

# Single target tracking using vector magnetometers

Niklas Wahlström, Jonas Callmer and Fredrik Gustafsson

**Linköping University Post Print**

N.B.: When citing this work, cite the original article.

Original Publication:

Niklas Wahlström, Jonas Callmer and Fredrik Gustafsson, Single target tracking using vector magnetometers, 2011, Proceedings of the 2011 IEEE International Conference on Acoustics, Speech, and Signal Processing, May 22–27, 2011, Prague Congress Center, Prague, Czech Republic, 4332-4335.

<http://dx.doi.org/10.1109/ICASSP.2011.5947312>

Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-73372>

# SINGLE TARGET TRACKING USING VECTOR MAGNETOMETERS

Niklas Wahlström, Jonas Callmer and Fredrik Gustafsson

Division of Automatic Control  
Linköping University  
Linköping, Sweden  
{nikwa, callmer, fredrik}@isy.liu.se

## ABSTRACT

With the electromagnetic theory as basis, we present a sensor model for three-axis magnetometers suitable for localization and tracking applications. The model depends on a physical magnetic dipole model of the target and its relative position to the sensor. Furthermore, the dependency between the magnetic dipole and the target orientation has been modeled enabling tracking of a maneuvering target. Due to multimodality, a bank of Extended Kalman Filters is proposed for tracking road vehicles. Results from field test data indicate excellent tracking of target position.

**Index Terms**— Target tracking, magnetic dipole

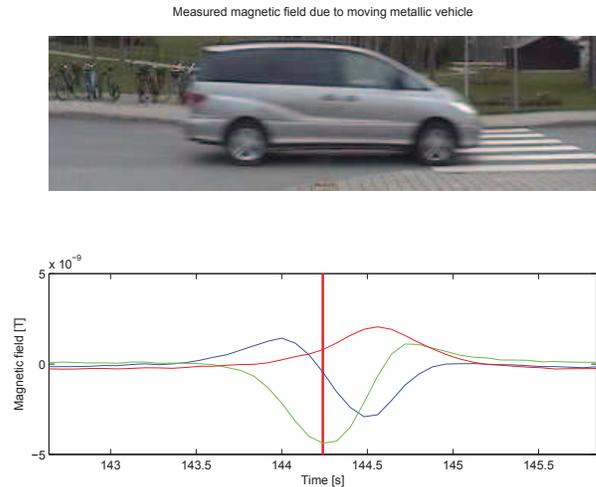
## 1. INTRODUCTION

Tracking and classification of targets are primary concerns in automated surveillance systems. The tracking and classification information can be used for statistical purposes, i.e. counting number of targets of a specific type and registration of their velocities and directions of arrival.

Today, the most common method used in traffic flow counting system is *pneumatic road tubes* lying on the road surface, sensing the pressure of passing vehicles. However, due to extensive physical strain, the equipment will wear approximately within one week at busy roads and can therefore only be used for shorter periods. Furthermore, the technique is mainly suited for detection and not for estimating vehicle or scenario specific parameters.

Other more sustainable solutions are based on measuring the magnetic properties of the vehicles. We know that ferromagnetic objects induce a magnetic field partly due to the permanently magnetized material in the vehicle and partly due to the deflection of earth magnetic field [1]. Most common sensors sensing metallic objects are *inductive loops*, which are excited with a periodic signal and work as an inductive element. Presence of metallic objects near the sensor decreases the inductance in the loop which is detected. This extended loop must be buried in the roadway, which results in higher installation costs. Furthermore, like the pneumatic tubes, inductive loops are mainly suitable for detection.

In this work the use of passive *vector magnetometers* [2, 3] for automated surveillance systems will be analyzed. These can be placed beside the road and have not to be buried in the roadway as the inductive loops. For moving metallic vehicles, the magnetometer measurements will vary in time, which results in a time dependent signal, see Figure 1. This signal depends on the position, velocity, orientation (relative to the



**Fig. 1:** A metallic vehicle gives rise to a magnetic field which can be measured with a stationary three-axis magnetometer. How can this be used to track and classify vehicles?

earth magnetic field) and the magnetic signature of the target. This work presents non-linear statistical signal processing methods to be used with this signal in order to estimate the position, heading and velocity of the vehicles.

The simplest far-field model for the vehicle is to approximate it with a moving magnetic dipole, which is parametrized by the magnetic dipole moment  $\mathbf{m}$  and can be interpreted as the magnetic signature of the vehicle. This dipole model has previously been used for classification and target tracking of ground targets in [4–12]. In a near-field scenario the signal structure is much more complex where nonparametric methods have been shown to be successful. In [13, 14] an extensive investigation of this approach is presented.

As in [4–12], this work will be based on the dipole model. Since the magnetic dipole moment  $\mathbf{m}$  heavily depends on the heading angle of the target, a constant velocity assumption can be used to get a fairly simple model [4–7]. Since this assumption does not hold for non-uniform motions the model has to be extended for such scenarios. One way of solving this is to marginalize the dipole moment  $\mathbf{m}$  as in [8–10]. However, this work will explicitly describe  $\mathbf{m}$  as a function of the heading angle. Furthermore, in contrast to [9, 10] and like [8] we combine the sensor model with a motion model and apply an *Extended Kalman Filter* (EKF) to estimate the target

trajectory.

We know that the EKF needs a good Gaussian estimate of the initial state to converge. Since we do not know the arrival direction of the vehicle this will be problematic. In [12], the problem has been solved by using a particle filter, which does not require the initial guess to be Gaussian. However, in this work, the vehicle is assumed to follow a given road net and only a finite number of arrival directions have to be considered. This multi-modality will be solved by initializing multiple EKFs. Furthermore, many works using the dipole model [5, 9–12] are based on simulations, whereas this work validates the theory with experimental data.

## 2. METHODOLOGY

This section describes the nonlinear estimation problem including the dynamic state space model and the estimation technique. All vectors are expressed in a world fixed coordinate system.

### 2.1. System description

In the statistical signal processing framework, a dynamic state space model is given by a motion model and a measurement model on the standard form

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{w}_k \quad (1a)$$

$$\mathbf{y}_{j,k} = \mathbf{h}_j(\mathbf{x}_k) + \mathbf{e}_{j,k} \quad \forall j = 1 \dots J, \quad (1b)$$

where  $\mathbf{y}_{j,k}$  is the measurement of the  $j$ th sensor,  $\mathbf{x}_k$  is the state of the system,  $\mathbf{w}_k$  is the process noise and  $\mathbf{e}_k$  is the measurement noise of the  $j$ th sensor, all at time instant  $kT_s$ ,  $T_s$  being the sample period.

#### 2.1.1. Motion model

The motion model (1a) describes the vehicle dynamics. There are quite complex vehicle models available, however, since we in this work have short observation intervals, the *constant velocity model* suffices

$$\mathbf{r}_{k+1} = \mathbf{r}_k + T_s \mathbf{v}_k + \frac{T_s^2}{2} \mathbf{w}_k \quad (2a)$$

$$\mathbf{v}_{k+1} = \mathbf{v}_k + T_s \mathbf{w}_k, \quad (2b)$$

where  $\mathbf{r}_k = [r_k^{(x)}, r_k^{(y)}, r_k^{(z)}]$  is the position of the target and  $\mathbf{v}_k = [v_k^{(x)}, v_k^{(y)}, v_k^{(z)}]$  its velocity. Furthermore,  $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$  is assumed to be white Gaussian noise.

#### 2.1.2. Sensor model

A stationary vector magnetometer placed beside a road measures a bias term due to the earth magnetic field  $\mathbf{B}_0$  and magnetic distortions due to vehicles passing by, see Figure 1. We can approximately model a magnetic target as a magnetic dipole. This approximation holds if the characteristic length of the target is small in comparison with its distance to the sensor [1]. Further, a magnetic dipole gives raise to a magnetic dipole field which decays cubically with the distance to

the dipole. With  $J$  magnetometers positioned at  $\{\boldsymbol{\theta}_j\}_{j=1}^J$  we get

$$\mathbf{h}_j(\mathbf{x}_k) = \mathbf{B}_0 + \frac{\mu_0}{4\pi} \frac{3(\mathbf{r}_{j,k} \cdot \mathbf{m}_k)\mathbf{r}_{j,k} - \|\mathbf{r}_{j,k}\|^2 \mathbf{m}_k}{\|\mathbf{r}_{j,k}\|^5}, \quad (3)$$

where  $\mathbf{m}_k$  is the magnetic dipole moment of the target and  $\mathbf{r}_{j,k} = \mathbf{r}_k - \boldsymbol{\theta}_j$  its position relative to the sensor  $j$ . Furthermore, the vacuum permeability  $\mu_0$  is a physical constant having an exact defined value  $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{V}\cdot\text{s}}{\text{A}\cdot\text{m}}$ . Since the dipole term is a good approximation if the target is small in comparison to the distance to it, this can be seen as a point target sensor model and thus a good far-field model. The bias  $\mathbf{B}_0$  will in theory be constant and equal to the earth magnetic field, however in practise it is mixed up with the bias of the sensor, which slightly depends on temperature, light etc.

The magnetic dipole moment  $\mathbf{m}_k$  will be constant if the target has a constant heading angle  $\Psi_k$  since the target is traveling through a homogeneous magnetic field. However, for a maneuvering target the dependency  $\mathbf{m}_k(\Psi_k)$  has to be found by describing the magnetization process in more detail.

We know that metallic objects induce a magnetic field partly due to the ferromagnetic content (hard iron) and partly due to the deflection of the earth magnetic field (soft iron). The magnetization due to the hard iron can be represented with a magnetic dipole moment  $\mathbf{m}_0$  which is independent of the external magnetic field and will thus always be constant in the reference frame of the target. Since the reference frame of the world is not the same as the one of the vehicle, a transformation between these two reference frames has to be found. Generally the roll  $\phi_k$ , pitch  $\theta_k$  and yaw  $\Psi_k$  angles are used to define the relative orientation of a vehicle with respect to world coordinates. Here, no roll and pitch is assumed and any slip is neglected, i.e. the direction of the velocity vector  $\mathbf{v}_k$  uniquely defines the orientation of the vehicle in that

$$\Psi_k = \arctan 2(v_k^{(y)}, v_k^{(x)}), \quad (4)$$

with  $\arctan 2$  being the four quadrant arc-tangent. Now with the rotation matrix

$$\mathbf{Q}(\Psi_k) = \begin{pmatrix} \cos \Psi_k & -\sin \Psi_k & 0 \\ \sin \Psi_k & \cos \Psi_k & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (5)$$

the magnetic dipole moment due to the hard iron can be described as

$$\mathbf{m}_{\text{hard}} = \mathbf{Q}(\Psi_k) \mathbf{m}_0 \quad (6)$$

Furthermore, the soft iron induces a magnetic dipole which will be parallel to the earth magnetic field  $\mathbf{B}_0$

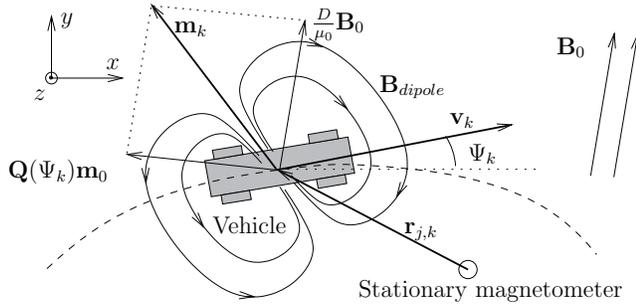
$$\mathbf{m}_{\text{soft}} = \frac{D}{\mu_0} \mathbf{B}_0 \quad (7)$$

with  $D$  being a target characteristic scalar constant. Since magnetization is additive, the total magnetic dipole moment can be modeled as

$$\mathbf{m}_k = \mathbf{Q}(\Psi_k) \mathbf{m}_0 + \frac{D}{\mu_0} \mathbf{B}_0. \quad (8)$$

In Figure 2 a graphical representation of the model is pre-

sented. We have now decomposed the magnetic dipole mo-



**Fig. 2:** A stationary magnetometer measures the earth magnetic field  $\mathbf{B}_0$  together with a magnetic dipole field  $\mathbf{B}_{dipole}$ . The magnetic dipole field is induced by a moving vehicle at position  $\mathbf{r}_{j,k}$  with velocity  $\mathbf{v}_k$  and magnetic dipole moment  $\mathbf{m}_k = \mathbf{Q}(\Psi_k)\mathbf{m}_0 + \frac{D}{\mu_0}\mathbf{B}_0$ . The yaw angle  $\Psi_k$  is defined by the velocity direction.

ment into two components, one representing the hard iron and one representing the soft iron. The corresponding model parameters  $D$  and  $\mathbf{m}_0$  will represent the real magnetic signature of the target and will thus be constant even in a target maneuvering scenario.

### 2.1.3. Measurement noise

The dipole sensor model (3) is only a good far-field model and can be interpreted as the first term  $\mathbf{h}^1$  in the multipole expansion  $\sum_{i=1}^{\infty} \mathbf{h}^i$ , [1]. The higher order terms are thus neglected in (3). This approximation is reasonable in far-field since the dipole term decays as  $\|\mathbf{h}^1\| \propto \tilde{r}^{-3}$ , whereas higher order terms decay more rapidly as  $\|\mathbf{h}^2\| \propto \tilde{r}^{-4}$ ,  $\|\mathbf{h}^3\| \propto \tilde{r}^{-5}$ ,  $\|\mathbf{h}^4\| \propto \tilde{r}^{-6}$ , etc. Here,  $\tilde{r} = \|\mathbf{r}\|/\lambda$  is the distance between the sensor and the target normalized by the characteristic length  $\lambda$  of the target. We can compensate for this model error by amplifying the measurement noise according to the first neglected term  $\|\mathbf{h}^2\|$ . From (3) we can conclude that the magnitude of the dipole term is  $\|\mathbf{h}^1\| \approx \frac{\mu_0}{4\pi} \frac{\|\mathbf{m}\|}{\|\mathbf{r}\|^3}$  and consequently  $\|\mathbf{h}^2\| \approx \|\mathbf{h}^1\| \cdot \tilde{r}^{-1} \approx \frac{\mu_0}{4\pi} \frac{\|\mathbf{m}\| \cdot \lambda}{\|\mathbf{r}\|^4}$ . Since the covariance is related to the signal energy, everything has to be squared giving

$$\mathbf{e}_{j,k} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{R}_j + \left(\frac{\mu_0}{4\pi}\right)^2 \frac{\|\mathbf{m}_k\|^2 \lambda^2}{\|\mathbf{r}_{j,k}\|^8} \mathbf{I}_3\right), \quad (9)$$

where  $\mathbf{R}_j$  is the covariance of the white Gaussian measurement noise of the  $j$ th sensor and  $\lambda$  is regarded as a tuning parameter. With (2), (3), (5), (4), (8), and (9) the description of the dynamic state space model in (1) is completed.

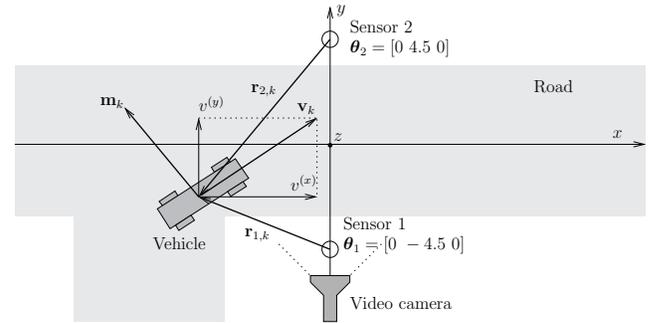
## 2.2. State Estimation

Our approach is to solve the state estimation problem by using EKF. An EKF must be initialized with a fairly good guess of the initial state which is quite difficult if one does not know

anything about the arrival direction of the target. However, with the prior knowledge that the vehicle follows the roads in a specific intersection, a better prediction of the initial states can be made. In order to track vehicles in the  $N$ -way intersection,  $N$  EKFs will be initialized. Each EKF will then assume that the vehicle is coming from one of the  $N$  different heading directions. With the filter bank methodology [15], the probability of each EKF can be calculated as a function of time. When a hypothesis becomes too unlikely, the calculation of the corresponding EKF will be dropped.

## 3. TARGET TRACKING EXPERIMENT

Real measurements have been collected at a 3-way intersection heading north, south and east. In accordance with the results in [7, 16], we have used two magnetometers to reach observability for the model presented in Section 2.1. The two sensors have been deployed at each side of the north-south going road with a relative distance of 9 meters (see Figure 3). The state of the system



**Fig. 3:** Sensor setup for target tracking experiment.  $\mathbf{r}_{j,k}$  is a vector from the  $j$ th sensor to the vehicle,  $\mathbf{v}_k$  is the velocity of the vehicle and  $\mathbf{m}_k$  is the magnetic moment of the vehicle.

$$\mathbf{x}_k = [\mathbf{B}_0, \mathbf{r}_k, \mathbf{v}_k, D, \mathbf{m}_0]$$

can now be computed with the methodology presented in Section 2. The tuning

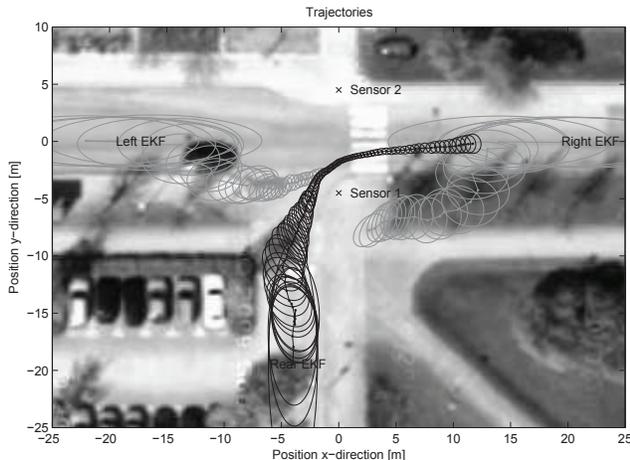
$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda = 1$$

has been used. Since we assume zero velocity in  $z$ -direction, the corresponding element in the process noise covariance matrix  $\mathbf{Q}$  has been set to zero. The magnitude of the tuning parameter  $\lambda$  can be justified by roughly assuming the characteristic length of the vehicles to be  $\lambda = 1$  [m]. The tracking result for one scenario is found in Figure 4, results for more scenarios can be found in [16].

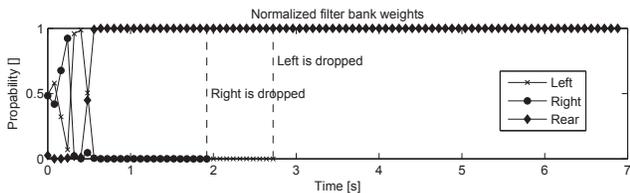
In the scenario a correct determination of arrival and heading direction has been accomplished. The two incorrect hypotheses that the vehicle is coming from the left and the right respectively, could be excluded quite early. Furthermore, covariances are of reasonable size being big at the beginning, decreasing when the vehicle enters the range field of the magnetometers, and finally increasing the uncertainty as the vehicle leaves the magnetometers.



(a) The vehicle is coming from the rear turning right.



(b) The trajectories according to the three different EKFs.



(c) The probabilities that the vehicle is coming from the left, the right and the rear at time instance  $t = kT_s$ . Only the hypothesis that the vehicle is coming from the rear survives and the others are dropped after less than one second.

**Fig. 4:** Tracking experiment results with three differently initialized EKFs estimating the state of vehicle in Figure 4a. Estimated trajectory is presented with a 90% confidence interval.

#### 4. CONCLUSIONS AND FUTURE WORK

It has been found (theoretically and experimentally) that a moving metallic target can be modeled as a magnetic dipole if the distance to the object is large in comparison to its characteristic length. A dependency between magnetic dipole moment and the vehicle orientation has been found making a tracking implementation possible. Results from field test data indicate excellent tracking of target position.

There is still a lot to be investigated and improved. Here, a few suggestions are presented. In the future the robustness of the methods will be analyzed in more large scale experiments in the order of 10000 vehicles and experiments should also be performed with more accurate reference data. Furthermore, this work only deals with single target tracking. A future direction will be to investigate multi target tracking.

#### 5. REFERENCES

- [1] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. John Wiley and Sons, Inc., 1975.
- [2] J. Lenz, "A review of magnetic sensors," *Proc. IEEE*, vol. 78, pp. 973–989, Jun. 1990.
- [3] J. Lenz and S. Edelstein, "Magnetic sensors and their applications," *Sensors Journal, IEEE*, vol. 6, no. 3, pp. 631–649, Jun. 2006.
- [4] J. W. Casalegno, "All-weather vehicle classification using magnetometer arrays," in *Unattended Ground Sensor Technologies and Applications IV*, vol. 4743, no. 1. SPIE, 2002, pp. 205–212.
- [5] R. Kozick and B. Sadler, "Algorithms for tracking with an array of magnetic sensors," in *Sensor Array and Multichannel Signal Processing Workshop, 2008. SAM 2008. 5th IEEE*, Jul. 2008, pp. 423–427.
- [6] C. T. Christou and G. M. Jacyna, "Vehicle detection and localization using unattended ground magnetometer sensors," in *Proc. of 13th International Conference on Information Fusion*, 2010.
- [7] N. Wahlström, J. Callmer, and F. Gustafsson, "Magnetometers for tracking metallic targets," in *Proc. of 13th International Conference on Information Fusion*, 2010.
- [8] M. Rakijas, "Magnetic object tracking based on direct observation of magnetic sensor measurement," U.S. Patent US 6,269,324 B1, 2001.
- [9] L. Merlat and P. Naz, "Magnetic localization and identification of vehicles," in *Unattended Ground Sensor Technologies and Applications V*, vol. 5090, no. 1. SPIE, 2003, pp. 174–185.
- [10] A. S. Edelstein, "Magnetic tracking methods and systems," U.S. Patent US 6,675,123 B1, 2004.
- [11] M. Birsan, "Non-linear Kalman filters for tracking a magnetic dipole," in *Proc. of Intl. Conf. on Maritime Electromagnetics, MARELEC*, 2003.
- [12] —, "Unscented particle filter for tracking a magnetic dipole target," in *Proc. of MTS/IEEE OCEANS*, 2005.
- [13] S. Y. Cheung, S. Coleri, B. Dundar, S. Ganesh, C. W. Tan, and P. Varaiya, "Traffic measurement and vehicle classification with single magnetic sensor," *Journal of the Transportation Research Board*, 2005.
- [14] S. Y. Cheung and P. Varaiya, "Traffic surveillance by wireless sensor networks: Final report," Traffic surveillance, University of California, Berkeley, Tech. Rep., 2007.
- [15] F. Gustafsson, *Statistical Sensor Fusion*, 1st ed. Studentlitteratur, 2010, page 257-272.
- [16] N. Wahlström, "Target Tracking using Maxwell's Equations," Master's thesis, Linköping University, Automatic Control, 2010.