Development of Simulation Tool and New Tracking Algorithms for Radio Occultation Receivers

Examensarbete utfört i Kommunikationssystem vid Tekniska högskolan i Linköping

av

David Ahlsin och Oskar Rönnberg Sjödin

LiTH-ISY-EX--11/4473--SE

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Utveckling av Simuleringsverktyg och Nya Trackingalgoritmer för Radio-Ockultations-mottagare

When a radio signal traverses the atmosphere it will be delayed by not only the distance between transmitter and receiver, but also the atmosphere. Given knowledge of the characteristics of the sent signal the effect of the atmosphere can be obtained from the received signal. This concept is called radio occultation. Radio occultation can provide high accuracy profiles of temperature, pressure and water vapour throughout the atmosphere.

This report aims to present the work and results from a thesis performed at RUAG Space in Göteborg. The purpose of the thesis was to implement a simulator which with high accuracy could generate a signal as it would have been received had it propagated through the atmosphere.

We will show that the generated signal passes the requirements that have been set.
Abstract

When a radio signal traverses the atmosphere it will be delayed by not only the distance between transmitter and receiver, but also the atmosphere. Given knowledge of the characteristics of the sent signal the effect of the atmosphere can be obtained from the received signal. This concept is called radio occultation. Radio occultation can provide high accuracy profiles of temperature, pressure and water vapour throughout the atmosphere.

This report aims to present the work and results from a thesis performed at RUAG Space in Göteborg. The purpose of the thesis was to implement a simulator which with high accuracy could generate a signal as it would have been received had it propagated through the atmosphere.

We will show that the generated signal passes the requirements that have been set.

Sammanfattning


Denna rapport ämnar presentera det jobb och de resultat som uppnåtts genom ett examensarbete genomfört på RUAG Space i Göteborg. Examensarbetets syfte var att implementera en simulator som med hög noggrannhet kan generera en signal så som den hade sett ut då den propagerat genom atmosfären.

Vi kommer att visa att den genererade signalen uppnär de krav som ställts.
Acknowledgments

First of all, a big thank you to RUAG Space for the opportunity to work with this project. Magnus and Thomas, thank you for your valuable knowledge, commitment and guidance during our time at RUAG Space. Thank you, Jacob Christensen and Anders Carlström, for your input and interest in our work. Thank you, Jonas and Danyo, for your support at the university.

David Ahlsin and Oskar Rönnberg Sjödin. Göteborg, June 2011
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## Abbreviations

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<th>Description</th>
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<tbody>
<tr>
<td>ACF</td>
<td>Auto-Correlation Function</td>
</tr>
<tr>
<td>BOC</td>
<td>Binary Offset Carrier</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase-Shift Keying</td>
</tr>
<tr>
<td>C/A</td>
<td>Coarse/Acquisition</td>
</tr>
<tr>
<td>CCF</td>
<td>Cross-Correlation Function</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CSM</td>
<td>Coded Signal Module</td>
</tr>
<tr>
<td>DMI</td>
<td>Danish Meteorological Institute</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth-Centered Earth-Fixed</td>
</tr>
<tr>
<td>ECMWF</td>
<td>European Centre for Medium-Range Weather Forecasts</td>
</tr>
<tr>
<td>ETP</td>
<td>Earth Tangent Point</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GRAS</td>
<td>GNSS Receiver for Atmospheric Sounding</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>I/Q</td>
<td>In-Phase/Quadrature-Phase</td>
</tr>
<tr>
<td>IRM</td>
<td>Ideal Receiver Module</td>
</tr>
<tr>
<td>IRR</td>
<td>Ideal Reference Receiver</td>
</tr>
<tr>
<td>LEO</td>
<td>Low-Earth Orbit</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NCO</td>
<td>Numerically Controlled Oscillator</td>
</tr>
<tr>
<td>NRIP</td>
<td>Nominal Ray Impact Point</td>
</tr>
<tr>
<td>P</td>
<td>Precision</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudo-Random Number</td>
</tr>
<tr>
<td>RO</td>
<td>Radio Occultation</td>
</tr>
<tr>
<td>ROPP</td>
<td>Radio Occultation Processing Package</td>
</tr>
<tr>
<td>ROSIM</td>
<td>Radio Occultation Simulator</td>
</tr>
<tr>
<td>Rx</td>
<td>Receiver</td>
</tr>
<tr>
<td>SLTA</td>
<td>Straight Line Tangent Altitude</td>
</tr>
<tr>
<td>STP</td>
<td>Straight Line Tangent Point</td>
</tr>
<tr>
<td>Tx</td>
<td>Transmitter</td>
</tr>
<tr>
<td>WOP</td>
<td>Wave Optics Propagation</td>
</tr>
</tbody>
</table>
Definitions

azimuth angle  An angular measurement between a vector pointing to the North Pole and a vector towards an object of interest, projected onto the observer’s horizontal plane.

code epoch  One period of the PRN sequence used for CDMA in GNSS.

excess phase  Difference between the measured range and the geometrical distance.

geometric distance  Distance that an electromagnetic ray travels, i.e. the distance from the transmitter at the time of transmission to the receiver at the time of reception.

multipath  The radio transmission concept of the sent radio wave being reflected onto a number of scatterers, causing constructive and destructive interference at the receiver as the signal echoes are being superimposed onto the primary signal wave.

rising occultation  Occultation where the receiving satellite appears at the horizon as seen from the transmitting satellite.

setting occultation  Occultation where the receiving satellite disappears at the horizon as seen from the transmitting satellite.
## Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{v}$</td>
<td>Vector</td>
</tr>
<tr>
<td>$\dot{\vec{R}}$</td>
<td>Derivative with respect to time</td>
</tr>
<tr>
<td>$</td>
<td>\vec{v}</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

This report aims to present the work and results of a Master of Science thesis performed at Linköping University and RUAG Space, Göteborg during the spring of 2011.

1.1 Background

When a radio signal traverses the atmosphere it will be delayed by, not only the straight line distance between transmitter and receiver, but also the atmosphere. Given knowledge of the characteristics of the sent signal the effect of the atmosphere can be obtained from the received signal. This is the basic idea behind radio occultation (RO) [10].

![Figure 1.1. Basic principle of RO [1].](image)
RO has been used to probe the atmosphere of other planets since the sixties when the Martian atmosphere was probed by e.g. Mariner-4 in 1965. The technique used to determine temperature and pressure profiles of our own atmosphere is relatively new. The US launched the first test system in 1995 [10].

RO of the Earth’s atmosphere utilizes a signal sent from a Global Navigation Satellite System (GNSS) satellite, e.g. the Global Positioning System (GPS) or Galileo, to a satellite in low-earth orbit (LEO). The LEO satellite orbits faster than the GNSS satellite, so by measuring the received signal it is possible to obtain both setting and rising occultations, as the LEO satellite disappears or appears at the horizon. This is illustrated in Figure 1.1. Due to the bending of the ray path in the atmosphere, it is even possible to retrieve a sent signal at the receiving satellite when the transmitting satellite is below the horizon [10].

Measuring atmospheric profiles has previously mainly been done with radio sondes. The coverage of these sondes has been far from global, and the vast majority of measurements is concentrated to areas such as Europe, the US and China. To measure the atmosphere of the entire Earth is not possible. However, by using RO the measurements can be distributed all over the world. The benefits with RO are global coverage, high accuracy, all weather operation, good vertical resolution and that it is inexpensive [2].

RUAG Space is in cooperation with the Danish Meteorological Institute (DMI) and the European Centre for Medium-Range Weather Forecasts (ECMWF) performing a study for the European Organization for the Exploitation of Meteorological Satellites (EUMESAT) on optimization of tracking strategies used for radio occultation. The purpose of this study is to develop a radio occultation simulator. This thesis has been conducted in close relation with this study [9].

1.2 Purpose and Goals

The purpose of this thesis is to implement a software tool which simulates how modulated GNSS signals are affected by real atmospheric profiles. The output from the simulator is intended to be used for developing new receivers dedicated to RO.

1.3 Disposition

This report begins with a short introduction and continues with a theory section that explains some of the basic concepts of radio communication and radio occultation, along with some other theoretical prerequisites. It is followed by an description of the data set that has been used in the development and evaluation of the simulator. That section is followed by a description of the implementation and simulation results. Finally, our results will be presented and discussed.
1.4 Problem Formulation

An implemented simulator should model the received radio signal realistically, when the signal is sent according to certain GNSS standards, specifically GPS and Galileo. Figure 1.2 represents the different parts inside and outside the simulator on the highest level. The Wave Optics Propagation (WOP) and the Wave Optics Inversion (ROPP) blocks in the picture are provided by DMI. The WOP module calculates the propagation of a single frequency wave through the atmosphere, defined by a refractivity profile obtained from physical parameters such as temperature and dry and wet pressure. The ROPP module performs retrieval of refractivity from the received signal properties. The input to the ROPP module is the complex carrier’s phase and amplitude after the signal has been modulated by the atmosphere.

ROSIM consists of three modules, the Coded Signal Module (CSM), the Ideal Reference Receiver (IRR, which is built-in in CSM) and an Ideal Receiver Module (IRM). CSM generates the received radio signal in the time domain, down-converted to baseband and sampled at 30 Mhz. IRR aims at producing the truth in terms of the tracked signal’s amplitude and phase by estimating the correlation function and tracking its maximum peak. Oftentimes when we mention CSM throughout this report it is intended to include the processing of IRR. IRM tracks the baseband signal in the time domain. It is ideal and is hence aided by true code and carrier phase. IRR on the other hand works in the frequency domain [9]. Figure 1.2 shows a generic receiver module in place of IRM.

Note that ROSIM is in a prototype stage, and its current performance does not mirror its future capabilities.

For a full specification of the requirements, see Table 1.1. The different test quantities in Table 1.1 are defined in Chapter 4.2.
Introduction

<table>
<thead>
<tr>
<th>Req.</th>
<th>Property</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.a</td>
<td>GNSS Signals</td>
<td>GPS L1C/A</td>
</tr>
<tr>
<td>1.b</td>
<td></td>
<td>Galileo E1b</td>
</tr>
<tr>
<td>1.c</td>
<td></td>
<td>Galileo E5a</td>
</tr>
<tr>
<td>2.a</td>
<td>Realistic signal</td>
<td>Doppler shift</td>
</tr>
<tr>
<td>2.b</td>
<td></td>
<td>Time delay due to travel time</td>
</tr>
<tr>
<td>2.c</td>
<td></td>
<td>Navigation bit stream</td>
</tr>
<tr>
<td>2.d</td>
<td></td>
<td>Realizable filtering of the sent signal</td>
</tr>
<tr>
<td>3</td>
<td>Sampling frequency</td>
<td>30 MHz (specifiable)</td>
</tr>
<tr>
<td>4.a</td>
<td>Maximum relative amplitude error*</td>
<td>5 %</td>
</tr>
<tr>
<td>4.b</td>
<td>Moving average of relative amplitude error*</td>
<td>5 %</td>
</tr>
<tr>
<td>5</td>
<td>Maximum code delay error</td>
<td>40 mm</td>
</tr>
<tr>
<td>6</td>
<td>Maximum carrier delay error*</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>7.1</td>
<td>Maximum I/Q residual phase error span</td>
<td>3 mm</td>
</tr>
<tr>
<td>7.2</td>
<td>Moving average of residual I/Q phase</td>
<td>1 mm</td>
</tr>
<tr>
<td>7.3</td>
<td>Frequency bias of residual I/Q phase</td>
<td>0.01 mm/s</td>
</tr>
<tr>
<td>9.1</td>
<td>Maximum code phase estimate error</td>
<td>1 m</td>
</tr>
<tr>
<td>9.2</td>
<td>Maximum code phase estimate drift</td>
<td>0.3 m/s</td>
</tr>
</tbody>
</table>

* Measured as deviation from the center frequency

Table 1.1. Requirements for ROSIM

1.5 Unit Conventions

The units that are used in the field of radio occultation differ from the SI-standard, e.g. delay is by tradition expressed in meters. The reason for using non-standard units is that e.g. the phase represents a range. The conversions between units are done according to the following equations, where the superscript denotes the unit, \( f_c \) is the frequency of the carrier wave and \( c_0 \) is the speed of light in vacuum [3].

The conversion from Doppler frequency in Hz to Doppler velocity in m/s is done by multiplication with the wavelength

\[
D_{m/s} = D_{Hz} \lambda_0 = \frac{D_{Hz} c_0}{f_c} \quad (1.1)
\]

The carrier phase, which is normally expressed in radians, can correspondingly be converted to meters with aid from the wavelength

\[
\phi^m = \frac{\lambda_0}{2\pi} \phi^{rad} = \frac{c_0}{2\pi f_c} \phi^{rad} \quad (1.2)
\]

Code phase is normally expressed in seconds. Conversion to meters is done by multiplication with the velocity of the radio signal – the speed of light \( c_0 \).
\[ \tau^m = \tau^s c_0 \] (1.3)
Chapter 2

Theory

This chapter will give a brief introduction to some concepts related to radio communication, the Doppler effect and global navigation satellite system and a more detailed presentation of the field of radio occultation.

2.1 Basics of Radio Communication

This section presents a few basic concepts related to radio communication. It consists of a brief description of radio channels and the Code Division Multiple Access (CDMA) modulation technique.

2.1.1 The Radio Channel

The radio channel is an important aspect of any wireless communication system. When designing a communication systems it is important to have a good model of the channel so that good simulations of the system can be made [11]. A common way of modeling a channel is by treating it as a filter, and expressing its impulse response or transfer function. For radio communication the channel model is not simply an amplitude modulation and delay due to the travel distance. The multipath phenomenon needs to be represented as well. The transmission will be affected by multipath when a signal is reflected at one or more scatterers. The original wave and the reflected waves are superimposed which causes constructive and destructive interference since the reflected waves are phase shifted with respect to the direct ray path due to their longer travel distances [11].

An example of an impulse response which incorporates attenuation and delay, as well as multipath, is the following:

\[ h(t) = \sum_{k=0}^{N} a_k e^{j\phi_k} \delta(t - \tau_k) \]  \hspace{1cm} (2.1)

where \( a_k \) is the attenuation, \( \phi_k \) is the phase and \( \tau_k \) is the delay of the \( k \)th echo. \( \delta(t) \) is the Dirac impulse at \( t = 0 \). In general \( a_k, \phi_k \) and \( \tau_k \) are functions of time.
The attenuation is due to a number of phenomena, e.g. free space path loss and interference of the atmosphere.

### 2.1.2 Code Division Multiple Access

Code Division Multiple Access (CDMA) is a modulation technique which uses a bandwidth that is larger than the information rate. The basic concept is that the signal that is to be sent is spread by a binary spreading sequences. By doing this the spectrum of the signal is spread over a larger bandwidth. By using appropriate spreading sequences it is possible to have multiple users on the same channel if the users use different spreading sequences. The spreading sequences are often designed so that their auto-correlation and cross-correlation functions are small [11].

The benefits with CDMA is that frequency diversity is obtained, i.e. the signal can be received even if some part of the frequency spectrum is heavily attenuated, multiple users can use the same channel and it reduces the probability of interception and jamming [11].

Some important concepts related to the spreading sequences are chip rate and epoch. A chip is a bit of the spreading sequence and the chip rate is the number of chips per second. The term chip is used to distinguish it from bit since it carries no information [6]. The spreading sequences are finite and the period length is called epoch.

### 2.2 Radio Occultation

This subchapter introduces the concept of radio occultation (RO). It starts with a brief history of the subject and continues with explaining how the principle of RO can be applied to the Earth’s atmosphere. After that, some background reading is presented on how the retrieval process is performed, along with some information on current RO missions.

By accurately measuring the phase variation of a signal received by a satellite orbiting Earth, when those signals rises or sets through the atmosphere, it is possible to calculate a refractivity profile. The refractivity profile is dependent on temperature and pressure of the neutral atmosphere as well as the electron content of the ionosphere, which enables these parameters to be extracted. [1]

This concept was first utilized in the mid-sixties, as the interplanetary space sonde Mariner-4 made a flyby past Mars and transmitted signals through the Martian atmosphere. In 1969 Mariner-6 and Mariner-7 successfully probed the Martian atmosphere at four points above the planet’s surface (see [7] for detailed information). Since then NASA’s Pioneer and Voyager series have successfully applied RO to both Venus and the outer planets of our solar system [8]. The same remote sensing technique is now being applied to our own planet.
2.2 Radio Occultation

2.2.1 Probing Earth’s Atmosphere

By utilizing the GNSS satellites already in orbit, which continuously are transmitting well-defined radio signals, and receiving the signals with LEO satellites, accurate information on the atmospheric properties can be calculated.

A LEO can observe a GNSS satellite at high altitude. The LEO orbits Earth in about 1.5 hours, while the GNSS has an orbit time of 12 hours. When the LEO sees the GNSS set behind the horizon, the ray is successively immerged into the atmosphere and delayed and bent. The phase delay and corresponding Doppler shift is used to calculate the bending angle and the propagation altitude in the atmosphere [1].

Since the GNSS satellites for navigation purposes are transmitting a well-defined signal relying on an ultra stable internal clock, these signals can be received and processed for RO, see subchapter 2.4. These signals are modulated with a square wave pseudo-random noise (PRN) code. Since the PRN code is known for each specific GNSS satellite it is possible to determine the absolute delay by correlating the received signal with a code replica. This delay will be caused by both the actual distance the signal has travelled – which is known, given the position of both satellites – and the atmosphere. As the LEO satellites sets (or rises) one value for the delay for each epoch is obtained in the receiver. From these values a bending angle can be determined, which can be transformed into a refractivity profile and interpreted as temperature- and pressure profiles. For a full occultation we will in other words get a complete profile of the physical properties of the atmosphere.

2.2.2 Geometry

The LEO satellite orbits at 1.1 Earth radii with a geocentric angular rate of about 4 rad/h, while e.g. a GPS satellite orbits at 4.1 Earth radii at 0.5 rad/h [12]. This means that the geometry to a large part is determined by the movement of the LEO satellite, orbiting with a velocity at more than 7 km/s.

A number of vectors, points and angles are defined according to Figure 2.1. Note that the bending angle, which will be proven useful later in this chapter, is denoted $\alpha$. The straight line geometrical distance between the two satellites is $R_{LG}$.

The impact parameter, $a$, is defined as the perpendicular distance from the linear extension of the ray path outside the atmosphere to the Earth centre, see Figure 2.1. The Nominal Ray Impact Point (NRIP) is defined as the point where the extended ray meets the perpendicular radius from the Earth centre. Note that the LEO and GNSS satellites have one NRIP each. In Figure 2.1 these can be found as the point where $a$ is perpendicular to the ray path. Normally the ray is passing below NRIP due to the bending in the atmosphere. The Straight Line Tangent Point (STP) is defined as the point where the straight line vector between the two satellites is parallel to the surface of the Earth. The corresponding point at the surface is called the Earth Tangent Points (ETP). The difference between these two points is called the Straight Line Tangent Altitude, SLTA. SLTA, ETP and STP are defined in Figure 2.2.
The Doppler shift is a primary measurement quantity. It is determined by the GNSS satellite velocity, $\vec{v}_G$, as well as the LEO velocity, $\vec{v}_L$, projected onto the ray direction. The total Doppler velocity can hence be written, with vector and angle definitions according to Figure 2.1, as

$$D \text{ m/s} = \dot{R}_G \cos \chi \cos \phi_{az}^G - \dot{R}_L \cos \phi \cos \phi_{az}^L$$

$$= \dot{R}_G \frac{a}{R_G} \cos \phi_{az}^G - \dot{R}_L \frac{a}{R_L} \cos \phi_{az}^L$$

$$= a \left( \frac{\dot{R}_G}{R_G} \cos \phi_{az}^G - \frac{\dot{R}_L}{R_L} \cos \phi_{az}^L \right) = a\Omega \quad (2.2)$$

In Equation (2.2) above, $\Omega = \dot{\Theta}$ is the difference of angular velocities for the satellites, $\phi_{az}^G$ is the azimuth angle of the GNSS satellite and $\phi_{az}^L$ is the azimuth angle of the LEO satellite. The azimuth angle is an angular measurement defined as the angle between a vector towards the object of interest, projected onto the observer’s horizontal plane and the direction of the North Pole. Both $a$ and $\Theta$ are defined in Figure 2.1.

2.2.3 The Retrieval Procedure

The detailed retrieval processing will not be described in this thesis, but an overview of the basic principle will be presented (see [12], [8] for more details).

The starting point of a retrieval process is to estimate the phase of the received carrier wave at the RO instrument. The difference between the measured phase and the phase predicted for propagation through vacuum is called the excess phase. The excess phase $\phi$ can be calculated as
2.2 Radio Occultation

Figure 2.2. Definition of Straight Line Tangent Altitude and related points.

\[ \phi = \int_S \mu ds - R_{LG} \]  \hspace{1cm} (2.3)

In the expression above the integration is done along the ray path \( S \), integrating the refraction index profile \( \mu(r) \), at a certain height \( r \). The validity of the integral above requires the assumption of a spherically symmetric atmosphere.

The impact parameter, \( a \), is obtained from the measured total Doppler according to (2.2).

The excess Doppler shift can be calculated by differentiating the excess carrier phase with respect to time [5]. Some more details on Doppler calculations and its applications to RO can be found in chapter 2.3. By using the geometry defined in section 2.2.2 the bending angle can be determined as

\[ \alpha = \Theta - \cos^{-1} \left( \frac{a}{|R_L|} \right) - \cos^{-1} \left( \frac{a}{|R_G|} \right) \]  \hspace{1cm} (2.4)

By assuming a spherical symmetric atmosphere the bending angle can also be expressed as

\[ \alpha(a) = -2a \cdot \int_{\alpha}^{\infty} \frac{d\mu}{\mu} \frac{dr}{\sqrt{(\mu r)^2 - a^2}} \]  \hspace{1cm} (2.5)

which can be identified as an Abel transform.

By inverting Equation (2.5) it is possible to extract the refraction profile \( \mu(a) \) by using this inverse Abel transform along with the assumption of a spherically symmetrical atmosphere.

\[ \ln (\mu(a)) = \frac{1}{\pi} \cdot \int_{\alpha}^{\infty} \frac{\alpha(\xi)d\xi}{\sqrt{\xi^2 - a^2}} \]  \hspace{1cm} (2.6)
I.e. \( \mu(a) \) can be obtained from the bending angle \( \alpha(\xi) \). Using classical physics, the refractive index profile can be inverted to density, pressure and temperature.

The refractivity \( N \) is defined as

\[
\mu = 1 + N \cdot 10^{-6}
\]  

(2.7)

Refractivity depends on the physical parameters as

\[
N = k_1 \frac{P_d}{T} + k_2 \frac{P_w}{T^2} + k_3 \frac{P_w}{T} + 40.3 \frac{n_e}{f^2}
\]  

(2.8)

where

- \( P_d \) = dry air pressure \([hPa]\)
- \( P_w \) = water vapour pressure \([hPa]\)
- \( T \) = temperature \([K]\)
- \( n_e \) = electron content \([1/m^3]\)
- \( f \) = measured frequency \([Hz]\)
- \( k_1 = 77.60 \ K/hPa \)
- \( k_2 = 37.39 \cdot 10^4 \ K^2/hPa \)
- \( k_3 = 70.40 \ K/hPa \)

By measuring at two GNSS frequencies the neutral non-dispersive refractivity can be separated from the ionospheric dispersive refractivity. When we solve for the neutral atmosphere’s physical properties we also use the ideal gas law

\[
\frac{P}{\rho T} = R
\]  

(2.9)

where

- \( \rho \) = density
- \( R \) = the ideal gas constant

along with the hydrodynamic equation

\[
P(h) = \int_h^\infty \rho(h)g(h)dh
\]  

(2.10)

where

- \( h \) = altitude above Earth’s surface
- \( g \) = Earth’s gravitational acceleration \( \approx 9.81 \ m/s^2 \)
2.2.4 Current Missions

There are currently a number of RO missions flying. The first successful occultation measurement of the Earth’s atmosphere using GPS transmissions was done by the GPS/MET mission in 1995, see [12]. Since then, a number of missions have been launched, e.g. CHAMP, SAC-C and COSMIC [14]. Another RO system which is up and flying at the moment is the GNSS Receiver for Atmospheric Sounding (GRAS) instrument, onboard the meteorological Metop-A satellite, launched by EUMETSAT in 2006 [14]. A lot of detail on this exciting matter can be found in [4].

2.3 The Doppler Effect and Time Bases

If a receiver is moving relative to the transmitter of a wave a change in frequency of the observed wave will occur. This is called the Doppler effect. Since the LEO satellite moves with high velocities, typically around 7 km/s, the frequency will be shifted around 30 – 40 kHz\(^1\), where the shift direction depends on whether the satellites are moving closer or further apart. A consequence of this frequency shift is that the length of the received code epochs will differ over time, a typical difference is in the order of \(10^{-8} \text{s}\). As previously described, the atmosphere will introduce phase shifts in the transmitted signal which will give rise to an atmospheric Doppler shift. The derivative of the phase characteristics of the atmosphere corresponds to the relative velocities of the satellites, and the atmospheric Doppler shift can similarly be calculated as

\[
D_{m/s} = \frac{d}{dt} \phi^m(t) \tag{2.11}
\]

Note that this Doppler is given in m/s, which according to (1.1) can be transformed into Hz. The effect of the two Doppler shifts will be that the receiver experiences time in a different base than the transmitter. While the transmitted code epochs are of constant length and equidistantly transmitted, the received code epochs will vary in length and the receive time will generally not be linear. The transformation between the two time bases is mainly given by the propagation time of the wave. Let \(t_{Tx}\) be the transmitter’s linear time base, and \(t_{Rx}\) is the receiver’s time base. If the total distance the ray travels is denoted \(R(t)\) the full time transformation can be calculated as

\[
t_{Rx} = t_{Tx} + \frac{R(t_{Tx})}{c_0} \tag{2.12}
\]

The transmitter time is usually the GNSS satellite’s internal clock.

\(^1\)\(\dot{R}(t) \cdot \frac{1}{\lambda} \approx 7000\text{m/s} \approx 36.8\text{kHz}\) assuming a carrier frequency of 1575.42 MHz

\(^2\)Assume one code period is \(T = 1\) ms and a Doppler shift of \(f_D = 40\) kHz. A code epoch will then be extended by \(\Delta t_D = T f_D \frac{c_0}{\lambda} \approx 2.5 \cdot 10^{-8} \text{s}\), assuming a carrier frequency of 1575.42 MHz
2.4 Global Navigation Satellite Systems

Global navigation satellite systems (GNSS), like GPS or Galileo, use the fact that if the time it takes for a signal to reach the receiver is known it is possible to calculate the distance from the receiver to the transmitter. By using four satellites it is possible to calculate accurate position and time estimates for the receiver. Of importance for the system’s functionality is that the satellites can provide accurate time stamps for the sent data [6].

The development of the GPS system was initiated in the beginning of the 1960s by the US government. The desired attributes of the system were global coverage, all weather operation, ability to serve high-dynamic platforms and high accuracy. The GPS system is now operational. The satellite constellation is designed to consist of 24 satellites in six orbital planes with four satellites in each plane, but around 30 are in orbit to provide better accuracy and robustness. GPS was designed to have different accuracy for military users and civil users [6].

Galileo was initiated by EU in 1998. The objective was to design a system with high accuracy and integrity which was not dependent on other GNSS systems e.g. GPS. The system architecture is flexible and scalable so that it can be modified to fulfill future requirements. It will consist of 30 satellites in three orbital planes with ten satellites in each plane. Only nine will be active so there is one spare satellite per plane [6]. The launch is planned during 2015–2016 [3].

2.4.1 GPS Signals

GPS uses the CDMA modulation technique. There is a number of codes used in GPS, e.g. the C/A-, P- and Y-code. The Y-code is an encrypted version of the P-code. For more information on the different codes see Table 2.2. The codes are pseudo-random number-sequences, i.e. they look and behave as random sequences, but are in fact deterministic. The different satellites have a unique set of codes that are used for transmission, this makes it possible for a receiver to separate the signals received from different satellites. Navigation data (including the GPS almanac), which contains information like the satellite time, velocity and orbit parameters, is modulated onto the PRN sequence [6]. The different carrier frequencies that are used are defined in Table 2.1.

<table>
<thead>
<tr>
<th>Name</th>
<th>Carrier frequency [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>1,575.42</td>
</tr>
<tr>
<td>L2</td>
<td>1,227.6</td>
</tr>
<tr>
<td>L5</td>
<td>1,176.45</td>
</tr>
</tbody>
</table>

Table 2.1. GPS carrier frequencies.

In the GPS system Binary Phase Shift Keying (BPSK) is used together with CDMA to modulate the information. The general form for a BPSK modulated signal is given by:
where \( n(t) \) is the binary value to be sent and \( f_c \) is the carrier frequency. The power spectral density of a L1 C/A-code, which uses BPSK modulation, can be seen in Figure 2.3.

\[ s_{BPSK}(t) = \cos(2\pi f_c t + \pi(n(t) - 1)), \quad n(t) \in \{0, 1\} \]  
(2.13)

The auto-correlation function (ACF) of the GPS PRN-codes is an important aspect of the demodulation process. The power spectral density (PSD) of the GPS PRN codes determines the bandwidth required to transmit and receive the signals. \[6\] The ACF of an L1 C/A code can be seen in Figure 2.3. The characteristics of the ACF are important when it comes to design of tracking algorithms.

![Autocorrelation function of a L1 C/A-code](image1.png)

![Power spectral density of a L1 C/A-code](image2.png)

**Figure 2.3.** ACF and PSD for an L1 C/A-code.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 C/A</td>
<td>1.023</td>
<td>BPSK</td>
<td>50</td>
<td>2.046</td>
</tr>
<tr>
<td>L1 P(Y)</td>
<td>10.23</td>
<td>BPSK</td>
<td>50</td>
<td>20.46</td>
</tr>
<tr>
<td>L1 M</td>
<td>5.115</td>
<td>BOC(1,1)</td>
<td>N/A</td>
<td>4.092</td>
</tr>
<tr>
<td>L1 C</td>
<td>1.023</td>
<td>BOC(1,1)</td>
<td>N/A</td>
<td>4.092</td>
</tr>
<tr>
<td>L2 P(Y)</td>
<td>10.23</td>
<td>BPSK</td>
<td>50</td>
<td>20.46</td>
</tr>
<tr>
<td>L2 M</td>
<td>1.023</td>
<td>BOC(1,1)</td>
<td>N/A</td>
<td>4.092</td>
</tr>
<tr>
<td>L5</td>
<td>10.23</td>
<td>BPSK</td>
<td>50</td>
<td>20.46</td>
</tr>
</tbody>
</table>

**Table 2.2.** Properties for GPS signals.

### 2.4.2 Galileo Signals

Galileo uses the same principles for modulation as GPS, but the spreading sequences are different. There are a number of different signal types, but only those that have been relevant for this thesis will be discussed, i.e. E1-B, E5A-I and
E5A-Q. The modulation for the E1-B-code is Binary Offset Carrier-modulation (BOC).

A BOC signal can be seen as the product of a BPSK signal and a square wave. The number of half-periods in the square wave is usually selected as an integer, i.e. the frequency of the square wave is an integer multiple of two times the chip rate. The modulated signal is given by the following expression:

\[ s_{BOC}(t) = s_{BPSK}(t) \ominus (t) \]  

(2.14)

where is the \( \ominus(t) \) aforementioned square wave, see Figure 2.4. BOC signals are often written as \( BOC(m, n) \) where \( mf_0 \) is the frequency of the square wave, \( nf_0 \) is the chip rate and \( f_0 = 1.023 \) MHz. The PSD and ACF of an E1-B-code, which uses \( BOC(1, 1) \) modulation, can be seen in Figure 2.5.

The modulation for E5A is Alternative BOC (AltBOC), which has superior performance in terms of measurement accuracy and multipath suppression [13]. The modulation used is AltBOC(15,10). The E5A code has different spreading sequences for the I- and Q-components, but they are similar, hence only the I-component will be described. The signal generation is complicated and is not
2.4 Global Navigation Satellite Systems

The PSD and ACF of the I-component can be seen in Figure 2.6.

Figure 2.5. ACF and PSD for an E1-B-code.

Figure 2.6. ACF and PSD for an E5A-I-code.
Chapter 3

Description of Data Set

To develop the simulator in parallel to establishing the data interface with DMI, temporary data delivered from and simulated by the University of Graz has been used in the initial phase of the study. This section presents the data that has been used for the development and evaluation of the performance of the simulator. This data includes data from the University of Graz, data from DMI and ideal reference cases.

![Graphs showing amplitude and excess phase](image)

**Figure 3.1.** Data from the University of Graz, labeled as Case 5. The left figure shows the amplitude in volts for all supplied frequencies, while the right figure shows the excess phase for all frequencies.

The data is sampled in 1 kHz, and contains information on the receiver time, positions and velocities for both satellites in Earth-centered coordinates and phase and amplitudes for a number of frequencies. The phase is delivered as excess phase in meters, while the amplitude is normalized to one. Excess phase is the increased ray path length that the signal has travelled due to the bending that is caused by the atmosphere. The amplitude and phase matrices form the filtering caused by the atmosphere over time, expressed in the frequency domain in terms of the transfer function.
3.1 University of Graz Data

The data set labeled Case 5 was selected due to some interesting characteristics. Especially the amplitude curve in Figure 3.1 displays the characteristics of the data set. At around 28 seconds into the occultation a multipath region appears where a reflected and delayed version of the ray interference with the direct ray path. During some time periods there is destructive interference, causing the amplitude to approach zero.

Furthermore, from Figure 3.1 it is obvious that the data set describes a setting occultation. The LEO satellite starts receiving at a high altitude where there is no atmosphere present, and descends eventually beyond the horizon and the signal is lost.

3.2 Wave Optics Propagation Data

![Figure 3.2.](image)

One example of the data received from DMI can be seen in Figure 3.2. This case is a relatively simple case with no apparent multipath area and a gentle attenuation. The fact that the occultation is long, around 80 s, indicates that this case models a tropical atmosphere. In the Tropic Zone the atmosphere is denser, which will result in an increased bending of the ray path which in turn results in the receiver being able to receive the signal at a lower SLTA.

A problem with the simulated atmospheric profiles which is apparent both for WOP-data and data from the university of Graz, is that there is considerable numerical noise due to the resolution used when generating the profiles. This is mainly a problem at the end of occultations when the amplitude is small, since the relative effect of the numerical noise will have a greater impact. Another potential problem with the input data is that in some cases, at the end of the occultation
when the atmosphere is dense and the amplitude is low, there are errors with the unwrapping of the phase, i.e. the error will be a multiple of $\lambda_0$ m.

### 3.3 Reference Cases

![Amplitude and excess phase for Test Case 2](image)

**Figure 3.3.** Amplitude and excess phase for Test Case 2. The left figure shows the amplitude in volts for all supplied frequencies, while the right figure shows the excess phase for all frequencies.

To be able to fully evaluate the performance of the simulator in a controlled manner well-defined test cases can be a valuable aid. For this reason a number of interesting reference cases were defined. The test cases are simplified in comparison to real data. Since they are mathematically expressed they are completely noise-free, and have frequency-independent amplitude characteristics. An increasing test case number implies a more complex test case.

The most simple case is only a simple attenuation of the signal, a zero excess phase. The second case introduces a multipath region which affects both the amplitude and the phase, see Figure 3.3. The multipath region begins around the first second and has a duration of about one second. The third test case introduces an S-shaped phase variation over time. There is also a linear frequency separation of the phases which results in a delay. The fourth test case has an exaggerated phase variation, see Figure 3.4. The exaggerated phase makes it easier to find errors since time mismatches and other problems might have a larger impact during fast dynamics.
Figure 3.4. Amplitude and excess phase for Test Case 4. The left figure shows the amplitude in volts for all supplied frequencies, while the right figure shows the excess phase for all frequencies.
Chapter 4

Implementation

This chapter describes the implementation of ROSIM, the Radio Occultation SIMulator. It aims to describe the fundamental principles and ideas behind the performed processing.

4.1 ROSIM

4.1.1 Coded Signal Module

The Coded Signal Module (CSM) serves the purpose of simulating the modulated signal as filtered through the atmosphere. The received signal can either be delivered to the Ideal Reference Receiver, IRR, in the frequency domain or to the Ideal Receiver Module, IRM, in the time domain. IRR works as a built-in module in CSM, and is incorporated in the CSM iteration process.

CSM works under the assumption that the atmosphere is approximately stationary during one millisecond, and will for that reason produce millisecond batches.

![High level block diagram of the Coded Signal Module.](image)

Figure 4.1. High level block diagram of the Coded Signal Module.
of baseband data. Due to the Doppler elongation, this will not necessarily correspond to epochs at the receiver side, even in the case of millisecond epochs. Furthermore, the epochs in the received data stream will be offset from the millisecond time indices in the transmission stream. Since a receiver module works on epoch batches of data, CSM will be forced to work in several time bases to ensure continuity with regard to Doppler elongation of code epochs and geometrical range.

The time views of CSM and IRM will differ according to Figure 4.2. Note that the initial sample of the received baseband signal coincides with the first received epoch. This implies that we have a global clock zeroed at the time of the first received epoch. This time will be denoted as $t_{WOP}$.

CSM will produce evenly spaced milliseconds, independent of epoch offsets, while IRM processes epochs of varying length. The time spacing will primarily be of importance when the receiver down-converts the signal from the Doppler frequency, since any minor mismatch between the added Doppler shift in CSM and the down-conversion frequency in IRM would cause a drifting residual I/Q phase on the receiver side.

### 4.1.1.1 Geometrical Range

The input data is defined in receiver time for a received millisecond, which coincides with the CSM output for each iteration. The geometrical range used in the calculations corresponds to the straight line path between satellites for the full transmission, i.e. the straight line signal ray path. In the case of a stationary transmitter the geometrical range is calculated as
\[ R_{LG} = |\bar{R}_G - \bar{R}_L| \]  

(4.1)

The ray travel time, \( \Delta t \), is then calculated as

\[ \Delta t = \frac{R_{LG}}{c_0} \]  

(4.2)

### 4.1.1.2 Atmosphere

As previously stated, the atmosphere is assumed to be stationary during 1 ms. DMI delivers the complex transfer function for each ms as the phase and amplitude for a number of frequencies, typically seven, evenly spaced with 5 MHz steps in a frequency band symmetric around the carrier frequency. The complex values for the seven frequencies need to be interpolated to reach the same frequency resolution as the spectrum of the transmitted signal. Internally in CSM the original signal is represented as a vector, using \( U_S \) samples/bit. The frequency resolution of the signal spectrum is hence given as

\[ \Delta f = \frac{L}{LU_S} = \frac{1}{T} \]  

(4.3)

where \( L \) is the length of the code in bits and \( T \) is the epoch length of the code in seconds. Note that this is independent of \( U_S \) and is in the kHz order of magnitude.

The spectrum needs to be interpolated from seven points to several thousands. Due to the fast dynamics of the atmosphere over frequency the complex vector representing the filter in each frequency component would spin several cycles in the complex plane. To be able to interpolate between the frequencies the transfer function needs to be down-converted so that only the slow dynamics remain. For down-conversion an excess range model is estimated from the given input data as the mean value of absolute phase of the frequencies for each time instant. By removing the excess range model from the transfer function as a linear trend (i.e. delay) the spectrum is down-converted and can then be interpolated. The excess range model can then be re-added as a linear trend to each of the new frequency components to yield the original filter in a higher frequency resolution.

Two modes are available in CSM, as demonstrated in Figure 4.1. A frequency spectrum of the (excess) received signal can be delivered to IRR or a 30 MHz signal in the time domain can be delivered to IRM. The frequency spectrum which is the input to IRR is given as the following multiplication:

\[ U_0(f) = H(f) \cdot U(f) \]  

(4.4)

where \( H(f) \) is the frequency spectrum of the transfer function of the atmosphere, and \( U(f) \) is the frequency spectrum of the original signal.

A first approximation of the signal in the time domain as modulated by the atmosphere can then be retrieved as the inverse Fourier transform below. The \( n \) signifies the iteration number.
The exponential term will introduce the ray travel delay. However, since the output of each iteration is 1 ms of data, the delay introduced will be modulo 1 ms. $R_{\text{LG}}$ is constant for the specific iteration.

### 4.1.1.3 Doppler

In the simulation so far, everything has been stationary during the processed ms. However, in a more realistic scenario the satellites will move in a continuous motion and the phase shift introduced by the atmosphere will change during the transmission, which will cause the signal to have an increasing I/Q phase and spin in the complex plane. To model this, the effect will be represented as a linear increase in phase. First, we calculate the full range as the sum of the geometric range and excess range introduced by the atmosphere, $R[n]$:

$$R[n] = R_{\text{LG}}[n] + R_{\text{excess}}[n] \quad (4.6)$$

The Doppler velocity is given as

$$V_D[n] = \frac{R[n+1] - R[n]}{10^{-3}} \quad (4.7)$$

The Doppler frequency, $f_D$ is given according to

$$f_D[n] = V_D[n] \frac{f}{c_0} \quad (4.8)$$

By turning the phase linearly throughout the 1 ms batch of data, the second and final approximation is completed:

$$u_{0,n}(t) = u_{0,n}^{(1)}(t) \cdot e^{-j2\pi f_D[n]t} \quad (4.9)$$

We iterate over all the samples of the atmosphere and can produce the full received signal in the time domain as the composite of all the previously produced ms batches of data:

$$u_0(t_{\text{WOP}}) = \left[ u_{0,0}(t) \quad u_{0,1}(t) \quad ... \quad u_{0,N}(t) \right] \quad (4.10)$$

where $t_{\text{WOP}}$ spans the full occultation and $t$ is one ms long.

### 4.1.1.4 Navigation Bits

Figure 4.3 displays how the navigation bit stream is added to the generated time domain signal. For each iteration in CSM three milliseconds of data are processed. For a code of epoch length 1 ms all three milliseconds will be identical. As the signal is delayed due to geometric range and the delay introduced by the atmosphere millisecond $n - 1$ will be shifted into the interval we are considering. I.e., the output will be a composite of a fraction of the previous millisecond, as well as a
Figure 4.3. Implementation of navigation bits. a) For each ms of data output, three ms of data is processed. The three corresponding samples of navigation bits are added. Here a flank occurs at the beginning of the nth ms. b) The processing introduces a delay, which pushes the flank into the interval. Due to the cyclic properties of the FFT the last ms wraps around to the beginning. c) The middle interval is cutout and output, containing the navigation bit flank.

fraction of the millisecond we are processing. By using this algorithm the flanks will appear at the correct positions throughout the generated baseband time domain signal.

4.1.1.5 Longer Codes

Codes with a longer epoch length are implemented in the same way as navigation bits above. A three milliseconds long signal is cut out from the code and sent through the filter, after which the center millisecond is cropped out from the received signal. The code is then cyclically shifted so that the next iteration will transmit the next millisecond.

4.1.2 Ideal Reference Receiver (IRR)

The Ideal Reference Receiver (IRR) aims to track the signal in the frequency domain and estimate what henceforth will be regarded as the truth. For each millisecond of CSM processing IRR will estimate a sampled version of the correlation function, using the approximate complex CCF in Equation (4.11). See Appendix A for derivation.

\[ C[k] \approx \frac{1}{N} \mathcal{F}^{-1}\{X[i]Y^*[i]\} \] (4.11)

The sample with the maximum amplitude is called the punctual (P) value, the one sample before it is the early (E) value, and the one after is the late (L) value. The punctual value does not normally correspond to the theoretical maximum of the correlation function, which means that the theoretical maximum has to be
Implementation

(a) Theoretical correlation function (dashed line) and sampled correlation function (solid)

(b) Minimum Square Method estimation of the theoretical maximum. $\tau$ is the estimated code phase.

Figure 4.4. Absolute value of the sampled complex correlation function. P stands for punctual value, E is early and L is late.

estimated. Note figure 4.4a. The theoretical maximum is the top of the dashed triangle, while the solid line represents the sampled correlation function, as it would be processed.

The difference between the amplitudes of E and L will determine on which side of the theoretical peak the punctual value is. Two straight lines are then estimated based on the Least Squares Method. The time index, $\tau$, of the intersection between these two lines is the code phase. This concept is demonstrated in Figure 4.4b. The angle of the punctual correlator corresponds to the excess I/Q phase of the carrier.

An alternative method of performing the approximation of $\tau$ is to up-sample the correlation peak. An interval around the peak is cut out from the correlation function and smoothly windowed to reach zero at its borders. The correlation function is then interpolated by a factor of several thousands in the frequency domain to reach a higher resolution in time. After the interpolation the punctual value can simply be picked as the new maximum of the peak.

A necessary condition for the up-sampling to work is that the original correlation peak has a high enough sample rate. This is not the case for e.g. the 10 MHz E5 code. The required up-sampling of the code, $U_S$, would significantly slow down both the CSM and IRR processing. Instead, a code independent approximation of the truth is generated.

Additional functionality for peak tracking in IRR is the lock on peaks. For each iteration the last found index of the punctual value is saved. The next maximum is only allowed to lie in a designated interval around that value. If another value is found it is assumed that the lock has either jumped to a different peak due to a multi-peak correlation function or multipath interference, or that the interpolation has somehow failed. If this happens the last found value will be used instead. This will reduce the noise of the resulting curve. It also serves the purpose of reducing the simulation time, since the module only needs to search for the maximum in the
predetermined, smaller interval, instead of over the entire correlation function. If the interpolation fails to find an allowed maximum during two or more iterations in sequence, the interval we search in will be linearly expanded.

Since IRR is part of CSM iteration process, truth data will be estimated on a millisecond basis, defined in received millisecond time.

4.1.3 Ideal Receiver Module
The Ideal Receiver Module, IRM, serves the purpose of tracking the generated baseband signal in the time domain. Since the module works in an ideal scenario it has aid from the true code and carrier phases, delivered by IRR, as well as the full geometry for the occultation. IRM works in a receive time basis, defined by the epoch reception instants.

IRM estimates the Doppler shift from the input data and down-converts the signal, epoch by epoch. Note that the Doppler shift was applied millisecond by millisecond in CSM. After the down-conversion the signal is correlated with a code replica in the time domain. The output values from the correlators are integrated and dumped for each epoch.

As in a real receiver, two numerically controlled oscillators (NCOs) are used. The NCOs control the code and carrier phases in the receiver and are aided by the IRR truth data.

Figure 4.5 shows a block diagram of the IRM processing.

![Figure 4.5. High level block diagram of the IRM processing.](image-url)

4.2 Testbench
To be able to verify the results of CSM and IRM a testbench was designed, implemented and in addition a number of well-known test scenarios were devised. These
test scenarios serve the purpose of being mathematically expressed (i.e. noise-free) and aims at testing certain aspects of the different modules separately, see chapter 3.3 for more details on the test cases. The testbench is essentially a framework that runs CSM and IRM and compares their results to a mathematically derived truth data. By investigating the error characteristics it is possible to get a better understanding of the simulator and how certain aspects of the input data can propagate to the output.

The test quantities that are related to amplitude, code phase and carrier phase can be used to verify the functionality of both CSM and IRM whereas the residual I/Q phase and residual code phase are only observable from IRM.

![Framework for the ROSIM testbench.](image)

The test quantities can be divided in five subgroups and will be described in the following subsections. \( T_{ij} \) will be used as a label for the test quantity \( j \) in subgroup \( i \). The test quantities will be evaluated mathematically by comparison with an error threshold.

### 4.2.1 Amplitude

The true carrier amplitude can easily be found directly from the input data. For CSM the carrier amplitude is given as the amplitude of the complex carrier, i.e. the amplitude of the peak of the correlation function. IRM uses the absolute value of the punctual correlator as the amplitude.

To evaluate the amplitude a relative error is used, i.e. the output amplitudes from CSM and IRM are divided by the true amplitude from the input data. Denote the relative amplitude \( A_{Rel} \). The maximum error span is then given as

\[
T_{11} = \max(A_{Rel}) - \min(A_{Rel}) \quad (4.12)
\]

To counter-act the effects of noise a weighted moving average over 0.1 s of \( A_{Rel}^2 \), called \( A_{Rel,MA} \), is used. A measurement can then be e.g.

\[
T_{12} = \max(|1 - A_{Rel,MA}|) \quad (4.13)
\]

### 4.2.2 Code Phase

The true code phase for a single carrier can be mathematically derived as the derivative of the phase with respect to angular frequency, i.e. the group velocity:
\[ \tau^m(t) = c_0 \tau^s(t) = c_0 \frac{\partial \phi^{rad}(t, \omega)}{\partial \omega} = \frac{\partial f \phi^m(t, f)}{\partial f} \] (4.14)

In CSM the generated signal is correlated with a signal replica to find the code phase. In IRM the code phase is calculated by deriving the geometrical range plus the true code phase from IRR and integrating it during 1 ms. The code phase is evaluated as the maximum difference from the true code phase.

\[ T_{21} = \max(\tau - \tau_{\text{truth}}) \] (4.15)

4.2.3 Carrier Phase

The true carrier phase is taken from the input data as the phase characteristic of the carrier frequency. CSM derives this as the angle of the complex correlation peak. The set of values, one for each millisecond, is then unwrapped. In IRM the carrier phase is calculated by deriving the geometrical range plus the true carrier phase from IRR and then integrating it during 1 ms.

The carrier phase can be evaluated as the difference from the true carrier phase.

\[ T_{31} = \max(\phi - \phi_{\text{truth}}) \] (4.16)

4.2.4 Residual I/Q phase

The residual I/Q phase is the remaining phase after the down-conversion with regard to Doppler shift in IRM. If the down-conversion was entirely correct and there are no numerical errors the residual I/Q phase should be constant. However, since noise is added in the conversion to uint8 in CSM the expected result is independent noise with zero mean.

Through IRM the residual I/Q phase can be evaluated. The first interesting test quantity is the absolute error span. Let \( \phi_{IQ} \) be the residual I/Q phase.

\[ T_{41} = \max(\Phi_{IQ}) - \min(\Phi_{IQ}) \] (4.17)

Just as for the amplitude a weighted moving average over each 0.1 s period of \( \phi_{IQ} \), resulting in \( \phi_{IQ,MA} \), can be used to even out larger peaks. A test quantity is then given as

\[ T_{42} = \max(|\Phi_{IQ,MA}|) \] (4.18)

The last characteristic of the residual I/Q phase to be evaluated is the frequency bias. This can be determined as the absolute value of the mean value of the discrete derivative of the residual I/Q phase:

\[ T_{43} = \frac{1}{N - 1} \sum_{k=1}^{N-1} \left| \frac{\Phi_{IQ}[k] - \Phi_{IQ}[k-1]}{\Delta t} \right| \] (4.19)

The test quantities are then converted to m and m/s respectively.
4.2.5 Residual Code Phase

In IRM it is possible to calculate an approximative code phase error by using the following discriminator:

$$\tau_\epsilon = \frac{1}{2} \cdot \frac{I_P(I_E - I_L) + Q_P(Q_E - Q_L)}{I_P^2 + Q_P^2}$$  \hspace{1cm} (4.20)

The equation above uses the early, late and punctual correlator, expressed as an I- and Q-component. E.g. the punctual correlator is given as $I_P + iQ_P$ and the others follow the same pattern. Ideally we should have approximated the correlation peak exactly, which means that $I_E = I_L$ and $Q_E = Q_L$, since the early and late correlators have the same amplitude. I.e. the code phase estimate should be constant zero. The residual code phase, $\tau_\epsilon$ has two test quantities of interest: the offset and the drift. The maximum offset can simply be determined as:

$$T_{51} = \max(|\tau_\epsilon|)$$  \hspace{1cm} (4.21)

The drift can be determined as:

$$T_{52} = \max\left(|\frac{d\tau_\epsilon}{dt}|\right)$$  \hspace{1cm} (4.22)

The test quantities are then converted to m and m/s respectively.
Chapter 5

Results

This chapter will begin by presenting the results from ROSIM and that will be followed by a discussion and the conclusions of this thesis. The graphical simulation results are available in Appendix B.

<table>
<thead>
<tr>
<th>Property (Criterion)</th>
<th>GPS L1C/A</th>
<th>Galileo E1b</th>
<th>Galileo E5a-I</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{11}(&lt; 5%)$</td>
<td>8.1</td>
<td>427</td>
<td>7.4</td>
<td>%</td>
</tr>
<tr>
<td>$T_{12}(&lt; 5%)$</td>
<td>2.1</td>
<td>40</td>
<td>20</td>
<td>%</td>
</tr>
<tr>
<td>$T_{21}(&lt; 40 \cdot 10^{-3} \text{ m})$</td>
<td>$36 \cdot 10^{-3}$</td>
<td>$36 \cdot 10^{-3}$</td>
<td>$36 \cdot 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$T_{31}(&lt; 0.2 \cdot 10^{-3} \text{ m})$</td>
<td>$0.17 \cdot 10^{-3}$</td>
<td>$42 \cdot 10^{-3}$</td>
<td>$0.22 \cdot 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$T_{41}(&lt; 3 \cdot 10^{-3} \text{ m})$</td>
<td>$1.6 \cdot 10^{-3}$</td>
<td>$13 \cdot 10^{-3}$</td>
<td>$2.1 \cdot 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$T_{42}(&lt; 1 \cdot 10^{-3} \text{ m})$</td>
<td>$0.06 \cdot 10^{-3}$</td>
<td>$15 \cdot 10^{-3}$</td>
<td>$0.08 \cdot 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$T_{43}(&lt; 0.01 \cdot 10^{-3} \text{ m/s})$</td>
<td>$10^{-6}$</td>
<td>$8 \cdot 10^{-3}$</td>
<td>$10^{-6}$</td>
<td>m/s</td>
</tr>
<tr>
<td>$T_{51}(&lt; 1 \text{ m})$</td>
<td>$0.77$</td>
<td>$1.45$</td>
<td>$0.15$</td>
<td>m</td>
</tr>
<tr>
<td>$T_{52}(&lt; 0.3 \text{ m/s})$</td>
<td>$0.13$</td>
<td>$0.11$</td>
<td>$0.09$</td>
<td>m/s</td>
</tr>
</tbody>
</table>

Table 5.1. Performance results for IRM

<table>
<thead>
<tr>
<th>Property (Criterion)</th>
<th>GPS L1C/A</th>
<th>Galileo E1b</th>
<th>Galileo E5a-I</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{11}(&lt; 5%)$</td>
<td>$6 \cdot 10^{-4}$</td>
<td>$2 \cdot 10^{-4}$</td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>$T_{12}(&lt; 5%)$</td>
<td>1.9</td>
<td>0.07</td>
<td>999.96</td>
<td>-</td>
</tr>
<tr>
<td>$T_{21}(&lt; 40 \cdot 10^{-3} \text{ m})$</td>
<td>$20 \cdot 10^{-3}$</td>
<td>$16 \cdot 10^{-3}$</td>
<td>3009</td>
<td>m</td>
</tr>
<tr>
<td>$T_{31}(&lt; 0.2 \cdot 10^{-3} \text{ m})$</td>
<td>$10^{-10}$</td>
<td>$10^{-10}$</td>
<td>1.91</td>
<td>m</td>
</tr>
</tbody>
</table>

Table 5.2. Performance results for CSM

The data that has been used in the simulations is Test Case 4, which is described in Chapter 3. The testbench was used to verify the performance of the simulator according to the test quantities mentioned in Chapter 4.2. The results can be seen in Table 5.1 and 5.2. A graphical representation of some the test quan-
tities for GPS L1C/A, Galileo E5a-I and Galileo E1b can be seen in Appendix B, while the figures in this chapter mainly will focus on problem areas.

The method of determining the code delay in CSM is the alternative method described in Subsection 4.1.2. This is due to its superior performance for narrowband signals.

It is important to notice that these criterions are extremely tough. The performance reached by the ROSIM prototype is extraordinary, since the error magnitude in most cases are in the fraction of mm region, while the excess phase is several thousand meters, and the geometrical distance between the satellite is thousands of km. However, there are discrepancies, and these will be discussed in the chapter below.

5.1 Discussion

In this section the results will be analyzed. The main focus will be on the GPS L1C/A signal, Galileo E5A-I and E1-B. The comparisons that are made are the outputs of CSM and IRM to the truth. As previously mentioned the mathematically defined Test Case 4 is used and the baseband data is saved in double precision floating-point numbers, preventing errors from drowning in noise. In its working form ROSIM will have noise in the input data from WOP, as well as introduced noise in the conversion to uint8. By looking at Table 5.1 and 5.2 it is possible to see some of the weaknesses of ROSIM. An initial comparison of the two tables shows that IRM has larger problems tracking the signal in the time domain than CSM has with tracking the signal in the frequency domain. However, this might be due to discrepancies in the generated baseband signal. Since CSM will output a constant delay of the code for each millisecond, there is an ambiguity for when the code phase is valid. In reality it would continuously vary throughout the epoch. There might also be discontinuities when the Doppler shift is applied, which would have a negative impact on IRM’s performance.

A quick inspection of the amplitude and carrier phase, which are the primary measurement quantities in RO, in Figure B.1, B.2, B.3, B.4, B.5, B.6, B.7 and B.8 indicates that the performance of CSM is within the requirements, delivering results with the accuracy needed for the retrieval procedure.

By looking at the performance of the Galileo E5a-I code, see Figures B.5 and B.6, it is obvious that CSM’s built-in IRR does not work properly for this case. This is due to the fact that the bandwidth of the signal is large, around 20 MHz (see Figure 2.6), so the correlation function is sampled with too low frequency and the method for estimating the code delay does not work. Because of this, the results of the simulations with the Galileo E5a-I will not be discussed in detail. The sample frequency of the correlation peak could be made arbitrarily high by increasing the internal upsampling of the original code. However, this would significantly slow down the simulation process.
5.1 Discussion

5.1.1 Amplitude

By comparing the relative amplitude of CSM to that of IRM relative the truth it is easy to see that CSM clearly agrees better with the truth than IRM does in this aspect. A comparison of the graphs for the absolute amplitude and that of the relative amplitude for IRM one notices that at the time when the relative amplitude has its maximum deviation from one the amplitude is close to zero and a small error might be scaled by a large factor.

The noise-like characteristics of the relative amplitude for CSM comes from the interpolation method used. The offset from one could have several explanations. E.g. could our approximation of the CCF lose signal energy and since the signal is band-limited at $\pm$ 10 MHz around the carrier frequency even more of the signal energy is lost. In general the amplitude modulation of the atmosphere is not constant over frequency, so we might also drop in amplitude in the received signal as some of the frequency components are scaled more than others.

In Figure 5.1 the absolute deviation from the true amplitude for both GPS L1C/A and Galileo E1-B are shown. GPS L1C/A clearly shows a smaller deviation from the truth. This can be explained by E1-B’s wider spectrum, and will merely cause a scaling of the entire occultation.

![Figure 5.1. Absolute amplitude errors.](image)

5.1.2 Code Phase

The method used to determine the code delay is described in Subsection 4.1.3. It introduces some errors when there is a change in the punctual value of the CCF.
Since the set of points used for the least-squares method estimation of the straight lines changes. I.e. the change in punctual value results in a code delay that is not continuous. However, these errors can be reduced by using the alternative method for approximating $\tau$. The difference between the two methods is about 7 cm, which can be seen in Figure 5.2. This is a big improvement. Another major improvement is that the value of the complex carrier is also interpolated so the delivered amplitude and carrier delay will consist of more accurate values.

![Figure 5.2. Absolute code phase errors.](image)

5.1.3 Carrier Phase

The estimation of the carrier phase done by CSM has very small errors, in the 0.1 nm order of magnitude. This can be explained by the fact that the phase of the CCF varies very little. The Galileo E1-B shows a larger error but well within the requirements. For the GPS L1C/A code the maximum error in IRM is around 0.2 mm, but otherwise it is on a $\mu$m scale. The Galileo E1-B code has larger maximum error which is around 4 cm. Regardless of the GNSS code IRM exhibits an impulse-like error characteristics around the mutlithpath region. This might be due to a time mismatch between CSM and IRM which becomes extra prominent when the phase varies quickly. This effect would not have been noticed with more realistic input data.
5.2 Conclusion

![Residual I/Q Phase, GPS L1C/A (zoomed)](image.png)

**Figure 5.3.** Residual I/Q phase, zoomed in to uncover variations that were hidden due to a large peak.

5.1.4 Residual I/Q Phase

The fact that the carrier phase for IRM is close to the truth indicates that the residual I/Q phase should be close to 0. At around 1.65 s the same impulse-like error as was noted for the carrier phase appears, which hides all the smaller variations of the residual I/Q phase in e.g. Figure B.2. A zoomed in version of the curve is plotted in Figure 5.3, where it is possible to see that there is no linear trend. This indicates that the Doppler down-conversion lacks a systematic error.

5.2 Conclusion

To fully verify the performance of ROSIM a full iteration, see Figure 1.1, has to be performed. I.e. an atmospheric profile in the form of temperature, pressure and water vapour should be transformed into refractive indices and input to WOP, which generates time-varying multi-frequency transfer functions. ROSIM uses the WOP output to generate the complex carrier at two frequencies. ROPP can then retrieve the atmospheric profiles from the ROSIM output. A comparison between the ROPP output and the WOP input will be able to provide the complete error characteristics. However, the test cases permit a detailed verification of ROSIM, and they show that it accomplished very good results. The error analysis performed in the chapter above is for an extreme test case, where discrepancies and errors will be extra noticeable. For a more realistic scenario most of what has previously been mentioned will completely disappear in noise.
Bibliography


Appendix A

Approximative Correlation

To reduce the processing time when calculating the cross-correlation function (CCF) an approximative CCF, which is valid when the input have a small bandwidth, is used.

\[
C[k] \approx \frac{1}{N} \mathcal{F}^{-1}\{X[i]Y^*[i]\} \tag{A.1}
\]

The CCF is defined as follows for a periodic signal with period \(N\):

\[
C[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n + k]y^*[n] \tag{A.2}
\]

By replacing \(x[n+k]\) and \(y^*[n]\) by their respective Fourier transform (A.2) becomes:

\[
C[k] = \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{N} \sum_{l=0}^{N-1} X[l]e^{\frac{2\pi j(n+l)}{N}} \frac{1}{N} \sum_{m=0}^{N-1} Y^*[m]e^{-\frac{2\pi jmn}{N}} \tag{A.3}
\]

By changing the order of the summations and manipulating the exponential terms (A.3) becomes:

\[
C[k] = \frac{1}{N^3} \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} X[l]Y^*[m]e^{\frac{2\pi jlk}{N}} \sum_{n=0}^{N-1} e^{\frac{2\pi j(n-l)m}{N}} \tag{A.4}
\]

The last summation can be calculated directly as:

\[
\sum_{n=0}^{N-1} e^{\frac{2\pi jn(l-m)}{N}} = \frac{1 - e^{\frac{2\pi j(l-m)}{N}}}{1 - e^{\frac{2\pi j(l-m)}{N}}} = \frac{\sin(\pi(l-m))}{\sin(\pi(l-m))} e^{\frac{\pi j(N-1)(l-m)}{N}} \tag{A.5}
\]

By using the fact that \(X[l]\) and \(Y^*[m]\) have a small bandwidth one can make the approximation that

\[
X[l]Y^*[m] = \begin{cases} X[l]Y^*[m] & \text{if } l = m \\ 0 & \text{otherwise} \end{cases}
\]
With this approximation (A.5) becomes:

$$\lim_{k \to 0} \frac{\sin \pi k}{\sin \frac{\pi k}{N}} e^{\pi j(N-1)k} = N$$

(A.6)

By then inserting (A.6) in (A.4) the following is obtained:

$$C[k] \approx \frac{1}{N^2} \sum_{l=0}^{N-1} X[l] Y^*[l] e^{2\pi jlk/N}$$

(A.7)

(A.7) can easily be identified as an inverse Fourier transform, which gives the final expression for the approximative CCF:

$$C[k] \approx \frac{1}{N} \mathcal{F}^{-1} \{ X[i] Y^*[i] \}$$
Appendix B

Simulation Results

ROSIMTB provides an excellent method of evaluating the CSM and IRM processing chain. On the following pages a graphical representation of a number of simulations has been run, primarily on Test Case 4 using different GNSS signals, is presented. The figures are evaluated and discussed in Chapter 5.

The last two figures show how ROSIM works in a more realistic scenario. Due to the noise in the input data and the added noise in the uint8 conversion many of the errors we have pointed out drowns. This implies that the performance requirements has been reached. No real truth is available in such a scenario. However, the carrier phase should correspond to the phase characteristics of the carrier frequency in the input data, and similarly should the signal amplitude correspond to the amplitude of the carrier frequency component in the input data. The code phase can not accurately be estimated since the phase curve is not linear over frequency.
Figure B.1. Signal: GPS L1C/A, Data: Test Case 4.
Figure B.2. Signal: GPS L1C/A, Data: Test Case 4.
Figure B.3. Signal: Galileo E1B, Data: Test Case 4.
Figure B.4. Signal: Galileo E1b, Data: Test Case 4.
Figure B.5. Signal: Galileo E5A-I, Data: Test Case 4.
<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Residual I/Q-phase</th>
<th>Residual code phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0 \times 10^{-3}</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>0 \times 10^{-3}</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0 \times 10^{-3}</td>
</tr>
<tr>
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<td>0</td>
<td>0 \times 10^{-3}</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0 \times 10^{-3}</td>
</tr>
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<td>0 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Figure B.6. Signal: Galileo E5A-I, Data: Test Case 4.
Figure B.7. Signal: GPS L1C/A, Data: University of Graz Data, case 5. Baseband data was saved in uint8.
Figure B.8. Signal: GPS L1C/A, Data: University of Graz Data, case 5. Baseband data was saved in uint8.