Passive Control for a Human Power Amplifier, providing Force Amplification, Guidance and Obstacle Avoidance

Examensarbete utfört i Reglerteknik vid Tekniska högskolan vid Linköpings universitet av

Fredrik Eskilsson

LiTH-ISY-EX--11/4531--SE

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In this master thesis a control strategy for a Human Power Amplifier (HPA) is presented. An HPA can be described as a machine that amplifies a force exerted by a human operator. The HPA in this thesis can best be described as a mechanical ore with two degrees of freedom.

The approach for the control strategy presented here is to look at the control problem not directly as a force amplifying problem, but as coordination problem between the real system and a virtual system, where the virtual system is used as a reference. If the systems are synchronized then desired force amplification will naturally follow from that.

Furthermore is the possibility to implement guidance and obstacle avoidance on the machine investigated. The guidance is performed by using velocity fields, i.e., vector fields where a vector represents the desired velocity for each point in the plane. For the obstacle avoidance potential fields are used, where the idea is that a high potential should repel the machine from restricted areas.
Abstract

In this master thesis a control strategy for a Human Power Amplifier (HPA) is presented. An HPA can be described as a machine that amplifies a force exerted by a human operator. The HPA in this thesis can best be described as a mechanical ore with two degrees of freedom.

The approach for the control strategy presented here is to look at the control problem not directly as a force amplifying problem, but as coordination problem between the real system and a virtual system, where the virtual system is used as a reference. If the systems are synchronized then desired force amplification will naturally follow from that.

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Sammanfattning

I detta examensarbete har en reglerstrategi för en Human Power Amplifier (HPA) tagits fram. En HPA är en maskin som förstärker den krafter som en mänsklig operatör påverkar systemet med. Den HPA som har använts kan bäst beskrivas som en mekanisk åra med två frihetsgrader.

Den reglerstrategi som presenteras går ut på att istället för att direkt försöka reglera krafterna från den motor eller cylinder som skall utföra rörelsen, så introduceras ett nytt virtuellt system som används som en referens. Reglerproblemet blir nu att synkronisera rörelse av det verkliga systemet med det virtuella och på så vis uppnå kraftförstärkningen.

Vidare undersöks även möjligheterna att implementera guidning och en funktion för att undvika hinder, på maskinen. Guidning görs med hjälp av hastighetsfält, vilket är vektorfält där vektorerna representerar en önskad hastighet i varje punkt i planet. För att undvika hinder används potentialfält, där idén är att ett område med hög potential skall stöta bort maskinen.
Acknowledgments

I would like to thank Professor Perry Y Li, for giving me the opportunity to work in his lab and for helping me learn more about automatic control and robotics. I would also like to thank graduate student Venkat Durbha for guidance and support during my time at the University of Minnesota, graduate student Patrik Axelsson and my examiner, Assistant Professor Johan Löfberg, for help and advice during the report writing process. Finally thanks to Professor David A Bernlohr and Professor Maria Sumnerhagen, for all there help and for making my stay at the University of Minnesota possible.
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Chapter 1

Introduction

1.1 Background

This thesis was performed at the Mechanical Engineering department of the University of Minnesota (UoM), in Minneapolis, USA. It will treat a study of the control system for a human power amplifier (HPA). An human power amplifier is here defined as a machine that helps a human operator perform a task of some kind, by amplifying the force that the operator exerts on the machine.

The most well known type of HPA is probably the exoskeleton, which is an external skeleton with some type of actuators helping the human to perform a desired motion without requiring as much effort as otherwise needed, e.g., when doing heavy lifts or carrying heavy burdens. There is a wide range of applications where such machines could be of great use, e.g., a fireman who carries out people from a burning building, helping people who have reduced ability to move, or for rehabilitation purposes.

The HPA at UoM is not an exoskeleton, instead it can best be described as a hydraulic assisted mechanical oar with two degrees of freedom (DoF), more details are given in Chapter 2. The benefits with this system, compared to an exoskeleton, is that it is safer as it is possible to let go of the machine if something goes wrong. Furthermore it is actuated with hydraulic actuators, which give it high power density but make it quite unattractive to carry. This HPA is therefore more suited to use as crane to lift heavy burdens e.g., lift the engine out of a car or similar.

At UoM the HPA is in many ways an experimental set up for testing different theories, that in a later state might be used on other machineries. It can also be used as a demonstration tool for educational purposes.

One great benefit of using an HPA when performing a task is that the operator obtains direct feedback from the system in terms of force and displacement. An example of an application where a system such as the one presented here could be used is a backhoe where the operator will get feedback from the environment and can, e.g., feel when he hits the ground, if the ground is soft or hard and if he hits any obstacles while digging.
1.2 Aim of Study

The aim of the study in this thesis is to construct a controller for the HPA that works in two degrees of freedom. The controller should be able to amplify the desired force from the human operator with a fix gain and thereby making it possible to perform heavy tasks with low effort from the human. The aim is also that the system should be strictly passive. When satisfying properties for the controller are achieved the aim is to add some extra features to the system, guidance and obstacle avoidance. The guidance should help the operator follow a predetermined path and the obstacle avoidance should help the operator avoid predefined virtual obstacles.

1.3 Related Work

Human amplification

Mush work has been done in the area human power amplification. For this particular case there is a couple of studies made on the machine of subject in this thesis, [16] and [17]. In [16] a first attempt to construct a controller for this system is presented. The approach here was to use a PI-controller and a feed forward term to directly control the force error. This controller worked satisfactory during static amplification but not during free motion. The main problem for the controller was sensitivity to the feed forward term. In [17], a new approach is presented, here the problem is not seen directly as a force amplification problem, but as a velocity tracking problem between a virtual and the real system. This theory has successfully been implemented for one DoF on the machine. This theory will be the base for the controller design in this thesis.

Furthermore there is a lot written about power amplifiers of different kind, a lot of them are some kind of exoskeleton, e.g., [4] and [21]. Most of the applications use some kind of electrical actuator, but there are examples that, similar to this thesis, uses hydraulics [8]. In most cases a force sensor is used for the power amplifications but there are also ideas how to perform power assistance without any sensors at all [20]. In Section 1.1 it is mentioned that a power amplifier can be used for rehabilitation, [26] gives a good example on hand rehabilitation.

Guidance

For doing guidance control one approach is to use velocity fields. Velocity fields or vector fields is also an area where a lot of research has been done. In [14], [13] and [12] Li and Horowitz present a theory, Passive Velocity Field Control (PVFC), to use velocity fields for tracking and guidance problem. In [11] and [10] Lee present a way to use PVFC to create guidance for mechanical teleoperators. Chen and colleagues presents a way to create more complex shapes for the guidance path, using velocity fields [3]. Others that also worked with velocity fields in a similar ways are Moreno and Kelly [19] and Yamakita et.al. [28].
Obstacle avoidance

Obstacle avoidance often mean that an obstacle should first be detected and then avoided e.g. [7]. In this thesis that is not the aim, i.e., the position of the obstacle is considered known. Given the position of the obstacle there is however many ways to avoid it. One approach proposed in [25] and [9] is to use potential functions and potential fields for doing the avoidance. In [9] potential fields are used for real time avoidance by changing the field on line. Potential fields can also be used for path planning and for guiding a robot to a certain point [25]. Theories for implementation of obstacle avoidance using potential fields are also proposed in [11] and [10].

1.4 Methodology

Initially a literature on previous work in related areas is made. Specially literature concerning previous work on the machine that is to be studied in this thesis. The methods of interest chosen to test on the machine was a coordination formulation to create the force amplification, using velocity field (vector fields) for constructing the guidance and using potential field to create the obstacle avoidance.

To be able to test the theories in simulation before testing them on the machine, a model is build and system identification is made, to estimate unknown parameters.

After that, the first step is to create a working force amplification controller with both DoF working simultaneously. Once the force amplification is working and gives satisfactory results, the system is extended with guidance and obstacle avoidance control.

1.5 Outline

The system is described in detail in Chapter 2. In Chapter 3 the modulation of the system is presented and in Chapter 4 the system identification results. The first approach is to see the two DoF as separate systems and construct a independent controller for each of them. This approach is presented in Chapter 5. A new system formulation where the influence of the DoF:s on each other is accounted for is presented in Chapter 6, and the controller using that architecture is described in Chapter 7. The machine is then equipped with some extra features, guidance and obstacle avoidance, and these extensions are described in Chapter 8 and Chapter 9. Results from implementation on the real system are presented in Chapter 10. Chapter 11 gives a discussion about the possible conclusions that can be made and future work for the HPA.
Chapter 2

System description

In this chapter a short introduction on the properties of the system is given. A picture of the human amplifier is shown in Figure 2.1. As mentioned earlier the system can be seen as a hydraulic assisted oar with two Degrees of freedom (DoF).

![Figure 2.1: Hydraulic assisted mechanical ore.](image)

2.1 Mechanical system

When talking about the mechanical system, it is the mechanical skeleton that is moved by the operator and the hydraulic actuators that is referred. Simplified the mechanical system can be described by Figure 2.2. As mentioned the system has
two DoF, these two degrees will be called *reach* and *pitch*. The reach motion is the motion obtained when moving beam A (see figure) back and forward inside beam B. The pitch motion comes from rotating both beams around the rotation point.

### 2.2 Hydraulic system

To perform the power amplification, hydraulic actuators are used. The hydraulic system contains of two hydraulic actuators, one hydraulic motor and one hydraulic cylinder, each actuator is controlled by a servo valve. A hydraulic pump provides the system with a constant pressure (65 bar), see Figure 2.3 for hydraulic scheme and Figure 2.1 for a picture of the components.

![Hydraulic Scheme](image)

Figure 2.3: Hydraulic Scheme, where a hydraulic pump provides the system with a constant pressure. Two servo valves controls the two actuators, a hydraulic cylinder and a hydraulic motor. The cylinders cap side area is denoted $A_c$ and the piston side area $A_p$. The motor has displacement $D_m$ and $r_m$ is the radius for the pulley.

The task for the hydraulic components is to amplify the forces from the human
operator exerted on the handle. The cylinder performs the pitch motion and the motor the reach motion. The motor is connected to a belt and pulley application where the belt pulls the beam in each direction.

2.3 Sensors

To be able to control the system, information about forces acting on it and its position is required. This information will be obtained from in total five force sensors and two potentiometers, see Figure 2.4. Two force sensors on the handle measures the force from the human operator in each DoF. The remaining force sensors measures the corresponding forces from the actuators. The reach motion needs two sensors one for each direction, where the sensor are calibrated so that the reach force is the difference between the two sensors. To keep track of the position a potentiometer is used for each DoF, see Figure 2.4.

2.4 Peripherals

To be able to make measurement and construct a controller for the system the sensor signals are fed to the computer via operational amplifiers and a DAQ-card. MATLAB Simulink is used for constructing the controller and controlling the system.
Chapter 3

Modelling

To get to know the system and to be able to test different control strategies, before they are implemented on the real system, a model for the system is developed. For this model to be useful it is important that it has the same interface with the controller as the real system. The inputs can be seen in Table 3.1 and the outputs in Table 3.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit and Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_p$</td>
<td>Control signal for pitch motion</td>
<td>[-5 - 5 V]</td>
</tr>
<tr>
<td>$u_r$</td>
<td>Control signal for reach motion</td>
<td>[-5 - 5 V]</td>
</tr>
</tbody>
</table>

Table 3.1: Input Signals

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>Torque for pitch motion actuator</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$x_p$</td>
<td>Position for pitch motion</td>
<td>[rad]</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Force for reach motion actuator</td>
<td>[N]</td>
</tr>
<tr>
<td>$x_r$</td>
<td>Position for reach motion</td>
<td>[m]</td>
</tr>
</tbody>
</table>

Table 3.2: Output Signals

The model will be divided into a hydraulic and a mechanic model, where the hydraulic model gives the torque/force and the mechanical model gives the position. See Figure 3.1 for a basic idea of the system.

3.1 Mechanical Model

The mechanical model should describe the dynamics of the mechanical skeleton of the system when reaching back and forward and pitching up and down, see Figure 3.2 for a sketch of the mechanical system. When reaching it is the mass, in the figure denoted $m_r$, that is moved. When pitching both $m_r$ and the mass denoted $m_p$ are rotated, i.e., there is one linear and on rotational motion in the system.
Newton laws gives that the mechanical system for the two DoF:s will be on the following form,

\[
\begin{align*}
J \ddot{x}_p &= T, \quad (3.1) \\
\dot{m} \ddot{x}_r &= F.
\end{align*}
\]

Where \( J \) and \( T \) are the inertia and torque for the rotation and \( m \) and \( F \) are the mass and force for the linear movement.

Let’s start investigating the linear motion, where \( m = m_r \) as explained earlier. The forces that act on the system are assumed to be a human force \( F_h \) from the human operator, the force from the hydraulic actuator \( F_r \) which will be issued in the hydraulic model section below and a force \( F_g \) from gravity acting on the mass.

In the model the value or the function describing \( F_h \) can be whatever desired for the particular simulation. \( F_g \) will be dependent of the pitch position as

\[
F_g = m_r g \cos(x_p).
\]

For the rotational motion the inertia \( J \) becomes some more complicated than the mass in the linear case. The inertia will differ depending on the reach position. If the center of gravity (CoG) of the beam with mass \( m_r \) is placed in the rotational
point the inertia has one value but as moving the CoG away from the rotation point the inertia increases, i.e., one part of the inertia is constant and one part is dependent on the reach position \(x_r\). The constant part is denoted \(J_c\) and by using the parallel-axis theorem \([22]\), the inertia is given by

\[
J = J_c + m_r l_{cog}(x_r)^2,
\]

where \(l_{cog}(x_r)\) is the distance from the CoG to the rotation point. The torques acting on this inertia are similar to before denoted \(T_h, T_p\) and \(T_g\), where \(T_h\) and \(T_p\) is the human and the actuator force. \(T_g\) is the gravitational force and is for this case

\[
T_g = m_r g l_{cog}(x_r) \sin(x_p).
\]

To encounter for the damping a damping term with damp constants \(b_p\) and \(b_r\) was added in both subsystems and the total mechanical system is given by

\[
\begin{align*}
(J_c + m_r l_{cog}(x_r)^2)\ddot{x}_p &= T_p + T_h + m_r g l_{cog}(x_r) \cos(x_p) + b_p \dot{x}_p, \\
m_r \ddot{x}_r &= F_r + F_h + m_r g \sin(x_p) + b_r \dot{x}_r.
\end{align*}
\]

Unknown parameters in equation (3.6) and (3.7) are estimated in Chapter 4.

### 3.2 Hydraulic Model

A scheme of the hydraulic system is shown in Figure 2.3. As described in Chapter 2 the hydraulic system contains of a hydraulic pump providing the system with a constant pressure, two hydraulic valves and two hydraulic actuators, all connected via hydraulic hoses.

**Hydraulic Valve**

A fair assumption is that the bandwidth of the valve is much higher then what is required for the system. Making this assumption the dynamics of the hydraulic valve can be neglected. If the leakage in the valve also is neglected, the valve can be seen as a variable orifice and can be modelled as follows, see e.g. \([24]\),

\[
Q = K_q A \sqrt{\frac{2}{\rho}} (P_1 - P_2),
\]

where \(K_q\) is the flow gain constant, \(\rho\) denotes the density of the fluid, \(P_1\) and \(P_2\) are the pressures on each side of the orifice and \(A\) is the area of the orifice. In this valve \(A\) will depend on the input signal \(u\). If assuming that the relation between \(A\) and \(u\) is linear, i.e., \(A = ku\) where \(k \in \mathbb{R}\) is a constant and by including \(\sqrt{\frac{2}{\rho}}\) and \(K_q\) in this constant, but still denote the entire constant \(K_q\), the flow equation for the valve can be written on the following form

\[
Q = u K_q \sqrt{P_1 - P_2}.
\]
The valve does however contain two orifices one leading the flow $Q_1$ to the actuator and one leading the flow $Q_2$ from the actuator, see Figure 2.3. So the format for the total valve model will be

$$Q_1 = uK_q\sqrt{P_s - P_1},$$  

(3.8)

$$Q_2 = uK_q\sqrt{P_2 - P_t}.$$  

(3.9)

In the pitch case $K_q$ will depend on the sign of $u$ since the cylinder actuating this motion is asymmetric, more details on this in Chapter 4. The system and tank pressure $P_s$ and $P_t$ will be considered as known, while $P_1$ and $P_2$ will be states for the model.

**Hydraulic actuator**

The equation for the states, $P_1$ and $P_2$ can be obtained for the two different actuators by using the continuity equation form [24]

$$Q_{in} = \frac{dV}{dt} + \frac{V}{\beta_e} \frac{dP}{dt},$$  

(3.10)

where $Q_{in}$ is a flow (positive or negative) into a volume $V$. $V$ can be divided into a constant part $V_c$ and a part that changes with the actuator position $V(x)$. $V_c$ is here the constant volume inside the hoses and actuators and is large in relation to $V(x)$, therefore the following assumption is made $V_c + V(x) \approx V_c$. Equation (3.10) can then be rewritten as

$$\dot{P} = (Q_{in} - \dot{V}) \frac{\beta_e}{V_c}. $$

(3.11)

Using this the state equations for the pressures $P_1$ and $P_2$ is given by

$$\dot{P}_1 = (Q_1 - \dot{V}_1) \frac{\beta_e}{V_{c,1}},$$  

(3.12)

$$\dot{P}_2 = (-Q_2 + \dot{V}_2) \frac{\beta_e}{V_{c,2}}.$$  

(3.13)

$\dot{V}_{1,2}$ is the volume flow due to motion of the actuator, the difference between this flow and the flow from the valve $(Q_{1,2})$ will decide the compression in the liquid. How $\dot{V}$ is calculated will depend on the actuator. For the cylinder case

$$\dot{V}_1 = A_p \dot{x}_p,$$

(3.14)

$$\dot{V}_2 = A_c \dot{x}_p,$$

(3.15)

where $A_p$ and $A_c$ are the cylinder areas for the piston and cap side. In the motor case $\dot{V}$ is the same for both $P_1$ and $P_2$ since the motor is symmetric and will be given by

$$\dot{V} = \frac{D_m}{2\pi r_m} \dot{x}_r$$

(3.16)
where $D_m$ is the motor displacement and $r_m$ is the radius of the pulley in the belt and pulley construction. Finally the torque and the force from the actuators are given by the pressure state according to

$$T_p = P_{1,p}A_c - P_{2,p}Ap,$$  \hspace{1cm} (3.17)

for the hydraulic cylinder and

$$F_r = (P_{1,r} - P_{2,r}) \frac{D_m}{2\pi r_m}$$  \hspace{1cm} (3.18)

for the motor.

### 3.3 Complete Model

To summarize the complete model is given by combining the two submodels from above.

$$\ddot{x}_p = \frac{T_p + T_g(x_p, x_r) + T_h - b_p \dot{x}_p}{J_p(x_r)}$$ \hspace{1cm} (3.19)

$$\ddot{x}_r = \frac{F_r + F_g(x_p) + F_h - b_r \dot{x}_r}{m_r}$$ \hspace{1cm} (3.20)

$$\dot{P}_{1,p} = (Q_{1,p} - \dot{V}_{1,p}) \frac{\beta_e}{V_{c,1,p}}$$ \hspace{1cm} (3.21)

$$\dot{P}_{2,p} = (-Q_{2,p} + \dot{V}_{2,p}) \frac{\beta_e}{V_{c,2,p}}$$ \hspace{1cm} (3.22)

$$\dot{P}_{1,r} = (Q_{1,r} - \dot{V}_{1,r}) \frac{\beta_e}{V_{c,1,r}}$$ \hspace{1cm} (3.23)

$$\dot{P}_{2,r} = (-Q_{2,r} + \dot{V}_{2,r}) \frac{\beta_e}{V_{c,2,r}}.$$ \hspace{1cm} (3.24)

Where

$$T_p = P_{1,p}A_c - P_{2,p}Ap$$ \hspace{1cm} (3.25)

$$F_r = (P_{1,r} - P_{2,r}) \frac{D_m}{2\pi r_m}$$ \hspace{1cm} (3.26)

$$T_g(x_p, x_r) = m_r gl_{cog}(x_r) \sin(x_p)$$ \hspace{1cm} (3.27)

$$F_g(x_p) = m_r g \sin(x_p)$$ \hspace{1cm} (3.28)

$$J_p(x_r) = J_c + m_r l^2_{cog}(x_r)$$ \hspace{1cm} (3.29)

$$Q_{1,p} = u_p K_{qp} \sqrt{P_s - P_{1,p}}$$ \hspace{1cm} (3.30)

$$Q_{2,p} = u_p K_{qp} \sqrt{P_{2,p} - P_t}$$ \hspace{1cm} (3.31)

$$Q_{1,r} = u_r K_{qr} \sqrt{P_s - P_{1,r}}$$ \hspace{1cm} (3.32)

$$Q_{2,r} = u_r K_{qr} \sqrt{P_{2,r} - P_t}$$ \hspace{1cm} (3.33)

Unknown parameters will be determined in Chapter 4.
3.4 Simplified model

When constructing the control system a simplified version of the hydraulic model will be used, mainly because this model corresponds better to the available sensor signals in the system. The hydraulic cylinder is used as an example for how this simplified model is constructed, but the same approach is used for the hydraulic motor actuating the reach motion.

Figure 3.3: Mass spring system with ideal actuator.

Here the model is simplified to a mass spring system connected to a idealized hydraulic actuator and servo valve, see Figure 3.3. As before the mass $m$ resembles the inertia of the system but here the spring accounts for the compressibility of the hydraulic fluid as well as for mechanical compressibility. Other than that the actuator is still seen as ideal where $x_I$ give its position and $x$ is the real, measured, position.

The compression of the modelled spring is given as $\Delta = x_I - x$. The forces acting on the system is, as before, the human force from the operator $F_h$, the environmental forces are represented by $F_{env}$ and the actuator force measured by a sensor will be denoted $F_s(\Delta)$, i.e., the actuator force will be a function of the spring compression.

This give that the system for one DoF can be written as

\begin{align}
    m\ddot{x} &= F_h + F_{env} + F_s(\Delta) \quad (3.34) \\
    \dot{\Delta} &= -\dot{x} + \dot{x}_I. \quad (3.35)
\end{align}

As described in Chapter 2 $F_s(\Delta)$, $F_h$ and $x_r$ is measured from the system, while $F_{env}$ and $x_I$ is unknown.

The idea to use a spring can be derived from the following reasoning. Assuming that the relation between the flow and the areas of the hydraulic cylinder can be
expressed as
\[ \frac{Q_1}{Q_2} = \frac{A_c}{A_p}, \]  
(3.36)
then the ideal actuator velocity becomes
\[ \dot{x}_I = \frac{Q_1}{A_c} = \frac{Q_2}{A_p}. \]  
(3.37)
Consistently with (3.25) the load force is
\[ F_L = A_c P_1 - A_p P_2, \]  
(3.38)
from which follows the derivative
\[ \dot{F}_L = A_c \dot{P}_1 - A_p \dot{P}_2. \]  
(3.39)
Similar to earlier the change in pressure is given by
\[ \dot{P}_1 = \frac{\beta}{V_{c,1}(x)} (A_c \dot{x}_I - A_c \dot{x}) \]  
(3.40)
\[ \dot{P}_2 = \frac{\beta}{V_{c,2}(x)} (A_p \dot{x}_I - A_p \dot{x}) \]  
(3.41)
Given all this the change in load force is described by
\[ \dot{F}_L = \beta \left( \frac{A_c^2}{V_1(x)} + \frac{A_p^2}{V_2(x)} \right) (\dot{x}_I - \dot{x}), \]  
(3.42)
\[ \approx K_s \]  
where the first term can be seen as the spring coefficient and the second term as the spring displacement. If a linear spring is assumed \( K_s \) will be a constant and the load force is described by \( F_L = K_s \Delta \) and since this is what is measured \( F_s = F_L \Rightarrow F_s(\Delta) \).
Chapter 4

System Identification

To estimate the parameters for the modulation system identification is performed. Since the system is naturally divided in two parts (pitch and reach motion) it is natural to try to keep these two parts separated as long as possible when doing this identification. Since the model from Chapter 3 is purely built on physical relations the identification method used here is, what is called *white-box identification*.

Some component parameters such as motor displacement $D_m$ and the cylinder areas $A_c$ and $A_p$ can be found in data sheets. But, for example, masses and inertias ($m_r$ and $J_p$) and the flow gain $K_q$ are unknown. Flow gains for the valves could probably be found from data sheets but due to assumptions that have been made, when constructing the model, and because other components may effect this number a system identification experiment is made.

Parameters needed for constructing the model and later on also for the control system are given in Table 4.1, where parameters that can be found in a data sheet, or similar, are marked with *D.S* and parameters estimated by system identification are marked with *S.I*.

This chapter will first go through the methods used for the system identification and in the end of the chapter, values for the parameters and a validation of the model is presented.
Table 4.1: System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>D.C/S.I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{qp}$</td>
<td>Constant gain from input signal $u_p$ to speed $\dot{x}_p$</td>
<td>S.I</td>
</tr>
<tr>
<td>$K_{qr}$</td>
<td>Constant gain from input signal $u_r$ to speed $\dot{x}_r$</td>
<td>S.I</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Cylinder area cap side</td>
<td>D.S</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Cylinder area piston side</td>
<td>D.S</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Dead volumes for the hoses and actuators</td>
<td>D.S</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>Bulk modulus for the hydraulic fluid</td>
<td>D.S</td>
</tr>
<tr>
<td>$P_s$</td>
<td>System pressure</td>
<td>D.S</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Motor displacement</td>
<td>D.S</td>
</tr>
<tr>
<td>$r_m$</td>
<td>Pulley radius</td>
<td>D.S</td>
</tr>
<tr>
<td>$J_p(x_r)$</td>
<td>Rated inertia in pitch motion dependent on $x_r$</td>
<td>S.I</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Rated mass reach motion</td>
<td>S.I</td>
</tr>
<tr>
<td>$x_{r,cog}$</td>
<td>$x_r$ position for center of gravity</td>
<td>S.I</td>
</tr>
</tbody>
</table>

4.1 Mass $m_r$ and Inertia $J_p$

Since the reach motion is a linear motion a mass is to be decided, while the pitch motion is a rotation and therefore an inertia is required. Both parameters will however be determined in a similar way. The mass and the inertia is here found by giving the actuator a sinusoidal command signal, i.e., letting the input signal $u$ be a sin wave and measuring the force $F$ and the position $x$. According to Newton the force is given by

$$F = \hat{m}_r \ddot{x}. \quad (4.1)$$

where $\hat{m}_r$ denotes the mass estimation. In this case $x$ will also be a sin wave and is described by

$$x = X \sin(\omega t), \quad (4.2)$$

where the frequency $\omega$ is the same as for the input signal and thereby known. The amplitude $X$, is available from measurements. Differentiate (4.2) two times gives

$$\ddot{x} = -\omega^2 X \sin(\omega t). \quad (4.3)$$

By substituting (4.3) into (4.1), and introducing $\alpha = -\hat{m}_r \omega^2$ the measured force can be described by

$$F = \alpha X \sin(\omega t), \quad (4.4)$$

i.e., $F$ is a sin wave with measurable amplitude $\alpha X$ and since $X$ is known it can be cancelled out, and $\alpha$ is obtained. By plotting $\alpha$ against $\omega^2$ for different frequencies, a line where the estimation mass $\hat{m}_r$ is the slope is obtained. Since the slope varies between different data points, the least square method is used for determine one value for $\hat{m}_r$, see Figure 4.1, where this is done by using the MATLAB function Basic Fitting.

As mentioned the inertia for the pitch motion $J_p$ is calculated in a similar way, however as already discussed in Chapter 3, the distance between the center of gravity and the rotating point will change as the arm is pushed back and forward,
4.2 Flow Coefficient $K_q$

In (3.8) $K_q$ is the flow coefficient, which is a constant describing the relation between the input signal $u$ and the flow $Q$. The flow in the system can not be measured, but since there is a direct connection between $Q$ and the velocity $\dot{x}$, which can be measured, this will be used when calculating $K_q$.

$K_q$ will be decided during a constant speed condition. This will be achieved by controlling the input signal via a relay, Figure 4.4 shows such an experiment for the pitch motion. The slope of the line in the Figure 4.2b will give the speed and
from this a relation between the input signal $u$, shown in Figure 4.2a and velocity of the machine $\dot{x}$ can be determined.

By doing the approximation that the tank pressure $P_t \approx 0$, the flow equation becomes

\begin{align}
Q_1 &= uK_q\sqrt{P_s - P_1} \\
Q_2 &= uK_q\sqrt{P_2}
\end{align}

For the reach motion $K_q$ will be the same in both direction, but for the pitch motion this will not be the case, because of the asymmetric cylinder.

### 4.2.1 Reach Flow Gain $K_{qr}$

Because of symmetry, the assumption that $Q_1 = Q_2$ can be made. From this follows that $P_s - P_1 = P_2$ and since the load pressure $P_L = P_1 - P_2$, $P_1$ is given by

$$P_1 = \frac{P_s - P_L}{2}.$$  \hfill (4.8)

The load pressure $P_L$ can be calculated from the measurable force as $P_L = F_L 2\pi r_m / D_m$, where as before $F_L$ is the load force, $D_m$ the motor displacement and $r_m$ the pulley radius. Furthermore the flow is given from the velocity by $Q = (D_m)/(2\pi r_m)\dot{x}$. Using these relations in (4.6), makes it possible to estimate $K_{qr}$ by using

$$K_{qr} = \frac{\dot{x}D_m}{2\pi r_m\sqrt{P_s - P_1}}.$$  \hfill (4.9)

### 4.2.2 Pitch Flow Gain $K_{qp}$

The pitch case will however be somewhat more complicated since the cylinder is not symmetric. This means that $K_{qp}$ will depend on the sign on $u$, i.e., if the
4.2 Flow Coefficient $K_q$

cylinder is being extracted or detracted. By looking closely at Figure 4.2a, it can be seen that it takes longer time going up than going down (positive $u$ gives negative velocity).

The cylinder will here be defined as described in Figure 4.3. Since $Q_1 \neq Q_2$ in (4.6) and (4.7), the following assumption is made instead [17]

$$
\frac{Q_1}{Q_2} = \frac{A_c}{A_p} = z, \quad (4.10)
$$

where $z$ is a constant used for representing this relation. If first looking at the extraction case, $P_1$ is connected to $P_s$ and $P_2$ to the tank pressure, using (4.10) in (4.6) and (4.7) gives

$$
P_2z^2 = P_s - P_1. \quad (4.11)
$$

Since $P_1$ and $P_2$ can not be measured, the measurable load force, $F_L$, will be used for doing the system identification. $F_L$ is given by

$$
P_1 A_c - P_2 A_p = F_L. \quad (4.12)
$$

By denoting $F_L = P_L A_c$ and us (4.11) and (4.12) the pressures can be expressed as

$$
P_{1,ext} = \frac{1}{1 + z^3} (P_s + P_L z^3), \quad (4.13)
$$

$$
P_{2,ext} = \frac{z}{1 + z^3} (P_s - P_L). \quad (4.14)
$$

In the retraction case it is $P_2$ that is connected to $P_s$, so for this case the relation between the pressures can be expressed as

$$
(P_s - P_2)z^2 = P_1. \quad (4.15)
$$

From this follows that the expression for the pressures in the retraction case becomes

$$
P_{1,ret} = \frac{P_s z^2 + P_L z^3}{1 + z^3}, \quad (4.16)
$$

$$
P_{2,ret} = \frac{P_s z^3 - P_L z}{1 + z^3}. \quad (4.17)
$$

Figure 4.3: schematic picture of the hydraulic cylinder, showing the flows, pressures in the system and the areas in the cylinder.
Finally by e.g using \( Q_1 = \dot{x}_{ext}/A_c \) for the extraction and \( Q_2 = \dot{x}_{ret}/A_p \) for the retraction, the flow coefficients \( K_{qp,ext} \) and \( K_{qp,ret} \) can be estimated from (4.6) as

\[
K_{qp,ext} = \frac{\dot{x}_{ext}A_c}{u\sqrt{P_s - P_{1,ext}}},
\]

and

\[
K_{qp,ret} = \frac{\dot{x}_{ret}A_p}{u\sqrt{P_s - P_{2,ret}}}. \tag{4.19}
\]

### 4.3 Validation

The result of the system identification described above is presented in Table 4.2. To validate the values presented in Table 4.2 and the model presented in Chapter 3, data from a the real system is compared with data from the model, when they are both given the same control signal. The system and the model are both given a sin wave signal, the results are shown in Figure 4.5 and Figure 4.5. In all plots there is some initial data that does not match the model. This data is measured before the input signal to the system is initiated can therefore be neglected.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_r )</td>
<td>4.8</td>
<td>[kg]</td>
</tr>
<tr>
<td>( J_c )</td>
<td>2.5</td>
<td>[kgm^2]</td>
</tr>
<tr>
<td>( K_{qp,ext} )</td>
<td>(-1.57 \cdot 10^{-8})</td>
<td>[-]</td>
</tr>
<tr>
<td>( K_{qp,ret} )</td>
<td>(-1.44 \cdot 10^{-8})</td>
<td>[-]</td>
</tr>
<tr>
<td>( K_{qr} )</td>
<td>(1.25 \cdot 10^{-8})</td>
<td>[-]</td>
</tr>
</tbody>
</table>
4.3 Validation

(a) Position.

(b) Force.

Figure 4.4: Validation of the model for the reach motion. Comparison between data from model and from the real system.

(a) Position.

(b) Force.

Figure 4.5: Validation of the model for the pitch motion. Comparison between data from model and from the real system.
Chapter 5

Uncoupled system control

As described in earlier chapters the Human Power Amplifier (HPA) has two actuated degrees of freedom (DoF:s). In this chapter an initial approach for constructing a control law where each DoF are controlled separately is proposed. The objective of the controller is to amplify the force exerted on the machine by a human operator and to keep the controller stable at all times. In earlier paper [15] such a controller have successfully been implemented on the pitch DoF [17], so what this chapter will focus on is to explain the theories used in [17] and present how they are applied on the reach degree of freedom.

5.1 Passivity

One way to ensure that a system remains stable is to check that it is passive. A passive component can be described as a component that either consumes energy or is incapable of power gain, i.e., a system that does not generates any energy, but only stores dissipates or releases it [15], such a system will therefore remain stable.

Consider a system with input \( u \) and output \( y \). Then the supply rate for that system is some function \( f(u, y) \). Such a system is passive with respect to the supply rate, if there for an initial condition exists a constant \( c \) such that for all time and for all input \( u(\cdot) \),

\[
\int_0^t f(u(\tau), y(\tau)))d\tau \geq -c^2. \tag{5.1}
\]

If \( f(u, y) \) is the power input to the system, then (5.1) gives, that no matter how the input \( u(t) \) changes, the amount of energy extracted from the system is limited by the constant \( c^2 \). This term can therefore in some way be seen as the initial energy of the system, [15]. So if a system can be proved to satisfy (5.1) it is passive and thereby stable.
5.2 Modelling

To describe the system the simplified model presented in 3.4 is used, that gives

\[ m_r \ddot{x}_r = F_h + F_{env} + F_s(\Delta), \quad (5.2) \]
\[ \dot{\Delta} = -\dot{x}_r + u - u_{loss}, \quad (5.3) \]

where \( m_r \) is the mass acting in the reach direction, \( F_h \) is the force put input from the human operator, \( F_{env} \) is forces from the environment and \( F_s(\Delta) \) is the actuator force. \( u \) and \( u_{loss} \) gives the speed of the ideal actuator, \( \dot{x}_I \) with losses, i.e., \( \dot{x}_I = u - u_{loss} \). Since \( u \) is proportional to the valve input, \( u \) will here be seen as the control input instead.

This system is passive see Appendix A for proof.

5.3 Coordination formulation

As described in the introduction, there may be problems with trying to control the force right away, e.g. with a PI and feed forward control.

The approach here is therefore not to look at the problem directly as a force amplifying problem, but rather as a coordination problem between two masses. Figure 5.1 can give a basic idea for how this coordination control is supposed to work. A new, virtual, system with the mass \( m_v \) and the velocity \( \dot{x}_v \) is introduced.

![Figure 5.1: Basic idea for the coordination control. The desired force \( F_d \) acts on the virtual mass \( m_v \) and the actuator force \( F_s \) is used to coordinate the motion between the virtual and real mass \( m_v \) and \( m_r \).](image)

This system is given by the following equations

\[ m_v \dot{x}_v = F_d - F_s(\Delta), \quad (5.4) \]
\[ u = \dot{x}_v + u_1, \quad (5.5) \]

where \( F_d \) is the desired amplified force and is pushing on the virtual system instead of the real. \( F_d \) is obtained from \( F_d = \rho F_h \), i.e., scaling the force from the human
operator by a factor $\rho$. The real system is in turn affected by $F_h$ and other environmental forces $F_{env}$. The connection between the system is reassembled by a spring that pushes on both systems with the actuator force $F_s(\Delta)$, where $\Delta$ as before is the spring compression. $F_s(\Delta)$ is then via the new control signal $u_1$ used for synchronizing the motion of the two systems.

If the two systems are successfully synchronized the coordinated system is described by Figure 5.2 As can be seen in the figure, $F_d$ is now pushing on the system as desired. The only problem is that the additional virtual mass, $m_v$, in reality should not be a part of this system, therefore $m_v$ needs to be small, $m_v << m_r$. If $m_v$ is sufficiently smaller then $m_r$, the coordinated system approximately resembles the actual system.

The extended system from Figure 5.1 is given by,

\begin{align}
    m_r\ddot{x}_r &= F_h + F_{env} + F_s(\Delta), \quad (5.6) \\
    m_v\ddot{x}_v &= F_d - F_s(\Delta), \quad (5.7) \\
    \dot{\Delta} &= -\dot{x}_r + \dot{x}_v - u_{loss} + u_1. \quad (5.8)
\end{align}

### 5.4 Locked and Shaped system

To make the control of the extended system (5.6)-(5.8) easier and to better see the impact from the control signals, it will be transformed into a new system.

This new system is divided into two parts which will be called the shaped and the locked system. These two new systems are to be designed so that the locked system describes the extended system once it is coordinated, i.e., when it looks like it does in Figure 5.2, and the shaped system describes the coordination between the virtual and real system. The new systems will therefore have the following requirements [11]:

1. Locked system mass $m_L = m_r + m_v$
2. Shaped system velocity $v_E = \dot{x}_r - \dot{x}_v$

3. The kinetic energy is preserved through the transformation

Requirement 1 and 2 gives the system the desired control properties described above, while requirement 3 is necessary to make sure that the system remains passive through the transformation. To fulfil these requirements let us look at it as a transformation of the system states, i.e., the transformation $(\dot{x}_r, \dot{x}_v) \rightarrow (v_L, v_E)$, where $v_L$ and $v_E$ are the velocity of the locked and shaped system respectively. This transformation can be done as follows

$$
\begin{bmatrix}
 v_L \\
 v_E 
\end{bmatrix} = S \begin{bmatrix}
 \dot{x}_p \\
 \dot{x}_v 
\end{bmatrix},
$$

(5.9)

where $S$ is the transformation matrix. To achieve requirement 2 ($v_E = \dot{x}_r - \dot{x}_v$), $S$ need to have the following form

$$
S = \begin{bmatrix}
 a & b \\
 1 & -1 
\end{bmatrix},
$$

(5.10)

(5.9) also gives

$$
S^{-1} \begin{bmatrix}
 v_L \\
 v_E 
\end{bmatrix} = \begin{bmatrix}
 \dot{x}_p \\
 \dot{x}_v 
\end{bmatrix}.
$$

(5.11)

If the inertia matrix for the untransformed system is

$$
M = \begin{bmatrix}
 m_r & 0 \\
 0 & m_v 
\end{bmatrix}
$$

(5.12)

and $x = [x_r, x_v]$, then requirement 3, to preserve the kinetic throughout the transformation can be satisfied by looking at the energy transformation

$$
\kappa_e = \frac{1}{2} x^T M x = \frac{1}{2} [v_L^T, v_E^T] S^{-T} M S^{-1} \begin{bmatrix}
 v_L \\
 v_E 
\end{bmatrix}.
$$

(5.13)

The new inertia matrix is then given by

$$
S^{-T}(x)MS^{-1} = \begin{bmatrix}
 m_r + m_v & -m_r b + m_v a \\
 -m_r b + m_v a & m_r b^2 + m_v a^2 
\end{bmatrix}.
$$

(5.14)

This matrix can be made diagonal by choosing $a$ and $b$ wisely. To get a diagonal matrix $m_v a - m_r b = 0$ is a requirement and because the locked system is supposed to describe the behaviour of the systems when they are coordinated, i.e., when $v_E = 0$ and $\dot{x}_v = \dot{x}_r = v_L$, $a + b$ needs to be equal to one. This gives

$$
a = \frac{m_r}{m_r + m_v} \quad \text{and} \quad b = \frac{m_v}{m_r + m_v}.
$$

(5.15)
With \( m_L = m_r + m_v \) and \( m_E = m_r b^2 + m_v a^2 \) the transformation becomes

\[
\begin{bmatrix}
v_L \\
v_E
\end{bmatrix} = \begin{bmatrix}
m_r & m_v \\
m_L & -1
\end{bmatrix} \begin{bmatrix}
\dot{x}_p \\
\dot{x}_v
\end{bmatrix},
\]

(5.16)

\[ m_L = m_r + m_v, \]

(5.17)

\[ m_E = \frac{m_r m_v}{m_r + m_v}. \]

(5.18)

Finally the right hand side of (5.6) and (5.7) also needs to be transformed

\[
S^{-T} \begin{bmatrix}
F_h + F_{env} + F_s(\Delta) \\
F_d - F_s(\Delta)
\end{bmatrix} = \begin{bmatrix}
\frac{m_r}{m_L} (F_h + F_{env}) - \frac{m_r}{m_L} F_d + F_s \\
m_L F_h + F_{env} + F_d
\end{bmatrix}.
\]

(5.19)

By lumping all uncontrollable forces in the shaped system into \( F_E = m_v m_L (F_h + F_{env}) - \frac{m_r}{m_L} F_d \) the locked and shaped system is given by

\[
\begin{align*}
m_L \dot{v}_L &= F_h + F_{env} + F_d, \\
m_E \dot{v}_E &= F_E + F_s, \\
\dot{\Delta} &= -v_E + u_1 + u_{loss}.
\end{align*}
\]

(5.20) \hspace{1cm} (5.21) \hspace{1cm} (5.22)

### 5.5 Shaped System Control

The control objective for this system is, as mentioned above, to make \( v_E \to 0 \). Here two different controllers will be presented, one PI-controller and one controller with Innovation feedback [18], the later is what was proposed for the pitch direction in [17]. In both cases the control signal is modified in order to implement some damping. If \( u_{loss} \) is seen as disturbances and \( F_E \) is assumed to be constant and assuming that the spring is linear, i.e., letting \( F_s(\Delta) = K_s \Delta \) the transfer function \( u_1 \to v_E \) is

\[
G(s) = \frac{K_s / m_E}{s^2 + K_s / m_E},
\]

(5.23)
i.e., an undamped system with natural frequency \( \omega_n = \sqrt{\frac{K_s}{m_E}} \). Damping could be added by letting

\[
u_1 = -\gamma_1 v_E - \gamma_2 K_s \Delta + (1 + \gamma_1) u_2.
\]

(5.24)
The transfer function from \( u_1 \to v_E \) now becomes

\[
G(s) = \frac{\frac{K_s}{m_E} (\gamma_1 + 1)}{s^2 + s \gamma_2 K_s + \frac{K_s}{m_E} (\gamma_1 + 1)}.
\]

(5.25)

So by doing this a damping factor \( \zeta = \frac{\gamma_2}{2 + \omega_n} \) is added and the natural frequency is increased by a factor \( 1 + \gamma_1 \). This transfer function have a steady state gain one and the dynamics of the system is considered fast enough to make the following approximation \( G(s) \approx 1 \).

What is left now is to design a controller to determine \( u_2 \). As mentioned above two different versions of the controller is tried out on the system, a PI-controller and a controller using innovation feedback.
PI-Controller

The first approach was to just try a regular PI-controller, where the transfer function $u_2 \rightarrow v_E$ is

$$u_2 = v_E \left( K_p + \frac{K_i}{s} \right), \quad (5.26)$$

where $K_p$ is the proportional gain and $K_i$ the integrator gain.

Innovation Feedback

The innovation is the difference between an estimation and the actual output from a system [5], i.e., $y - C\hat{x}$, where $y$ is the output and $\hat{x}$ is the estimated state and $C$ is the $C$-matrix in state space formulation see e.g. [6]. Since $G(s) \approx 1$ the control signal $u_2$ can be seen as the state estimation and the innovation can thereby be seen as $v_E(s) - u_2(s)$.

If a controller with the following transfer function for $v_E \rightarrow u_2$ is constructed

$$R(s) = \frac{p^2}{s(s + 2p)} \quad (5.27)$$

where $p > 0$ is a constant, the characteristic function for the close loop system will be $(s + p)^2$, i.e., stable. However since the dynamics of the system is ignored, $p << \omega_n$ is a requirement to ensure stability.

And then by including the innovation feedback from above the following expression can be obtained

$$u_2(s) = -R(s)v_E - Q(s)(v_E(s) - u_2(s)) = -\frac{R(s) + Q(s)}{1 - Q(s)}v_E, \quad (5.28)$$

where $Q(s)$ can be used to achieve desired properties for the system, see Figure 5.3 for details. Here $Q(s)$ will be used to modified the complementary sensitivity function $T(s)$ to increase the bandwidth of the system. $T(s)$ is given by

$$T(s) = \frac{R(s) + Q(s)}{1 + R(s)}. \quad (5.29)$$

If

$$Q(s) = p^2 \frac{(p/\omega_n^2) - 1}{(s + 2p)^2(s + p)} \quad (5.30)$$

the complementary transfer function becomes

$$T(s) = \frac{p^3}{\omega_n^2 (s + p)^2(s + p)}. \quad (5.31)$$

This $T(s)$ is close to one for low frequencies, i.e., the sensitivity function close to zero as desired.
5.6 Locked System Control

As described the locked system describes the system when it is coordinated, that is, when $v_E = 0$ and from (5.20) the locked system is

$$m_L \dot{v}_L = F_h + F_{env} + F_d.$$  \hfill (5.32)

The objective for the locked system control is therefore that the force pushing on the locked system should be the desired force $F_d$ as described in Figure 5.2, and since it in reality is the actuator force that pushes the system the objective for the controller becomes $F_s(\Delta) = F_d$.

Since $F_d = \rho F_h$ (5.32) can be written as

$$m_L \dot{v}_L = F_h (\rho + 1) + F_{env}.$$  \hfill (5.33)

Letting $F_h (\rho + 1) + F_{env} = F_{tot}$ and substitute that into (5.21) gives

$$m_E \dot{v}_E = \frac{m_v}{m_L} F_{tot} + F_s - F_d.$$  \hfill (5.34)

Whit $v_E \to 0$ and with $F_{tot}$ being scaled by $\frac{m_v}{m_L}$ where, as described in chapter 5.3, $m_v << m_L$ making this term relatively small the relation will be

$$F_s(\Delta) \approx F_d$$  \hfill (5.35)

as desired.
Chapter 6

Coupled System

Until now the human amplifier have been seen as two decoupled systems, with one system for each DoF, i.e., the control for the pitch and the reach motion have been constructed separately, see Chapter 5. Although this approach works fairly well, see Chapter 10, also when working in two degrees of freedom, it seems to be some cross talk between the controllers for each degree. In this chapter a way to describe the system as a coupled system is presented. In the coming chapter a new approach for controlling this system is developed. The aim when constructing this coupled system is to get a better idea of how the two DoF:s effect each other and thereby make it possible to compensate for these influences in the controller.

6.1 Euler Lagrange approach

To create the coupled system it is mainly the mechanical model that needs to be changed. The approach here is to use the Euler-Lagrange equations [27]

$$\frac{d}{dt} \frac{\partial L(\dot{q}, q)}{\partial \dot{q}} - \frac{\partial L(\dot{q}, q)}{\partial q} = \psi, \quad (6.1)$$

where $q$ is the generalized coordinate. For a mechanical system $q$ is typically the position. $\psi$ are the generalized forces acting on the system and $L(\dot{q}, q)$ is the Lagrangian describing the total energy of the system, including the kinetic energy $T(\dot{q}, q)$ and the potential energy $V(q)$ and is given by

$$L(\dot{q}, q) = T(\dot{q}, q) - V(q). \quad (6.2)$$

6.1.1 Kinetic Energy

For a mechanical system the kinetic energy is

$$T(\dot{q}, q) = \frac{1}{2} \dot{q}^T M(q, q) \dot{q}. \quad (6.3)$$

For our system $q = [x_p, x_r]^T$, i.e., the position for the pitch and the reach motion and $M(q)$ is the inertia matrix. To get an expression for the kinetic energy an
expression for $M(q)$ first needs to be determined. This can be done by investigating the kinetic energy and the motion of the center of gravity (CoG). Figure 6.1 gives a picture for the motion of the center of gravity in the system. XY is a earth fix Cartesian coordinate system, $v_{cog}$ is the velocity vector for the CoG, where $\dot{x}_p$ and $\dot{x}_r$ is the velocity for each DoF. $l_{cog}(x_r)$ is the distance from the rotation point to the CoG.

For derivation of $J$ see Appendix A.2. Using this transformation in the kinetic equation (6.3) the inertia matrix becomes

$$M(q) = J^T m_{cog} J = \begin{bmatrix} m_{cog} l_{cog}(x_r)^2 & 0 \\ 0 & m_{cog} \end{bmatrix}, \quad (6.5)$$

where $m_{cog} = m_r$ is assumed, $m_r$ is estimated in Chapter 4. Since there is a rotation motion there will also be an constant inertia component

$$I = \begin{bmatrix} I_c & 0 \\ 0 & 0 \end{bmatrix} \quad (6.6)$$

affecting the pitch motion. The value of $I_c$ is estimated in Chapter 4 (where it is called $J_c$). By adding these two components together the following inertia matrix is obtained

$$M(q) = \begin{bmatrix} I_c + l_{cog}(x_r)^2 m_r & 0 \\ 0 & m_r \end{bmatrix}. \quad (6.7)$$
Note that this is consistent with what was concluded in Chapter 3.1. To avoid clutter in future calculations the inertia matrix is denoted.

\[ M(q) = \begin{bmatrix} m_p(x_r) & 0 \\ 0 & m_r \end{bmatrix} \]  \hfill (6.8)

### 6.1.2 Potential energy

To determine a function for the potential energy of the system study Figure 6.2. The figure is a sketch to describe how potential energy changes with the angle \( x_p \)

![Figure 6.2: Sketch to determine a function for potential energy. \( V_c \) is a constant and describe the potential energy when the CoG is in level with the rotation point and \( l_{cog}(x_r) \) is the distance to the rotation point.](image)

and the length between the rotation point and the CoG, \( l_{cog}(x_r) = x_{r,cog} - x_r \). \( V_c \) is a constant and resembles the potential energy when the CoG is in level with the rotation point and is given by

\[ V_c = |(x_{r,cog} - x_{r,max}) \sin(x_{p,max} - x_{p,min})| \]  \hfill (6.9)

Given this the potential energy can be described as

\[ V(q) = V_c + m_r g l_{cog}(x_r) \sin(x_{p,max} - x_{p,min}). \] \hfill (6.10)

### 6.1.3 Euler-Lagrange calculation

With \( T(\dot{q}, q) \) and \( V(q) \) known it is now possible to generate an equation describing the system from (6.1). The first term will be

\[ \frac{d}{dt} \frac{\partial L(\dot{q}, q)}{\partial \dot{q}} = \begin{bmatrix} m_p(x_r) \ddot{x}_p \\ m_r \ddot{x}_r \end{bmatrix} + \begin{bmatrix} \frac{m_p(x_r)}{\partial x_r} \ddot{x}_p & 0 \\ 0 & 0 \end{bmatrix} \] \hfill (6.11)

and the second term becomes

\[ \frac{\partial L(\dot{q}, q)}{\partial q} = \begin{bmatrix} 0 \\ \frac{1}{2} m_p(x_r) x_p^2 \end{bmatrix} - \begin{bmatrix} \frac{\partial V(x_p, x_r)}{\partial x_p} \\ \frac{\partial V(x_p, x_r)}{\partial x_r} \end{bmatrix}. \] \hfill (6.12)
The total system is then given by

\[
\begin{bmatrix}
m_p(x_r)\dddot{x}_p \\
m_r\dddot{x}_r
\end{bmatrix} + \begin{bmatrix}
\frac{m_p(x_r)}{\partial x_r} \frac{\partial^2}{\partial x_r^2} \frac{\partial}{\partial x_r} \\
0
\end{bmatrix} - \begin{bmatrix}
0 \\
\frac{1}{2} \frac{m_p(x_r)}{\partial x_r} \frac{\partial^2}{\partial x_r^2}
\end{bmatrix} + \begin{bmatrix}
\frac{\partial V(x_p,x_r)}{\partial x_p} \\
\frac{\partial V(x_p,x_r)}{\partial x_r}
\end{bmatrix} = \dot{C}(\dot{q},q) \dot{q} + \frac{\partial V(x_p,x_r)}{\partial x_r} \dot{x}_r.
\]

(6.13)

As hinted in (6.13) the two middle terms can be used for defining the Coriolis matrix \( C(\dot{q},q) \). Doing this the system will look as follows,

\[
\begin{bmatrix}
m_p(x_r) & 0 \\
0 & m_r
\end{bmatrix} \begin{bmatrix}
\dddot{x}_p \\
\dddot{x}_r
\end{bmatrix} + \begin{bmatrix}
\frac{1}{2} \frac{m_p(x_r)}{\partial x_r} \frac{\partial^2}{\partial x_r^2} \\
-\frac{1}{2} \frac{m_p(x_r)}{\partial x_r} \frac{\partial^2}{\partial x_r^2}
\end{bmatrix} \begin{bmatrix}
\dddot{x}_p \\
\dddot{x}_r
\end{bmatrix} + \begin{bmatrix}
\frac{\partial V(x_p,x_r)}{\partial x_p} \\
\frac{\partial V(x_p,x_r)}{\partial x_r}
\end{bmatrix} = \dot{C}(\dot{q},q) \dot{q} + \frac{\partial V(x_p,x_r)}{\partial x_r} \dot{x}_r.
\]

(6.14)

Finally by putting the force \( \psi \) back in the right hand side and by representing the matrices with capital letters the system is described by

\[
M_c(q)\dddot{q} + C_c(\dot{q},q) \dot{q} + G_c(q) = \psi,
\]

(6.15)

where \( M_c(q) \) is the inertia matrix, \( C_c(\dot{q},q) \) is the Coriolis matrix and \( G_c(q) \) resembles the forces due to potential energy. For details on all the component of equation (6.15) see Appendix A.2. If the inertia and the Coriolis matrices are correct then the following matrix should be skew symmetric [27].

\[
\dot{M}_c(q) - 2C_c(\dot{q},q)
\]

(6.16)

This nice property will be used when constructing the controller in Chapter 7. For proof that (6.16) is skew symmetric see Appendix A.2.

### 6.2 External forces

The external forces until now represented by \( \psi \) will basically be seen in the same way as in Chapter 5.3. However now they are represented by \( 2 \times 1 \) matrices, where each row represent one of the DoF:s, i.e., the dynamics of the system is modelled by

\[
M_c(q)\dddot{q} + C_c(\dot{q},q) \dot{q} + G_c(q) = F_h + F_{env} + F_s(\Delta),
\]

(6.17)

\[
\dot{\Delta} = -\dddot{q} + u - u_{loss},
\]

(6.18)

where, as before, the matrices \( F_h, F_{env} \) represent the human and environmental force and \( F_s(\Delta) \) is the actuator force given as a function of the equivalent spring compression \( \Delta \). This new coupled system is also passive, for proof see Appendix A.2.
6.3 Coordination formulation

The idea to introduce a virtual system and look at the control problem as a coordination problem between this virtual system and the real system, presented in Chapter 5, will still be the idea here, i.e., the virtual system will still be given by

\[ M_v \ddot{x}_v = F_d - F_s(\Delta), \]  
\[ u = \dot{x}_v + u_1, \]  
\[ F_d = \rho F_h, \]  
(6.19), (6.20), (6.21)

only now it is on matrix form, The total coupled system can now be described as

\[ M_c(\dot{q}_1)\ddot{q}_1 + C_c(\dot{q}_1, q_1)\dot{q}_1 + G_c(q_1) = F_h + F_{env} + F_s(\Delta), \]  
\[ M_v \ddot{q}_2 = F_d - F_s(\Delta), \]  
\[ \dot{\Delta} = \dot{q}_2 - \dot{q}_1 - u_{loss} + u_1, \]  
(6.22), (6.23), (6.24)

where the generalized coordinates \( q \) for this coupled system becomes \( q_1 = [x_p, x_r] \) and \( q_2 = [x_v, p, x_v, r] \), i.e., the position of the real system and the virtual system respectively.

6.4 Locked and Shaped System

For this system the transformation into the locked and shaped system as was done in Chapter 5.4 is also desired. The requirements for these systems will be the same as earlier and therefore the transformation will be pretty similar to what was done in previous chapter. The transformation matrix \( S(q) \) is defined and calculated in the same way as in Chapter 5.4 but will be a function of \( q \) since \( m_p(q) \) depends on the reach position (6.15).

\[ \begin{bmatrix} v_L \\ v_E \end{bmatrix} = S(q) \begin{bmatrix} \dot{x}_c \\ \dot{x}_v \end{bmatrix} \quad \text{and} \quad S(q) = \begin{bmatrix} a(q) & b(q) \\ I & -I \end{bmatrix} \]

where

\[ a(q) = [M_c(q) + M_v]^{-1} M_c(q), \]
\[ b(q) = [M_c(q) + M_v]^{-1} M_v \]

As expected also the former constants \( a \) and \( b \) will now be functions of \( q \). Using \( S(q) \), the locked and shaped system velocities \( v_L \) and \( v_E \) are similar to before

\[ v_L = a(q)\dot{q}_1 + b(q)\dot{q}_2, \]  
\[ v_E = \dot{q}_1 - \dot{q}_2 \]  
(6.25), (6.26)

Recall that the expression for \( v_E \) was one of the requirement when constructing the locked and shaped systems and that \( v_E = 0 \Rightarrow v_L = \dot{x}_a = \dot{x}_v \). Furthermore the inertias of the system become

\[ M_L = M_c(q) + M_v, \]  
\[ M_E = b(q)^T M_c(q) b(q) + a(q)^T M_v a(q), \]  
(6.27), (6.28)
where the expression for $M_L$ was one of the other requirements for constructing the systems.

From here on the calculations differs from what was done in Chapter 5. The transformation from $\ddot{q} \rightarrow \dot{v}_{L,E}$, will be different due to dependence on $q$. Differentiating both sides of (6.25) and (6.26) gives

\[
\ddot{v}_L = \dot{a}(q)\dot{q}_1 + a(q)\ddot{q}_1 + \dot{b}(q)\dot{q}_2 + b(q)\ddot{q}_2, \tag{6.29}
\]

\[
\ddot{v}_E = \ddot{q}_1 - \ddot{q}_2. \tag{6.30}
\]

By using the relation that $b(q) = I - a(q)$ (which also is a design requirement) this can be written as

\[
\begin{bmatrix}
\dot{v}_L \\
\dot{v}_E
\end{bmatrix} = \begin{bmatrix}
-a(q)v_E \\
0
\end{bmatrix} + S(q) \begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}, \tag{6.31}
\]

By putting $G_c(q)$ from (6.22) on the right hand side, the total transformation of the remaining left hand side for (6.22)-(6.24) will be

\[
S(q)^{-T} \begin{bmatrix}
M_c(q) \\
0
\end{bmatrix} S(q)^{-1} \begin{bmatrix}
\dot{v}_L \\
\dot{v}_E
\end{bmatrix} +
S(q)^{-T} M_v \begin{bmatrix}
\dot{b}(q)v_E \\
0
\end{bmatrix} + S(q)^{-T} C_c(q, q) \begin{bmatrix}
0 \\
0
\end{bmatrix} S(q)^{-1} \begin{bmatrix}
v_L \\
v_E
\end{bmatrix},
\]

where the fist terms are the transformation for $M_E$ and $M_L$. The remaining terms can be represented by the matrix proposed in the equation. Where $C_L(q, q)$ and $C_E(q, q)$ are the Coriolis matrix for the locked and shaped system, while $C_{EL}(q, q)$ and $C_{EL}(q, q)$ are coupling terms between the systems. If letting the forces on the right hand side (plus the potential energy term) be $\psi'$ and adding the dynamics from the hydraulic model, the total system becomes

\[
M_L(q)\ddot{v}_L + C_L(q, q)v_L + C_{EL}(q, q)v_E = \psi'_L, \tag{6.33}
\]

\[
M_E(q)\ddot{v}_E + C_E(q, q)v_E + C_{EL}(q, q)v_L = \psi'_E, \tag{6.34}
\]

\[
\dot{\Delta} = -v_E - u_{loss} + u_1. \tag{6.35}
\]

The right hand side of (6.22) and (6.23) is transformed as they were in (5.19), which gives

\[
\psi'_L = F_h + F_{env} + F_d - G_c(q) \tag{6.36}
\]

\[
\psi'_E = b(q)(F_h + F_{env} - G_c(q)) - a(q)F_d + F_s(\Delta). \tag{6.37}
\]
Chapter 7

Coupled System Control

In this chapter a new control strategy for the HPA, using the coupled system presentation in the previous chapter, is presented. The aim with this controller is to improve, compared with the controllers presented in Chapter 5, the control of the HPA when operating both DoF:s simultaneously. This will be done by better taking care of the cross couplings in the system and also by trying to cancel out disturbances that earlier were ignored. The controller will be proven stable using the Lyapunov criteria.

7.1 Shaped System Control

The objective for the shaped system is, like before, to coordinate the real and virtual system, i.e., trying to achieve $v_E \rightarrow 0$. From (6.34), (6.35) and (6.37) the shaped system is given by

$$M_E(q)\ddot{v}_E + C_E(\dot{q}, q)v_E + C_{EL}(\dot{q}, q)v_L = F_E + F_s(\Delta),$$

$$\dot{\Delta} = -v_E - u_{loss} + u_1,$$

where $F_E = b(q)(F_h + F_{env} - G_c(q) - a(q)(F_d)$. In the previous approach, the effects of $F_E$ were seen as a disturbance and neglected. Here a controller that will cancel out these effects is presented. Since it is possible to indirectly control $F_s(\Delta)$ by controlling $\Delta$, the approach will be to construct a controller that works in two steps. First a control term $F_s(\Delta)'$ that denotes the desired $F_s(\Delta)$ which cancel out $F_E$ and make $v_E \rightarrow 0$ is constructed. The second step is to design a controller that controls $\Delta$ so that $F_s(\Delta)'$ is achieved, i.e., a backstepping controller.

Accordingly to above the first step is to determine $F_s(\Delta)'$. This is done by first introducing an integration term for the error $\int v_E = v_{E,int}$, which has the desired property to cancel out the disturbance $F_E$, i.e., $F_E = K_I v_{E,int}'$, where $K_I$ is the integration gain and $v_{E,int}'$ is a constant. The error between the desired property and the actual is given as $\tilde{v}_{E,int} = v_{E,int}' - v_{E,int}$. Using this the following
Coupled System Control

PI-controller can be constructed to meet the requirements for $F_s(\Delta)'$,

$$F_s(\Delta)' = -K_p v_E - K_I v_{E,int}.$$  \hspace{1cm} (7.3)

Furthermore the cross term from the locked system $C_{EL}(\dot{q}, q)$ also needs to be handled. Since both $C_{EL}$ and $v_L$ are known it can be cancelled out. To handle this a new control term is introduced

$$T_E = C_{EL}(\dot{q}, q) v_L + F_s(\Delta)'.$$  \hspace{1cm} (7.4)

With this term the difference between the real and desired control force control is given by

$$\tilde{F}_s(\Delta) = T_E - F_s(\Delta).$$  \hspace{1cm} (7.5)

By substituting (7.3), (7.4) and (7.5) into (7.1) the system can be rewritten as

$$M_E(\dot{q}) v_E + C_E(\dot{q}, q) v_E = -\tilde{F}_s(\Delta) - K_p v_E + K_I \tilde{v}_{E,int}.$$  \hspace{1cm} (7.6)

This completes the first step in the backstepping controller, now let us look at (7.2), where $\dot{\Delta}$ describes the difference in velocity between the virtual and real system. $\Delta$ can be seen as the displacement of a spring connecting the two systems (see Figure 5.1). If this spring is assumed to be linear the desired control force from (7.3) can be seen as a constant times a desired $\Delta'$. Let the errors between desired ($\Delta'$ and $\dot{\Delta}'$) and actual properties be

$$\tilde{\Delta} = \Delta' - \Delta,$$ \hspace{1cm} (7.7)

$$\hat{\Delta} = \dot{\Delta}' - \dot{\Delta}.$$ \hspace{1cm} (7.8)

Assuming $u_{loss} = 0$, then (7.2) and (7.8) gives

$$\dot{\hat{\Delta}} = \dot{\Delta}' + v_E - u_1$$ \hspace{1cm} (7.9)

Furthermore if the spring coefficient for the assumed linear spring is denoted $K_s$ then the force is $F_s(\Delta) = K_s \Delta$. This means that the derivative of the force error can be written as

$$\dot{\tilde{F}_s}(\Delta) = K_s \dot{\tilde{\Delta}} = K_s (\dot{\Delta}' + v_E - u_1).$$ \hspace{1cm} (7.10)

So the question now is how to choose $u_1$ to cancel out the control error in a good way. For now consider the second term in (7.10) $K_s \dot{\tilde{\Delta}}$ as the error, and rewrite (7.6) to

$$M_E(\dot{q}) v_E + C_E(\dot{q}, q) v_E = -K_s \tilde{\Delta} - K_p v_E + K_I \tilde{v}_{E,int}$$ \hspace{1cm} (7.11)

The objective is to construct an exponential stable controller for the second step in the Backstepping controller. Such a requirement can be proved by showing that there exist a Lyapunov function $V(q)$ such that $-\dot{V}(q)$ is positive definite, see e.g [5]. One approach to find such a function, presented in [18], is to use the total
energy of the system as the Lyapunov function. So by choosing the total energy from (7.11) the following function is obtained

\[ V = \frac{1}{2}(v_E^T M_E v_E + \tilde{v}_{E,\text{int}}^T K_I \tilde{v}_{E,\text{int}} + \tilde{\Delta}^T K_s \tilde{\Delta}) + \epsilon \tilde{v}_{E,\text{int}}^T M_E v_E. \]  

(7.12)

To make the equations more viewable the \( q \)-dependency is from now on left out.

The last term is added to insure that \(-\dot{V}\) becomes positive definite [18] this will be explained in detail below. The derivative of (7.12) with respect to time is,

\[ \dot{V} = v_E^T M_E \dot{v}_E + \frac{1}{2} v_E^T \dot{M}_E v_E - \tilde{v}_{E,\text{int}}^T K_I v_E + \tilde{\Delta}^T K_s \dot{\Delta} + \epsilon (-v_E^T M_E v_E + \tilde{v}_{E,\text{int}}^T \dot{M}_E v_E). \]  

(7.13)

Substituting (7.11) and (7.9) into (7.13) and use the fact that \( \dot{M}_E - 2C_E \) is skew symmetric, see Appendix A.2, gives

\[ \dot{V} = -v_E^T (\epsilon M_E + K_p) v_E + \tilde{\Delta}^T K_s (-u_1 + \dot{\Delta}') - \epsilon (\tilde{v}_{E,\text{int}}^T K_I \tilde{v}_{E,\text{int}} + \tilde{v}_{E,\text{int}}^T \dot{M}_E v_E). \]  

(7.14)

If \( \epsilon \) is sufficient small and positive, the only problem in (7.14) is the underlined component, since \( K_s \) is unknown and since \( \dot{\Delta}' \) can't not be controlled. One approach to solve this is to use a gain \( \gamma \) big enough to dominate the \( \dot{\Delta}' \), i.e., pick

\[ u_1 = \gamma \tilde{\Delta}. \]  

(7.15)

By doing this the underlined term will approximately be reduced to \(-\tilde{\Delta}^T K_s (\gamma \dot{\Delta})\) and (7.14) becomes.

\[ \dot{V} \approx -v_E^T (\epsilon M_E + K_p) v_E - \tilde{\Delta}^T K_s (\gamma \dot{\Delta}) - \epsilon (\tilde{v}_{E,\text{int}}^T K_I \tilde{v}_{E,\text{int}} + \tilde{v}_{E,\text{int}}^T \dot{M}_E v_E). \]  

(7.16)

Now it is possible to show that \(-\dot{V}\) is positive definite, see Appendix A.3 and that the controller thereby is exponentially stable.

Above it is claimed that the desired control signal is \( u_1 = \gamma \dot{\Delta} \). But since it is the force and not some spring displacement that is available from feedback, it is the force that is to be controlled, see (7.5). \( K_s \) is however unknown but assumed to be constant, therefore \( K_s \) can be included in the control gain \( \gamma \) and \( u_1 \) can be expressed as \( u_1 = \gamma \tilde{F}_s(\Delta) \).
7.2 Control law summary

The control law presented above is summarized by the following equations:

\[ v_{E,int} = \int v_E \]  \hspace{1cm} (7.17)

\[ K_I v'_{E,int} = F_E \]  \hspace{1cm} (7.18)

\[ \ddot{v}_{E,int} = v'_{E,int} - v_{E,int} \]  \hspace{1cm} (7.19)

\[ F_s(\Delta)' = -K_p v_E - K_I v_{E,int} \]  \hspace{1cm} (7.20)

\[ T_E = C_{EL} + F_s(\Delta)' \]  \hspace{1cm} (7.21)

\[ \ddot{F}_s(\Delta) = T_E - F_s(\Delta) \]  \hspace{1cm} (7.22)

\[ u_1 = \gamma \ddot{F}_s(\Delta) \]  \hspace{1cm} (7.23)

Result form the implementation of this controller is presented in Chapter 10.2.
Chapter 8

Passive Velocity Field Control PVFC

To expand the usage area for the HPA, a control strategy using velocity fields to create guidance for the HPA, while still keeping the system passive, is introduced in this section. A fictive velocity field is placed in the constrained working area of the machine and will thereby help the human operator to follow a given path, this strategy is called Passive Velocity Field Control (PVFC) and was first introduced by Horowitz and Li [12]. How the desired trajectory looks is of course dependent on the particular task to be performed. For this reason a framework which makes it possible to introduce arbitrary velocity fields on the system will be presented. Two different velocity fields are proposed and these are used for test and verification of the controller.

The strategy presented here differs from the common approach for a tracking problem. In both cases there is a path to be followed. Usually the machine or robot is expected to be at a certain position at a certain time, and the control error will be the difference between the actual and desired position at that time, while using PVFC the error will only be the shortest distance to the desired path, see Figure 8.1. The PVFC is therefore advantages in applications where timing is not important, which is the case for this HPA.

Because the operator is directly physically connected to the machine the PVFC should work as a feedback to the operator telling him if he is on the right track or not. For safety and driveability it is however still important that the machine remains passive, i.e., not extracting any energy that have not been put in by the operator or that are caused by possible initial potential energy. It is also desired that the velocity field becomes stronger the more energy the operator puts in to the system.

The structure of the chapter is to first go into details on how the velocity field is constructed and then present the control law. The source for the theories presented in this chapter are [14],[13],[12], [10] and [11].
8.1 Velocity Field

The velocity field presented in this thesis is a vector field where the coordinates of the vector field is the position of a mechanical system given in generalized coordinates and the vector are the velocity for the system. Since the machine has two degrees of freedom the velocity fields are given in the plane.

8.1.1 Designing Velocity Field

As mentioned earlier the velocity field has the role of helping an operator perform a certain task. Since there still is no designated use for HPA the velocity fields design here are made in order to satisfy possible future usage. Two different fields are constructed, these will be called the linear field and the circular field. The linear field is design so that the operator can make a strait lift up and down. The circular field will guide the operator and help him make a circular motion.

Linear field

The purpose of this field is to help the operator perform a straight lift. The natural way to lift something with this HPA is to just use the pitch DoF, which results in an arc shaped motion. This might be a problem if the machine is used for lifting something from the ground and place it on a table, see Figure 8.2. The field will be denoted as

\[ V_{XY} = \begin{bmatrix} V_X \\ V_Y \end{bmatrix}, \]

where \( V_X \) and \( V_Y \) are the components of the velocity field in the \( X \) and \( Y \) coordinates from Figure 8.2.

The general idea is to construct the field in such way that it from all undesired positions, points toward the desired path. Here this is made by first introduce an error coefficient \( \tilde{X} \), that gives a value of the distance to the desired path

\[ \tilde{X} = X - X_d, \]
where $X$ is the actual position and $X_d$ is the desired position. Because the desired path is a straight vertical line there is not really any desired $Y$ coordinate, the demand for this direction is instead to push the machine up when $\tilde{X} \approx 0$. Given these conditions, $V_X$ should be big as long as $|\tilde{X}| > 0$ and $V_Y$ big if $\tilde{X} \approx 0$.

These conditions can be fulfilled e.g. by using exponential functions

$$V_X = \text{sgn}(\tilde{X})(1 - e^{-k_X|\tilde{X}|})\dot{X}_d, \quad (8.3)$$

$$V_Y = e^{-k_Y|\tilde{X}|}\dot{Y}_d, \quad (8.4)$$

where $k_X$ and $k_Y$ are constants, $\dot{X}_d$ and $\dot{Y}_d$ will be the magnitude of the field in each point and can in some sense be seen as the desired velocity for the machine. All these parameters can be used for tuning when implementing the PVFC on the machine. Figure 8.3 shows how the functions behave around the desired $X$ value $X_d$ and Figure 8.4 shows how that field looks in the $X$ and $Y$ coordinates.

If the load is to be lowered instead of lifted, it can simply be done by changing the sign on $\dot{X}_d$. Furthermore it is likely that the operator want to do this during operation. There are several way how to solve this, one solution is proposed in Chapter 10.3.

Circular field

This field will guide the operator into a circular motion. The same basic idea is used to creating the field as in the linear case, but with polar coordinates instead. The desired position now is exchanged for a desired radius $R_d$ and when that radius is reached a circular motion with the angular velocity $\dot{\theta}_d$ is performed. This gives the following functions for creating the circular field

$$V_R = \text{sgn}(\dot{R})(1 - e^{-k_R|R|})\dot{R}_d, \quad (8.5)$$

$$V_{\theta} = e^{-k_{\theta}|R|}\dot{\theta}_d. \quad (8.6)$$
Figure 8.3: Behaviour for the linear field functions around the desired $X$ value $X_d$.

Similar to before $\tilde{R}$ is the difference between actual and desired position and $k_R$ and $k_\theta$ are constants. The circular field is showed in Figure 8.5.

**Coordinate transformation**

The field described in the two previous sections are both created in the coordinate system most suited for that particular field, but neither of these is the system that the machine naturally works in. Because of this, the field has to be transformed to the correct working system, see Figure 8.6.

The linear system just have to make one transformation from the $XY$ – system to the $pr$ – system. This transformation is done by using the Jacobian $J_{XY}$ according to

$$V_{pr} = J_{XY}^{-1} V_{XY},$$  \hspace{1cm} (8.7)

where

$$J_{XY} = \begin{bmatrix} \cos(x_p) & -x_r \sin(x_p) \\ \sin(x_p) & x_r \cos(x_p) \end{bmatrix}. \hspace{1cm} (8.8)$$

For details on the derivation of $J_{XY}$ see Appendix A.4.

As illustrated in Figure 8.6 the circular field need an additional transformations from the polar $R\theta$ – system to the $XY$ – system

$$V_{XY} = J_{R\theta} V_{R\theta},$$ \hspace{1cm} (8.9)

where the Jacobian $J_{R\theta}$ is used as transformation matrix see Appendix A.4 for details.

Furthermore in Figure 8.6 a coordinate system $xy$ with the origin in the circle center has been added. The purpose for this system is to place the circle somewhere in $XY$, the transformation $xy \mapsto XY$ is simply

$$x = X - X_k,$$
$$y = Y - Y_k,$$
where $X_k$ and $Y_k$ is the desired position for the circle. But since $\dot{x} = \dot{X}$ and $\dot{y} = \dot{Y}$ this transformation does not effect the transformation of the velocity field, just the position.

8.2 PVFC

Once the velocity field is constructed, some control law to implement it on the machine and make it useful for control is needed. In this section a control strategy given some arbitrary velocity field is presented, this strategy is called Passive Velocity Field Control (PVFC) [12],[13],[14]. Because the operator is directly physically connected to the machine the PVFC should work as a feedback to the operator telling him if he is on the right track or not. For safety and driveability it is however still important that the machine remains passive, i.e., not extracting any energy that have not been put in by the operator or that are caused by possible initial potential energy. It is also desired the the velocity field becomes stronger the more energy the operator puts in to the system.

In Chapter 6 the virtual system was introduced. Here this system will now be used for implementing the PVFC. By adding a control term $T_{gad}$ in (6.19) the virtual system is modified to

$$M_v = F_d - F_s(\Delta) + T_{gad}.$$ (8.10)

$T_{gad}$ will work as an additional force pushing on the virtual mass, see Figure 8.7. Including this extra term when doing coordinate transformation from Chapter 6.4
the locked and shaped system is changed to
\[
M_L(q)\ddot{v}_L + C_L(\dot{q}, q)v_L + C_{LE}(\dot{q}, q)v_E = F_h + F_{env} + F_d + T_{gad},
\]
\[
M_E(q)\dot{\dot{v}}_E + C_E(\dot{q}, q)v_E + C_{EL}(\dot{q}, q)v_L =
\]
\[
b(q)(F_h + F_{env} - G_a(q)) - a(q)(F_d + T_{gad}) + F_s(\Delta),
\]
\[
\dot{\Delta} = -v_E - u_{loss} + u_1.
\]

When doing the guidance it is the behaviour of the locked system that is to be controlled and therefore it is assumed that the control presented in Chapter 7 is good enough to keep \(v_E \approx 0\) and thereby keep the virtual and real mass synchronized at all times. By doing that the desired behaviour for the machine will be obtained.

\(T_{gad}\) does not necessarily have to be used for guidance control as will be shown in Chapters 9. But for now this will be the only scope of use, i.e,
\[
T_{gad} = T_{PVFC}.
\]

Now the only problem left is to design \(T_{PVFC}\).

**Augment the mechanical system**

As mentioned earlier it is significant to obtain the passivity of the system at all times. In order to do that the system is augmented with an additional state. This additional state can be resembled by a flywheel see e.g. [12].
\[
M_{fw} \dot{q} = T_{fw}
\]

Where \(M_{fw}\) is the mass, \(q\) the position in general coordinates and \(T_{fw}\) is the torque from the fly wheel. This flywheel is then used for keeping track of the
energy in the system. If the operator put in energy to the system the fly wheel will accelerate and if the controller demands torque it will decelerate.

This gives that the state for the flywheel, i.e., the flywheel velocity will determine how much energy that can be exerted from the system. Augmenting the locked system with the flywheel gives

\[
M = \begin{bmatrix}
M_L(q) & 0 \\
0 & M_{fw}
\end{bmatrix}, \quad 
C = \begin{bmatrix}
C_L(q, \dot{q}) & 0 \\
0 & 0
\end{bmatrix}.
\] (8.16)

**Augment the velocity field**

To be able to control and to use the fly wheel in the controller the velocity field \( V_{pr} \) has to be augmented as well. From the kinetic energy of the augmented system
(8.16) the flywheel velocity field is given by

$$V_{fw} = \sqrt{\frac{2}{M_w} (E - V_{pr}^T M_L(q)V_{pr})},$$  \hspace{1cm} (8.17)

where $E$ is the total energy of the system and has to be chosen as a large enough value to ensure that (8.17) has a real solution. $V_{pr}$ is now augmented with the flywheel field

$$\mathbf{V} = [V_{pr}^T, V_{fw}]^T.$$  \hspace{1cm} (8.18)

### Constructing the control law

With the augmented system and velocity field in hand the controller can be constructed. First a couple of parameters needs to be defined.

$$\omega = M(q) \dot{V} + C(q, \dot{q}) V,$$  \hspace{1cm} (8.19)

$$p = M(q) \dot{q},$$  \hspace{1cm} (8.20)

$$P = M(q) V,$$  \hspace{1cm} (8.21)

where $\omega$ describes the dynamics of the system when effected by the velocity field, $p$ is the momentum for the real system and $P$ is the desired momentum, i.e., the momentum the system has if it tracks the desired velocity field. The time derivative of $\mathbf{V}$ is given by

$$\dot{\mathbf{V}}_i = \sum_{k=1}^{n} \frac{\partial \mathbf{V}(q)}{\partial q_k} \dot{q}_k,$$  \hspace{1cm} (8.22)

where $i = [p, r, f_w]$. For details on the derivative calculations of the two augmented fields (the linear and the circular from above) see Appendix A.4.

The parameters defined in (8.19) to (8.21) can now be used to form the following control law

$$T_r = k_{Tr} (Pp^T - \bar{p}P^T) \dot{q},$$  \hspace{1cm} (8.23)

$$T_g = \frac{1}{2E} (\omega P^T - P\omega^T) \dot{q},$$  \hspace{1cm} (8.24)

$$T_{PVFC} = T_r + T_g,$$  \hspace{1cm} (8.25)

where $T_r$ can be seen as a feedback control since it depends on a relation between the desired and actual momentum, $k_{Tr}$ is thereby the feedback gain and can be used as a tuning parameter when implementing the control law. $T_g$ can on the other hand be seen as a feed forward control, possessing information about the system dynamics. One important property of $T_r$ and $T_g$ is that the matrices

$$R(\dot{q}, q) = (Pp^T - \bar{p}P^T),$$  \hspace{1cm} (8.26)

$$G(\dot{q}, q) = \frac{1}{2E} (\omega P^T - P\omega^T),$$  \hspace{1cm} (8.27)

are skew symmetric, which is a requirement for keeping the controller passive, for proof see Appendix A.4.
Extending the controller with PVFC, the middle step in the backstepping controller from equation (7.21) now becomes

$$T_E = C_{EL}(\dot{q}, q)v_L + F_s(\Delta)' + a(q)T_{PVFC}, \quad (8.28)$$

where the transformation $a(q)$ is needed since the backstepp controller operates on the shaped system. For implementation and validation of the PVFC control law see Chapter 10.3.
Chapter 9
Obstacle Avoidance

When operating a robot there might be certain areas in the work area that should be avoided. There could be a numerous reasons why to avoid these areas, but typically it is in order to protect the environment or the machine from physical damage. The areas to avoid can be seen as obstacles and can of course be real physical obstacles as well.

In this thesis a control law using potential fields to implement obstacle avoidance on the HPA is presented. This chapter will first go in to detail on the potential field, what it is, how it can be used and how it is constructed for this particular case. Finally the controller using these fields is presented. The sources for this chapter is mainly [9] and [10].

9.1 Potential Field

The aim is to be able to repel the machine from certain areas by placing an artificial potential field at that location. One can loosely say that the potential field should be big, have a high potential, where the obstacle is, and small, have low potential, everywhere else.

The potential field is created from a potential differentiable function $U_{oa}$ as the negative gradient of that function, $-\Delta U_{oa}$, e.g. [23]. The function $U_{oa}$ is to be designed so that it is big when an obstacle is present and small otherwise. Such a function is

$$U_{oa,XY} = e^{-k_{oa}\lambda}U_d,$$  \hspace{1cm} (9.1)

where $k_{oa}$ and $U_d$ are constants and can be used for changing the characteristics of the function and thereby making the field more or less aggressive. $\lambda$ denotes the distance to the obstacle and is defined as

$$\lambda = \sqrt{(X_{ob} - X)^2 + (Y_{ob} - Y)^2}.$$  \hspace{1cm} (9.2)

where $[X_{ob}, Y_{ob}]$ is the position of the obstacle. The potential field is derived from

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the partial derivative of $U_{oa,XY}$ according to

$$T_{oa,XY} = U_{oa,XY} k_{oa} \lambda \frac{\partial \lambda}{\partial q}, \quad (9.3)$$

where

$$\frac{\partial \lambda}{\partial q} = \left[ \frac{X-X_{oa}}{\lambda} \quad \frac{Y-Y_{oa}}{\lambda} \right]^T. \quad (9.4)$$

A plot of the potential field is shown in Figure 9.1

![Potential field plot](image)

Figure 9.1: Potential field using the potential function (9.1).

In the same way as with the PVFC from Chapter 8 the potential field is constructed in the $XY$ system, see e.g. Figure 8.6, and therefore needs to be transformed to the $pr$ system according to

$$T_{oa,pr} = J_{XY}^{-1} T_{oa,XY}. \quad (9.5)$$

A modification inspired by [9] is to include the mass of the locked system $M_L(q)$ in (9.5), that is,

$$T'_{oa,pr} = J_{XY}^{-1} M_L(q) T_{oa,XY}. \quad (9.6)$$

This way the magnitude of the potential field and ultimately the control signal will depend on how big the mass it is trying to repel is. The benefit with this modification can be realised, e.g., if $m_r << I_c + m_r l_{cog}^2$ and if either just doing a pitch or reach motion when trying to enter the obstacle, the repelling force will be the same in both cases, but since the masses are different the acceleration will be much higher in the reach case, for this example.
9.2 Rectangular obstacle

The potential field obtained in the previous section will only work if the obstacle is a point in the work area. In this section a method to avoid a rectangular obstacle put into the area with an arbitrary angle is proposed, see Figure 9.2.

The same potential function as earlier, \((9.1)\), is used for each point of the contour of the rectangle. But roughly speaking only one point can be active at the time. Therefore one wants to figure out what point on the rectangular contour that is closes to the present position, i.e., the projection on the rectangle.

![Figure 9.2: Rectangular obstacle inside the working area. Three possible present positions are proposed (blue) with a corresponding points on rectangular representing the closest point (green). The closest distance to each point is denoted \(\lambda\).](image)

In Figure 9.2 the obstacle and the working area are shown. Three points showing possible present positions are proposed, each of these have corresponding points on the rectangle representing the closest point. The distance to the closest point is as before \(\lambda\). By finding the closest point and determine \(\lambda\) the rectangular obstacle can be obtained. As seen in the figure the working area has been divided into five subareas \(A - E\). In subareas \(B\) and \(D\) the closest point is always the corner of the rectangle and \(\lambda\) is as before, given in \((9.2)\). In the other areas \((A, C\) and \(E)\) is \(\lambda\) the perpendicular distance to the rectangle. Using the notations from Figure 9.2, and using area \(A\) as an example \(\lambda\) is

\[
\lambda = \sqrt{\lambda_1^2 - \gamma_{11}^2},
\]

\((9.7)\)
where (9.2) gives $\lambda_1$ and $\lambda_2$, and $\gamma_{11}$ is

$$
\gamma_{11} = \frac{\lambda_1^2 - \lambda_2^2 + \gamma_1^2}{2\gamma_1}.
$$

(9.8)

The potential field from (9.3) also requires an expression for the partial derivative. As before areas $B$ and $D$ won’t change from the point field case. In the other areas, ones again with $A$ as an example, the derivative becomes

$$
\frac{\partial \lambda}{\partial q} = \frac{1}{\lambda} \left[ (X - X_{ob1}) - \frac{\gamma_{11}}{\gamma_1} (X_{ob2} - X_{ob1}) \right]
\left[ (Y - Y_{ob1}) - \frac{\gamma_{11}}{\gamma_1} (Y_{ob2} - Y_{ob1}) \right].
$$

(9.9)

The notations $ob1$ and $ob2$ corresponds to the corners off the rectangle. For more details on the calculation see Appendix A.5. Using these theories the potential field in Figure 9.3 can be designed.

![Rectangular potential field, for obstacle avoidance.](image)

Figure 9.3: Rectangular potential field, for obstacle avoidance.

### 9.3 Conditional Field

The fields presented above can be designed to be very small everywhere else but close to the obstacle. However to get rid of any unwanted influence from the obstacle, it can be beneficial to add conditions that makes $U_{oa} = 0$ when positioned far from the obstacle.

Once having information about $\lambda$, i.e., the distance to the obstacle, such a condition is easily obtained just by putting a fixed value $\lambda_0$ at which the field is to be enabled.

$$
U_{oa} = \begin{cases} 
U_{oa,XY} = e^{-k_{oa}\lambda}U_d & \text{if } \lambda \leq \lambda_0 \\
0 & \text{if } \lambda > \lambda_0
\end{cases}
$$

(9.10)
9.4 Obstacle Avoidance Control

To implement the obstacle avoidance, exactly the same approach as for the PVFC in Chapter 8.2 is used. The control term $T_{gad}$ is changed to

$$T_{gad} = T'_{oa,pr},$$

where $T'_{oa,pr}$ is given by (9.6). For further information see the Chapter 8. For results and implementation see Chapter 10.
Chapter 10

Results

In this chapter different results from test on the machine will be presented. The aim of this section is to show how the different controllers work. For the amplification part the objective is to coordinate the virtual and the real system by trying to get the control error \( \dot{x}_a - \dot{x}_v = v_E \to 0 \), and thereby achieve a good tracking of the desired force, for more details see Chapters 5 and 7. For the PVFC and obstacle avoidance the aim is to show that the theories work and try to give a picture of how they work.

To implement the theories on the machine, MATLAB Simulink is used to build the control system.

10.1 Uncoupled Controllers

Here results for the controllers issued in Chapter 5 will be presented. For these controllers it is mainly the reach direction that have been in focus since there already existed a controller for the pitch direction.

10.1.1 PI-Controller

The first approach was to use a PI-controller to achieve coordination. To check the performance for the reach controller the pitch degree of freedom is locked \( (x_p = 80 \text{ deg}) \) and the HPA is being moved back and forth, see Figure 10.1.

As shown in the figure both the velocity and the force tracks the desired state pretty well. What is really desired is to achieve a good force tracking since that is the purpose for the machine, the velocity tracking is just a step in achieving this. It is however an important step because if a good velocity tracking between the real and virtual mass is maintained, then theoretically, the force tracking should be good as well. The figure also shows that the sensor signals are pretty noisy, especially the force sensor. There also seems to be outliers in the curves. To explain these outliers let us just look at the desired force (since this signal is cleaner) and examine at what position the outliers occurs, see Figure 10.2.
Figure 10.1: Results for the PI-controller in the reach direction, moving the arm back and forth.

(a) Velocity Tracking.

(b) Force Tracking.
Figure 10.2: Investigation of peaks in the force data. The green circles show that the peaks occur at a certain position and when going in the backward direction.
Looking at the force peaks in Figure 10.2a and then checking the current position at that time in Figure 10.2b (this is represented by the green circles), it becomes clear that these peaks take place at a certain point and when going in a certain direction. This can be explained by the construction of the machine. The beam performing the reach can be seen as a telescope device with two stages. When going in the negative direction the inner car of the telescope devise hits a heel (typically around $x_r = 0.354$) to pull the outer car along. When going in the positive direction the two cars are initially connected and then released from each other and therefore there is no impact and now peak in the force sensor signals.

You can feel a light impact when operating the machine but since one of the fundamental ideas for the HPA is that you should get feedback from the environment this is quite in order.

### 10.1.2 Innovation Controller

The other control strategy presented in Chapter 5 was the Innovation feedback controller. The same test as for the PI-controller was made, and the result is shown in Figure 10.3

The performance of the innovation controller is also satisfactory, it is hard to see any significant differences between the results for the two strategies.

### 10.1.3 Pitch Controller

As mentioned the pitch controller was already working at the start of this project. Here, to demonstrate the performance for this controller, a similar test as for the reach case is done. The reach motion is locked at $x_r = 0.4$ and the arm is moved up and down. The result from the experiment is shown in Figure 10.4.

### 10.1.4 Pitch and Reach Uncoupled Controller

Finally a test was made to see how these controllers work when running them at the same time. For this test the PI-controller is used for the reach direction and the original innovation controller for the pitch. The result is presented in Figures 10.5 and 10.6.

The uncoupled controllers work pretty well. As before, the force signal is very noisy but is tracked reasonably well. For the pitch case, e.g., around ten seconds, the controller does not track well. This is mainly due to saturation in the actuator.

One problem with the controller that does not show in the plots but which you can feel when operating the machine is that it can feel a little bit “jumpy“ at some point. This does not occur unless using both DoF simultaneously. A reasonably explanation is that it is some interactions between the two DoF that is not accounted for in this controller.
Figure 10.3: Results for the Innovation controller in the reach direction, moving the arm back and forward.
Figure 10.4: Results for the pitch controller, moving the arm up and down.
10.1 Uncoupled Controllers

Figure 10.5: Results for two DoF, moving the arm up and down and back and forth simultaneously. Using uncoupled system with PI controller for the reach motion and Innovation control for the pitch motion. Results for the pitch motion.
Figure 10.6: Results for two DoF, moving the arm up and down and back and forth simultaneously. Using uncoupled system with PI controller for the reach motion and Innovation control for the pitch motion. Results for the reach motion.
10.2 Coupled Controller

Here the results for the coupled controller introduced in Chapter 7 are presented. This controller was made, to see if it was possible to improve the performance from the earlier approaches and especially for the case when operating both DoF at the same time. This should be obtained by 1) make a more accurate system description 2) create a controller that compensates for disturbance and couplings that were not considered before, for more details see Chapter 6 and 7.

The test presented here is made running both DoF, that is, moving the arm up and down and back and forth at the same time.

As can be seen in Figure 10.7 and 10.8 both the velocity and the force tracking works well for both DoF:s. The jumpy feeling from earlier is gone and the force signals looks less noisy. Other than that, the performance is not significantly better than what was achieved with the previous controllers. Since the backstepping controller works in two steps it might also be interesting to see how well the force tracking is for the middle stage. How well this tracking works does not directly affect the operator but it is still important to achieve a good final result. So if not for other reasons, this tracking is a great help in the design process and when tuning the gains for the controller. Result for the force tracking of the middle stage is shown in Figure 10.9

10.3 PVFC

As been explained in earlier chapters, the Passive Velocity Field Control (PVFC) is supposed to help a human operator follow a given path or contour by creating a velocity field. The two fields presented in this thesis are a linear and a circular field. In this section some results from the simulation and the real system are presented

10.3.1 Simulation

Here the simulation is used to make sure that the velocity field and the controller is correct by verifying that the simulation converges to the desired path from an arbitrary starting position.

Figure 10.10 and 10.11 show results from the simulation. Figure 10.10a and 10.11a are plots of the functions used for creating the fields, making sure that they look as intended. Figure 10.10b and 10.11b are results from simulations where the model is given an initial velocity in some direction from an arbitrary position. The test is a check to verify that the PVFC control make the model converge to the desired path, regardless of starting position.

10.3.2 Real system

Here the results from implementation on the real system is presented.
Figure 10.7: Result from using the coupled system, with the Backstepping control. Both DoF are moved at the same time. Results for the pitch motion,
Figure 10.8: Result from using the coupled system, with the Backstepping control. Both DoF are moved at the same time. Results for the reach motion,
Figure 10.9: Force tracking for the middle stage in the backstepping controller.
(a) Linear velocity field.  (b) Convergence of simulation from arbitrary starting points.

Figure 10.10: Design of linear velocity field and result from simulation.

(a) Circular velocity field.  (b) Convergence for simulation from arbitrary starting points.

Figure 10.11: Design of circular velocity field and result from simulation.
Linear field

The linear field can e.g. be used for making linear lifts as proposed in Chapter 8.1.1. In the simulation the field is only going in one direction. To make the machine useful an extra feature is added when the controller is implemented on the machine. An outer loop making it possible to change the direction of the field is implemented. When getting close to the top or bottom the direction of the field can be changed by applying a big enough force in the opposite direction.

![Position tracking linear field](image)

Figure 10.12: Position tracking linear velocity field, up and down. Green line actual position and blue line desired position.

Figure 10.12 shows the position of the machine and the desired path. To visualize the change in the fields direction, $X_d$ (the desired $X$ value) is also changed as seen in the figure. By turning off the field one can also see the path if only using the pitch to do the lift.

Circular field

The circular field should help the operator track the contour of a circle. Figure 10.13a shows the contour tracking of the circle. Figure 10.13b can give an idea of how the same circle looks placed in the working area. In these two figures the data from the initial part of the experiment is removed to give a good picture of the actual tracking. In Figure 10.13c the entire data set is shown. Here it can be seen that when the energy in the system is low, i.e., before the fly wheel is charged the control signal is not large enough to track the circle perfectly, but once the
(a) Tracking circular field close up.

(b) Tracking circular field placed in the work area.

(c) Tracking circular field with uncharged flywheel.

Figure 10.13: Position tracking for circular velocity field.
operator puts in more energy the tracking works as desired.

**Fly wheel**

The fly wheel is a fictive state introduced in Chapter 8.2, to ensure that the system remains passive, by making sure that energy used by the PVFC is either put in by the operator or is initial energy in the system.

In Figure 10.14 it is shown that the fly wheel work as intended. Both plots are from runs with the circular field. In Figure 10.14a the flywheel velocity is initially zero but increases as the operator puts in energy to the system. After about 40 seconds the operator let go of the handle and the fly wheel begin to discharge. In Figure 10.14b an experiment with an initial speed of 10 rad/s is performed. Here the operator does not interact with the machine at all. As can bee seen the energy is constantly decreasing in the fly wheel. The pattern that can be seen in both curves comes from the circular motion the machine is making. When going down the fly wheel is charged, but it needs to be discharged even more to go up again, this is most clearly seen when there is no interaction with the operator.

**10.4 Obstacle Avoidance**

Here experiments for testing the obstacle avoidance from Chapter 9 is presented. The tests are performed on the real system. First the point obstacle avoidance is tested.

Figure 10.15a shows a close up on the obstacle, it shows that the machine bounces away when trying to enter the forbidden area. In Figure 10.15b the same thing is shown with the obstacle placed in the working area.

Figure 10.16 demonstrates the implementation of the rectangular obstacle. In Figure 10.16a the obstacle is put straight down into the working area, while Figure 10.16b shows that it is also possible to place the obstacle with an arbitrary angle into the area. Both figures also prove that the rectangular obstacle avoidance works as desired.

In the PVFC section above it was proposed that the linear velocity field could be used to make a straight lift e.g. to lift something to the top of a table. Figure 10.17 illustrate that it is possible to achieve a similar functionality by placing a rectangular obstacle as a straight wall where the considered table is standing. In Figure 10.17a the entire experiment is plotted. As before the obstacle repels the machine when it tries to enter the restricted area. At the end of the experiment the operator makes a straight lift by pushing against the obstacle and track the contour of the obstacle, this part is more clearly shown in Figure 10.17b and as can be seen here, this method also works well for doing the straight lift.
10.4 Obstacle Avoidance

(a) Flywheel charge and discharge.

(b) Flywheel discharge with initial speed.

Figure 10.14: Flywheel velocity during operation.
Figure 10.15: Point obstacle, where the green line shows the motion of the machine when it is trying to access the forbidden area and where the blue line shows where the field is initiated and the red dot is the center of the obstacle.
10.4 Obstacle Avoidance

(a) Straight obstacle.

(b) Leaning obstacle.

Figure 10.16: Rectangular obstacle, where the green line shows the motion of the machine and how it is repelled from the obstacle, marked by blue and red.
Figure 10.17: Straight wall obstacle, showing that the obstacle avoidance can be used for preventing the machine to access a certain area, but also that it can be used for tracking the edge of the obstacle.
Chapter 11

Discussion

11.1 Conclusions

Human amplification

Chapter 10 gives results from the experiments, and shows that the controller and the different features that were added to it works fairly well. However there are things that do not work perfectly and one can always wish that the force tracking would have been even better.

What can be seen in the result plots, and maybe even more felt when operating the machine, is that the coupled backstepping controller improves the performance of the force amplification. If it is due to the new system description or the new controller design is uncertain. But since the new controller, at least theoretically, better takes care of the disturbances and since the new model description better resembles the system, it is most likely a combination of both.

Furthermore as seen in Chapter 10 the sensor signals from the force sensors were quite noisy and this causes problems when implementing the controller because the control gains will also amplify the influence of the noisy. Attempts to filter these signal where made but in order to get a good enough filtering, i.e., to get a filtered signal that made any different for the result, the cut of frequency for the filters had to be very low and the dynamics of the controller thereby became to slow.

During the work on the machine there were no devices for lifting any loads which means that the machine only lifts its own weight. Apart from that it becomes more useful if such a device is mounted, at present state it basically just moves around and does not perform any real task, the influence from the noisy force signal becomes significantly reduced.

One other problem is that the pitch actuator, the hydraulic cylinder, saturates if the operator tries to move the machine fast up and down. This is maybe not a big problem and especially not if the machine will be used for moving heavy burdens, because then one might not want to move the load to fast anyway. It does however reduce the driving experience when moving the machine without
any load. When it comes to the force tracking this is not good since the operator tends to push even more if he is not getting the velocity he desired, and thereby the desired force $F_d$ increases and the actuator force $F_a$ can not keep up. This is however not anything that can be solved with a controller since it is a system constraint, but since there is a PI-controller in one of the steps of the algorithm some action has to be taken to prevent integration wind up. In this thesis this is, as suggested in e.g. [1], handled with conditional integration, i.e., the integration part stops updating when the saturation level is reached.

**PVFC**

As can be seen in Chapter 10, this feature works as desired. But, as for the amplification problem the noisy force signal can become a problem, if a very strong field is desired. The field can on the other hand not be made too strong even if the force signal would be perfect. Because with a very strong field the machine will have a high velocity when reaching the desired path and overshoot it. The field pushing it in the other direction will then throw it back and forth and the controller thereby becomes unstable.

The gains in functions (8.3),(8.4),(8.5) and (8.6), can on the other hand be designed so that if a strong field is desired one can make the field less aggressive close to the desired path.

**Obstacle avoidance**

This controller also works as desired, again see Chapter 10, the machine is repelled from the restricted areas.

The idea to introduce conditional fields, see (9.10), seems to work. It takes away any disturbance from the field and even if it might be small it is still unwanted. And using this together with the function parameters it is possible to form the obstacle as desired. It could either be made pretty soft and bouncy or really hard, depending on what is desired.

One can argue that by including the mass for the control force as done in (9.6) the control signal will not have the correct unit. Maybe it is therefore not ultimate to use the mass notation but rather see it as a unitless control gain that varies depending on the direction of the motion.

### 11.2 Future work

There is a lot of things that can be done to improve the HPA and extend the usage of it. One obvious thing is to add one more DoF so that the machine can move in three dimensions. This can e.g. be done by making it possible to rotate the machine and actuate this motion with one more hydraulic motor. This will of course increase the difficulty to control the machine in many ways. Additional force and position sensors need to be implemented to read the new desired force and the new actuator force and position. The most complicated task when it comes to hardware is probably to implement sensors in the handle that just register force in
the desired direction and avoids mechanical cross talk to the other sensors. When it comes to software there is of course a lot of different challenges. Above all, the system description will become more complicated. However with the description in hand hopefully the control strategy presented here will still work.

One other thing that can be done in future work is to add some gripping device, so that the machine actually will be able to lift and move things. Such a device would of course increase the usage for the machine significantly. Also as the HPA works today the machine amplifies the human force with a constant gain $\rho$. If a gripping device were to be mounted on the machine one nice feature would be to add some kind of load sensing and thereby be able to adjust $\rho$ continuously so that the machine helps more when there is a load present and so on.

Furthermore, one thing that might be interesting to investigate is gravity compensation. Gravity is not necessarily an unwanted property for this machine, since it should be an amplification of the real world. But there are situations where it might be desirable to cancel out the influences of gravity. Theoretically this compensation could be implemented in the same way as the obstacle avoidance and the PVFC, via $T_{\text{gad}}$.

When it comes to future work on PVFC and obstacle avoidance for this particular machine, it is probably some implementation on a real problem that would be most interesting. Say for example that the HPA can be used as a lift when changing engine in a car, PVFC and obstacle avoidance can then be use to guide the machine and to avoid hitting fragile parts. Moreover, something that was never tested in this thesis was to implement multiple obstacles.

Since the machine is placed on a university one obvious future usage is to use it for educational purposes. One example could be a lab where students design a velocity field and then uses the PVFC on the machine to see if it works.

In a bigger scope the theories used here can hopefully continue to be developed and be used on other types of human amplifiers or power assisting machines.
Bibliography


Appendix A

Appendix

A.1 Uncoupled System

Passivity uncoupled system

Recall the definition for a passive system given in (5.1), that says that if there is a constant \( c \) such that,

\[
\int_0^t f(u(\tau), y(\tau))d\tau \geq -c^2,
\]

for all \( u \) then the system is passive, is given.

Here a proof that the system,

\[
m_r\ddot{x}_r = F_h + F_{env} + F_s(\Delta), \quad (A.1)
\]

\[
\dot{\Delta} = -\dot{x}_r + u - u_{\text{loss}}, \quad (A.2)
\]

presented in Chapter 5.2 is passive.

If considering the components that can store energy in the system, it is the mass and the spring, the mass stores energy in kinetic energy and the spring in accumulated compression, i.e., the stored energy in the system is expressed by,

\[
W_s = \int_0^\Delta F_s(\delta)d\delta + \frac{1}{2}m_r\dot{x}_r^2. \quad (A.3)
\]

Differentiating this gives

\[
\dot{W}_s = F_s(\Delta)\dot{\Delta} + m_r\ddot{x}_r\dot{x}_r \quad (A.4)
\]

and by using (A.2) this can be rewritten to

\[
\dot{W}_s = F_s(\Delta)u - F_s(\Delta)u_{\text{loss}} + (F_h + F_{env})\dot{x}_r. \quad (A.5)
\]

Assuming \( F_s(\Delta)u_{\text{loss}} \geq 0 \) for all \( u \) gives,

\[
\dot{W}_s \leq F_s(\Delta)u + (F_h + F_{env})\dot{x}_r. \quad (A.6)
\]
Integration with respect to time gives

\[-W_s(0) \leq 0 \leq \int_0^t (F_s(\Delta)u + (F_h + F_{env})\dot{x}_r) d\tau, \quad (A.7)\]

and thereby it is proven that the system is passive with respect to the supply rate, where the supply rate is the environmental, human, and actuator power.

**Passivity coordinated system**

For the coordinated system an additional mass is added

\[
m_r\ddot{x}_r = F_h + F_{env} + F_s(\Delta), \quad (A.8)
\]

\[
m_v\ddot{x}_v = F_d - F_s(\Delta), \quad (A.9)
\]

\[
\dot{\Delta} = -\dot{x}_r + \dot{x}_v - u_{loss} + u_1. \quad (A.10)
\]

Here is a proof that the system still remains stable.

The new storage function becomes

\[
W_{s,c} = \Delta \int_0^\Delta F_s(\delta) d\delta + \frac{1}{2}m_r\dot{x}_r^2 + \frac{1}{2}m_v\dot{x}_v^2. \quad (A.11)
\]

Using the same approach and steps as above give,

\[-\dot{W}_{s,c}(0) \leq 0 \leq \int_0^t (F_s(\Delta)u_1 + (F_h + F_{env})\dot{x}_r + F_d\dot{x}_v) d\tau, \quad (A.12)\]

i.e., still passive.

**A.2 Coupled System**

**Jacobian J**

Derivation of Jacobian $J$ for transferring $\dot{x}_p, \dot{x}_r \rightarrow \dot{X}, \dot{Y}$.

Trigonometry gives that the positions of the center of gravity can be transformed $x_p, x_r \rightarrow X, Y$ with

\[
X = l_{cog}(x_r) \sin(x_p), \quad (A.13)
\]

\[
Y = -l_{cog}(x_r) \cos(x_p). \quad (A.14)
\]

Differentiate (A.13) and (A.14), where $l_{cog}(x_r) = x_r - x_{r,cog}$ gives,

\[
\dot{X} = \dot{x}_r \sin(x_p) + \dot{x}_p l_{cog}(x_r) \cos(x_p), \quad (A.15)
\]

\[
\dot{Y} = -\dot{x}_r \cos(x_p) + \dot{x}_p l_{cog}(x_r) \sin(x_p). \quad (A.16)
\]
From this the Jacobian $J$ is created and the expression can be written on matrix form as,

$$
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix} =
\begin{bmatrix}
l_{\text{cog}}(x_r) \cos(x_p) & \sin(x_p) \\
l_{\text{cog}}(x_r) \sin(x_p) & -\cos(x_p)
\end{bmatrix}
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix}.
\tag{A.17}
$$

### Details on matrices in Equation 6.15

$$M_c(q) = \begin{bmatrix} I_c + m_r l_{\text{cog}}(x_r)^2 & 0 \\ 0 & m_r \end{bmatrix}, C_c(\dot{q}, q) = \begin{bmatrix} -m_r l_{\text{cog}}(x_r) \dot{x}_r & -m_r l_{\text{cog}}(x_r) \dot{x}_p \\ m_r l_{\text{cog}}(x_r) \dot{x}_p & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} -m_r g l_{\text{cog}}(x_r) \cos(x_{p,\text{max}} - x_p) \\ -m_r g \sin(x_{p,\text{max}} - x_p) \end{bmatrix}$$

### Skew Symmetry

Definition, the matrix $A$ is skew symmetric if

$$A = -A^T, \tag{A.18}$$

[18]. Proof that $\dot{M}_c(q) - 2C_c(\dot{q}, q)$ is skew symmetric:

$$\dot{M}(q) = \begin{bmatrix} -2m_r l_{\text{cog}}(x_r) \dot{x}_r & 0 \\ 0 & 0 \end{bmatrix} \tag{A.19}$$

this gives

$$\dot{M}_c(q) - 2C_c(\dot{q}, q) = 2 \begin{bmatrix} 0 & -m_r l_{\text{cog}}(x_r) \dot{x}_p \\ m_r l_{\text{cog}}(x_r) \dot{x}_p & 0 \end{bmatrix} \tag{A.20}$$

which according to (A.18) is skew symmetric.

### Passivity coupled system

Here a proof that the coupled system is passive is presented. The coupled system is according to (6.17) given by

$$M_c(q) \ddot{q} + C_c(\dot{q}, q) + G_c(q) = F_h + F_{\text{env}} + F_s(\Delta), \tag{A.21}$$

$$\dot{\Delta} = -\dot{q} + u - u_{\text{loss}}. \tag{A.22}$$

This system can be proven passive in a very similar way to what was presented for the uncoupled system. Lets again look at the energy stored in the system

$$W_s = \frac{1}{2} \dot{q}^T M \dot{q} + \int_0^{\Delta} F_s(\delta) d\delta, \tag{A.23}$$
Dependency of \( q \) is omitted to make the equations more readable. Differentiating (A.23) gives,

\[
\dot{W}_s = \dot{q}^T M \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M} \ddot{q} + F_s(\Delta) \dot{\Delta}.
\]  
(A.24)

Using (A.21) and the fact that \( M - 2C = 0 \) gives,

\[
\dot{W}_s = (-G + F_h + F_{env}) \dot{q} + F_s(\Delta)(u - u_{loss}).
\]  
(A.25)

If \( G \dot{q} \geq 0 \) and \( F_s(\Delta)u_{loss} \geq 0 \) at all points then,

\[
\dot{W}_s \leq (F_h + F_{env}) \dot{q} + F_s(\Delta)u.
\]  
(A.26)

Integrating this w.r.t. time the following expression can be obtained,

\[
-W_s(0) \leq \int_0^t ((F_h + F_{env}) \dot{q} + F_s(\Delta)u) dt,
\]  
(A.27)

i.e., the system is passive w.r.t. supply rate according to (5.1). Using the same method as in Appendix A.1 the system can be proven to remain stable also when it is extended with a virtual system.

### A.3 Coupled System Control

**Proof, \(-\dot{V}\) Positive Definite**

Proof that \(-\dot{V}\) from Chapter 7 is positive definite. \( \dot{V} \) is given by (7.16) and \(-\dot{V}\) thereby becomes

\[
-\dot{V} \approx v_E^T (\epsilon M_E + K_p) v_E + \dot{\Delta}^T \gamma K_s \dot{\Delta} + \epsilon \left( \tilde{v}_{E,int}^T K_I \tilde{v}_{E,int} + \tilde{v}_{E,int}^T K_p \tilde{v}_{E,int} + \frac{1}{2} \tilde{v}_{E,int}^T M_E \tilde{v}_{E,int} \right)
\]  
(A.28)

If \(-\dot{V}\) can be expressed on the form \(-\dot{V} = Z^T AZ\) where \( A \) is a symmetric matrix, then \(-\dot{V}\) is positive definite if \( A \) positive definite [2]. Therefore \(-\dot{V}\) is expressed on the above given form

\[
-\dot{V} = \begin{bmatrix}
    v_E^T & \dot{\Delta}^T & \tilde{v}_{E,int}^T
\end{bmatrix}
\begin{bmatrix}
    (K_p + \epsilon M_E) & 0 & 1/2\epsilon(K_p - 1/2M_E) \\
    0 & \gamma K_s & 1/2\epsilon K_s \\
    1/2\epsilon(K_p - 1/2M_E) & 1/2\epsilon K_s & \epsilon K_I
\end{bmatrix}
\begin{bmatrix}
    v_E \\
    \dot{\Delta} \\
    \tilde{v}_{E,int}
\end{bmatrix}.
\]  
(A.29)

Using the Sylvester’s criterion, \( A \) is positive definite if \( A \) and all leading under matrices have positive determinant. The determinant of the first leading matrix is

\[
|K_p + \epsilon M_E| > 0,
\]  
(A.30)
and the determinant of the second leading matrix becomes
\[
\begin{vmatrix}
(K_p + \epsilon M_E) & 0 & \gamma K_s \\
0 & \gamma K_s & 0 \\
\end{vmatrix} = (K_p + \epsilon M_E)\gamma K_s > 0.
\] (A.31)

Finally the determinate of A is
\[
\begin{vmatrix}
(K_p + \epsilon M_E) & 0 & 1/2\epsilon(K_p - 1/2\dot{M}_E) \\
0 & \gamma K_s & 1/2\epsilon K_s \\
1/2\epsilon(K_p - 1/2\dot{M}_E) & 1/2\epsilon K_s & \epsilon K_I \\
\end{vmatrix} = \epsilon(K_p + \epsilon M_E)\gamma K_s K_I - \\
\frac{1}{4}\epsilon^2((K_p - 1/2\dot{M}_E)^2\gamma K_s + K_s^2(K_p + M_E\epsilon)) > 0.
\] (A.32)

It might not be obvious that \(\det(A) > 0\), as claimed above, but this can be shown by comparing the positive and the negative part of the answer, if letting \(\epsilon > 0\) have a sufficient small value
\[
(K_p + \epsilon M_E)\gamma K_I > \frac{1}{4}\epsilon((K_p - 1/2\dot{M}_E)^2\gamma + K_s(K_p + M_E\epsilon)),
\] (A.33)

and thereby is \(\det(A) > 0\), i.e., A is positive definite which gives that \(-\dot{V}\) is positive definite.

\section*{A.4 PVFC}

\subsection*{Coordinate transformation}

Trigonometry gives the following transformation when changing coordinate systems:
\[
x_r x_p \rightarrow XY
\]
\[
X = x_r \sin(x_p) \tag{A.34}
\]
\[
Y = -x_r \cos(x_p) \tag{A.35}
\]
\[
XY \rightarrow xy
\]
\[
x = X - X_k \tag{A.36}
\]
\[
y = Y - Y_k \tag{A.37}
\]
\[
xy \rightarrow R\theta
\]
\[
R = \sqrt{x^2 + y^2} \tag{A.38}
\]
\[
\theta = \arctan\left(\frac{y}{x}\right) \tag{A.39}
\]
Jacobians for Velocity Field coordinate transformation

The transformation for the velocity fields is obtained, similar to (A.15) and (A.16), by differentiate the position transformations, which gives the following Jacobians:

\[ x_p, x_r \mapsto XY \]

\[
J_{XY} = \begin{bmatrix}
\cos(x_p) & -x_r \sin(x_p) \\
\sin(x_p) & x_r \cos(x_p)
\end{bmatrix}
\]

(A.40)

\[ R \theta \mapsto XY \]

\[
J_{R\theta} = \begin{bmatrix}
\cos(\theta) & -R \sin(\theta) \\
\sin(\theta) & R \cos(\theta)
\end{bmatrix}
\]

(A.41)

Derive \( \dot{V} \)

Here all the steps for calculation of the time derivative for the augmented velocity field \( \dot{V} \). The derivative is given by

\[
\dot{V}_i = \sum_{k=1}^{n} \frac{\partial \dot{V}(q)}{\partial \tilde{q}_k} \dot{q}_k.
\]

(A.42)

Here \( \dot{V} = [V_p, V_r, V_{fw}] \), but since the fields are given in other coordinate systems it is desirable to derive the derivative in these systems. For the linear field the \( XY – system \) and for the circular the \( R\theta – system \) the corresponding velocity filed will be denoted \( V_{XY, Lin} \) and \( V_{R\theta, Cir} \).

\[ V_{XY, Lin} = \begin{bmatrix}
V_{X, Lin} \\
V_{Y, Lin}
\end{bmatrix}
\]

\[
\dot{V}_{XY, Lin} = \begin{bmatrix}
sgn(\dot{X})(1 - e^{-k|\dot{X}|})\dot{X}_d \\
\dot{X}_d e^{-k|\dot{X}|} \dot{X}
\end{bmatrix}
\]

(A.43)

\[ V_{R\theta, Cir} = \begin{bmatrix}
V_{R, Cir} \\
V_{\theta, Cir}
\end{bmatrix}
\]

\[
\dot{V}_{R\theta, Cir} = \begin{bmatrix}
sgn(\dot{R})(1 - e^{-k|\dot{R}|})\dot{R}_d \\
\dot{R}_d e^{-k|\dot{R}|} \dot{R}
\end{bmatrix}
\]

(A.45)

\[ \dot{X} \] and \( \dot{R} \) is obtained using the Jacobians from (A.40) and (A.38).

To get the field derivative to the machine fix system \( pr \), the Jacobians have to be used ones again, the chain rule gives,
\[ R\theta \mapsto XY \]
\[
\dot{V}_{XY} = \dot{J}_{R\theta} V_{R\theta} + J_{R\theta} \dot{V}_{R\theta}
\]
\[ XY \mapsto pr \]
\[
\dot{V}_{pr} = J^{-1}_{XY} V_{XY} + J^{-1}_{XY} \dot{V}_{XY}
\]
where
\[
\dot{J}^{-1}_{XY} = \begin{bmatrix}
-x_r \dot{x}_r \sin(x_p) - x_r \dot{x}_r \cos(x_p) & -x_r \dot{x}_p \cos(x_p) & -x_r \dot{x}_r \sin(x_p) \\
x_r \dot{x}_r \cos(x_p) & -x_r \dot{x}_p \sin(x_p) & x_r \dot{x}_r \cos(x_p) \\
\end{bmatrix}
\]
and
\[
\dot{J}_{R\theta} = \begin{bmatrix}
-\dot{\theta} \sin(\theta) & -\dot{R} \sin(\theta) - R \dot{\theta} \cos(\theta) \\
\dot{\theta} \cos(\theta) & \dot{R} \cos(\theta) - R \dot{\theta} \sin(\theta) \\
\end{bmatrix}
\]

The derivative of the fly wheel field \( V_{fw} \) becomes
\[
V_{fw} = \sqrt{\frac{2}{M_{fw}}} (E - V_{pr}^T M_L(q) V_{pr})
\]
\[
\dot{V}_{fw} = -\frac{1}{2V_{fw} M_{fw}} (2V_{pr}^T M_L \dot{V}_{pr} + V_{pr}^T M_L \dot{V}_{pr})
\]
where
\[
\dot{M}_L = \begin{bmatrix}
-2m_r l_{tp} \dot{x}_r & 0 \\
0 & 0 \\
\end{bmatrix}
\]
and \( l_{tp} \) and \( m_r \) are constant see Chapter 4. Equations (A.48) and (A.52) finally gives
\[
\dot{V} = \begin{bmatrix}
\dot{V}_{pr} \\
\dot{V}_{fw} \\
\end{bmatrix}
\]

**Passivity of PVFC control**

Here a proof is presented, showing that the system remains passive, when the PVFC control is added. If the error between the desired and actual control is zero, i.e., if \( \tilde{F}_s(\Delta) = 0 \), then the shaped system can be described as,
\[
M_E(q) \dot{v}_E + C_E(\dot{q}, q)v_E + C_{EL}(\dot{q}, q)v_L = T_E + F_E,
\]
\[
\dot{\Delta} = -v_E - u_{loss} + u_1,
\]
where \( T_E = C_{EL} v_L + F_S(\Delta)' + k_T R(\dot{q}, q) + G(\dot{q}, q) \) are the control forces and \( F_E \) are the external forces. Using the same logic as for the proofs of uncoupled and coupled system passivity a storage function is given as
\[
W_s = \frac{1}{2} v_E^T M_E v_E + \int_0^\Delta F_s(\Delta)'.
\]
Diffrentiating this and using (A.55) and (A.56) this becomes

$$\dot{W}_s = v_E^T(-C_E v_E + k_T R + G + F_E) + \frac{1}{2} v_E^T M_E v_E + F_s(\Delta)'(u_1 + u_{loss}),$$  \hspace*{0.5cm} (A.58)

where the \(q\)-dependency is omitted to avoid clutter. Using the skew symmetric properties for \(G, R\) and \(M_E - 2C_E\) and that \(F_s(\Delta)'u_{loss} \geq 0\) this can be expressed as,

$$\dot{W}_s \leq F_E v_E + F_s(\Delta)'u_1.$$  \hspace*{0.5cm} (A.59)

By integrating this w.r.t. passivity can be shown by the following expression,

$$\dot{W}_s(0) \leq \int_0^t (F_E v_E + F_s(\Delta)'u_1)dt,$$  \hspace*{0.5cm} (A.60)

i.e., the system is passive w.r.t. time the supply rate consisting of command power and external power.

### A.5 Obstacle avoidance

**Differentiate \(\lambda\)**

Differentiation of \(\lambda\) for rectangular obstacles in areas where the shortest distance to the obstacle has to be determined. The notations are from Figure 9.2 and area A is used as an example. The partial derivative is denoted

$$\frac{\partial \lambda}{\partial q} = \lambda'. \hspace*{0.5cm} (A.61)$$

And with this notation follows

$$\lambda = \sqrt{\lambda_1^2 - \gamma_{11}^2}, \hspace*{0.5cm} (A.62)$$

$$\lambda' = \frac{1}{\lambda}(\lambda_1 \lambda_1' - \gamma_{11} \gamma_{11}'). \hspace*{0.5cm} (A.63)$$

The first term \(\lambda_1 \lambda_1'\) is given by

$$\lambda_1 = \sqrt{(X_{ob} - X)^2 + (Y_{ob} - Y)^2},$$  \hspace*{0.5cm} (A.64)

$$\lambda_1' = \frac{1}{\lambda_1} [X_{ob} - X \hspace*{0.1cm} Y_{ob} - Y]^T, \hspace*{0.5cm} (A.65)$$

$$\lambda_1 \lambda_1' = [X_{ob} - X \hspace*{0.1cm} Y_{ob} - Y]^T, \hspace*{0.5cm} (A.66)$$

and the second term \(\gamma_{11} \gamma_{11}'\)

$$\gamma_{11} = \frac{\lambda_1^2 - \lambda_2^2 + \gamma_1^2}{2\gamma_1}, \hspace*{0.5cm} (A.67)$$

$$\gamma_{11}' = \frac{1}{\gamma_1}(\lambda_1 \lambda_1' - \lambda_2 \lambda_2' + \gamma_1 \gamma_1'). \hspace*{0.5cm} (A.68)$$
Since $\gamma_1$ is a constant $\gamma_1 \gamma'_1 = 0$, which gives

$$\gamma'_1 = \frac{1}{\gamma_1} (\lambda_1 \lambda'_1 - \lambda_2 \lambda'_2).$$

(A.69)

Using (A.62) this becomes

$$\gamma_{11} = \frac{1}{\gamma_1} [X_{ob2} - X_{ob1} \ Y_{ob2} - Y_{ob1}]^T$$

(A.70)

and

$$\gamma_{11} \gamma'_{11} = \frac{1}{\lambda} [X_{ob2} - X_{ob1} \ Y_{ob2} - Y_{ob1}]^T.$$  

(A.71)

Finally using (A.66) and (A.71) in (A.63) gives

$$\lambda' = \frac{1}{\lambda} \left[ (X - X_{ob1}) - \frac{\gamma_{11}}{\gamma_1} (X_{ob2} - X_{ob1}) \right] \left[ (Y - Y_{ob1}) - \frac{\gamma_{11}}{\gamma_1} (Y_{ob2} - Y_{ob1}) \right]^T$$

(A.72)