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Efficient Realizations of Wide-Band and Reconfigurable FIR Systems

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To the memories of my mother and father...

Abstract

Complexity reduction is one of the major issues in today's digital system design for many obvious reasons, e.g., reduction in area, reduced power consumption, and high throughput. Similarly, dynamically adaptable digital systems require flexibility considerations in the design which imply reconfigurable systems, where the system is designed in such a way that it needs no hardware modifications for changing various system parameters. The thesis focuses on these aspects of design and can be divided into four parts.

The first part deals with complexity reduction for non-frequency selective systems, like differentiators and integrators. As the design of digital processing systems have their own challenges when various systems are translated from the analog to the digital domain. One such problem is that of high computational complexity when the digital systems are intended to be designed for nearly full coverage of the Nyquist band, and thus having one or several narrow don't-care bands. Such systems can be divided in three categories namely left-band systems, right-band systems and mid-band systems. In this thesis, both single-rate and multi-rate approaches together with frequency-response masking techniques are used to handle the problem of complexity reduction in non-frequency selective filters. Existing frequency response masking techniques are limited in a sense that they target only frequency selective filters, and therefore are not applicable directly for non-frequency selective filters. However, the proposed approaches make the use of frequency response masking technique feasible for the non-frequency filters as well.

The second part of the thesis addresses another issue of digital system design from the reconfigurability perspective, where provision of flexibility in the design of digital systems at the algorithmic level is more beneficial than at any other level of abstraction. A linear programming (minimax) based technique for the coefficient decimation FIR (finite-length impulse response) filter design is proposed in this part of thesis. The coefficient decimation design method finds use in communication system designs in the context of dynamic spectrum access and in channel adaptation for software defined radio, where requirements can be more appropriately fulfilled by a reconfigurable channelizer filter. The proposed technique provides more design margin compared to the existing method which can in turn can be traded off for complexity reduction, optimal use of guard bands, more attenuation, etc.

The third part of thesis is related to complexity reduction in frequency selective filters. In context of frequency selective filters, conventional narrow-band and wide-band frequency response masking filters are focused, where various optimization based techniques are proposed for designs having a small number of non-zero filter coefficients. The use of mixed integer linear programming (MILP) shows interesting results for low-complexity solutions in terms of sparse and non-periodic subfilters.

Finally, the fourth part of the thesis deals with order estimation of digital differentiators. Integral degree and fractional degree digital differentiators are

used in this thesis work as representative systems for the non-frequency selective filters. The thesis contains a minimax criteria based curve-fitting approach for order estimation of linear-phase FIR digital differentiators of integral degree up to four.

Populärvetenskaplig sammanfattning

En trend inom digital signalbehandling (DSP), t ex för kommunikation, är att inkludera mer och mer funktionalitet, samtidigt som beräkningskomplexiteten måste hållas på en rimlig nivå. Det är därför viktigt att fortsätta att utföra grundläggande forskning rörande principer för att reducera komplexiteten hos signalbehandlande algoritmer. I denna avhandling presenteras ett flertal sådana principer för olika ändamål.

En del av avhandlingen handlar om att reducera beräkningskomplexiteten hos olika bredbandiga DSP-system. Ju mer bredbandigt ett system är, desto mer nyttig information kan behandlas per tidsenhet. Traditionella algoritmer är dock beräkningsmässigt väldigt dyra när man ökar bandbredden. I avhandlingen presenteras tekniker som erbjuder betydligt lägre beräkningskostnad jämfört med traditionella lösningar. I exemplen betraktas genomgående sk differentiatorer som används för att beräkna derivator av underliggande analoga signaler. De föreslagna teknikerna kan dock användas även för många andra typer av funktioner.

I en annan del av avhandlingen presenteras en designteknik för att reducera beräkningsbördan i avstämbara filter, främst med användningsområde inom interpolering och decimering som används för att öka respektive reducera data-takten. Detta behövs tex inom framtida kommunikationssystem där en trend är att systemen ska kunna hantera många olika standarder samtidigt som kostnaden för detta måste hållas låg. Den föreslagna tekniken består i att man delvis använder sig av samma filterparametrar i flera olika filter där värdena på parametrarna bestäms med hjälp av optimering.

En tredje del av avhandlingen behandlar metoder för att reducera beräkningskostnaden hos sk frekvensselektiva filter vilka också används flitigt i kommunikationssystem. Speciellt studeras en metod som utnyttjar sk glesa filter. Detta motsvarar många multiplikationer med noll vilka därigenom kan elimineras.

Slutligen, i en fjärde del av avhandlingen, presenteras matematiska uttryck som uppskattar den sk systemordningen hos en differentiator som krävs för att uppnå önskvärd bandbredd och acceptabelt approximationsfel. Systemordningen svarar i detta fall direkt mot beräkningskostnaden. Denna del av avhandlingen knyter an till den första delen som diskuterades ovan.

Preface

The thesis comprises research publications which were produced as results of research work carried out at Electronics Systems Division, Department of Electrical Engineering at Linköping University, Sweden. The work has been done between January 2008 and December 2011 and consists of following publications.

Paper A

The complexity reduction problem for left-band system is formulated using combination of frequency response masking and a two-rate approach. A left-band system here implies a digital system for which the computational complexity grows substantially high when the passband edge approaches the digital Nyquist frequency π . A class of digital differentiators is introduced in this context and realizations for all four types of linear-phase FIR differentiators are demonstrated. Design examples show that differentiators in this class can achieve substantial savings in arithmetic complexity in comparison with conventional direct-form linear-phase FIR differentiators. These design problems show typical break-even points in terms of frequency bandwidth beyond which the proposed technique gives substantial saving in computational complexity. However this reduction in computational complexity is achieved at the cost of a moderate increase in delay and number of delay elements. Further, in terms of structural arithmetic operations, the proposed filters are comparable to filters based on piecewise-polynomial impulse responses. However, the proposed filters can be implemented using non-recursive structures as opposed to polynomial-based filters which are implemented with recursive structures.

This work resulted in the following publication:

1. Z. U. Sheikh and H. Johansson, “A class of wide-band linear phase FIR differentiators using a two-rate approach and the frequency response masking technique,” *IEEE Trans. Circuits Syst. I*, vol. 58, no. 8, pp. 1827–1839, Aug., 2011.

A preliminary version of the above work resulted in the following publication (not included in the thesis):

- ★ Z. U. Sheikh and H. Johansson, “Wideband linear-phase FIR differentiators utilizing multirate and frequency response masking Techniques,” in *Proc. IEEE Int. Symp. Circuits Syst.*, Taipei, Taiwan, May 24–27, 2009.

Paper B

This work focuses on mid-band systems, where the computational complexity of the digital systems tends to be intolerably high as the left and right passband edges are chosen for nearly full coverage of the Nyquist band, i.e., the overall

desired system is wide-band and therefore having narrow don't care bands towards both ends of the Nyquist band. Fractional differentiators are used as an example of such mid-band systems. The technique consists of dividing the overall frequency region into three subregions through lowpass, bandpass, and highpass filters realized in terms of only one filter. The actual function to be approximated is in the low- and high-frequency regions realized using periodic subsystems. In this way, one can realize an overall wide-band LTI function in terms of three low-cost subblocks, leading to a reduced overall arithmetic complexity as compared to the regular realization. Design examples illustrate the savings in multiplication and additions. Moreover, a design example shows that the savings increase/decrease with increased/decreased bandwidth.

This work resulted in the following publication:

2. Z. U. Sheikh and H. Johansson, "A Technique for efficient realization of wide-band FIR LTI systems," *IEEE Trans. Signal Process.*, accepted.

Paper C

This work again addresses the problem of complexity reduction for mid-band specification systems as in paper B. However, the approach here is different in that efficient single-rate structures are derived via multi-rate techniques and sparse bandpass filters. Typical examples of fractional degree differentiator are chosen for demonstrating substantial complexity savings as compared to conventional minimax optimization based direct-form realizations.

This work resulted in the following publication:

3. Z. U. Sheikh and H. Johansson, "Efficient wide-band FIR LTI systems derived via multi-rate techniques and sparse bandpass filters," *IEEE Trans. Signal Process.*, under review.

Paper D

In this paper, a minimax optimization based linear programming approach is applied for the design of reconfigurable channelizer filters, typically targeting software defined radio applications. The coefficient decimation technique for reconfigurable FIR filters was recently proposed as a filter structure with low computational complexity. We propose to design these filters using linear programming taking into consideration all the configuration modes. Results based on minimax solutions show significantly less approximation errors compared to the conventional design method.

This work resulted in the following publication:

4. Z. U. Sheikh and O. Gustafsson, "Linear programming design of coefficient decimation FIR filters," *IEEE Trans. Circuits Syst. II*, vol. 59, no. 1, pp. 60–64, Jan. 2012.

Paper E

Frequency-response masking filters inherently contain at least one periodic model filter. However, as the main purpose of this filter is to produce a sharp transition band, we have looked into producing this with a sparse filter, but without constraints on periodicity. This problem has been studied for narrow-band and wide-band frequency-response masking techniques. As these consist of two cascaded filters, leading to a non-convex optimization problem, the problem is solved iteratively, designing one filter at a time. In this initial work, a standard masking filter is used as the start. It is shown that while the model filter still often ends up to be periodic, computational savings are obtained compared to the standard design technique.

This work resulted in the following publication:

5. Z. U. Sheikh and O. Gustafsson, "Design of narrow-band and wide-band frequency response masking filters using sparse non-periodic sub-filters," in *Proc. European Signal Process. Conf.*, Aalborg, Denmark, Aug. 23–27, 2010.

Paper F

This work builds on Paper E and contributes a better initial guess of the masking filter. Further savings are obtained and the resulting model filters exhibit a sharp transition band, while the rest of the filter does not exhibit any clear form of periodicity.

This work resulted in the following publication:

6. Z. U. Sheikh and O. Gustafsson, "Design of sparse non-periodic narrow-band and wide-band FRM-like FIR filters," in *Proc. IEEE Int. Conf. Green Circuits Syst.*, Shanghai, China, June 21–23, 2010.

Paper G

Commonly used procedures for digital differentiators design are based on various optimization techniques and are also iterative in nature. The order estimation, for differentiators is important from design point of view as it can help in reducing the design time by providing a good initial guess of the order to the iterative design procedures. Moreover, order estimation helps in giving a fairly good estimation of the computational complexity in the overall design. A non-linear optimization problem based on minimax criteria was formulated for curve fitting between the modeled and the actual design data to estimate the best model for various types of differentiators. This work presents linear-phase, finite-length impulse response (FIR) filter order estimation for integral degree differentiators of up to fourth degree.

This work resulted in the following publication:

7. Z. U. Sheikh, A. Eghbali and H. Johansson, "Linear-phase FIR digital differentiator order estimation," in *Proc. European Conf. Circuit Theory Design*, Linköping, Sweden, Aug. 29–31, 2011.

In addition to these, the following papers were also produced but are not included in the thesis either due to being out of scope of the thesis or due to being preliminary version of work which afterwards were extended for publications in journals.

8. M. Abbas, F. Qureshi, Z. U. Sheikh, O. Gustafsson, H. Johansson, and K. Johansson, "Comparison of multiplierless implementation of nonlinear-phase versus linear-phase FIR filters," in *Proc. Asilomar Conf. Signals Syst. Comp.*, Pacific Grove, CA, Oct. 26–29, 2008.
9. Z. U. Sheikh, H. Johansson and O. Gustafsson, "Multiplierless realization of wideband linear-phase FIR differentiators utilizing multirate and frequency-response masking techniques," *Swedish System on Chip Conf.*, Rusthållargården, Arild, Sweden, May 4–5, 2009.

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DIGITAL FILTERS

“If I have seen further, it’s by standing on shoulders of giants.”, Issac Newton

1.1 Introduction

Digital filters can be classified into two major classes, finite impulse response (FIR) filters and infinite impulse response (IIR) filters. Both types of filters have their own characteristics. However, FIR filters are often preferred due to their stability, better phase characteristics, and more flexible implementation capabilities. This thesis focuses on FIR filters only and this chapter discusses some fundamental aspects of finite-length impulse response (FIR) digital filters. Two forms of FIR filters, direct and transposed are stated along with various classes like complementary filters, M th band filters, linear-phase filters and their different types. Some design methods, especially optimization based design techniques are discussed. Various FIR filter transformations are also discussed.

1.2 FIR Filters

A causal FIR filter of order N has an impulse response $h(n)$ with $N + 1$ coefficients $h(0), h(1), \dots, h(N)$. In the time-domain, with an input sequence $x(n)$, the output sequence is given by the convolution sum

$$y(n) = \sum_{k=0}^N h(k)x(n - k). \quad (1.1)$$

The transfer function of an N th-order FIR filter is

$$H(z) = \sum_{n=0}^N h(n)z^{-n}. \quad (1.2)$$

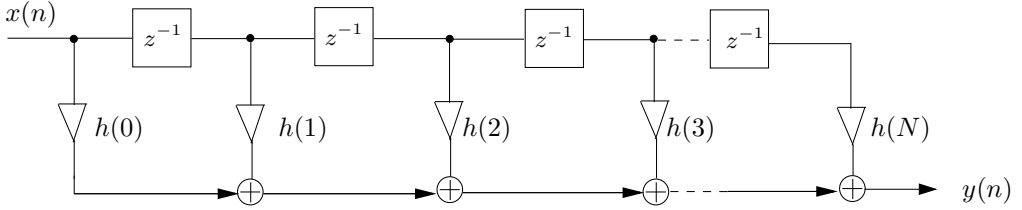


Figure 1.1: Direct-form FIR filter structure.

The frequency response is obtained for $z = e^{j\omega T}$ and thus

$$H(e^{j\omega T}) = \sum_{n=0}^N h(n)e^{-j\omega T n}, \quad (1.3)$$

where ωT is the digital frequency.

In the design, it is often convenient to use non-causal filters. The non-causal filter frequency response can be described as

$$G(e^{j\omega T}) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} g(n)e^{-j\omega T n}. \quad (1.4)$$

The causal FIR filter impulse response can be obtained from the non-causal impulse response by a shifted version of $g(n)$ as

$$h(n) = g\left(n - \frac{N}{2}\right). \quad (1.5)$$

The causal frequency response of an FIR filter $H(e^{j\omega T})$ can be represented as product of a delay term and the non-causal frequency response, according to

$$H(e^{j\omega T}) = e^{-j\omega T N/2} G(e^{j\omega T}). \quad (1.6)$$

1.3 Realizable Forms of FIR Digital Filters

A transfer function can be realized using many different algorithms. They may differ with respect to the number of arithmetic operations, throughput etc. [1]. A class of structures is the direct form which is the most straight forward but nevertheless attractive for the realization of (1.1). Figure 1.1 shows the direct-form structure consisting of a delay line for the input signal $x(n)$, resulting in delayed versions $x(n - k)$. The delay line is tapped at various positions, where the delayed input is scaled by the appropriate impulse response coefficients $h(k)$, and all products are then summed together to form the output $y(n)$.

The other form, called transposed direct-form structure, can be obtained from the direct-form structure using the transpose operation. The transposed

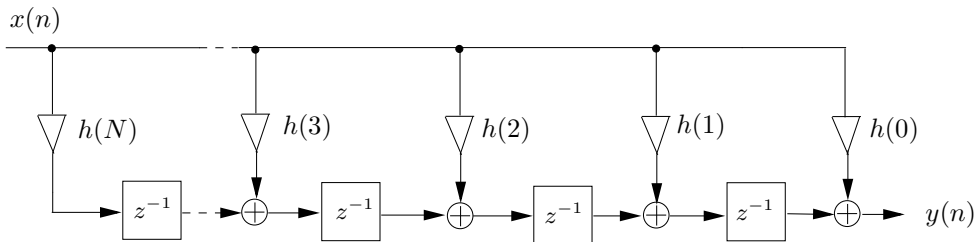


Figure 1.2: Transposed direct-form FIR filter structure.

direct-form structure is shown in Fig. 1.2. The transposed direct-form structure may result in better implementation for high-speed applications, as the long chain of adders in the direct-form may result in a longer critical path and therefore, may require additional pipelining registers [2].

In the thesis, we propose several new classes of FIR filter realizations composed of several subfilters. These subfilters are typically realized using direct-form structures and/or transposed direct-form structures.

1.4 Some Classes of Digital Filters

1.4.1 Linear-Phase FIR Filters

Many applications, e.g., spectral analysis, speech processing, image processing and digital communication require extensive use of digital filtering but cannot tolerate nonlinear-phase distortion. These applications use linear-phase FIR filters for retaining the linearity of phase and these filters exhibit symmetry or antisymmetry in their impulse responses around $n = N/2$, i.e.,

$$h(n) = \pm h(N - n), \quad n = 0, 1, \dots, N. \quad (1.7)$$

As the order N is either even or odd, this leads to four different types of FIR filters with linear phase. It is usually convenient to express linear-phase FIR filters in terms of the non-causal real-valued frequency response $H_R(\omega T)$ as

$$H(e^{j\omega T}) = e^{j\Theta(\omega T)} H_R(\omega T) \quad (1.8)$$

The function $H_R(\omega T)$ in (1.8) is called the zero-phase frequency response of $H(z)$, whereas $\Theta(\omega T)$ is given by

$$\Theta(\omega T) = -\frac{N\omega T}{2} + c \quad (1.9)$$

where $c = 0$ for filters with a symmetric impulse response (Type I and Type II) and $c = \pi/2$ for filters with an antisymmetric response (Type III and Type IV). The zero-phase response is also referred to as non-causal filter response and the

$N/2$ term in (1.9) as the group delay. The real zero-phase frequency response $H_R(\omega T)$ can be written as follows for the different filter types:

$$H_R(\omega T) = \begin{cases} h\left(\frac{N}{2}\right) + 2 \sum_{n=1}^{N/2} h\left(\frac{N}{2} - n\right) \cos(\omega T n), & \text{for Type I} \\ 2 \sum_{n=1}^{(N+1)/2} h\left(\frac{N+1}{2} - n\right) \cos\left(\omega T \left[n - \frac{1}{2}\right]\right), & \text{for Type II} \\ 2 \sum_{n=1}^{N/2-1} h\left(\frac{N}{2} - n\right) \sin(\omega T n), & \text{for Type III} \\ 2 \sum_{n=1}^{(N+1)/2} h\left(\frac{N+1}{2} - n\right) \sin\left(\omega T \left[n - \frac{1}{2}\right]\right), & \text{for Type IV.} \end{cases} \quad (1.10)$$

Some important inferences can be made from (1.10). Type I filters have no restrictions on the positions of the zeros in the frequency plane. Type II filters always have a zero at $\omega T = \pi$ and therefore cannot be used for realization of highpass filters. Type III filters always have zeros at $\omega T = \pi$ and $\omega T = 0$ and therefore cannot realize lowpass or highpass filters. Type IV filters always have a zero at $\omega T = 0$ and therefore cannot realize lowpass filters.

The direct-form linear-phase FIR filter structures imply reduction in multiplications. The number of distinct coefficients are $\lfloor \frac{N+2}{2} \rfloor$ for Type I, II and IV. Similarly the number of distinct coefficients are $\frac{N}{2}$ for Type III. However, direct-form linear-phase implementations do not change the numbers of adders.

In various structures proposed in the thesis, one or several subfilters are linear-phase filters.

1.4.2 Complementary Filters

Various transformations are available which can be used for generation of different frequency responses from a given filter. One such transformation generates complementary filters which are used to generate several frequency responses from a single filter.

The complementary transfer function $H_c(z)$ is defined by

$$|H(e^{j\omega T}) + H_c(e^{j\omega T})| = 1. \quad (1.11)$$

This requirement defines that the two FIR filters are related as

$$H(z) + H_c(z) = z^{-N/2}. \quad (1.12)$$

Hence, the two transfer functions $H(z)$ and $H_c(z)$ are complementary. The output of the complementary filter $H_c(z)$ can be obtained by subtracting the ordinary filter output from the central value $x(n - N/2)$, which reduces the arithmetic work load significantly. The complementary FIR filters considered

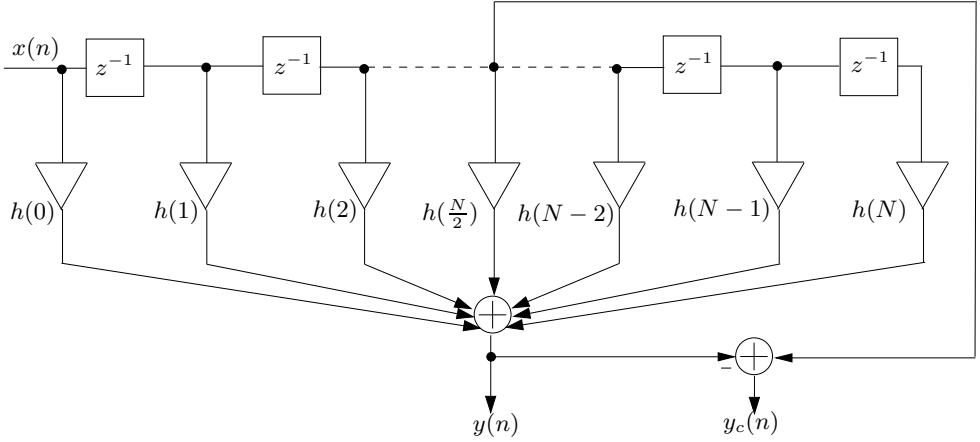


Figure 1.3: Complementary FIR filter.

here are always of even order as shown in Fig. 1.3, because otherwise the center value of the filter is not available. A typical example can be considered of a lowpass filter $H(z)$ with $\omega_c T$ as passband edge and $\omega_s T$ as the stopband edge. Then $H_c(z)$, represents a highpass filter with stopband edge at $\omega_c T$ and passband edge at $\omega_s T$. Also the stopband and pass band ripples will undergo a similar interchange in the highpass filter $H_c(z)$.

The transformation (1.12) refers to a complementary filter transformation. This transformation is used in papers E and F for narrow-band to wide-band filters transformations.

Another interesting transformation is for bandpass filters which can be obtained by subtracting a lowpass and a highpass filter from a pure delay i.e.,

$$H_{BP}(z) = z^{-N/2} - H_{LP}(z) - H_{HP}(z). \quad (1.13)$$

The bandpass filter described by (1.13) has the passband edges, $\omega_{c1}^{BP}T$ and $\omega_{c2}^{BP}T$ and the stopband edges $\omega_{s1}^{BP}T$ and $\omega_{s2}^{BP}T$ given by,

$$\begin{aligned} \omega_{c1}^{BP}T &= \omega_s^{LP}T, & \omega_{c2}^{BP}T &= \omega_s^{HP}T \\ \omega_{s1}^{BP}T &= \omega_c^{LP}T, & \omega_{s2}^{BP}T &= \omega_c^{HP}T, \end{aligned} \quad (1.14)$$

where $\omega_s^{LP}T$ and $\omega_c^{LP}T$ represent the stopband and passband edges of the lowpass filter, respectively, and $\omega_s^{HP}T$ and $\omega_c^{HP}T$ denote the stopband and passband edges of the highpass filter, respectively. This transformation is used in paper B.

1.4.3 Frequency Transformed Filters

Two types of Type I FIR transfer functions with different frequency responses can be generated as

$$F(z) = (-1)^{N/2} H(-z) \quad (1.15)$$

$$G(z) = z^{-N/2} - (-1)^{N/2} H(-z). \quad (1.16)$$

The zero-phase responses of the transfer functions $E(z)$, $F(z)$ and $G(z)$ are related to that of $H(z)$ according to

$$\begin{aligned} F_R(\omega T) &= H_R(\omega T - \pi) \\ G_R(\omega T) &= 1 - H_R(\omega T - \pi), \end{aligned} \quad (1.17)$$

where $H_R(\omega T)$ denotes the zero-phase frequency response.

The transformations (1.15) and (1.16) refer to shifted versions of the original filter. These transformations are used in papers B and C for deriving efficient filter structures. In paper B, for example, these transformations are used for realizing various filters in terms of one filter only. Similarly, in paper C, these transformations are used for transforming a lowpass filter into a highpass filter for a subsequent transformation into the required bandpass filter.

1.4.4 M th-Band Filters

M th-band filters form a special type of filters with an impulse response containing certain zero-valued coefficients at specific positions. These types of filters are computationally efficient due to presence of fewer non-zero coefficients than other filters of the same order. Typical examples of applications for M th-band filters are interpolators, decimators, Hilbert transformer and quadrature-mirror filter banks. The impulse response of a non-causal, lowpass M th-band filter satisfies

$$h(n) = \begin{cases} 0, & \text{for } n = \pm M, \pm 2M, \dots \\ \frac{1}{M}, & \text{for } n = 0, \end{cases}$$

where M is an integer. The transition band of a lowpass M th-band filter always includes π/M . The above equations can be represented for causal M th-band filter as

$$h\left(\frac{N}{2} \pm kM\right) = \begin{cases} \frac{1}{M} & \text{for } k = 0 \\ 0, & \text{for } k = 1, 2, 3, \dots \end{cases}$$

where N is an even number and denotes the order of the filter.

These M th band filters are used in the thesis for derivation of efficient structures. In paper A, where two-rate approach is used i.e., $M = 2$, half-band filters are employed. Similarly in paper C, where a three-rate approach is used i.e., $M = 3$, third-band filters are used.

1.5 FIR Filter Design

There are various approaches for the design of FIR digital filters, but unlike IIR digital filters, which are usually designed using their analog prototypes, FIR filter designs are based on direct approximation of the specified frequency response, often with a requirement on the linearity of the phase response. In this thesis, only frequency selective filters and digital differentiators are considered from the design perspective.

1.5.1 Frequency Selective Filters

The frequency response of an ideal non-causal lowpass digital filter is equal to unity in the passband(s) and zero in the stopband(s). In others words,

$$D(e^{j\omega T}) = \begin{cases} 1 & \text{in passband(s)} \\ 0 & \text{in stopband(s)} \end{cases} \quad (1.18)$$

where D stands for desired frequency response. To get a realizable filter, the ideal transfer function needs to be approximated in the passband(s) and stopband(s) by allowing transition band(s) as well as some ripples. The realizable approximation for a digital filter can be represented as

$$\begin{aligned} 1 - \delta_c &\leq |H(e^{j\omega T})| \leq 1 + \delta_c & \omega T \in \Omega_c \\ |H(e^{j\omega T})| &\leq \delta_s, & \omega T \in \Omega_s. \end{aligned} \quad (1.19)$$

Here, δ_c and δ_s are the passband and stopband ripples, whereas Ω_c and Ω_s are the passband and stopband regions. As an example, in a lowpass filter, $\Omega_c \in [0, \omega_c T]$ whereas $\Omega_s \in [\omega_s T, \pi]$, where $\omega_c T$ and $\omega_s T$ denote the passband and stopband edges respectively. However, digital filters can also be designed with multiple passband and stopband regions. Similarly, additional requirements on e.g., phase response, group delay etc. can also be specified.

A commonly used formula to estimate the order of a linear-phase FIR filter is the following due to Bellanger [3].

$$N \approx -\frac{2}{3} \log_{10}(10\delta_s\delta_c) \frac{2\pi}{\omega_s T - \omega_c T}. \quad (1.20)$$

The equation above provides a reasonably good estimation for frequency selective filters. More accurate estimations can be found in literature e.g., [4]. It can be noted from this equation that the order of the filter is inversely proportional to the quantity $(\omega_s T - \omega_c T)$, which shows the transition bandwidth. Although the equation represents frequency selective filters but the same general conclusion about the transition bandwidth holds for non-frequency selective filters as well. Such formulations of order estimation for non-frequency selective filters do not exist generally. However, such formulations are presented for linear-phase differentiators of integral orders in Paper G.

1.5.2 Digital Differentiators

Digital differentiators form an important block in various digital processing systems, where time rate derivatives of the underlying signals are required. Common examples are that of displacement to velocity or displacement to acceleration conversions, where first and second order time derivatives of the displacement signal are taken, respectively. Such applications of digital differentiators are common in control engineering, target tracking, various signal analysis applications etc. The digital differentiator can be considered as non-frequency selective filter and its design is discussed in the literature e.g. [5–16]. The desired function for a causal digital differentiator is,

$$D(j\omega T) = e^{-Nj\omega T/2}(j\omega T)^k, \quad (1.21)$$

where k denotes the degree of the differentiator [4, 17–19].

1.6 Design Methods

After estimating the filter order, the impulse response of an FIR filter, i.e., $h(n)$, must be determined such that e.g. (1.19) is satisfied for the prespecified values of Ω_c , Ω_s , δ_c , and δ_s .

1.6.1 Window-Based Design

Window-based approaches for design of linear-phase FIR filters are based on truncating the ideal infinite length impulse response. Various fixed window functions and adjustable window functions have been proposed in the literature for reducing the errors introduced by the truncation. An extensive account of these can be found in e.g., [17, 20]. Although the windowing method is a simple method available for designing FIR filters, it results in sub-optimal designs and therefore should normally not be used in practice.

1.6.2 McClellan-Parks-Rabiner's Design

This is one of the widely used algorithms for linear-phase FIR filter design. This algorithm requires the specification of the order N of the filter, all of the passband and stopband edges, and the ratios between the values of the peak passband and stopband errors. The algorithm then minimizes the sizes of the ripples simultaneously subject to the specified ratios using Remez's exchange algorithm [21, 22]. The McClellan-Parks-Rabiner's algorithm finds a unique set of filter coefficients that minimizes a weighted error function. The algorithm solves a minimax optimization problem which is formulated as

$$\text{minimize } E_{\max} = \max|E(\omega T)|, \quad \omega T \in \Omega, \quad (1.22)$$

where $E(\omega T)$ is a weighted error function expressed as

$$E(\omega T) = W(\omega T) [H_R(\omega T) - D(\omega T)], \quad \omega T \in \Omega, \quad (1.23)$$

with Ω being the union of the passband and stopband regions and where $W(\omega T)$ is a weighting function. In Matlab, the `firpm` command is available for the McClellan-Parks-Rabiner design.

1.6.3 Least Squares Design

Filter specifications are generally given in the frequency domain, and, since the energy of a signal is related to the square of the signal, a squared error approximation criterion for the filter design is often appropriate. Therefore, a commonly used approach is to design on the basis of minimization of the energy of the signal. The design problem is formulated by defining an error measure as an integral of the squared differences between the actual and desired frequency response i.e., $H(e^{j\omega T})$ and $D(e^{j\omega T})$. The energy of the signal P is therefore represented as

$$P = \frac{1}{2\pi} \int_{\omega T \in \Omega} |E(e^{j\omega T})|^2 d\omega T \approx \sum_{i=0}^K |H(e^{j\omega_i T}) - D(e^{j\omega_i T})|^2. \quad (1.24)$$

Linear-phase FIR filter design by least squares has several obvious advantages e.g., optimality with respect to square error and non-iterative solution.

1.6.4 Linear Programming (Minimax) Optimization Based Design

Linear programming problems are constrained optimization problems [23–29]. The goal is to minimize (maximize) an objective function subject to a finite number of constraints. The approximation problem is the same here as in Section 1.6.2. However, the difference is that the linear programming is more general and flexible than as defined in Section 1.6.2 and additional constraints can be added. Defining the weighted error $E(\omega T)$ again as in (1.23), the approximation problem can be stated as

$$\begin{aligned} & \text{minimize } \delta \\ & \text{subject to } |E(j\omega_i T)| \leq \delta, \quad i = 1, \dots, K \end{aligned} \quad (1.25)$$

where, $\omega_i T \in \Omega$ and δ represents the maximum approximation error. Linear programming based designs are used in papers D, E, and F using GLPK (GNU Mathprog).

1.6.5 Mixed Integer Linear Programming Design

A linear programming based optimization problem is said to be mixed integer linear programming problem when some but not all variables are restricted

to be integers, and is called a pure integer linear programming problem when all variables are restricted to be integers. The linear programming for such problems are usually based on branch-and-bound or branch-and-cut algorithms. Linear-phase FIR filter design problems can be also formulated using MILP approach for minimizing the non-zero coefficients and hence resulting in sparse FIR filters [30]. The constraints for MILP based linear-phase FIR filter design problems can be formulated as

$$|H_R(\omega T) - D(\omega T)| \leq \delta(\omega T), \quad (1.26)$$

where $H_R(\omega T)$, $D(\omega T)$ and $\delta(\omega T)$ denote the usual variables. When the aim is to minimize the number of non-zero coefficients, a set of binary variables $x_i \in \{0,1\}$ are introduced as

$$x_i = \begin{cases} 0, & h_i = 0 \\ 1, & h_i \neq 0. \end{cases} \quad (1.27)$$

Equation (1.27) can be written using linear constraints as

$$h_i \leq k_i x_i, \forall i \quad (1.28)$$

$$-h_i \leq k_i x_i, \forall i \quad (1.29)$$

where k_i is a constant defining the largest possible absolute value of h_i .

Hence, a mixed integer linear programming problem is formulated as

$$\text{minimize } \sum_{i=0}^M x_i \quad (1.30)$$

subject to (1.26), (1.28), (1.29).

The above problem can be solved using standard MILP solvers e.g., GLPK, SCIP or CPLEX. The MILP based optimizations are used in the thesis in papers E and F.

1.6.6 Non-Linear Optimization Based Design

Non-linear problems can be stated in the same form as that of linear programming, i.e., these are constrained optimization problems where the goal is to minimize (maximize) an objective function subject to a finite number of constraints. The difference between linear programming and non-linear programming in general is that for linear programming the objective function and constraints are linear functions of independent variables, whereas for non-linear programming these can be non-linear functions of the independent variables. In the thesis, various structures are derived using cascaded subfilters which imply non-convex optimization problems. These non-convex optimization problems can result in a local optimum, whereas convex optimization problems guarantee the global

optimum. As non-convex optimization problems are solved using non-linear optimization, the solutions obtained strongly depend on start-up solutions. Therefore, good start-up solutions are important for non-linear optimization. These start-up solutions are obtained usually by designing individual subfilters via convex optimization.

1.6.7 Real Rotation Theorem

Non-linear problems and the linear problems can be formulated with the help of the real-rotation theorem. By using the real-rotation theorem, an infinite-dimensional problem can be converted into a finite-dimensional one [31]. The real-rotation theorem states that minimizing $|f|$ is equivalent to minimizing $\Re\{f(e^{j\Theta})\}$, $\Theta \in [0, 2\pi]$. Therefore, the optimization problems can be formulated with the help of the real-rotation theorem as

$$\begin{aligned} \text{minimize } \delta \\ \text{subject to } |E(j\omega_i T)| \leq \delta \quad \equiv \quad \text{subject to } \Re\{E(e^{j\Theta})e^{j\Theta}\}, \forall \Theta \in [0, 2\pi] \end{aligned} \quad (1.31)$$

In the special case of linear-phase filters, it can be viewed as the real-rotation theorem is used with $P = 2$ which corresponds to $+1$ and -1 on the unit circle, thus only the real part of $H(z)$ are considered. In case of $P = 4$ (for the nonlinear-phase filters), the real and imaginary parts are optimized separately. In the thesis, ordinary integer-degree differentiators k corresponds to an integer, and therefore $(j\omega T)^k$ is either j times a real function or a real function. As, it is a linear-phase design problem, the regular linear programming can be used by taking the non-causal frequency response of the desired function and the non-causal frequency response of linear-phase FIR filter response in the standard minimax problem formulation as described in [1]. However, when k is not an integer, the term j^k is a general complex constant with both real and imaginary parts. Thus, a linear-phase FIR filter design approach cannot be used, though the design problem is still convex and a minimax solution is possible. This problem can be well handled using the real-rotation theorem.

In the thesis, real-rotation based designs have been used in papers B and C, for the formulation of constraints of the fractional degree differentiators.

MULTI-RATE SIGNAL PROCESSING

“Discovery consists of seeing what everybody has seen and thinking what nobody has thought.”, Albert Szent

2.1 Multi-Rate Systems

Digital systems that use multiple sampling rates in the processing of digital signals are termed as multi-rate digital signal processing systems. Different parts of such a system work at different sampling frequencies and therefore require sampling rate conversion between these parts for their proper interoperability. Multi-rate signal processing systems employ the fundamental operations of interpolation and decimation for sampling rate conversions. Sampling rate conversions of a discrete-time signal by an integral factor is basically carried out using two fundamental operators, up-sampler and down-sampler. The L -fold up-sampling generates an output sequence with a sampling rate that is L times larger than that of the input sequence. A down-sampler with a down-sampling factor M creates an output sequence with a sampling rate that is $(1/M)$ th of the input sequence. These operators can be used in cascade, i.e., up-sampling by a factor of L , followed by down-sampling by a factor M to achieve a rational factor sampling rate change of L/M [32–36].

2.1.1 Interpolation

The upsampler can be represented in the time domain as

$$x_1(m) = \begin{cases} x\left(\frac{m}{L}\right), & \text{for } m = 0, \pm L, \pm 2L \dots \\ 0, & \text{otherwise.} \end{cases} \quad (2.1)$$

In the z -domain, (2.1) becomes

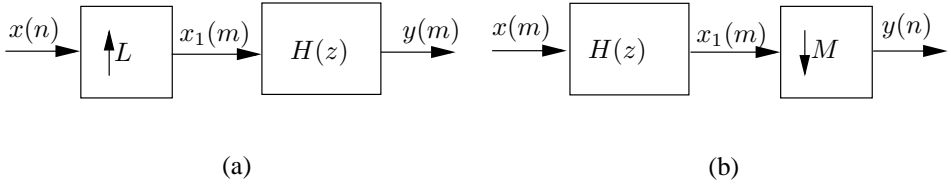


Figure 2.1: (a) Interpolator and (b) decimator.

$$X_1(z) = X(z^L). \quad (2.2)$$

As up-sampling by an integer factor L causes periodic repetitions of the basic spectrum, the basic interpolator structure for integer-valued sampling rate increase consists of an upsampler followed by a lowpass filter $H(z)$ with a cutoff at π/L , as indicated in Fig. 2.1(a). The lowpass filter $H(z)$, called the interpolation filter, removes the $L - 1$ unwanted images in the spectra of the up-sampled signal. Typical spectra of an interpolator are shown in Fig. 2.2.

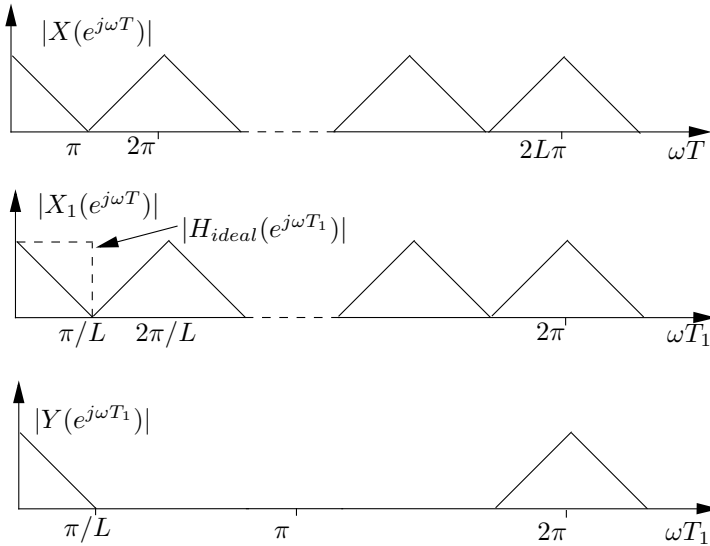


Figure 2.2: Spectra of original, intermediate, and interpolated output sequences.

2.1.2 Decimation

The down-sampling operation is implemented by keeping every M th sample of the input sequence and removing $M - 1$ samples in between to generate the

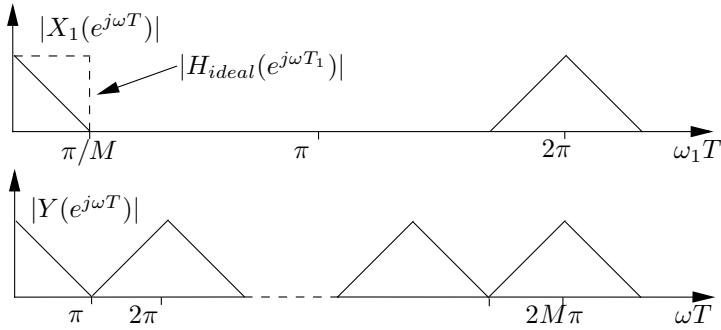


Figure 2.3: Spectra of the intermediate and decimated sequence.

output sequence according to the relation

$$y(n) = x_1(nM) \quad (2.3)$$

The above relation can be represented in the z -domain as

$$Y(z) = \frac{1}{M} \sum_{k=0}^{M-1} X_1(z^{1/M} W_M^k) \quad (2.4)$$

where $W_M = e^{-j\frac{2\pi}{M}}$. As down-sampling by an integer factor M may result in aliasing, the basic decimator structure for integer-valued sampling rate decrease consists of a lowpass filter $H(z)$ with a cutoff at π/M , followed by the down-sampler, as indicated in Fig. 2.1(b). Here, the lowpass filter $H(z)$, termed as decimation filter, band-limits the input signal of the down-sampler to $\omega T \leq \pi/M$ prior to down-sampling in order to avoid aliasing. Typical spectra of a decimator are shown in Fig. 2.3.

2.1.3 Sampling Rate Conversions by Rational Factor

Sampling rate conversion by a rational factor can be implemented by the scheme shown in Fig. 2.4, where the input signal is upsampled by a factor of L followed by an interpolation filter $H_I(z)$. This interpolated signal is passed through an anti-aliasing filter $H_D(z)$ before downsampling by a factor of M . Since the interpolation filter and decimation filter are working in a cascade and operate at the same sampling rate, both can be combined in a single lowpass filter $H(z)$ as shown in Fig. 2.5. The filter $H(z)$ acts both as an interpolation and decimation filter at the same time. The cut-off frequency of the ideal filter $H(z)$ should be

$$\omega_s T = \min\left(\frac{\pi}{L}, \frac{\pi}{M}\right). \quad (2.5)$$

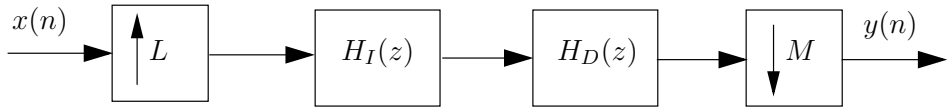


Figure 2.4: Cascade of an interpolator and decimator for sampling rate conversion by a factor of L/M .

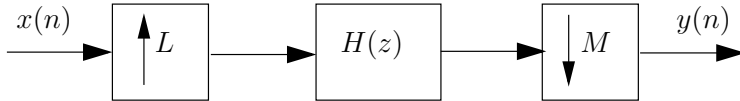


Figure 2.5: An implementation of sampling rate conversion by a factor of L/M by combining the interpolation and decimation filters.

2.2 Noble Identities

If $H(z)$ is a rational function, i.e., a ratio of polynomials in z or in z^{-1} , the noble identities can be represented as in Fig. 2.6. Such interconnections arise when polyphase representations of decimators and interpolators are utilized for realization. The use of noble identities makes it possible to do the filtering operations at the low sampling rate by moving the downsampler/up-sampler appropriately.

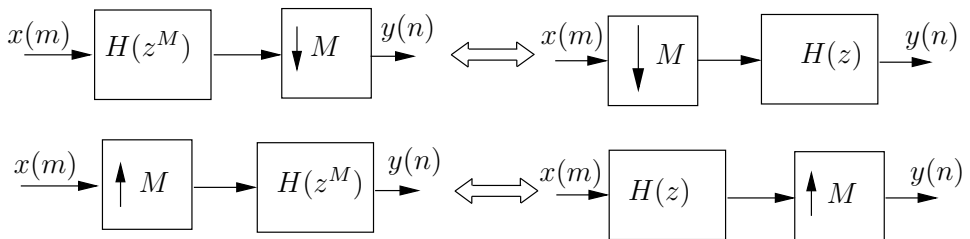


Figure 2.6: Noble identities.

2.3 Polyphase Decomposition

The transfer function in (1.2) can be decomposed as

$$\begin{aligned}
 H(z) &= \sum_{n=-\infty}^{\infty} h(nL)z^{-nL} \\
 &+ z^{-1} \sum_{n=-\infty}^{\infty} h(nL+1)z^{-nL} \\
 &\dots \\
 &+ z^{-(L-1)} \sum_{n=-\infty}^{\infty} h(nL+L-1)z^{-nL},
 \end{aligned} \tag{2.6}$$

which can be rewritten as [2, 20, 34, 37, 38]

$$H(z) = \sum_{i=0}^{L-1} z^{-i} H_i(z^L). \tag{2.7}$$

Here, $H_i(z)$ are the polyphase components whose impulse responses are given by

$$h_i(n) = h(nL+i), \quad i = 0, 1, \dots, L-1. \tag{2.8}$$

This decomposition is frequently referred to as the Type I polyphase decomposition. The Type II polyphase decomposition of (1.2) is [34]

$$H(z) = \sum_{i=0}^{L-1} z^{-(L-1-i)} R_i(z^L), \tag{2.9}$$

where $R_i(z) = H_{L-1-i}(z)$. The Type I and II polyphase decompositions allow one to efficiently realize interpolators and decimators.

The polyphase equivalent forms of the decimator and interpolator are shown in Figs. 2.7 and 2.8, respectively and it can be observed that all filtering operations are done at the lower sampling rate.

2.3.1 M th-Band Filters

The output-input relation of an interpolator is given by

$$Y(z) = H(z)X(z^M). \tag{2.10}$$

M th-band filters receive special attention from complexity savings point of view when used as interpolators and decimators [39, 40]. Moreover, if used as interpolators and represented in polyphase form, it can be observed that they preserve

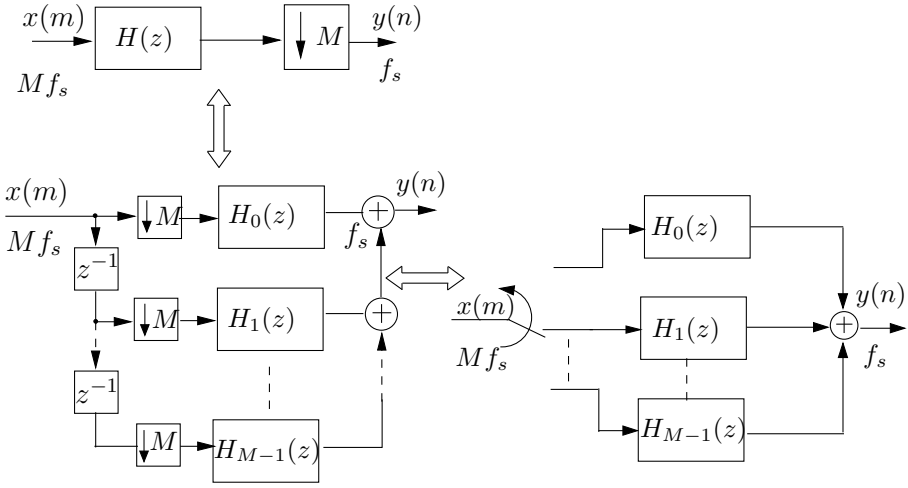


Figure 2.7: Decimation with polyphase decomposition and noble identities.

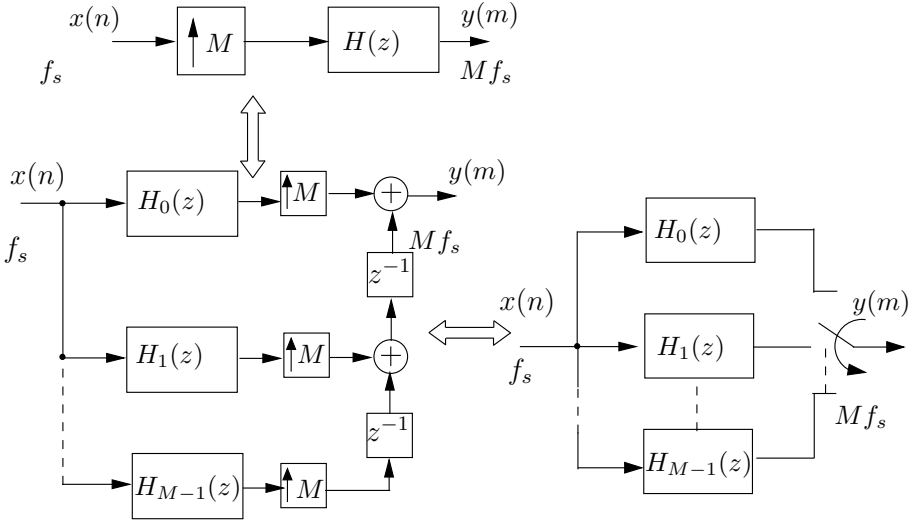


Figure 2.8: Interpolation with polyphase decomposition and noble identities.

the nonzero samples of the up-sampler output i.e., the interpolation filter $H(z)$ can be realized in M -band polyphase form as,

$$H(z) = E_0(z^M) + z^{-1}E_1(z^M) + z^{-2}E_2(z^M) + \dots + z^{-(L-1)}E_{L-1}(z^M). \quad (2.11)$$

Assume that the k th polyphase component consists of a single non-zero element (the rest of the elements are zeros) i.e., $E_k(z) = 1/M$. This corresponds to the fact which can be recalled from Section 1.4.4 for the causal M th-band

lowpass filter

$$\begin{aligned} h\left(\frac{N}{2}\right) &= \frac{1}{M} \\ h\left(\frac{N}{2} \pm pM\right) &= 0, \text{ for } p = 1, 2, 3, \dots \end{aligned} \quad (2.12)$$

Using (2.11) and (2.12), (2.10) can be expressed as

$$Y(z) = \frac{1}{M} z^{-k} X(z^M) + \sum_{l=0, l \neq k}^{L-1} z^{-l} E_l(z^M) X(z^M). \quad (2.13)$$

The above result implies $y[Ln + k] = \frac{1}{M} x[n]$ for the non-causal structure, which corresponds to the fact that input samples appear at the output without any distortion at time instants $Ln + k, -\infty < n < \infty$, whereas the in-between $(L-1)$ output samples are determined by interpolation. If $H(z)$ is a zero-phase transfer function satisfying (2.11) with $k = 0$, i.e., $E_0(z) = 1/M$, then it can be shown that [20]

$$\sum_{k=0}^{L-1} H(zW_M^k) = c, \quad W_M = e^{-j\frac{2\pi}{M}}, \quad c > 0. \quad (2.14)$$

Since the frequency response of $H(zW_M^k)$ is the shifted version $H(e^{j(\omega T - 2\pi k/M)})$ of $H(e^{j\omega T})$, the sum of all of these M uniformly shifted versions of $H(e^{j\omega T})$, add up to a constant.

2.4 Polyphase Identity

The scheme presented in Fig. 2.9 finds interesting usage for simplifying the complex multi-rate networks, e.g., transmultiplexers and various other applications [41, 42]. An important aspect of this scheme is that inspite of down-sampler and up-sampler being time varying building blocks, the overall structure is a time-invariant system. The transfer function of the overall system can be represented by the zeroth polyphase component of $H(z)$ i.e., $H_0(z)$. In the thesis, this scheme is used in paper A and C for the derivation of efficient single-rate structures. The proof of this identity is given below.

2.4.1 Proof of the Polyphase Identity

The output $y(n)$ of the down-sampler can be represented in the z -domain as

$$Y(z) = \frac{1}{M} \sum_{k=0}^{M-1} Y_1(z^{\frac{1}{M}} e^{-j2\pi k/M}). \quad (2.15)$$

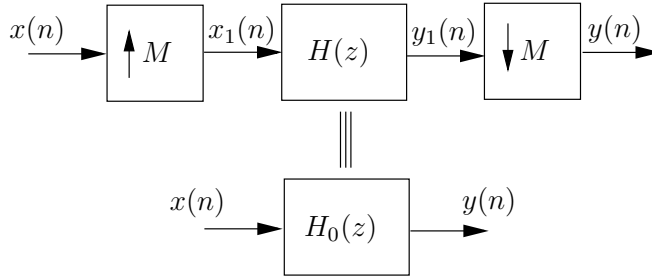


Figure 2.9: Polyphase identity.

It can then be rewritten utilizing (2.10) as

$$\begin{aligned}
 Y(z) &= \frac{1}{M} \sum_{k=0}^{M-1} Y_1(z^{\frac{1}{M}} e^{-j2\pi k/M}) \\
 &= \frac{1}{M} \sum_{k=0}^{M-1} X_1(z^{\frac{1}{M}} e^{-j2\pi k/M}) H(z^{1/M} e^{-j2\pi k/M}) \\
 &= \frac{1}{M} \sum_{k=0}^{M-1} X(z) H(z^{\frac{1}{M}} e^{-j2\pi k/M}) \\
 &= \frac{X(z)}{M} \sum_{k=0}^{M-1} \sum_{m=0}^{M-1} (z^{\frac{1}{M}} e^{-j2\pi k/M})^{-m} H_m(z) \\
 &= \frac{X(z)}{M} \sum_{m=0}^{M-1} z^{-m/M} H_m(z) \sum_{k=0}^{M-1} e^{j2\pi km/M} \\
 &= X(z) H_0(z).
 \end{aligned} \tag{2.16}$$

FREQUENCY RESPONSE MASKING TECHNIQUES

“Genius is the ability to reduce the complicated to the simple.”, C.W. Ceram

3.1 Introduction

The frequency response masking approach was introduced initially as a means of generating narrow transition band linear-phase FIR filters with a low arithmetic complexity. The method is applied to linear-phase FIR filters for realization of sharp transition narrow-band and wide-band frequency response masking filters based on a cascade of two filters referred to as model and masking filters. The technique originally attributed to Neuvo [43–52], and has undergone various improvements e.g., multi-stage FRM Approach [53, 54], combination of IFIR and FRM approaches [55, 56], single filter frequency masking FIR filters [57, 58], low-delay FRM filters [59], pre-filter based structures [60–64].

3.2 Narrow-Band FRM Filters

In a narrow-band FRM filter, a model filter $F(z)$ is cascaded with a masking filter $G(z)$ according to Fig. 3.1, where $F(z)$ is modified to have periodicity of $2\pi/L$ instead of 2π . This is equivalent to replacing all the delay elements in the realization of $F(z)$ by L delay elements in cascade, which in the time domain is equivalent to insertion of $L - 1$ zeros between the consecutive values of the impulse response $f(n)$. The overall transfer function is

$$H(z) = F(z^L)G(z). \quad (3.1)$$

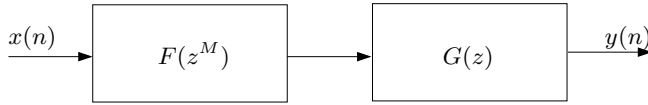


Figure 3.1: Narrow-band frequency response masking filter.

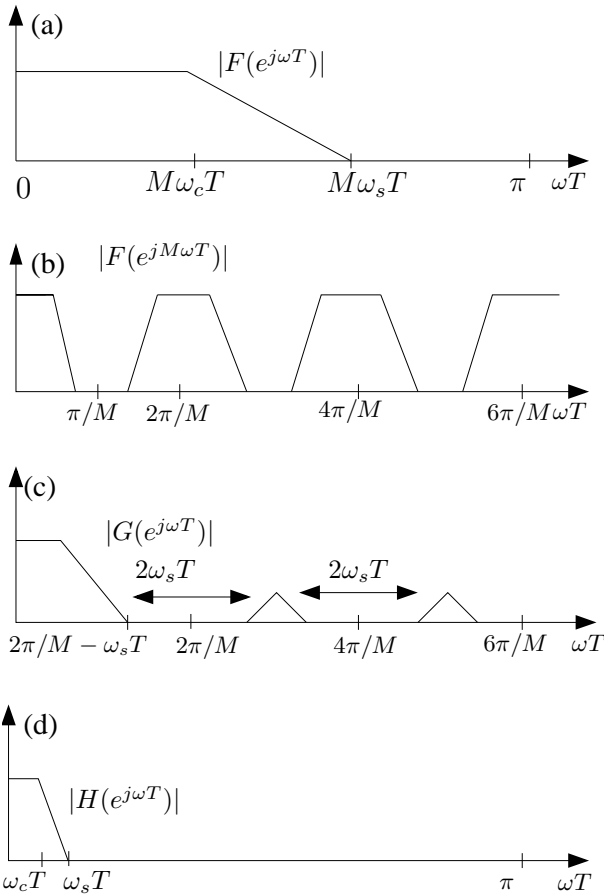


Figure 3.2: Typical magnitude response of the (a) model, (b) periodic model, (c) masking, and (d) overall filters for narrow-band lowpass frequency response masking filter.

Typical magnitude responses of the subfilters and the overall filter $H(z)$ can be seen in Fig. 3.2. The overall filter $H(z)$ is however restricted to have a narrow bandwidth. For instance, for a lowpass filter, the stopband edge is restricted as

$$\omega_s T < \pi/L \quad (3.2)$$

The filter order of $F(z)$ is reduced by a factor of L since the transition bandwidth is increased by a factor of L , but the complexity of $G(z)$ is increasing with L . This dictates for an optimal choice of L .

3.3 Wide-Band FRM Filters

If the specification at hand has a passband edge $\omega_c T$ that is larger than $\pi(L - 1)/L$ for some integer $L \geq 2$, it is possible to synthesize the filter as a wide-band FRM filter. To obtain a wide-band lowpass filter, a narrow-band highpass filter is subtracted from a pure delay as shown in Fig. 3.3. The transfer function is

$$H(z) = z^{-K} - F(z^L)G(z), \quad (3.3)$$

where $K = LK_G + K_F$ and K_G and K_F are the delay of the subfilters $F(z)$ and $G(z)$, respectively. For even values of L , $F(z)$ is a lowpass filter with bandedges

$$\omega_c^{(F)} T = L(\pi - \omega_s T) \quad (3.4)$$

$$\omega_s^{(F)} T = L(\pi - \omega_c T), \quad (3.5)$$

and $G(z)$ is a highpass filter with

$$\omega_s^{(G)} = (L - 2)\pi/L + \omega_s^{(F)} T/L \quad (3.6)$$

$$\omega_c^{(G)} = \omega_s T. \quad (3.7)$$

For odd values of L , both subfilters are of highpass type with edges

$$\omega_c^{(F)} T = L(\omega_s T - \pi) + \pi.$$

$$\omega_s^{(F)} T = L(\omega_c T - \pi) + \pi.$$

$$\omega_s^{(G)} T = (L - 1)\pi/L - \omega_s^{(F)} T/L.$$

$$\omega_c^{(G)} T = \omega_s T. \quad (3.8)$$

It is to be noted that due to involvement of complementary filtering, $K = N_F/2$ should be an integer, i.e., the model filter must be of even order as mentioned in Section 1.4.2. The magnitude responses of various sub-filters and the overall wide-band filter for a case of even L are shown in Fig. 3.4. In papers E and F a method based on sparse filters is presented for designing such narrow-band and wide-band filters using non-periodic sub-filters.

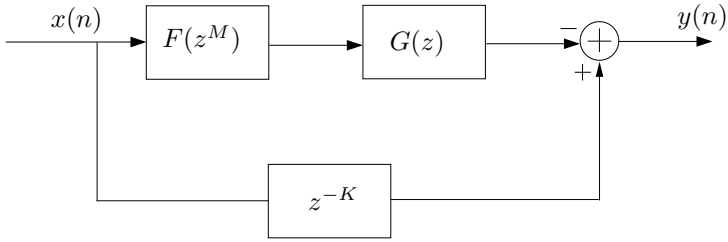


Figure 3.3: Wide-band frequency response masking filter.

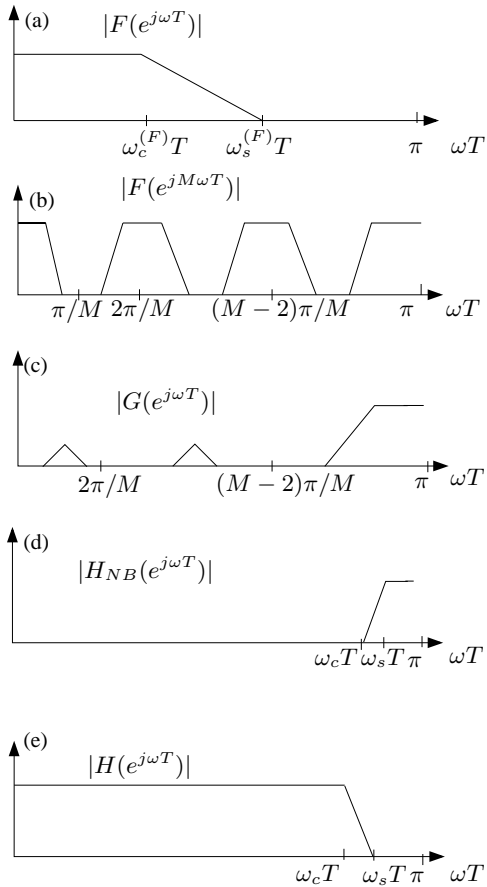


Figure 3.4: Typical magnitude responses of the (a) model, (b) periodic model, (c) masking, and (d) overall filters for a wide-band lowpass frequency masking filter.

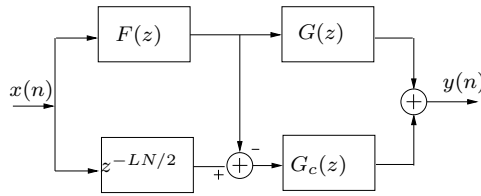


Figure 3.5: Arbitrary bandwidth frequency response masking filter.

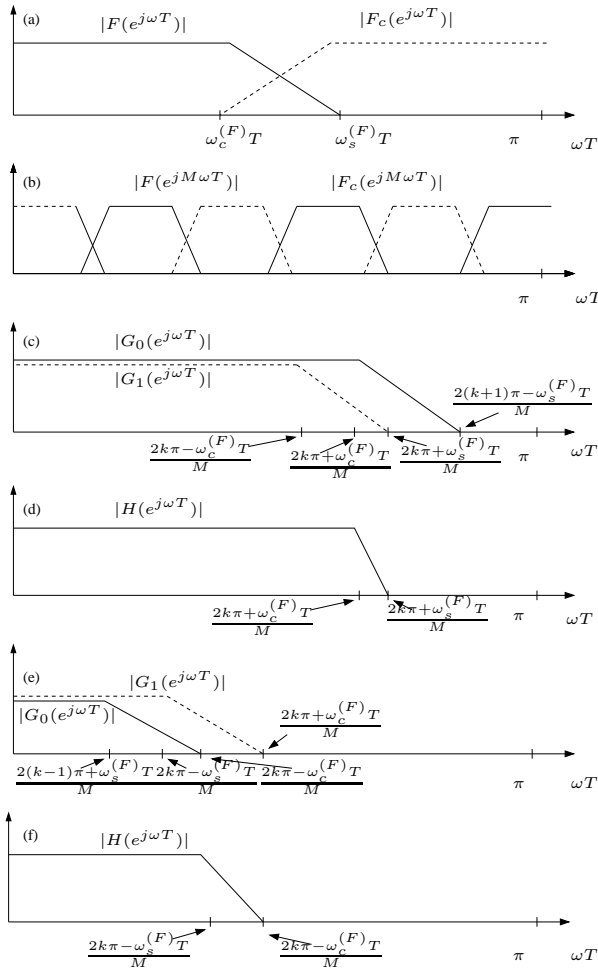


Figure 3.6: Arbitrary bandwidth frequency response masking filter responses of the (a) model and complementary filters (b) periodic model and complementary filters (c) masking filters for a Case 1 design, (d) overall filter for a Case 1 design, (e) masking filters for a Case 2 design, (f) overall filter for a Case 2 design.

3.4 Arbitrary Bandwidth FRM Filters

A more general FRM approach that can handle filters with any bandwidth is referred to as arbitrary-bandwidth FRM. The technique employs a complementary filter $F_c(z)$ of the model filter $F(z)$. This structure is shown in Fig. 3.5.

$$H(z) = F(z^L)G_1(z) + F_c(z^L)G_2(z) \quad (3.9)$$

Two masking filters $G_1(z)$ and $G_2(z)$ are required here. This scheme leads to two cases, Case I or Case II, depending on whether the overall transition band equals the transition band of one of either $F(z)$ or $F_c(z)$, respectively. Typical responses are shown in Fig. 3.6 for both cases

3.5 FRM Based M th-Band Interpolator

The proposed techniques in papers A and C make use of the structures for the FRM M th-band filter classes proposed in [65]. It can be recalled that the impulse response $h(n)$ of a non-causal M th-band linear-phase FIR filter satisfies

$$h(n) = \begin{cases} 1/M, & n = 0 \\ 0, & n = \pm M, \pm 2M, \dots \end{cases} \quad (3.10)$$

The above definition of M th band filters implies that the zeroth polyphase component in the polyphase representation of its transfer function $H(z)$ is $1/M$. The proposed classes of linear-phase FRM FIR filters, as usual consist of the model filter $A(z)$ and masking filters $B_0(z)$ and $B_1(z)$. The overall transfer function can be represented as

$$H(z) = A(z^L)B_0(z) + [1 - A(z^L)]B_1(z). \quad (3.11)$$

These FRM based interpolators/decimators can be categorized into two classes depending upon some restrictions imposed on an inter-relationship of masking filters, the periodicity factor L , and model filter or complementary model filter to be M th-band. Details can be found in [65], but in short it can be described that for the Class I case, the two masking filters are related as

$$B_1(z) = B_0(z) - \frac{M}{M-1}B_{00}(z^M) + \frac{1}{M-1}, \quad (3.12)$$

where $B_{00}(z)$ denotes the 0th polyphase component in the polyphase representation of $B_0(z)$. Similarly, for the Class II filters, the masking filters are related as

$$B_0(z) = B_1(z) - \frac{M}{M-1}B_{10}(z^M) + \frac{1}{M-1}, \quad (3.13)$$

where, $B_{10}(z)$ denotes 0th polyphase component in the polyphase representation of $B_1(z)$. It may be noted from the polyphase representation of $A(z)$ that the

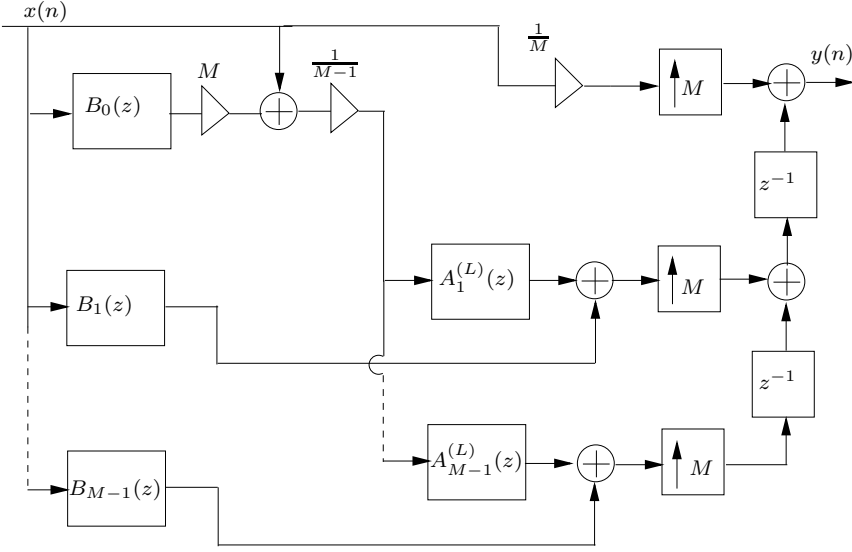


Figure 3.7: FRM interpolator structure from [65].

periodic model filter $A(z^L)$ can be written in polyphase form as

$$A(z^L) = \sum_{m=0}^{M-1} z^{-m} A_m^{(L)}(z^M) \quad (3.14)$$

where $A_0^{(L)}(z) = A_0(z^L)$ and $A_m^{(L)}(z)$, $m = 1, 2, \dots, M-1$, are for Class I filters:

$$A_m^{(L)}(z) = \begin{cases} z^{-2pm} A_m(z^L), & \text{Case I} \\ z^{-(2p(M-m)-1)} A_{M-m}(z^L), & \text{Case II} \end{cases} \quad (3.15)$$

Class II filters:

$$A_m^{(L)}(z) = \begin{cases} z^{-(2(p+1)(M-m)-1)} A_{M-m}(z^L), & \text{Case I} \\ z^{-(2p-1)m} A_m(z^L), & \text{Case II} \end{cases}$$

where p is related to L and M as given in [65]. It may also be noted that $A_0^{(L)}(z) = 1/M$ for the Class I case and that the structure for an interpolator can be obtained as in Fig. 3.7, where the polyphase components of $H(z)$ can be expressed as

$$H_0(z) = B_0(z) + \frac{(1 - MB_0(z))(1 - A_0^{(L)}(z))}{M-1}. \quad (3.16)$$

$$H_m(z) = B_m(z) - \frac{(1 - MB_0(z))A_m^{(L)}(z)}{M-1}. \quad (3.17)$$

It can be noted that the factor $[1 - MB_0(z)]/(M - 1)$ is shared by all the polyphase components. Furthermore, it can be noted that all filtering operations are performed at the lowest sampling rate. Moreover, only one copy of each filter part is needed, yielding a low overall complexity. All polyphase components $B_m(z)$ have the same input, and this is true also for all the polyphase components $A_m^L(z)$ as well. This makes it possible to exploit the coefficient symmetries of the linear-phase model and masking filters, if $B_m(z)$ and $A_m^L(z)$ are realized using transposed FIR filter structures. Hence, these classes of FRM filters provide an efficient solution for the design of M th band interpolation or decimation filters.

3.5.1 Pair-Wise Polyphase Implementation Using Coefficient Symmetry

Polyphase decomposition of filters for a multi-rate system results in unsymmetric polyphase components, although the original FIR filter has a symmetric (or anti-symmetric) impulse response. In the polyphase decomposition of a non-causal M th band interpolation filter into M polyphase components, when M is odd, only the zeroth-component is symmetric while the remaining polyphase components result in $(M - 1)/2$ mirror image pairs. These mirror-image pairs can be synthesized using pairs of symmetric and anti-symmetric impulse response filters [66]. In the thesis, this coefficient symmetry is used in paper C, where a three-rate approach is used. In the three-fold polyphase decomposition of an even-order non-causal $B_0(z)$, $B_{00}(z)$ ¹ represents the zeroth-polyphase component and exhibits the coefficient symmetry, while the other two polyphase components, i.e., $B_{01}(z)$ and $B_{02}(z)$ are reversed versions of each other. That is,

$$b_{01}(n) = b_{02}(-n). \quad (3.18)$$

These impulse responses can be represented in terms of a symmetric and an anti-symmetric filter to be exploited for complexity reduction as

$$\begin{aligned} C_1(z) &= \frac{B_{01}(z) + B_{02}(z)}{2}, \\ C_2(z) &= \frac{B_{01}(z) - B_{02}(z)}{2}, \end{aligned} \quad (3.19)$$

and therefore

$$\begin{aligned} B_{01}(z) &= C_1(z) + C_2(z) \\ B_{02}(z) &= C_2(z) - C_1(z). \end{aligned} \quad (3.20)$$

¹Small letters represent impulse response, where as capital letters denote frequency response of corresponding filters, e.g., $b_{01}(n)$ and $B_{01}(z)$.

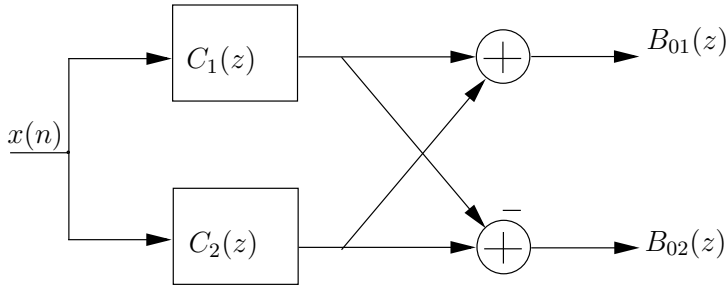


Figure 3.8: Pair-wise polyphase components implementation using coefficient symmetry.

Equations (3.19) and (3.20) can be represented by the structure shown in Fig. 3.8, where $C_1(z)$ and $C_2(z)$ are the filters with symmetric and antisymmetric coefficients, respectively. It may be noted that a similar coefficient symmetry use can also be applied on the model filter $A(z^L)$ for the complexity reduction. This scheme results in some 50% reduction in number of multipliers. However, the number of additions is nearly the same.

RECONFIGURABLE FIR FILTER DESIGN

“Coming together is a beginning. Keeping together is progress. Working together is success”, Henry Ford

4.1 Dynamically Reconfigurable Hardware

The flexibility in the design of digital systems is usually obtained in terms of reconfigurability where the same hardware platform is reusable for different system parameters configurations without requiring any hardware changes. An important example in this regard is contemporary communication systems design, where the objective is to utilize different telecommunication standards using the same hardware. Successful design of such flexible communication systems requires high level of flexibility in digital signal processing structures [67–70].

The rapidly evolving technology of software defined radio in the area of wireless communication is based on the concept of flexibility, where the idea is to replace most of the analog signal processing part of a typical transceiver with digital signal processing in order to gain the benefits of reconfiguration of the hardware through software programability. This flexibility in the design of various systems brings advantages of reconfigurability functions on demand during the run time but on the other hand it also encounters various challenging issues like, real time response, dynamic control and design complexity. Some of the digital filter design issues in this context are

- ★ Dynamic sampling rate conversion
- ★ Fractional delay filtering
- ★ Design of flexible channelizers
- ★ Dynamic spectrum allocation

Various digital signal processing architectures and algorithms have evolved in order to deal with the above mentioned design issues. In this context of reconfigurability, worth mentioning are the Farrow structure based digital filtering architectures and the coefficient decimation techniques of digital filter design.

4.2 Reconfigurable FIR Filter Design using The Farrow Structure

A brief introduction to the concept of reconfigurable design using Farrow structure based architectures is presented here. Design details can be found in [71–83]. An important application requiring reconfigurable hardware is the variable sampling rate conversion where the conventional sampling rate converters require different interpolation of decimation filters for each sampling ratio. However, much flexibility can be incorporated in the system by using the Farrow structure based interpolation and decimation filters. The use of the Farrow structure makes it possible to realize polyphase components of general interpolation/decimation filters (with the Nyquist filter being a special case), using one set of fixed subfilters and several sets of variable multipliers [84]. The Farrow structure is composed of linear-phase finite-length impulse response (FIR) subfilters $S_k(z)$, $k = 0, 1, 2, \dots, L$, with either a symmetric (for k even) or antisymmetric (for k odd) impulse response. The impulse response of the Farrow structure is

$$h(n, \mu) = \sum_{k=0}^L s_k(n) \mu^k, \quad (4.1)$$

where $s_k(n)$ represents the impulse response of the k th subfilter and μ is the fractional delay value which defines the time difference between each input sample and its corresponding output sample. Therefore, the transfer function of the Farrow structure can be represented as

$$H(z, \mu) = \sum_{k=0}^L S_k(z) \mu^k. \quad (4.2)$$

Similarly, there are various applications requiring variable fractional delay, i.e., delays which are not generally multiples of the sampling interval used. Farrow structure based systems find their application in such situations too. The frequency response of a pure delay is $e^{-j\mu\omega T}$ and can be expanded using the Taylor's series as

$$e^{-j\mu\omega T} \approx \sum_{K=0}^L \frac{(-j\mu\omega T)^K}{K!} = \sum_{K=0}^L \frac{(-j\omega T)^K}{K!} \mu^K. \quad (4.3)$$

Comparing (4.2) and (4.3), it can be seen that one way to obtain a fractional delay filter is to determine the filters $S_k(z)$ so that they approximate the re-

spective differentiators in k th branch of a typical Farrow structure whereas the overall Farrow structure approximates an allpass transfer function.

The Farrow structure based digital filter designs are known for reconfigurability point of view. However the computational complexity of these designs can be reduced substantially, as the proposed approach of using multi-rate techniques proposed in the thesis can be applied on Farrow structures as well. The proposed technique can be attractive for Farrow structure based designs from complexity saving perspective, e.g., fractional-delay FIR filters, because such filters utilize several subfilters in parallel with a common input to each branch.

4.3 Reconfigurable Design Using Coefficient Decimation

Coefficient decimation is a technique to implement reconfigurable FIR filters [85]. The technique is based on decimating and/or interpolating a fixed set of impulse response coefficients. In this way, several different frequency responses are obtained without changing the coefficient values. Hence, there is no need to store different filter coefficient sets and techniques for simplifying the multipliers, such as shift-and-add techniques, can be straightforwardly applied.

In the previous works [85–89], the design has been performed using the Parks-McClellan algorithm only for the initial filter specification (i.e., model filter). Interpolating the impulse response results in a compression of the frequency response. However, decimating it will deteriorate the frequency response to possibly violate the specifications. In Paper D, an offline design method based on linear programming is proposed that takes all required configuration modes into consideration and results in a minimax solution over all configurations. The resulting filter coefficients can replace the ones in previous work to either decrease the worst approximation error and/or decrease the filter order required for a given approximation error in the reconfiguration modes.

The frequency response obtained as a result of coefficient decimation produce images created during upsampling and can be explained as follows Let $h(n)$ be the original set of filter coefficients. If all coefficients except every D th are replaced by zeros, a new response $h'(n)$ is obtained as,

$$h'(n) = h(n)c_D \quad (4.4)$$

where c_D is defined as,

$$c_D = \begin{cases} 1 & n = mD, m = 0, 1, 2, \dots \\ 0 & \text{otherwise.} \end{cases}$$

The function $c_D(n)$ is periodic with period D and hence the Fourier series expansion is given by

$$c_D(n) = \frac{1}{D} \sum_{k=0}^{D-1} C(k) e^{j2\pi kn/D} \quad (4.5)$$

where $C(k)$ are the complex-valued Fourier series coefficients defined by

$$C(k) = \sum_{n=0}^{D-1} c_D(n) e^{-j2\pi kn/D}. \quad (4.6)$$

Substituting (4.6) into (4.5) it follows that $C(k) = 1, \forall k$. Hence,

$$c_D(n) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi kn/D}. \quad (4.7)$$

The Fourier transform of the modified coefficients $h_D(n)$, denoted as $H_D(e^{j\omega T})$, can be expressed as

$$\begin{aligned} H_D(e^{j\omega T}) &= \sum_{n=-\infty}^{\infty} h_D(n) e^{-j\omega T n} \\ &= \sum_{n=-\infty}^{\infty} h(n) c_D(n) e^{-j\omega T n} \\ &= \sum_{n=-\infty}^{\infty} h(n) \left(\frac{1}{D} \sum_{k=0}^{D-1} h(n) e^{j2\pi kn/D} \right) e^{-j\omega T n}. \end{aligned} \quad (4.8)$$

By further mathematical manipulations in (4.8), we have

$$\begin{aligned} H_D(e^{j\omega T}) &= \frac{1}{D} \sum_{k=0}^{D-1} \sum_{n=-\infty}^{\infty} h(n) e^{-jn(\omega T - 2\pi k/D)} \\ &= \frac{1}{D} \sum_{k=0}^{D-1} H \left(e^{j(\omega T - 2\pi k/D)} \right). \end{aligned} \quad (4.9)$$

It can be seen from (4.9) that the frequency response is scaled by D and the replicas of the frequency spectrum are introduced at integer multiples of $2\pi/D$. Thus, in order to obtain the desired result, the output of the filter needs to be scaled by D . For each value of D , different multi-band responses are obtained. This also forms the basic principle of reconfigurable FIR filters using the coefficient decimation. The reconfigurability can be easily achieved by changing the value of D with a given set of filter coefficients [85]. Therefore the proposed technique can be used to obtain various filter specifications, e.g., original, multi-band, and decimated frequency response. This, originally introduced technique had only one parameter D to control the transition bandwidth as well as bandwidth of the original model filter to get the desired response. However the response obtained is like that of a downsampler as mentioned in Section 2.3. A further flexibility in coefficient decimation technique is obtained by having two parameters instead of one i.e., decimation by D followed by interpolation by

M . The magnitude response after decimation has a bandwidth and transition bandwidth D times larger than that of model filter. The interpolation results in shrinking of bandwidth and transition bandwidth by a factor of $1/M$. Therefore, if $\omega_c T$ and $\omega_s T$ are the passband and stopband edges of the initial filter respectively, the corresponding first image bandedges of the resulting filter $\omega_c T'$ and $\omega_s T'$ can be expressed as

$$\omega_c T' = \frac{D}{M} \omega_c T, \quad (4.10)$$

$$\omega_s T' = \frac{D}{M} \omega_s T. \quad (4.11)$$

The desired channel can be extracted from these multiple bands which correspond to a model filter in regular FRM methods and, using the appropriate masking filter for desired channel selection.

The FIR systems based on coefficient decimation find applications in design of hardware for channelizer filter of SDR receivers and dynamic spectrum allocation and are discussed briefly as follows.

4.3.1 Channelizer Filters

Software defined radio (SDR) is evolving as a promising technology in the area of wireless communications. The basic idea of SDR is to replace most of the analog signal processing in the transceivers with digital signal processing in order to provide the advantage of flexibility through reconfiguration of programming. This will enable different air interfaces to be implemented on a single generic hardware platform. The SDR receiver typically employs a channelizer, which comprises digital filters/digital filter banks (FBs) to extract single/multiple radio channels (frequency bands) from the wideband receiver [90–95]. The channelizer is the most computationally intensive part of a wideband receiver since it operates at the highest sampling rate in the digital front end. The compatibility of channelizer with different communication standards is guaranteed by its reconfigurability [96, 97].

4.3.2 Dynamic Spectrum Allocation

Dynamic spectrum allocation refers, in a broad sense, to a method of dynamically allocating frequencies [98–100]. This method of allocating a portion of available frequency spectrum, in contrast to old methods of spectrum allocation based on separation of bands, offers an efficient method for utilization of available frequency spectrum in an opportunistic manner. Reconfigurability is the important potential enabling technology for effective dynamic spectrum allocation. The key concept for this is to use a single hardware implementation for more than one type of systems by reprogramming it for different standards. The main advantage of such a type of systems is to have on-the-fly reconfiguration.

CONCLUSIONS AND FUTURE WORK

“One never notices what has been done; one can only see what remains to be done.”, Madam Curie

5.1 Contributions of the Thesis

The governing idea through the thesis is to focus on design techniques of various digital FIR filter applications for low arithmetic complexity and a high degree of flexibility. The work resulted in the following publications: [41, 42, 101–105]. Much work has been done in recent years for complexity reduction in the area of frequency selective filters whereas less attention has been paid to non-frequency selective filters. A substantial part of the work in this thesis has focused on non-frequency selective filters, with digital differentiators as typical example applications. However, for frequency selective filters, sparse non-periodic subfilters based solutions are also proposed for narrow-band and wide-band filtering. Similarly, a linear programming based coefficient decimation technique for reconfigurable filters has also been described.

5.2 Conclusion

Various methods of complexity reduction in the digital systems with narrow don't care bands are discussed in this thesis. The digital systems with narrow don't care band can be categorized into left-, right-, and mid-band specification systems and techniques were developed using single-rate and multi-rate approaches along with frequency response masking techniques. The resulting realizations show substantial complexity reductions compared with the corresponding conventional realizations. The corresponding design problems are

solved using linear programming and non-linear programming based optimizations.

This thesis also covers a reconfigurability aspect of digital system design where linear programming techniques are applied for design of coefficient decimation based FIR filters, targeting mainly channelizer filters in software defined radio receivers. The proposed technique gives substantial design margins as compared to the conventional methods.

Similarly, the sparse non-periodic subfilter based solution for the conventional problems of narrow-band and wide-band frequency selective filters are proposed. It is demonstrated that the proposed methods result in substantial complexity reduction as compared to the conventional methods and that linear programming can still be utilized although the nature of the problem is non-convex due to cascaded subsystems.

Finally, equations for order estimation of linear-phase differentiators for all the four types of FIR filters are developed. These models are developed using minimax criteria based design and curve fitting. The equations show good agreement with the actual orders required.

5.3 Future Work

The following topics are identified as potential considerations for future research

1. Extension of the proposed wide-band system realization techniques to other systems and applications. Examples include multi-function systems, like the Farrow structure, time-varying systems, like time-interleaved analog-to-digital converter systems, and non-linear systems.
2. Extension of the sparse filtering methods to arbitrary bandwidth FRM-like filters.
3. Extension of the sparse filtering methods to other non-frequency selective filters.

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