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DOES NON-ORTHOGONAL SPECTRUM SHARING IN THE SAME CELL IMPROVE THE SUM-RATE OF WIRELESS OPERATORS?

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ABSTRACT

We study non-orthogonal spectrum sharing to determine under what circumstances operators can gain by such sharing. To model the spectrum sharing, we use the multiple-input single-output (MISO) interference channel (IC) assuming that the operators transmit in the same band. For the baseline scenario of no sharing, we use the MISO broadcast channel (BC) assuming that the operators transmit in disjoint bands. For both the IC and BC, we give achievable (lower) and upper bounds on the maximum sum-rate. While these bounds are well-known we also propose a new fast algorithm for finding a lower bound on the sum-rate of the BC using linear beamforming.

We use the bounds to numerically evaluate the potential gain of non-orthogonal spectrum sharing. In this study we assume that the operators efficiently utilize all their spatial degrees of freedom. We will see that the gains from spectrum sharing under these circumstances are limited.

Index Terms— Spectrum sharing, multiple-input single-output (MISO), interference channel, broadcast channel, beamforming

1. INTRODUCTION

A series of recent activities have promoted the idea that operators of cellular wireless networks would benefit from sharing spectrum resources. Activities reported in e.g., [1], suggest inter-operator sharing of spectrum on small timescales (comparable to the fading coherence interval), using advanced scheduling and multiple antenna techniques. The sharing concept makes technical sense since spectrum is a finite resource, so whenever one operator does not need all of its resources, another one may borrow from it. In the long run both could gain from doing so. Conceivably, since spectrum is expensive, larger revenues could be generated by better spectrum utilization.

Spectrum sharing comes in two basic forms; orthogonal sharing and non-orthogonal sharing [2]. Orthogonal sharing means that each operator owns and operates in an independent piece of spectrum, but that they can borrow spectrum from each other on a need basis. The borrower can use the borrowed piece of spectrum exclusively.

Non-orthogonal sharing, by contrast, refers to permitting operators to transmit concurrently in the same time-frequency resource. Fundamentally, a scenario with two transmitters (TX_1 , TX_2 herein, belonging to the two operators) using the same time-frequency resource to send data to two receivers (RX_1 , RX_2 herein, associated

with TX_1 and TX_2 respectively) constitutes an interference channel (IC) [3]. For the TX_1 - RX_1 and TX_2 - RX_2 pairs to achieve good performance, it is required that they coordinate their operations. Multiple antennas at the TXs is thereby a fundamental enabler. With multiple antennas at the TXs but a single antenna at the RXs, we have a multiple-input single-output (MISO) IC. It has been argued that non-orthogonal spectrum sharing can bring substantial benefits in terms of increased sum-rate. The paper [2], for example, reported gains close to a factor of two.

In non-orthogonal sharing, the two operators compete for resources. However, by cooperating in their choice of transmission strategies they can achieve rate pairs that are Pareto optimal [4]. An appropriate approach to analyze the trade-off involved here is to use game theory. Game theoretic analysis in the context of spectrum sharing and the MISO IC was probably first popularized by [4]. This work has inspired a surge of results on characterization of Pareto-optimal transmit strategies [5]; fast methods for computing such Pareto-optimal outcomes [6, 7]; and various beamforming techniques [7, 8] and extensions to frequency selective channels [9].

One major practical difficulty in making non-orthogonal sharing work is that channel state information (CSI) must be shared between the TXs that belong to different operators. This must be done on a timescale substantially smaller than the channel coherence time and with sufficient accuracy for coherent beamforming to work. This also includes the CSI of the interfering channels, i.e. TX_1 would have to know the channel $TX_1 \rightarrow RX_2$ and vice versa. This requires a coordinated approach to training, and schemes that facilitate over-hearing of training sequences. Therefore, for non-orthogonal inter-operator sharing to have much practical potential, the gains in communication performance must be substantial.

The objective of this paper is to examine, fundamentally, and assuming the CSI transfer problem can be solved, under what conditions non-orthogonal sharing can provide gains as compared to conventional operations. Specifically, 1) we model non-orthogonal spectrum sharing via a MISO IC, and the reference scenario as a MISO broadcast channel (BC); 2) we summarize known upper and achievable bounds on the sum-rate for these models in Secs. 3 and 4, respectively; 3) we evaluate the bounds for flat and frequency selective fading in Sec. 5; 4) we draw conclusions on what operating points where non-orthogonal spectrum sharing is likely to work at all. Sec. 6 summarizes these conclusions.

While most modeling techniques and algorithms used in this paper are known, to our knowledge their use in the context in question is new and provides considerable new insight. The fast method we use to compute achievable sum-rates for the MISO BC using linear transmitter processing (Sec. 4.1) extends the work for the MISO IC in [6] and represents a new algorithmic contribution; it is substantially faster than the method developed specifically for the same purpose in [10].

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2. MODELS

The system consists of a cell with two coexisting TXs belonging to different operators. The total spectrum is split into $2N_s$ subchannels. The bandwidth of each subchannel is the coherence bandwidth W_c . Hence, the total bandwidth for the system is $2B = 2W_c N_s$. For the non-sharing case, each operator owns a spectrum of bandwidth B , i.e., N_s subchannels. For the sharing case, both operators use the entire $2B$ band, see Fig. 1. We assume that each TX is equipped with $n_t = 2$ antennas. Hence, for the non-sharing and sharing scenarios, we can serve two users and one user, respectively, per TX.¹

For the spectrum sharing scenario, we assume that the operators have one RX (user) each to serve. These have different channels. Summing rates over the subchannels has the interpretation of ergodic capacity (coding over the subchannels) and summing over the users has the interpretation of sum-rate. Also, each TX can spend power P . Hence, the total power spent in the system is $2P$. Hence the average power per served user is P , i.e., $P/2N_s$ per user per subchannel. In order to meet the power constraint, we must have

$$\sum_{k=1}^{2N_s} P_{i,k} \leq P \quad (1)$$

for each TX_{*i*} where $P_{i,k}$ is the power sent by TX_{*i*} on subchannel k .

For the non-sharing scenario, each operator serves two users. Each operator can spend power P over the N_s subchannels. Hence, the average power per user and subchannel is $P/2N_s$.² In order to meet the power constraint for the non-sharing scenario, we must have

$$\sum_{i=1}^2 \sum_{k=1}^{N_s} P_{i,k} \leq P, \quad (2)$$

where $P_{i,k}$ is the power spent on user i on subchannel k .

As performance measure we use the average rate over users and subchannels. Maximizing the average rate is the same as maximizing the sum-rate. The motivation for this choice of performance measure is that it gives the highest possible throughput of the operators.

The analysis that follows will use information theoretic results for the MISO IC [5, 15] and BC [10, 16, 17]. For the MISO IC, we define $\mathbf{h}_{ij,k} \in \mathbb{C}^{n_t}$ to be the (conjugated) channel vector for the link TX_{*i*} → RX_{*j*} for $i, j \in \{1, 2\}$ on subchannel k . The channels for the MISO BC model can be obtained from the MISO IC model by setting $\mathbf{h}_{ij,k} = \mathbf{h}_{j,k}$, $i, j = 1, 2$ where $\mathbf{h}_{i,k}$ is the channel between the TX and RX_{*i*} [5]. For both scenarios we assume that the TXs have instantaneous CSI.

3. NON-ORTHOGONAL SHARING: THE MISO IC

The problem of finding the capacity region of the general IC is still open. For the single-antenna case, it was shown that interference does not reduce the capacity when interference is strong [3]. One good known achievable rate region for the general IC is given in [11]. Outer bounds of the capacity region were provided in [12]. During

¹As a variation, in the sharing scenario, four users in total could be served (like in the non-sharing setup), one in the first B band and one in the second B band. This however would not change the conclusion of our study, since the metric we will use is an average rate over all users and subchannels.

²We have chosen to keep the total emitted power per cell the same in the non-sharing and sharing scenarios. By contrast, in [2], it was assumed that the total emitted power per cell in the sharing scenario was twice that of the non-sharing scenario. It is not clear what future regulatory requirements would look like if non-orthogonal spectrum sharing becomes reality.

Fig. 1. Illustration of the spectrum sharing scenarios. The upper and lower parts of the figure show the non-orthogonal sharing and non-sharing scenarios, respectively.

the last decade, the multiple antenna IC has gained a lot of interest. The so-called vector Gaussian IC was introduced in [13]. Many of the existing achievable and upper bounds on the capacity region of the multiple-antenna IC are extensions from the single-antenna case. Recent results on the capacity region of the multiple-antenna IC are reported in [14, 15]. In this section, we summarize some lower and upper bounds on the sum-rate of the MISO IC.

3.1. Lower (Achievable) Bounds Using Linear Beamforming

Here, we first give the system model for the MISO IC using linear beamforming. Then, by using such a transmission scheme we obtain lower (achievable) bounds on the sum-capacity of the MISO IC. The matched-filtered symbol-sampled complex baseband data received by the RXs on subchannel k for the MISO IC is modeled as

$$\begin{aligned} y_{1,k} &= \sqrt{P_{1,k}} \mathbf{h}_{11,k}^H \mathbf{w}_{1,k} s_{1,k} + \sqrt{P_{2,k}} \mathbf{h}_{21,k}^H \mathbf{w}_{2,k} s_{2,k} + e_{1,k} \\ y_{2,k} &= \sqrt{P_{2,k}} \mathbf{h}_{22,k}^H \mathbf{w}_{2,k} s_{2,k} + \sqrt{P_{1,k}} \mathbf{h}_{12,k}^H \mathbf{w}_{1,k} s_{1,k} + e_{2,k}. \end{aligned} \quad (3)$$

Also, $P_{i,k}, \mathbf{w}_{i,k} \in \mathbb{C}^{n_t}$, $\|\mathbf{w}_{i,k}\| = 1$ are the power and beamforming vector allocated by TX_{*i*}, respectively, and $s_{i,k} \sim \mathcal{CN}(0, 1)$ is the transmitted symbol for RX_{*i*}. By $e_{i,k} \sim \mathcal{CN}(0, 1)$ we model the noise at RX_{*i*}.

Assuming linear beamforming using the beamforming vectors $\mathbf{w}_{i,k}$ and power allocation $P_{i,k}$, $i = 1, 2$, $k = 1, \dots, N_s$, the average rate in bits per channel use (bpcu) per served user is

$$\bar{R}^{\text{IC}} = \frac{1}{2} \sum_{i=1}^2 \frac{1}{2N_s} \sum_{k=1}^{2N_s} R_i^{\text{IC}}(P_{1,k}, P_{2,k}, \mathbf{w}_{1,k}, \mathbf{w}_{2,k}), \quad (4)$$

where³ [5]

$$R_1^{\text{IC}}(P_{1,k}, P_{2,k}, \mathbf{w}_{1,k}, \mathbf{w}_{2,k}) \triangleq \log \left(1 + \frac{P_{1,k} |\mathbf{h}_{11,k}^H \mathbf{w}_{1,k}|^2}{P_{2,k} |\mathbf{h}_{21,k}^H \mathbf{w}_{2,k}|^2 + 1} \right). \quad (5)$$

The inner sum of (4) is the ergodic rate for the i th user while the outer sum yields the average rate of the two users.

Zero-forcing (ZF) beamforming: ZF is a linear transmit beamforming strategy that cancels out the interference to the unintended

³Whenever an expression is given for only one user, the expression for the other is obtained by interchanging the indices. In this paper, we use the base-2 logarithm.

receiver. The ZF beamforming vectors are [5],

$$\mathbf{w}_{1,k}^{\text{ZF}} = \frac{\mathbf{\Pi}_{\mathbf{h}_{12,k}}^\perp \mathbf{h}_{11,k}}{\|\mathbf{\Pi}_{\mathbf{h}_{12,k}}^\perp \mathbf{h}_{11,k}\|}, \quad \mathbf{w}_{2,k}^{\text{ZF}} = \frac{\mathbf{\Pi}_{\mathbf{h}_{21,k}}^\perp \mathbf{h}_{22,k}}{\|\mathbf{\Pi}_{\mathbf{h}_{21,k}}^\perp \mathbf{h}_{22,k}\|}, \quad (6)$$

where $\mathbf{\Pi}_{\mathbf{x}}^\perp \triangleq \mathbf{I} - \mathbf{x}\mathbf{x}^H / \|\mathbf{x}\|^2$ is the orthogonal projection onto the orthogonal complement of the vector $\mathbf{x}$. Using the ZF beamforming vectors in (4), we get

$$\bar{R}^{\text{ICZF}} = \frac{1}{4N_s} \sum_{i=1}^2 \sum_{k=1}^{2N_s} \log \left(1 + P_{i,k} \left\| \mathbf{\Pi}_{\mathbf{h}_{ij,k}}^\perp \mathbf{h}_{ii,k} \right\|^2 \right). \quad (7)$$

We will consider two possible power allocations. The first is equal power allocation, i.e., $P_{i,k} = P/2N_s$. The second is to maximize (7) with respect to $P_{i,k}$, $i = 1, 2$ and $k = 1, \dots, 2N_s$ using water-filling.

Clearly, these schemes yield achievable bounds for the capacity. However, as we will see next, we can obtain better achievable bounds by also optimizing the beamforming vectors.

Maximum sum-rate using linear beamforming: Here, we maximize (4)–(5) with respect to the beamforming vectors, assuming equal power allocation over the subchannels. That is, for each subchannel, we solve⁴

$$\max_{\mathbf{w}_1, \mathbf{w}_2} \sum_{i=1}^2 R_i^{\text{IC}}(P/2N_s, P/2N_s, \mathbf{w}_1, \mathbf{w}_2) \quad (8)$$

$$\text{s. t.} \quad \|\mathbf{w}_i\|^2 = 1, \quad i = 1, 2. \quad (9)$$

We can equivalently write (8)–(9) as

$$\max_{\mathbf{w}_1, \mathbf{w}_2} \log \left(1 + \frac{|\mathbf{h}_{11}^H \mathbf{w}_1|^2}{|\mathbf{h}_{21}^H \mathbf{w}_2|^2 + \frac{2N_s}{P}} \right) + \log \left(1 + \frac{|\mathbf{h}_{22}^H \mathbf{w}_2|^2}{|\mathbf{h}_{12}^H \mathbf{w}_1|^2 + \frac{2N_s}{P}} \right) \quad (10)$$

$$\text{s. t.} \quad \|\mathbf{w}_i\|^2 = 1, \quad i = 1, 2. \quad (11)$$

The problem (10)–(11) is non-convex. However, by using our result from [6], we find the optimum of (10)–(11) in an efficient way. Specifically, we find the Pareto boundary of the two-user MISO IC in closed-form using the algorithm in [6]. In order to find the maximum sum-rate point, we then perform a one-dimensional search over the Pareto boundary.

3.2. Upper Bounds

Here, we give two upper bounds on the MISO IC sum-capacity. The first bound is a trivial bound, where interference is ignored. The second bound is more sophisticated and hence tighter.

Trivial upper bound assuming no interference: In order to find a trivial upper (TU) bound on the MISO IC capacity, we neglect the interference terms in (3). For this case, it is optimal to use the maximum-ratio transmission (MRT) beamforming vectors, i.e. $\mathbf{w}_{1,k}^{\text{MRT}} = \mathbf{h}_{11,k} / \|\mathbf{h}_{11,k}\|$. Inserting these in (4), we get

$$\bar{R}^{\text{ICTU}} = \frac{1}{4N_s} \sum_{i=1}^2 \sum_{k=1}^{2N_s} \log \left(1 + P_{i,k} \|\mathbf{h}_{ii,k}\|^2 \right). \quad (12)$$

To obtain an upper bound, we optimize the power using water-filling.

⁴In order to simplify notation, we omit the subchannel index when we focus on a single subchannel.

Non-trivial upper bound: As an improvement of the trivial upper bound given above, we give an upper bound based on [15]. From there, we have

$$\begin{aligned} R_1 + R_2 \leq & \min \{ \log(1 + P_1 \|\mathbf{h}_{11}\|^2) + \log(1 + P_2 \|\mathbf{h}_{22}\|^2), \\ & \log(1 + P_1 \|\mathbf{h}_{12}\|^2 + P_2 \|\mathbf{h}_{22}\|^2) + \log(1 + P_1 K_1), \\ & \log(1 + P_2 \|\mathbf{h}_{21}\|^2 + P_1 \|\mathbf{h}_{11}\|^2) + \log(1 + P_2 K_2), \\ & \log(1 + P_2 \|\mathbf{h}_{21}\|^2 + P_1 K_1) \\ & + \log(1 + P_1 \|\mathbf{h}_{12}\|^2 + P_2 K_2) \} \end{aligned} \quad (13)$$

where

$$K_1 \triangleq \|\mathbf{h}_{11}\|^2 - P_1 |\mathbf{h}_{12}^H \mathbf{h}_{11}|^2 / (1 + P_1 \|\mathbf{h}_{12}\|^2) \quad (14)$$

$$K_2 \triangleq \|\mathbf{h}_{22}\|^2 - P_2 |\mathbf{h}_{21}^H \mathbf{h}_{22}|^2 / (1 + P_2 \|\mathbf{h}_{21}\|^2). \quad (15)$$

We see that the first term in (13) corresponds to the trivial upper bound given above.

The bounds in [15] are derived for a single-channel MIMO IC. By formulating the multi-channel system as a single-channel MIMO system with block-diagonal channel matrices, we can extend (13) to also include optimal power allocation. However, this is not done herein. Hence, the bound in (13) is only valid for a given power allocation. As we will see in Sec. 5, power control appears to yield only a marginal difference for the conclusions of the paper.

4. NO SHARING: THE MISO BC

The MISO BC models the scenario without sharing and there is no interference between the operators. Here, no inter-operator CSI transfer is needed. The capacity region of the general BC is still an open problem. Nevertheless, the capacity region of the Gaussian BC is known. For the two-user MISO BC, the sum-capacity was derived in closed form in [16]. This result was later extended to the entire capacity region of the multiple-input multiple-output (MIMO) BC in [17]. Here, we give lower and upper bounds for the sum-capacity of the MISO BC.

4.1. Lower (Achievable) Bounds Using Linear Beamforming

Assuming linear beamforming using the beamforming vectors $\mathbf{w}_{i,k}$, and power allocation $P_{i,k}$, $i = 1, 2$, $k = 1, \dots, N_s$, the average rate over the users per subchannel is

$$\bar{R}^{\text{BC}} = \frac{1}{2} \sum_{i=1}^2 \frac{1}{N_s} \sum_{k=1}^{N_s} R_i^{\text{BC}}(P_{1,k}, P_{2,k}, \mathbf{w}_{1,k}, \mathbf{w}_{2,k}), \quad (16)$$

where [5]

$$R_1^{\text{BC}}(P_{1,k}, P_{2,k}, \mathbf{w}_{1,k}, \mathbf{w}_{2,k}) \triangleq \log \left(1 + \frac{P_{1,k} |\mathbf{h}_{11,k}^H \mathbf{w}_{1,k}|^2}{P_{2,k} |\mathbf{h}_{11,k}^H \mathbf{w}_{2,k}|^2 + 1} \right). \quad (17)$$

Zero-forcing beamforming: The ZF beamforming vectors for the MISO BC are similar to those of the MISO IC, namely [10],

$$\mathbf{w}_{1,k}^{\text{ZF}} = \frac{\mathbf{\Pi}_{\mathbf{h}_{2,k}}^\perp \mathbf{h}_{1,k}}{\|\mathbf{\Pi}_{\mathbf{h}_{2,k}}^\perp \mathbf{h}_{1,k}\|}, \quad \mathbf{w}_{2,k}^{\text{ZF}} = \frac{\mathbf{\Pi}_{\mathbf{h}_{1,k}}^\perp \mathbf{h}_{2,k}}{\|\mathbf{\Pi}_{\mathbf{h}_{1,k}}^\perp \mathbf{h}_{2,k}\|} \quad (18)$$

Using the ZF beamforming vectors, we get

$$\bar{R}^{\text{BCZF}} = \frac{1}{2N_s} \sum_{i=1}^2 \sum_{k=1}^{N_s} \log \left(1 + P_{i,k} \left\| \mathbf{\Pi}_{\mathbf{h}_{j,k}}^\perp \mathbf{h}_{i,k} \right\|^2 \right). \quad (19)$$

We consider two possible power allocations; either equal power allocation, i.e., $P_{i,k} = P/2N_s$ or maximizing (19) with respect to $P_{i,k}$, $i = 1, 2, k = 1, \dots, N_s$ using water-filling.

Maximum sum-rate using linear beamforming: We provide an algorithm for finding the maximum sum-rate of the MISO BC when equal power allocation over the subchannels is used. This method is an alternative to [10] where monotonic optimization using polyblocks was used. For each subchannel we solve

$$\max_{\mathbf{w}_1, \mathbf{w}_2} \sum_{i=1}^2 R_i^{\text{BC}}(P_1, P_2, \mathbf{w}_1, \mathbf{w}_2) \quad (20)$$

$$\text{s. t.} \quad \|\mathbf{w}_i\|^2 = 1, \quad i = 1, 2, \quad (21)$$

$$P_1 + P_2 = P/N_s. \quad (22)$$

For a given power allocation (P_1, P_2) that meets (22), we solve

$$\max_{\mathbf{w}_1, \mathbf{w}_2} \log \left(1 + \frac{P_1 |\mathbf{h}_1^H \mathbf{w}_1|^2}{P_2 |\mathbf{h}_1^H \mathbf{w}_2|^2 + 1} \right) + \log \left(1 + \frac{P_2 |\mathbf{h}_2^H \mathbf{w}_2|^2}{P_1 |\mathbf{h}_2^H \mathbf{w}_1|^2 + 1} \right) \quad (23)$$

$$\text{s. t.} \quad \|\mathbf{w}_i\|^2 = 1, \quad i = 1, 2 \quad (24)$$

If we use the effective channels $\tilde{\mathbf{h}}_{ij} \triangleq \sqrt{P_i} \mathbf{h}_j$, $i, j = 1, 2$, we can use the fast method of [6], to find the optimum of (23)–(24) for fixed P_1, P_2 . This requires a one-dimensional search. By traversing all feasible power allocations (P_1, P_2) with $P_2 = P/N_s - P_1$, we will find the optimum of (20)–(22). For example, we can search over P_1 , which also is a one-dimensional search. Hence, in order to solve (20)–(22), we have to perform a two-dimensional search, which is faster than the algorithm in [10].

4.2. Upper Bounds

As for the MISO IC in Sec. 3.2, we give two upper bounds; a trivial bound and the sum-capacity.

Trivial Upper Bound Assuming No Interference: We obtain a trivial upper bound by ignoring the interference and using MRT, i.e. $\mathbf{w}_{i,k}^{\text{MRT}} = \mathbf{h}_{i,k} / \|\mathbf{h}_{i,k}\|$. Then the upper bound on (16) is

$$\bar{R}^{\text{BCTU}} = \frac{1}{2N_s} \sum_{i=1}^2 \sum_{k=1}^{N_s} \log(1 + P_{i,k} \|\mathbf{h}_{i,k}\|^2). \quad (25)$$

We obtain the upper bound by performing power allocation using water-filling. We note that if $\mathbf{h}_{1,k}$ and $\mathbf{h}_{2,k}$ are orthogonal, then these trivial upper bounds will coincide with the achievable ZF bounds.

Sum-capacity of the MISO BC: The sum-capacity per sub-channel for the two-user MISO BC under the sum-power constraint P/N_s is found on closed form to be [16]

$$C_{\text{sum}}^{\text{BC}} = \begin{cases} \log \left(1 + \frac{P \|\mathbf{h}_1\|^2}{N_s} \right), & \frac{P}{N_s} \leq A, \\ \log \left(\frac{\left(\frac{P}{N_s} \det(\mathbf{H}\mathbf{H}^H) + \text{trace}(\mathbf{H}\mathbf{H}^H) \right)^2 - 4 \|\mathbf{h}_2^H \mathbf{h}_1\|^2}{4 \det(\mathbf{H}\mathbf{H}^H)} \right), & \frac{P}{N_s} > A, \end{cases} \quad (26)$$

where $A \triangleq (\|\mathbf{h}_1\|^2 - \|\mathbf{h}_2\|^2) / \det(\mathbf{H}\mathbf{H}^H)$ and $\mathbf{H} \triangleq [\mathbf{h}_1 \ \mathbf{h}_2]^T$. Without loss of generality, we have assumed that $\|\mathbf{h}_1\|^2 \geq \|\mathbf{h}_2\|^2$. In order to achieve the capacity (26), we need dirty-paper coding.

By using the duality property of the MIMO BC and MIMO multiple access channel (MAC) [18], we can include power control across the subchannels. The trick is to write the multi-channel system as an equivalent MIMO system. However, for finding the capacity for this case, numerical methods are needed.

Fig. 2. Achievable sum-rate bounds using ZF beamforming, with and without water-filling.

Fig. 3. Achievable sum-rate bounds using linear beamforming and equal power allocation across the subchannels.

Fig. 4. Upper sum-rate bounds for sharing and no sharing.

5. NUMERICAL COMPARISONS

In this section we provide numerical evaluations of the bounds provided in Secs. 3 and 4. We assume unit-variance Rayleigh-fading, i.e., $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. This corresponds to the scenario where the TXs are approximately co-located. We provide numerical results for the cases of $P/N_s = 7$ dB and $P/N_s = 17$ dB and $N_s = 4$ (the main conclusions seem to be insensitive of N_s). For each bound, we perform 10000 Monte Carlo runs and plot the cumulative density function (cdf) of the rate. Compared to time-average, the cdf gives us a better insight about extreme-case performance. As a general note, we see that the lower and higher choices of transmit power gives rates of approximately 1-3 and 3-6 bpcu and user, respectively.

In Fig. 2, we plot the cdfs of the bounds achievable using ZF. As expected, we get slightly higher rates with water-filling than without. Also, the spectrum sharing yields slightly higher rates than the non-sharing baseline. This observation holds for both choices of power.

For the achievable bounds using per subchannel sum-rate maximization and linear beamforming, we see in Fig. 3 that the non-sharing case yields rates that are approximately 0.2 bpcu and user higher than in the sharing case. This holds for both the high and low power. This difference could be explained by the fact that we for the MISO BC do power control among the users in one subchannel.

Fig. 4 shows the upper bounds. There, we can make a number of interesting observations. First, the trivial bounds for no-sharing and non-orthogonal sharing coincide. Second, we observe that the non-trivial upper bound (13) for the IC yields higher rates than the sum-capacity for the BC. The difference is approximately 0.1 bpcu and user for $P/N_s = 7$ dB and slightly more for $P/N_s = 17$ dB. Probably, the reason for this gap is the fact that (13) is an upper bound. However, we cannot for sure say that we can achieve higher rates with spectrum sharing than without, or vice versa. Third, for high transmit power, we notice that the trivial upper bounds yield rates that are approximately 0.7 bpcu and user higher than the sum-capacity bound for the MISO BC.

6. CONCLUSIONS

We have analyzed the potential gains of non-orthogonal spectrum sharing by using first principles based information theoretic models. In conclusion, the gain of non-orthogonal sharing appears to be very limited. The intuition behind this result is that without sharing, each operator has a bandwidth of B and two spatial degrees of freedom (DoF) per bandwidth unit. By contrast, with sharing each operator has a bandwidth of $2B$ and two spatial DoF per bandwidth unit of which one can be used for transmission and the other must be used to suppress interference.

The potential gain might be larger if, in the non-sharing scenario, highly suboptimal transmission schemes are used (not fully utilizing all spatial DoF); however, in this case, better utilization of spatial DoF per operator appears to be more attractive than non-orthogonal sharing between operators. Taking into account the extra CSI needed for non-orthogonal sharing, it is questionable whether non-orthogonal sharing can give any gain in practice.

In this study, channels were randomly drawn but given. This corresponds to the case of delay constrained traffic. For future studies, multi-user diversity, i.e., the possibility of selecting two users from a larger pool of users per subchannel, should be incorporated. However, results not shown herein indicate that this does not change the main conclusion.

7. REFERENCES

- [1] SAPHYRE, "Sharing physical resources," <http://www.saphyre.eu>.
- [2] E. A. Jorswieck, L. Badia, T. Fahldieck, M. Haardt, E. Karipidis, J. Luo, and R. Pisz, "Spectrum sharing improves the network efficiency for wireless operators," *IEEE Commun. Mag.*, 2012, submitted. Available at <http://www.saphyre.eu>.
- [3] A. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, pp. 60–70, Jan. 1978.
- [4] E. G. Larsson and E. A. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE J. Sel. Areas Commun.*, pp. 1059–1069, Sept. 2008.
- [5] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Trans. Signal Process.*, pp. 5292–5296, Oct. 2008.
- [6] J. Lindblom, E. Karipidis, and E. G. Larsson, "Closed-form parameterization of the Pareto boundary for the two-user MISO interference channel," in *Proc. IEEE ICASSP*, May 2011.
- [7] R. Mochaourab and E. Jorswieck, "Exchange economy in two-user multiple-input single-output interference channels," *IEEE J. Sel. Topics Signal Process.*, pp. 151–164, Apr. 2012.
- [8] R. Zakhour, Z. K. M. Ho, and D. Gesbert, "Distributed beamforming coordination in multicell MIMO channels," in *Proc. IEEE VTC*, Apr. 2009.
- [9] A. Leshem and E. Zehavi, "Game theory and the frequency selective interference channel," *IEEE Signal Process. Mag.*, pp. 28–40, Sept. 2009.
- [10] E. A. Jorswieck and E. G. Larsson, "Linear precoding in multiple antenna broadcast channels: Efficient computation of the achievable rate region," in *Proc. WSA*, Feb. 2008.
- [11] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, pp. 49–60, Jan. 1981.
- [12] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. Inf. Theory*, pp. 581–586, Mar. 2004.
- [13] S. Vishwanath and S. A. Jafar, "On the capacity of the vector Gaussian interference channel," in *Proc. IEEE ITW*, Oct. 2004.
- [14] X. Shang, B. Chen, G. Kramer, and H. V. Poor, "Capacity region and sum-rate capacities of vector Gaussian interference channels," *IEEE Trans. Inf. Theory*, pp. 5030–5044, Oct. 2010.
- [15] S. Karmakar and M. K. Varanasi, "Capacity of the MIMO interference channel to within a constant gap," in *Proc. IEEE ISIT*, Aug. 2011. Journal version available at <http://arxiv.org/abs/1102.0267>.
- [16] G. Caire and S. Shamai, "On the achievable throughput in the multiple antenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, pp. 1691–1706, July 2003.
- [17] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inf. Theory*, pp. 3936–3964, Sept. 2006.
- [18] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inf. Theory*, pp. 2658–2668, Oct. 2003.