate the computation of the MSE of each link, and the transmit powers of the sources and the relay nodes, we use the following lemma.

Lemma 1: For any set of zero-mean, independent and identically distributed random vectors with independent elements and individual variances of $E_{x_i} [x_i^* x_i] = \sigma_i^2$ we have

$$E_{\{x_i\}_i} \left[\left\| \sum_i A_i x_i \right\|_2^2 \right] = \sum_i \sigma_i^2 \|A_i\|_F^2 \quad (3)$$

Proof: The proof is omitted due to length constraints. ■

Based on this Lemma 1, the MSE of the links ($MSE_i, i = 1, 2$), and the transmit powers of the source ($TxP_{s_i}, i = 1, 2$) and the relay (TxP_r) nodes are defined as follows:

$$TxP_{s_i} \triangleq E_{s_i} [\|P_i s_i\|_2^2], \quad i = 1, 2 \quad (4a)$$

$$= \|P_i\|_F^2, \quad (4b)$$

$$TxP_r \triangleq E_{s_{1,2}, u} [\|W(E_1 P_1 s_1 + E_2 P_2 s_2) + W u\|_2^2] \quad (4c)$$

$$= \|W E_1 P_1\|_F^2 + \|W E_2 P_2\|_F^2 + \sigma_u^2 \|W\|_F^2, \quad (4d)$$

$$MSE_i \triangleq E_{s_{1,2}, u, e_{1,2}} [\|\hat{s}_i - s_i\|^2], \quad i = 1, 2 \quad (4e)$$

$$= \|D_i(G_i + F_i W E_i)P_i - I\|_F^2 + \sigma_u^2 \|D_i F_i W\|_F^2 + \|D_i(H_i + F_i W E_{-i})P_{-i}\|_F^2 + \sigma_{e_i}^2 \|D_i\|_F^2 \quad (4f)$$

Using these quantities, the problem formulation in its epigraph form will be to minimize the SMSE:

$$\underset{P_i, W, D_i, \tau_i \geq 0}{\text{minimize}} \quad \tau_1 + \tau_2 \quad \text{subject to} \quad (5)$$

$$TxP_{s_i} \leq P_{s_i}, \quad i = 1, 2,$$

$$TxP_r \leq P_r, \quad \forall \mathbf{X} \in \mathcal{X}, \mathbf{X} \leftrightarrow \{E_{i,-i}\}, i = 1, 2,$$

$$MSE_i \leq \tau_i, \quad \forall \mathbf{X} \in \mathcal{X}, \mathbf{X} \leftrightarrow \{E_{i,-i}, F_i, G_i, H_i\}, i = 1, 2,$$

where P_{s_i} and P_r are the power limits of the source and the relay nodes, and $\gamma_i, i = 1, 2$ are the MSE targets for each link.

Remark 1: In these robust problem formulations, the last constraints are semi-infinite constraints, i.e., they have infinitely many realizations, while the constraint functions are not simultaneously convex in the design variables. Both these features make the proposed problem very hard to solve (a NP-hard problem). Additionally, it should be mentioned that the last constraint is fivefold semi-infinite (in five different and independent variables). ♦

Remark 2: Clearly if we set $\delta_X = 0$, \mathcal{X} would become a singleton set, and this set is reduced to the nominal values of the CSI corresponding to each channel. This case is the perfect CSI scenario. \blacklozenge

In the following section we choose a two stage mechanism to simplify the problem (5) to an equivalent problem, and then we provide an iterative algorithm which solves the equivalent problem optimally.

III. SOLUTION

In this section, we deal with the nonconvex problem (5) and convert it into a convex equivalent problem, for which computationally efficient interior point methods exist. To do so, we employ a two stage process: first, we deal with the semi-infiniteness of the last two constraints of (5) using the generalized version of Petersen's lemma for complex valued matrices, and then we propose an iterative algorithm based on the Alternating Convex Search (ACS), to suboptimally solve the multilinear (nonconvex) problem. We start with the last constraint which, based on Remark 1, is multilinear in the design variables and threefold semi-infinite. The TxP_r constraint needs a similar procedure which is not repeated here. To deal with the MSE constraints, first using $\|\mathbf{A}_i\|_F = \|\text{vec}[\mathbf{A}_i]\|_2$, and after inserting the form of (2) into the above equation, and neglecting the higher order uncertainty terms², it is possible to recast the MSE_i as $\text{MSE}_i \triangleq \|\boldsymbol{\mu}_i\|^2$ where

$$\boldsymbol{\mu}_i = \tilde{\boldsymbol{\mu}}_i + \sum_{\mathbf{X} \leftrightarrow \mathcal{U}_i} M_X \text{vec}[\boldsymbol{\Delta}_X], \quad (6)$$

where $\mathcal{U}_i = \{\mathbf{E}_{i,-i}, \mathbf{F}_i, \mathbf{G}_i, \mathbf{H}_i\}$ and

$$\tilde{\boldsymbol{\mu}}_i = \begin{bmatrix} \text{vec} \left[D_i(\tilde{\mathbf{G}}_i + \tilde{\mathbf{F}}_i \mathbf{W} \tilde{\mathbf{E}}_i) \mathbf{P}_i - \mathbf{I} \right] \\ \text{vec} \left[D_i(\tilde{\mathbf{H}}_i + \tilde{\mathbf{F}}_i \mathbf{W} \tilde{\mathbf{E}}_{-i}) \mathbf{P}_{-i} \right] \\ \sigma_u \text{vec} \left[D_i \tilde{\mathbf{F}}_i \mathbf{W} \right] \\ \sigma_{e_i} \text{vec} \left[D_i \right] \end{bmatrix} \in \mathbb{C}^{m'_i}, \quad (7a)$$

and subsequently

$$M_{G_i} = \begin{bmatrix} \mathbf{P}_i^T \otimes D_i \\ \mathbf{0} \end{bmatrix} \quad M_{E_i} = \begin{bmatrix} \mathbf{P}_i^T \otimes D_i \tilde{\mathbf{F}}_i \mathbf{W} \\ \mathbf{0} \end{bmatrix} \quad (7b)$$

$$M_{H_i} = \begin{bmatrix} \mathbf{0} \\ \mathbf{P}_{-i}^T \otimes D_i \\ \mathbf{0} \end{bmatrix} \quad M_{E_{-i}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{P}_{-i}^T \otimes D_i \tilde{\mathbf{F}}_i \mathbf{W} \\ \mathbf{0} \end{bmatrix} \quad (7c)$$

$$M_{F_i} = \begin{bmatrix} (\mathbf{W} \tilde{\mathbf{E}}_i \mathbf{P}_i)^T \otimes D_i \\ (\mathbf{W} \tilde{\mathbf{E}}_{-i} \mathbf{P}_{-i})^T \otimes D_i \\ \sigma_u \mathbf{W}^T \otimes D_i \\ \mathbf{0} \end{bmatrix}, \quad (7d)$$

where $m'_i = S_i(S_i + S_{-i} + r + R_i)$. Using the Schur complement lemma [6], the MSE_i constraint can be recast as the following LMI:

$$\begin{bmatrix} \tau_i & \tilde{\boldsymbol{\mu}}_i^* \\ \tilde{\boldsymbol{\mu}}_i & \mathbf{I} \end{bmatrix} \succeq - \sum_{\mathbf{X} \leftrightarrow \mathcal{U}_i} \begin{bmatrix} 0 & (M_X \text{vec}[\boldsymbol{\Delta}_X])^* \\ M_X \text{vec}[\boldsymbol{\Delta}_X] & \mathbf{0} \end{bmatrix}$$

² $D_i \boldsymbol{\Delta}_{F_i} \mathbf{W} \boldsymbol{\Delta}_{E_i} \mathbf{P}_i$ and $D_i \boldsymbol{\Delta}_{F_i} \mathbf{W} \boldsymbol{\Delta}_{E_{-i}} \mathbf{P}_{-i}$ have a very small norm relative to the other terms and introduce a nonlinearity to the system which makes it mathematically intractable.

To proceed with this constraint, we formally generalize the Petersen's Lemma to multiple complex valued uncertainties [7]-[8]. It is noteworthy that the complex valued version of this lemma for a single uncertainty is proved in [9].

Lemma 2: Given matrices \mathbf{A} and $\{\mathbf{P}_i, \mathbf{Q}_i\}_{i=1}^N$ with $\mathbf{A} = \mathbf{A}^*$, the semi-infinite LMI of the form of

$$\mathbf{A} \succeq \sum_{i=1}^N (\mathbf{P}_i^* \mathbf{X}_i \mathbf{Q}_i + \mathbf{Q}_i^* \mathbf{X}_i^* \mathbf{P}_i), \quad \forall i, \mathbf{X}_i : \|\mathbf{X}_i\| \leq \varkappa_i;$$

holds if and only if there exist nonnegative real numbers $\epsilon_1, \dots, \epsilon_N$ such that

$$\begin{bmatrix} \mathbf{A} - \sum_{i=1}^N \epsilon_i \mathbf{Q}_i^* \mathbf{Q}_i & \text{MAT} [\{-\varkappa_i \mathbf{P}_i\}_{i=1}^N]^* \\ \text{MAT} [\{-\varkappa_i \mathbf{P}_i\}_{i=1}^N] & \text{blkdiag} [\{\epsilon_i \mathbf{I}\}_{i=1}^N] \end{bmatrix} \succeq 0. \quad (8)$$

Proof: The proof is omitted due to length constraints. \blacksquare Using Lemma 2, and by appropriately choosing its parameters as follows (we have included i to distinguish between two different constraints),

$$\mathbf{A}_i = \begin{bmatrix} \tau_i & \tilde{\boldsymbol{\mu}}_i^* \\ \tilde{\boldsymbol{\mu}}_i & \mathbf{I} \end{bmatrix} \in \mathbb{C}^{(1+m'_i) \times (1+m'_i)}$$

$$\mathbf{Q}_{i1} = \mathbf{Q}_{i2} = \mathbf{Q}_{i3} = \mathbf{Q}_{i4} = \mathbf{Q}_{i5} = \begin{bmatrix} -1 & \mathbf{0}^T \end{bmatrix} \in \mathbb{C}^{1 \times (1+m'_i)}$$

$$\mathbf{P}_{ij} = \begin{bmatrix} \mathbf{0} & M_Y^* \end{bmatrix}, \quad \mathbf{X}_{ij} = \text{vec}[\boldsymbol{\Delta}_Y] |_{Y \leftrightarrow \mathcal{U}_i}, \quad j = 1, \dots, 5$$

It is possible to rewrite the MSE_i constraint as the following finite(single) LMIs:

$$\begin{bmatrix} \left[\begin{array}{c|c} \tau_i - \sum_{j=1}^5 \epsilon_{ij} & \tilde{\boldsymbol{\mu}}_i^* \\ \hline \tilde{\boldsymbol{\mu}}_i & \mathbf{I} \end{array} \right] & \boldsymbol{\Upsilon}_i^* \\ \boldsymbol{\Upsilon}_i & \text{blkdiag} [\{\epsilon_{ij} \mathbf{I}\}_{j=1}^5] \end{bmatrix} \succeq 0, \quad (9a)$$

$$\text{diag} [\{\tau_i, \epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3}, \epsilon_{i4}, \epsilon_{i5}\}_i] \succeq 0, \quad (9b)$$

where

$$\boldsymbol{\Upsilon}_i = \text{MAT} \left[\left\{ -\delta_{\mathbf{X}} \mathbf{P}_{ij} \right\}_{\mathbf{X} \leftrightarrow \{\mathbf{G}_i, \mathbf{E}_{i,-i}, \mathbf{H}_i, \mathbf{F}_i\}}^{j=1, \dots, 5} \right]. \quad (9c)$$

Similarly, it is possible to replace the TxP_r constraint with the following single LMI:

$$\begin{bmatrix} \left[\begin{array}{c|c} P_r - \epsilon_6 - \epsilon_7 & \tilde{\pi}^* \\ \hline \tilde{\pi} & \mathbf{I} \end{array} \right] & \boldsymbol{\Pi}^* \\ \boldsymbol{\Pi} & \text{blkdiag} [\epsilon_6 \mathbf{I}, \epsilon_7 \mathbf{I}] \end{bmatrix} \succeq 0, \quad (10a)$$

$$\text{diag} [\epsilon_6, \epsilon_7] \succeq 0, \quad (10b)$$

where $\boldsymbol{\Pi} = \begin{bmatrix} \mathbf{0} & -\delta_{E_1} \mathbf{P}_{E_1}^* \\ \mathbf{0} & -\delta_{E_2} \mathbf{P}_{E_2}^* \end{bmatrix}$. Putting all these equivalent constraints together will result in the following LMI which replaces (5):

$$\underset{\mathbf{P}_i, \mathbf{W}, D_i, \tau_i}{\text{minimize}} \quad \tau_1 + \tau_2 \quad \text{subject to } (i = 1, 2) \quad (11)$$

$$\|\mathbf{P}_i\|_F^2 \leq P_{s_i}$$

$$\text{diag} [\{\tau_i, \epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3}, \epsilon_{i4}, \epsilon_{i5}\}_i, \epsilon_6, \epsilon_7] \succeq 0$$

$$\begin{bmatrix} \left[\begin{array}{c|c} P_r - \epsilon_6 - \epsilon_7 & \tilde{\pi}^* \\ \hline \tilde{\pi} & \mathbf{I} \end{array} \right] & \boldsymbol{\Pi}^* \\ \boldsymbol{\Pi} & \text{blkdiag} [\epsilon_6 \mathbf{I}, \epsilon_7 \mathbf{I}] \end{bmatrix} \succeq 0,$$

$$\begin{bmatrix} \left[\begin{array}{c|c} \tau_i - \sum_{j=1}^5 \epsilon_{ij} & \tilde{\boldsymbol{\mu}}_i^* \\ \hline \tilde{\boldsymbol{\mu}}_i & \mathbf{I} \end{array} \right] & \boldsymbol{\Upsilon}_i^* \\ \boldsymbol{\Upsilon}_i & \text{blkdiag} [\{\epsilon_{ij} \mathbf{I}\}_{j=1}^5] \end{bmatrix} \succeq 0.$$

This problem is not a semi-infinite problem, but it is still nonconvex. Due to biconvex and multilinear structure of the elements of $\mathbf{\Pi}$ and $\mathbf{\Upsilon}_i$, we resort to an iterative algorithms derived based on the ACS method, i.e., Algorithm 1.

Algorithm 1

Require: ε (the desired accuracy) and K_{max} (the maximum number of iterations)

1: *Initialization step:* set $k \leftarrow 0$, set the beamformer matrices randomly: $\mathbf{W} \leftarrow \mathbf{W}^{[0]}$, $\mathbf{D}_i \leftarrow \mathbf{D}_i^{[0]}$, $i = 1, 2$, and choose arbitrary $\tau_{i,new} \gg 0$, $i = 1, 2$.

2: **repeat**

3: $k \leftarrow k + 1$ and $\tau_{i,old} \leftarrow \tau_{i,new}$, $i = 1, 2$.

4: Solve (11) to find \mathbf{P}_i , $i = 1, 2$.

5: Solve (11) to update \mathbf{W} for fixed \mathbf{P}_i , $i = 1, 2$, found in the previous step.

6: Solve (11) to update \mathbf{D}_i and $\tau_{i,new}$, $i = 1, 2$ for fixed \mathbf{P}_i , \mathbf{F}_i and \mathbf{W} found in the previous steps.

7: **until** $k \leq K_{max}$ **or** $\sum_{i=1}^2 \tau_{i,new} - \tau_{i,old} \geq \varepsilon$

The convergence of the above algorithm is inherent from the ACS method. Since the original problem is not a convex problem, we may have different solutions due to different initialization processes. It is easily possible to show that if $\delta_{G_i} = \delta_{H_i} = \delta_{E_i} = \delta_{F_i} = 0$ the above problem reduces to problem with full (perfect) CSI. In that case, (11) becomes a simple SOCP as follows:

$$\begin{aligned} & \underset{\mathbf{P}_i, \mathbf{W}, \mathbf{D}_i, \mathbf{F}_i, \tau_i}{\text{minimize}} && \tau_1 + \tau_2 \\ & \text{subject to} && \|\mathbf{P}_i\|_F^2 \leq P_{s_i}, \quad i = 1, 2, \\ & && \|\tilde{\boldsymbol{\pi}}\|^2 \leq P_r, \\ & && \|\tilde{\boldsymbol{\mu}}_i\|^2 \leq \tau_i, \quad i = 1, 2. \end{aligned} \quad (12)$$

IV. SIMULATION RESULTS

To assess the performance of the proposed algorithm, the following simulation is done, and the results are summarized here in this section. The simulation setup is as follows: the system is used to transfer 2 streams of independent data between the source and the destination. The number of transmit and receive antennas in the source, relay and destination are equal to 4. Both source and relay power budgets are set to be equal to 1. The convergence parameters of the algorithm are set to $K_{max} = 1000$, and $\varepsilon = 10^{-4}$. The initial value of the relay precoder and the destination equalizer matrices are set to be some random matrices. The set of channels are generated randomly to model Rayleigh fading channels.

In Fig. 2 the SMSE of the relay system is depicted. As can be seen, the SMSE increases proportionally with the increase of the uncertainty size. It is expected since $\boldsymbol{\mu}_i$ is a linear combination of the uncertainty matrices, and the uncertainty size is the norm of these matrices. For smaller noise powers, the SMSE is mostly dominated by the uncertainty terms rather than the noise terms, and because of that the SMSE is more or less constant with respect to the noise power. However, for higher noise powers, the SMSE is dominated by the noise

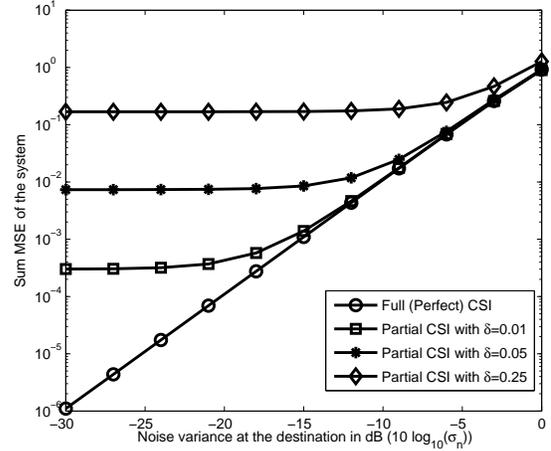


Fig. 2. System-wide SMSE

terms and on the rightmost part, the SMSE is only a function of the noise power but not the uncertainty size. Since the SMSE is proportional to the uncertainty size, the perfect (full) CSI case outperforms the other cases with uncertainty, because the full CSI case is a special case of the partial CSI case with $\delta_{G_i} = \delta_{H_i} = \delta_{E_i} = \delta_{F_i} = 0$.

V. CONCLUSION

In this paper the problem of the robust joint optimization of an interfering relay channel is studied. The system-wide SMSE-based problem formulation which is nonconvex in nature, is considered. Simulation results of the solution show that the SMSE of the system increases with the increase of the uncertainty size of the CSI. For the smaller noise powers, the SMSE is dominated by the uncertainty terms while for the larger noise powers SMSE is dominated by the noise terms.

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