

Parametric wing profile description for conceptual design.

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Introduction

A fundamental part of aircraft design involves the wing airfoil optimization, establishing an outer shape of the wing which has good aerodynamic performance, good internal volume distribution for fuel and systems and which also serves as an efficient structural member supporting the weight of the aircraft. As for all optimization tasks, the complexity of the problem is directly coupled to the parameterization of the geometry. Of highest relevance are the number of parameters and the number of additional constraints that are required to ensure valid modeling.

This paper proposes a parameterization method for two dimensional airfoils, aimed at providing a wide design space, while at the same time keeping the number of parameters low. With 15 parameters defining the wing profile, many of the existing airfoils can be modeled with close tolerance.

Several approaches to parameterization of wing profiles can be found in the literature. Airfoils can be described by point clouds as done in most airfoil libraries [1]. The number of parameters is twice as large as the number of points used (x and y coordinates) and in the case of aerodynamic optimization this parameterization will most certainly be not well behaved, since no smoothing function is included and must therefore be added. Other problems may arise for the fact that the airfoils sometimes are defined with too few coordinate points and/or too few decimals, a problem occurring especially with old airfoils. On the other hand, the design space that this kind of parameterization allows representing is extremely large, as any and all shapes can be reproduced, even degenerate ones.

Airfoils can also be represented by mathematical functions. Among the most famous representatives of this category are indeed the NACA 4-, 5- and 6-digits formulations [2] [3]. Compared to point clouds, they could be said to represent the opposite case: they are very well behaving parameterizations, but they cannot cover a very large design space, since they only provide four or more parameters respectively to be tuned. The NACA 4 digit series is particularly interesting as the parameters are a part of the name of the airfoil. In the case of the 5- and 6 digit series, the name is instead constructed from the airfoils aerodynamic characteristic and geometry.

Another known set of theoretically defined airfoils are the Joukowski profiles [4]. Using the conformal mapping method, airfoils with a round nose and sharp trailing edge can be represented. Sadly the method is not recommended for trying to match known airfoils and the design space it describes is quite confined to airfoils with often poor performances. More recently a very interesting and powerful representation method was presented by Kulfan [5]. The method is based on simple well behaved mathematical functions that allow for increasing the

number of parameters at will. Also the formulation is not restricted to round nose and sharp trailing edge profiles, but can describe almost any shape.

Problem

The authors identifies the need of a wing profile modeling method which shares the large design space with the point cloud method of representation, while maintaining a small number of parameters. By employing a functional parametric description with built-in smoothing and interpolation, coordinate resolution becomes infinite. Some basic assumptions regarding the shape of an airfoil are made: The airfoil is assumed to have one distinct top and bottom point, the chord line has a length of one, forming the x-axis of the profile, the trailing edge is assumed to be either sharp, i.e. with a y-coordinate of zero for both top and bottom sides, or blunt with the y coordinates of the top and bottom trailing edges being symmetric around the chord line.

Method

The airfoils top and bottom curves are both modeled by two cubic Beziér curves in parametric form being C^1 continuous at the defined top and bottom points. Figure 1 gives an example of a generic high speed profile and show the outer envelope, the camber line and the Beziér control points. Equation 1 shows the parametric Beziér curve.

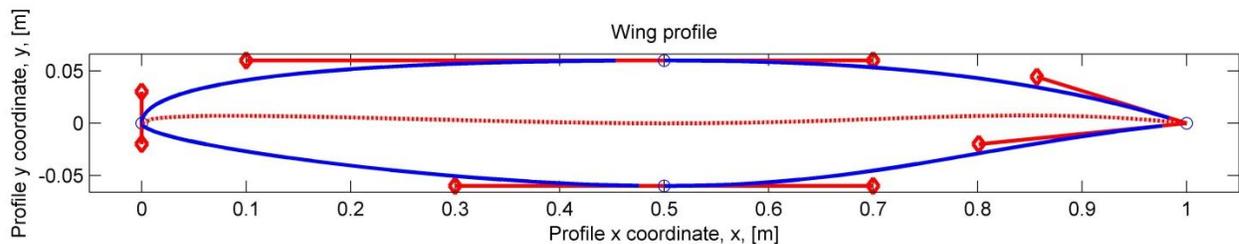


Figure 1: Generic model of a modern high speed airfoil. Diamonds are the Beziér control points. The dotted line is the camber line.

$$B(t) = (1 - t)^3P_0 + 3(1 - t)^2tP_1 + 3(1 - t)t^2P_2 + t^3P_3, t \in [0, 1]. \tag{1}$$

The method of representing a new airfoil with the proposed method is straight forward. As a baseline, a four part cubic Beziér curve, all 13 control points are needed to define the curve, giving 26 variables when both x and y coordinates are taken into account. However when some symmetries and simplifications are taken into account the number of independent parameters are reduced to 14. The Beziér control points for the wing profile are defined clockwise around the profile from the trailing edge bottom side, according to figure 2. Control point 1 and 13 always have the same x-coordinate, one, and the same absolute value of their y-coordinate. Control points 3,4 and 5 on the bottom side have the same y-coordinate. The same goes for the top side control points 9, 10 and 11. The leading edge points 6,7 and 8 all have an x-coordinate of zero. Table one show the dependent parameters for the control points.

Point #	Coordinate	
	X	Y
1	1	a
2	X_2	Y_2
3	X_3	b
4	X_4	b
5	X_5	b
6	0	Y_6
7	0	0
8	0	Y_8
9	X_9	c
10	X_{10}	c
11	X_{11}	c
12	X_{12}	Y_{12}
13	1	-a

Table 1: Coordinate dependency for the different control points. Some always take the value one or zero, while others (a,b,c) always shares the same numerical while (X_n, Y_n) are allowed any value.

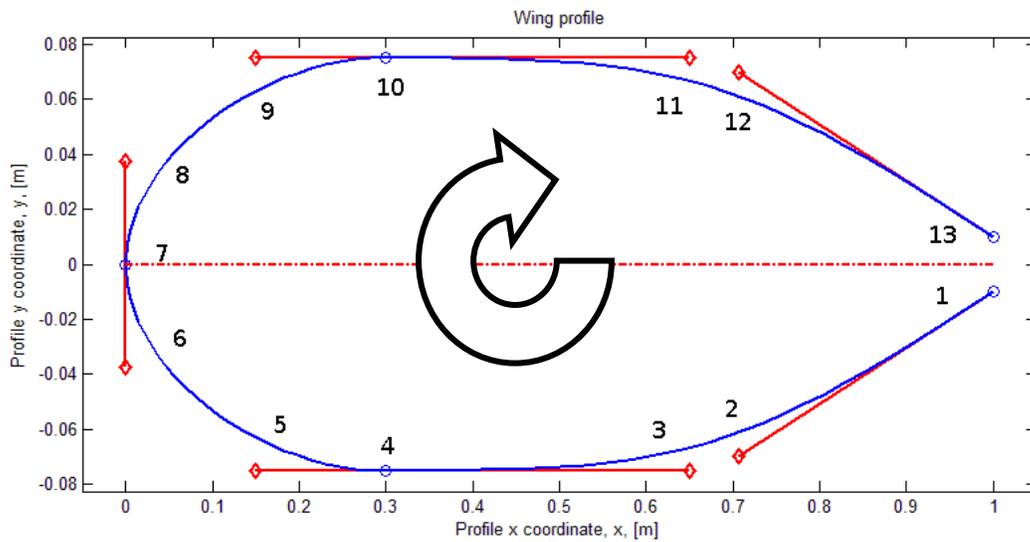


Figure 2: Control point distribution and numbering

Parameter reduction

Handling the control point coordinates directly when specifying an airfoil is not so user friendly. The coordinates themselves have no obvious relation to shape of the airfoil when reviewed alone and the possible range of the coordinates exceed the allowed range for non-degenerate airfoils. To alleviate this concern, and to reduce the number of free parameters, the following parameter transformation is done.

The lengths of the control vectors are limited to prevent degenerate cases. For example the upper leading edge control vector needs to be shorter than the distance between the top point and the chord line to prevent leading edge overshoot.

Thickness: The x and y coordinates of the top and bottom points (# 4 and #10) are kept as is and named upper thickness and upper thickness position and similarly for the lower side. The difference between the upper and lower thickness then becomes the ram thickness, which in case of the thickness positions being the same equals the thickness of the airfoil. If the upper and lower thickness positions are not equal, the ram thickness will be slightly larger than the thickness.

Control vectors:

The length of control the upper leading edge vector drawn by its start point, the leading edge which is point number 7, and point number 8, is defined as a fraction (k_1) of the upper thickness (h_u). The constant k is only allowed to be in the range $[0..1]$, se figure 3. This ensures that the part of the profile going from the leading edge to the top point does not overshoot the top points y value.

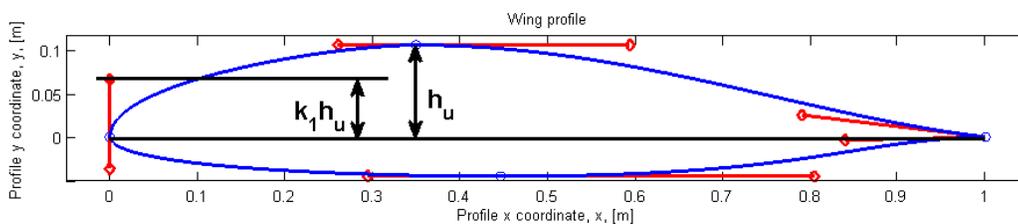


Figure 3: Definition of the leading edge control vector.

The top side of the airfoil is controlled by three parameters, shown in figure (4). The top point position, which is the x-coordinate with the largest y-value, and two fractions (k) which control the lengths of the control vectors. The control vector going from the top point towards the leading edge, i.e. between points #10->#9 has its length limited to a fraction of the top point position in the same way in order to prevent an overshoot of the profile into negative x-coordinates.

Likewise the control vector defined by points 10 to 11, the top to TE vector is a fraction (k_4) of the length between the upper thickness position (#10) and the trailing edge ($x=1$).

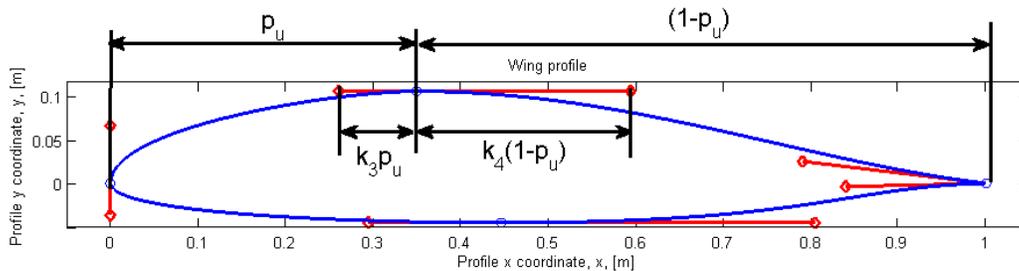


Figure 4: Parameters defining the top side of the airfoil, in this case a parameterized NACA747a415.

The trailing edge is defined by four parameters, shown in figure 5: The boat tail angle (β), which is the angle between tangents of the upper and lower sides of the airfoil at the trailing edge; The release angle (α), which is the angle between the camber line tangent and the chord line at the trailing edge; the trailing edge gap (g), which is the distance between the upper and lower side of the airfoil at the trailing edge; and finally the height of the trailing edge control vector which as the leading edge control vector lengths is defined as a fraction (k_7) of the upper thickness. To fully define the trailing edge we also need the lower side control vector height, which is defined in the same manner as the upper one.

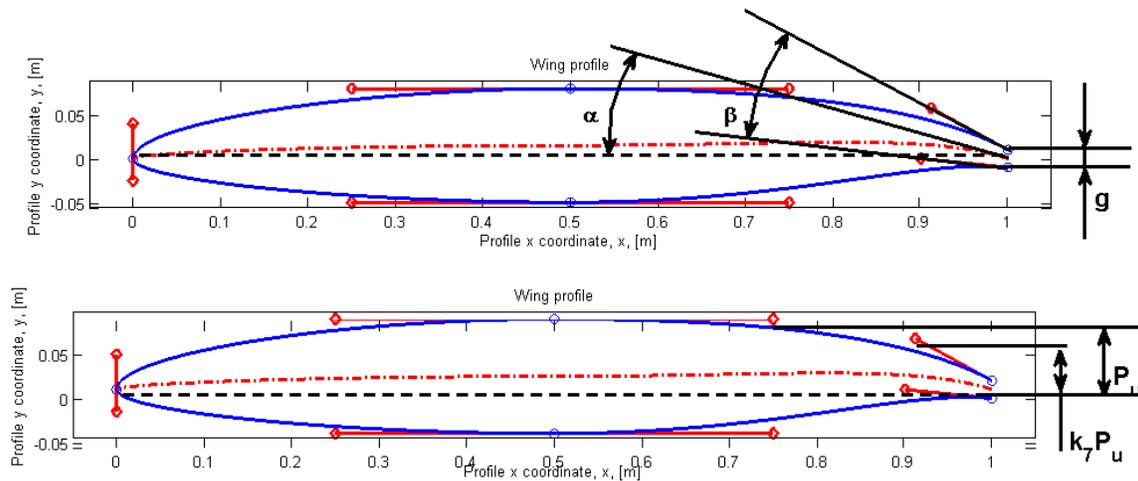


Figure 5: Definition of the parameters of the trailing edge.

The parameters of the lower side of the airfoil are defined in the same manner as the upper side parameters.

The number of parameters are now reduced to 15, see table 2, and by the limits of the fractions many degenerate airfoils are now impossible to generate. By using the chord length as normalizing agent, the parameters are also dimensionless and fully scalable.

Name	Variable	Range
Leading edge		
Upper Nose fraction	k_1	[0..1]
Upper Nose fraction	k_2	[0..1]
Upper side		
Upper thickness	H_U	[0..1]
Upper thickness position	P_U	[0..1]
Upper forward fraction	k_3	[0..1]
Upper rearward fraction	K_4	[0..1]
Lower side		
Lower thickness	H_L	[0..1]
Lower thickness position	P_L	[0..1]
Lower forward fraction	k_5	[0..1]
Upper rearward fraction	K_6	[0..1]
Trailing edge		
Trailing edge gap	g	[0..1]
Boat tail angle	β	$[-\pi..\pi]$
Release angle	α	$[-\pi..\pi]$
Upper trailing edge fraction	k_7	[0..1]
Lower trailing edge fraction	k_8	[0..1]

Table 2: Control parameters for the parameterized airfoil

Reproducing known airfoils

In many design cases it will not be appropriate to start with a generic airfoil for design optimization. Instead, by employing a known airfoil, whether it is a Clark-Y, NACA0012 or any other known airfoil there is a need of having the traditional airfoil shape expressed in these parameters. Modeling known airfoils require slightly more work than creating new airfoils.

Preprocessing

When fitting known wing profiles to the parameterization, the input data are sometimes in poor condition and in need of preprocessing. The point cloud have to have its leading edge at [0 0] and the trailing edge cord have to end at [1 0], if the profile has a trailing edge gap, this should be symmetrically distributed around the [1 0].

One example of wing profiles needing preprocessing is the Clark-Y profile, an older propeller profile today used frequently amongst model aircraft builders for its ease of manufacture rather than aerodynamic performance. The coordinate data for the Clark-Y profile, as presented in some databases [6] has the lower side flank parallel to the x-axis, rather than letting the chord line define the x-axis.

A starting guess of parameters were created by evaluating the profile radius at the leading edge, top and bottom, at the upper maximum y- position and at the lower minimum y-position. Together with the trailing edge gap and the slope of the upper and lower sides of the airfoil at the trailing edge. The radius at these points are computed with a three point circle algorithm from which the length of the control vectors can be computed according to equation 2, where a is the outgoing control vector length and h the perpendicular distance to the next control point, see figure 6.

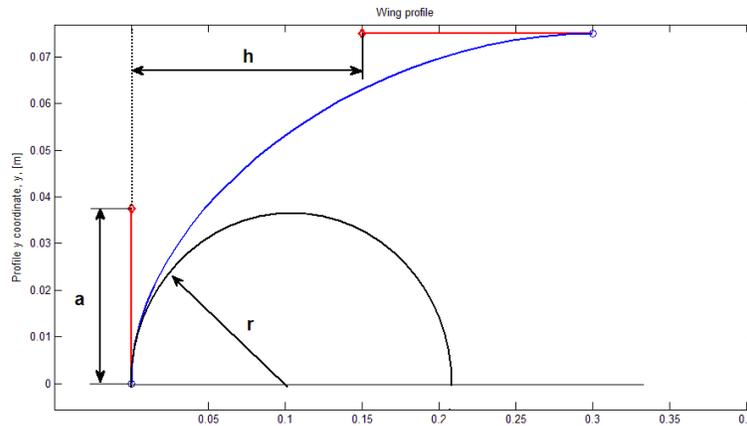


Figure 6: Radius definitions at the leading edge Beziér curve.

$$\frac{1}{r} = \frac{2}{3} \frac{h}{a^2} \quad (2)$$

Optimization

With the obtained starting guess, a standard optimization is run in MATLAB to fit the parameter curve to the corresponding point cloud. The mathematical method used is the built-in `fsolve` method, which in this case utilizes the Levenberg-Marquardt algorithm. The run is set to minimize the root mean square error of the vertical position of the coordinate points between parameter points and point cloud.

The optimization is nested in two layers in order to treat the parameter nature of the Beziér curve. The outer loop solves for the control parameters governing the curve segment in question and an inner loop solving for the curvilinear position parameter vector t , with respect to the x coordinate. –As the original data points may have any distribution along the x -axis, it can't be assumed that the t vector in equation 1 is evenly distributed.

Comparison with existing data

The optimization results can be evaluated both from a geometrical point of view, where the error of vertical position of each point can be plotted for the profile. The average RMS error for the profile is a general quality control. The average RMS error was usually in the order of 10^{-4} , necessitating the use of the unit “error counts” (ects), where 1 ects = 10^{-4} . Similarly, the curvature and error in curvature between were evaluated – Notably many of the available airfoils had very noisy curvature distributions, making an error plot of curvature fitting interesting but not a good indicator to whether the parametric interpolation was good or bad. Instead, as long as a good scatter is found any offset bias or systematic error can be assumed to be small.

To further investigate the correlation between the original data point cloud and the parametric wing profile, an aerodynamic study was performed utilizing the panel method XFOIL [7]. The original airfoil and the parameterized airfoil were both subjected to a C_l sweep ranging from -0.3 to 1 at a Reynolds number of 6×10^6 . The drag and moment coefficients were evaluated, as well as the boundary layer transition point for both profiles.

Results

The proposed parameterization method was tested on a set of different airfoils. Results for profiles: NACA 0012, Clark-Y, NASA747A415 and a Whitcomb supercritical profile are presented in this report. For the investigated airfoils the reported curvature distribution graph is truncated at the leading edge in order to give a better scale of the results for the rest of the profile. In the Clark-Y example, the curvature of the lower flank is zero, which gives an infinite error there as the parameterization curvature is non-zero.

Figure 7 show the comparison between a point cloud representation of the NACA0012 profile, and the parameterization. The RMS error of **0.51 ects**, which should be regarded as a good benchmark number when evaluating other airfoils. As the coordinates of the NACA0012 is a parametric curve in functional form in itself, there was a very close match between the two datasets. The geometrical similarity distribution shown in figure 8.

Maximum Thickness	0.12 @ 30 %C
Maximum Camber	0 @ 100 %C
Trailing Edge Gap	0 %C
Upper nose radius	1.5 %C
Lower nose radius	1.5 %C
Boat-Tail Angle	16.32 deg
Release Angle	0 deg

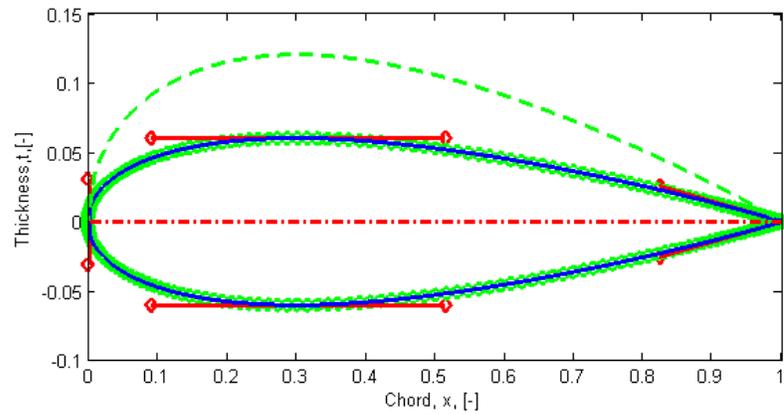


Table 3: NACA0012 geometrical properties.

Figure 7: NACA0012 profile. Original points and parametric curve overlaid. Height and width not to scale.

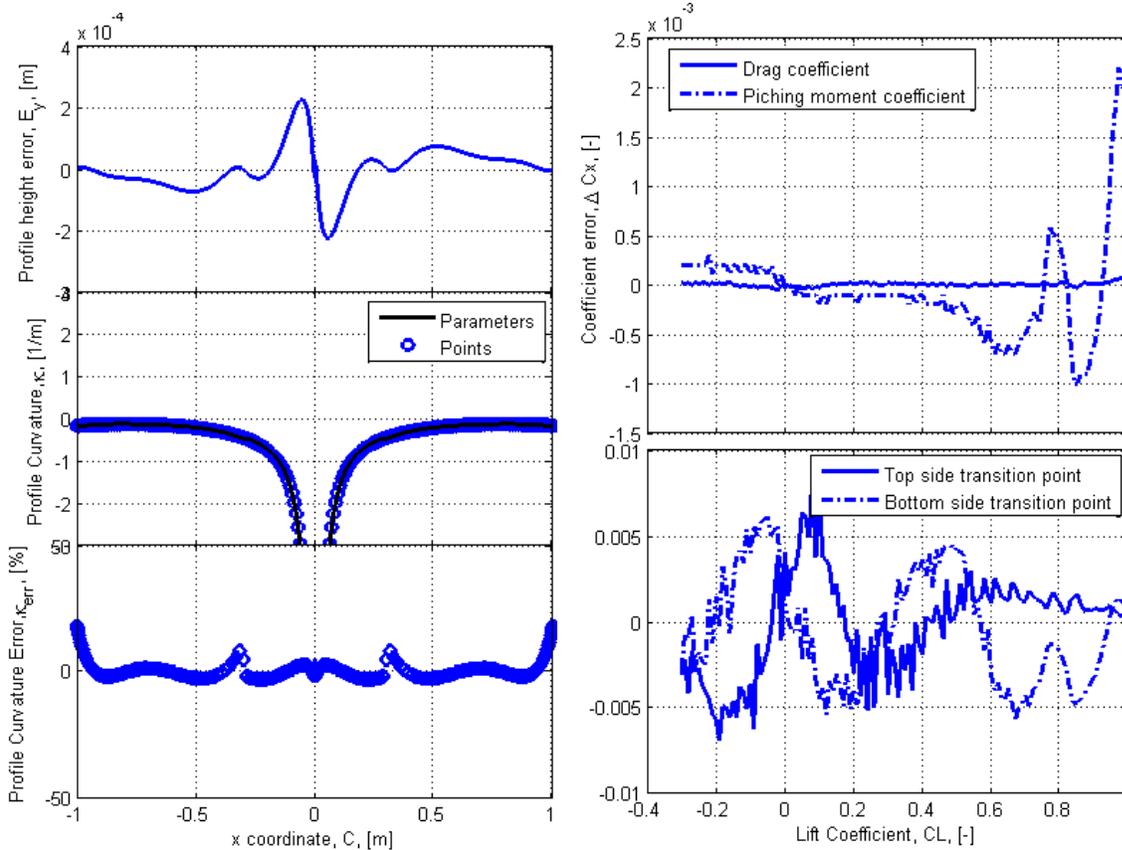


Figure 8: Geometrical and aerodynamic similarity errors between the two representations of the NACA 0012. To the left, comparisons between the vertical position of the points and the profile curvature. To the right a comparison between the aerodynamic properties of the two airfoils.

The Clark-Y airfoil shown in figure 9 rendered an error of **1.42 ects**, which should be also considered good, especially since this airfoil has a challenging straight lower side flank. The geometrical similarity distribution shown in figure 10

Maximum Thickness	0.12 @ 30 %C
Maximum Camber	0.03 @ 42 %C
Trailing Edge Gap	0.12 %C
Upper nose radius	0.2 %C
Lower nose radius	2.1 %C
Boat-Tail Angle	15.74 deg
Release Angle	5.72 deg

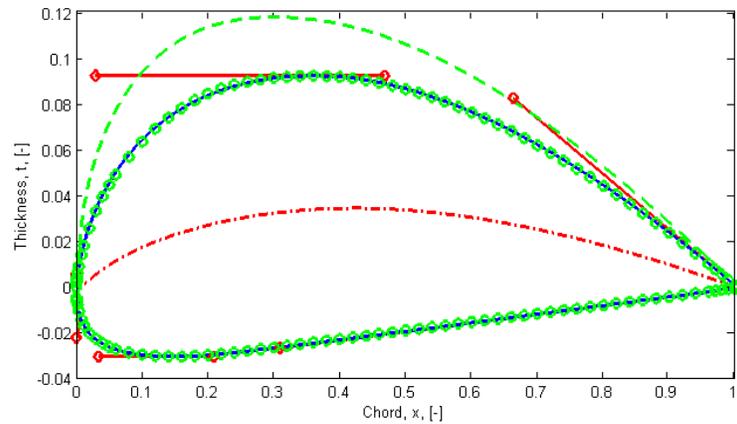


Table 4: Clark Y geometrical properties.

Figure 9: Clark Y profile. Original points and parametric curve overlaid. Height and width not to scale.

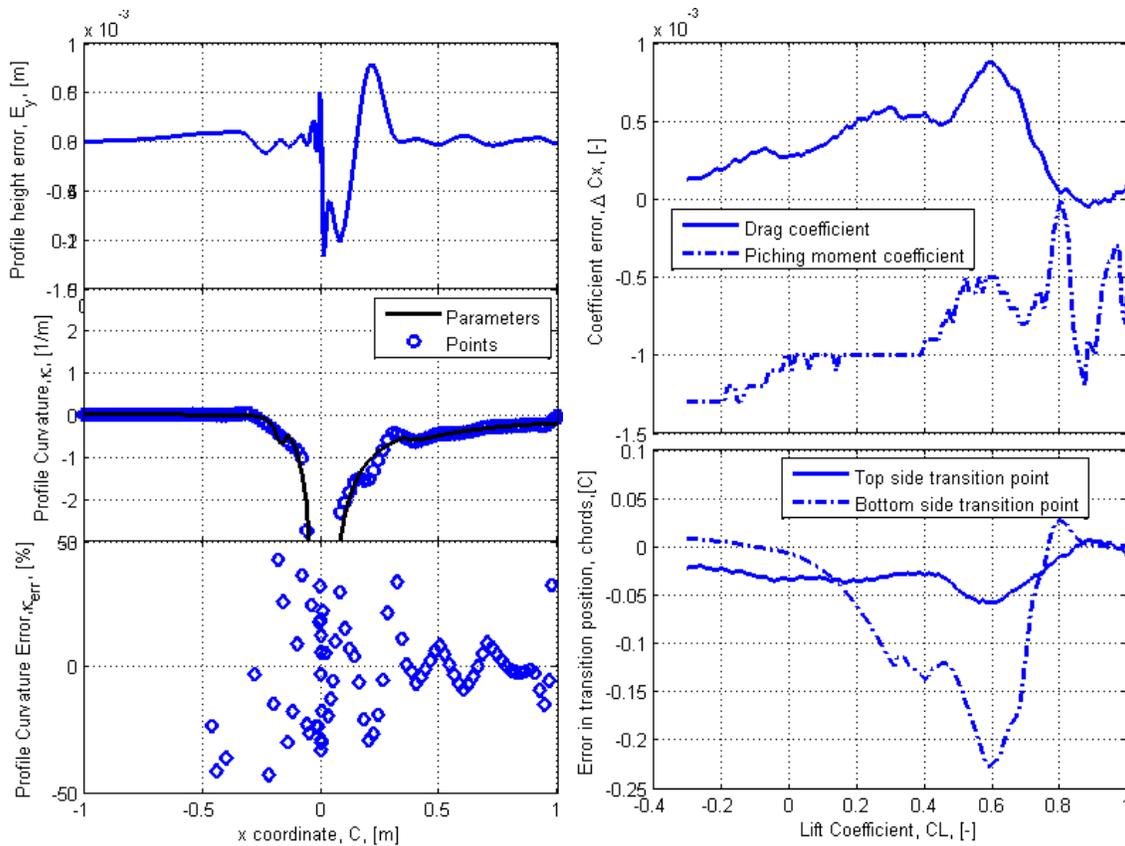


Figure 10: Geometrical and aerodynamic similarity errors between the two representations of the Clark-Y. To the left, comparisons between the vertical position of the points and the profile curvature. To the right a comparison between the aerodynamic properties of the two airfoils.

The NASA 747A415 profile shown in figure XX had fewer original datapoints than the earlier profiles, but was still possible to model with **3.98 ects**. The geometrical similarity distribution shown in figure 11 shows an error spike at the leading edge. This is where the optimization is the most sensitive to numerical errors.

Maximum Thickness	0.15 @ 37 %C
Maximum Camber	0.03 @ 34 %C
Trailing Edge Gap	0 %C
Upper nose radius	2.5 %C
Lower nose radius	0.7 %C
Boat-Tail Angle	8.13 deg
Release Angle	2.74 deg

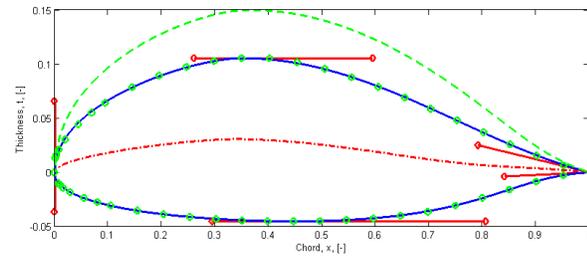


Table 5: 747A415 geometrical properties.

Figure 11: 747A415 profile. Original points and parametric curve overlaid. Height and width not to scale.

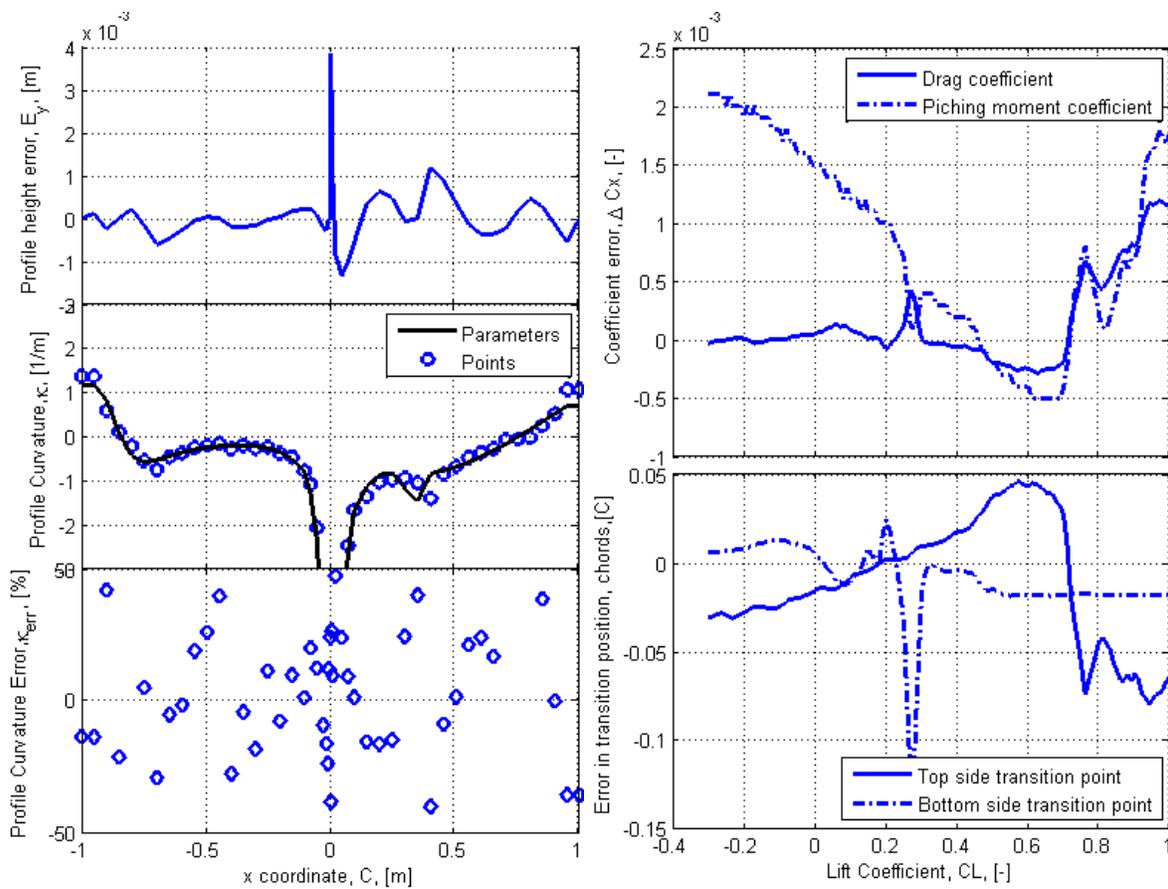


Figure 12: Geometrical and aerodynamic similarity errors between the two representations of the Nasa 747A415. To the left, comparisons between the vertical position of the points and the profile curvature. To the right a comparison between the aerodynamic properties of the two airfoils.

The supercritical Whitcomb profile was possible to emulate with the parameterization with **5.49** *ects*. As shown in figure 13, the largest error was at the trailing edge lower side where the original dataset a discontinuous curvature. Still the aerodynamic similarity was good with drag being within 5 drag counts for most CL's.

Maximum Thickness	0.11 @ 36 %C
Maximum Camber	0.02 @ 84 %C
Trailing Edge Gap	0.05 %C
Upper nose radius	2.4 %C
Lower nose radius	2.6 %C
Boat-Tail Angle	4.67 deg
Release Angle	17.95 deg

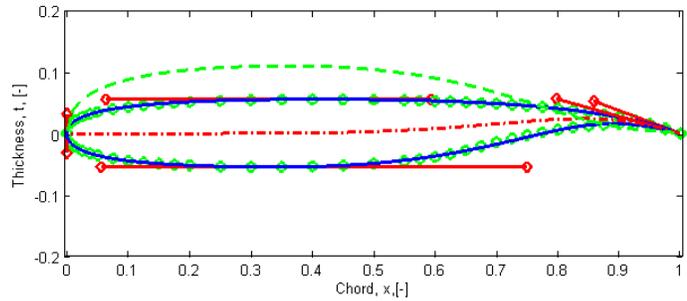


Table 6: Whitcomb geometrical properties.

Figure 13: Whitcombe profile. Original points and parametric curve overlaid.

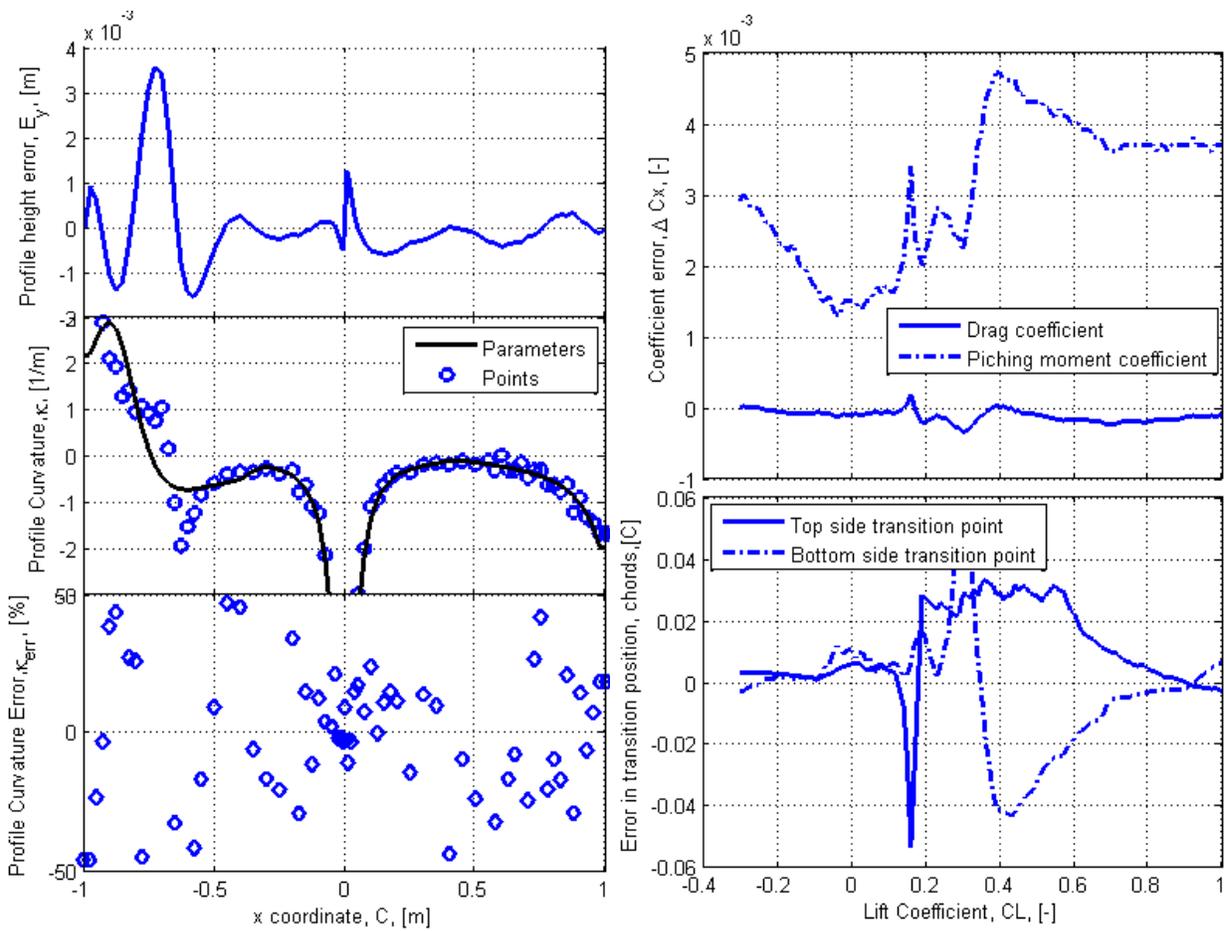


Figure 14: Geometrical and aerodynamic similarity errors between the two representations of the Whitcomb profile. To the left, comparisons between the vertical position of the points and the profile curvature. To the right a comparison between the aerodynamic properties of the two airfoils.

Figure 15 below show the pressure distribution on the top and bottom sides of the Naca747A415 profile at $C_L=0.3$ for both original data and parameterized airfoil .Additionally, the skin friction coefficient of the top side skin is shown for airfoil representations. For both aerodynamic properties a good agreement between the two representations can be found.

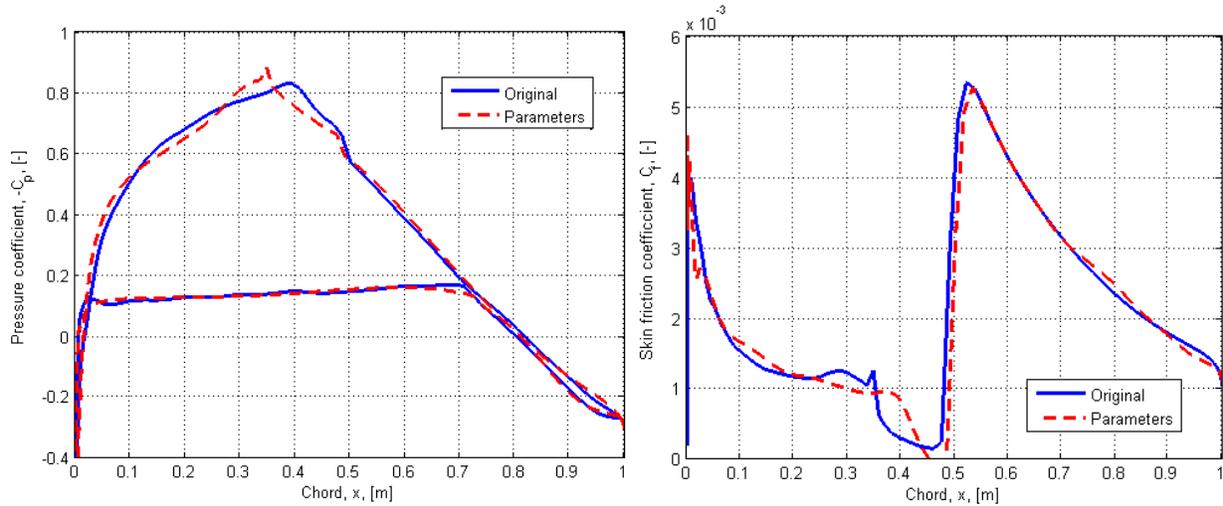


Figure 15: Pressure distribution and upper skin friction distribution of the two airfoil representations.

It's also interesting to look at a larger population of airfoils. 28 different classical airfoils have been parameterizes and analysed. The distribution of their geometrical model error is shown below in figure 16 for the profiles modeled so far. Most of the airfoils have an RMS error smaller than 0.0005, or half a millimeter to a meter chord.

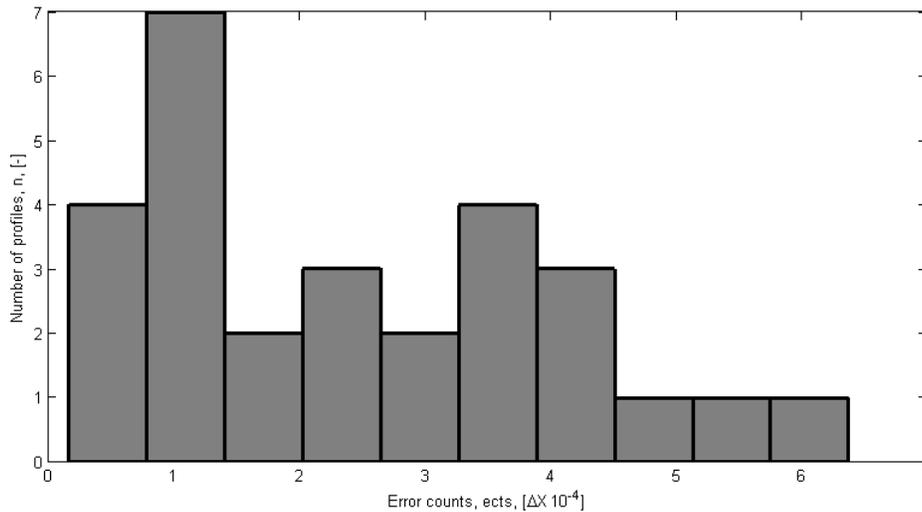


Figure 16: RMS error distribution for 28 parameterized profiles.

Conclusion

The proposed parameterization offers a stringent way of representing known airfoils without the problems of a finite number of coordinate points in a list or with the limitation of round-off and truncation errors in the number of decimals in the coordinates themselves.

Furthermore the proposed parameterization offers a way of continuously optimize an airfoil shape without the discrete changes occurring when going from one airfoil to the next.

Most known airfoils seem to be possible to parameterize with good geometrical and aerodynamic similarity with their original point-cloud representations.

Discussion

A further aerodynamic study should be performed to investigate any difference other aerodynamic phenomena. Notably stall angle of attack and post-stall behavior, which cannot be captured well with XFOIL. Likewise High speed shock formation similarities needs to be assessed before this parametric wing profile description can be adopted for wide scale airfoil interpolation.

The method as described does not require C2 continuity at the top and bottom points. Adding this constraint to the method will reduce the number of free parameters with two, and also give a more traditional curvature continuity. However, the geometrical fitting of known airfoils would suffer in having a higher geometrical error. –For most known airfoils tested, the optimized curvature is indeed discontinuous, but with a very small step. This discontinuity does not appear to have a significant impact on the aerodynamic performance of the parameterized airfoil.

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