Linköping Studies in Science and Technology
Dissertations, No. 1457

Spectrum Sensing Algorithms Based on Second-Order Statistics

Erik Axell

Division of Communication Systems
Department of Electrical Engineering (ISY)
Linköping University, SE-581 83 Linköping, Sweden
www.commsys.isy.liu.se

Linköping 2012

Spectrum Sensing Algorithms Based on Second-Order Statistics

© 2012 Erik Axell, unless otherwise noted.

ISBN 978-91-7519-876-7
ISSN 0345-7524
Printed in Sweden by LiU-Tryck, Linköping 2012

Pain is inevitable, suffering is optional

Haruki Murakami,
"What I Talk About When I Talk About Running"

Abstract

Cognitive radio is a new concept of reusing spectrum in an opportunistic manner. Cognitive radio is motivated by recent measurements of spectrum utilization, showing unused resources in frequency, time and space. Introducing cognitive radios in a primary network inevitably creates increased interference to the primary users. Secondary users must sense the spectrum and detect primary users' signals at very low SNR, to avoid causing too much interference. This dissertation studies this detection problem, known as spectrum sensing.

The fundamental problem of spectrum sensing is to discriminate an observation that contains only noise from an observation that contains a very weak signal embedded in noise. In this work, detectors are proposed that exploit known properties of the second-order moments of the signal. In particular, known structures of the signal covariance are exploited to circumvent the problem of unknown parameters, such as noise and signal powers or channel coefficients.

The dissertation is comprised of six papers, all in different ways related to spectrum sensing based on second-order statistics. In the first paper, we consider spectrum sensing of orthogonal frequency-division multiplexed (OFDM) signals in an additive white Gaussian noise channel. For the case of completely known noise and signal powers, we set up a vector-matrix model for an OFDM signal with a cyclic prefix and derive the optimal Neyman-Pearson detector from first principles. For the case of completely unknown noise and signal powers, we derive a generalized likelihood ratio test (GLRT) based on empirical second-order statistics of the received data. The proposed GLRT detector exploits the time varying correlation structure of the OFDM signal and does not require any knowledge of the noise or signal powers.

In the second paper, we create a unified framework for spectrum sensing of signals which have covariance matrices with known eigenvalue multiplicities. We derive the GLRT for this problem, with arbitrary eigenvalue multiplicities under both hypotheses. We also show a number of applications to spectrum sensing.

The general result of the second paper is used as a building block, in the third and fourth papers, for spectrum sensing of second-order cyclostationary signals received at multiple antennas and orthogonal space-time block coded (OSTBC) signals respectively. The proposed detector of the third paper exploits both the spatial and the temporal correlation of the received signal, from knowledge of the fundamental period of the cyclostationary signal and the eigenvalue multiplicities of the temporal covariance matrix.

In the fourth paper, we consider spectrum sensing of signals encoded with an OSTBC. We show how knowledge of the eigenvalue multiplicities of the covariance matrix are inherent owing to the OSTBC, and propose an algorithm that exploits that knowledge for detection. We also derive theoretical bounds on the performance of the proposed detector. In addition, we show that the proposed detector is robust to a carrier frequency offset, and propose another detector that deals with timing synchronization using the detector for the synchronized case as a building block.

A slightly different approach to covariance matrix estimation is taken in the fifth paper. We consider spectrum sensing of Gaussian signals with structured covariance matrices, and propose to estimate the unknown parameters of the covariance matrices using covariance matching estimation techniques (COMET). We also derive the optimal detector based on a Gaussian approximation of the sample covariance matrix, and show that this is closely connected to COMET.

The last paper deals with the problem of discriminating samples that contain only noise from samples that contain a signal embedded in noise, when the variance of the noise is unknown. We derive the optimal soft decision detector using a Bayesian approach. The complexity of this optimal detector grows exponentially with the number of observations and as a remedy, we propose a number of approximations to it. The problem under study is a fundamental one and it has applications in signal denoising, anomaly detection, and spectrum sensing for cognitive radio.

Populärvetenskaplig Sammanfattning

Ständigt ökade krav på högre datatakter i trådlösa kommunikationssystem, och begränsade eller outnyttjade frekvensband har lett till ett nytt koncept, kallat kognitiv radio. Traditionellt så tilldelas frekvensband, eller spektrum, under relativt långa tidsperioder och licensägaren har ensamrätt att utnyttja spektrumet för en specifik tillämpning och trådlös teknik. En övervägande majoritet av de spektra som är användbara för digital radiokommunikation är licensierade, och således till synes upptagna. Flertalet mätningar av det faktiska användandet av olika spektra har dock visat att de i praktiken är outnyttjade en stor del av tiden eller på vissa platser. Trots att de användbara frekvensbanden är tilldelade olika operatörer så finns det alltså outnyttjade resurser, så kallade "spektrumhål".

Den grundläggande idén med kognitiv radio är att återanvända dessa outnyttjade resurser. För att utnyttja dessa resurser måste en kognitiv radio därför kunna avgöra om någon redan använder ett frekvensband eller om det faktiskt finns ett spektrumhål. Denna detektionsprocess kallas spektrumavkänning (eng. spectrum sensing).

Denna avhandling föreslår algoritmer för spektrumavkänning som utnyttjar kända strukturer, eller korrelation, hos den signal som man vill detektera. Moderna trådlösa system använder olika sätt att representera och skydda data, till exempel genom modulation och kodning. För att skydda informationen mot olika typer av störningar så införs redundans till signalen på ett kontrollerat sätt. Denna redundans kan införas till exempel i olika tidpunkter av signalen (tidskorrelation) eller mellan signaler som skickas och tas emot på flera antenner samtidigt (rumskorrelation). I allmänhet är dessa metoder strikt reglerade i den standard som bestämmer hur ett visst trådlöst system är uppbyggt och fungerar. Eftersom frekvensband är tilldelade till operatörer endast för en viss tillämpning, så är det oftast känt vilken typ av radiosignal som eventuellt har sänts på ett visst frekvensband. Redundans kan också skapas hos mottagaren genom att använda flera antenner, och därmed ta emot flera olika versioner av

samma sända signal. Denna kunskap om den mottagna signalens struktur kan utnyttjas för att bättre kunna avgöra om det finns någon sändare som skickar en sådan signal eller inte. I avhandlingen föreslås metoder som utnyttjar både tids- och rumskorrelation. Några metoder kan tillämpas på ganska generella signalstrukturer, medan andra är designade mer specifikt för några typer av signaler som är väldigt vanliga i moderna trådlösa system.

Acknowledgments

First of all I would like to express my gratitude to my supervisor, Professor Erik G. Larsson, for guiding me in the right direction and always taking time for my scientific matters. Without your gentle pushes to keep me on track I would not have reached the final goal. Thank you Erik for your great help and encouragement!

I would also like to thank Professor emeritus Thomas Ericson, who ones gave me this opportunity of a research career in the first place.

Special thanks go also to my co-supervisors Jan-Åke Larsson and Danyo Danev. You bring structure to my sometimes confused thoughts in all mathematical issues. I am also very grateful to my other co-authors Geert Leus, H. Vincent Poor and Anton Blad for great cooperation.

Many thanks go to all my colleagues in the Communication Systems group and the former Data Transmission group, and to the neighboring research group Information Coding, formerly known as Information Theory and Image Coding. In particular, I express my sincere gratidue to my former colleagues and office neighbors Jonas Eriksson and Lasse Alfredsson. Thank you Jonas for always keeping your door open and for bringing clarity to all my concerns, and Lasse for being a great mentor in all teaching related matters and always spreading cheerfulness at work! I would also like to thank my friends and former colleagues Tobias Ahlström and Björn Hofström for all the great times we had during my early years as a Ph.D. student and ever since.

Finally I would like to thank my friends and family. In particular, I thank my parents Gunilla and Lars-Åke Axell for always supporting and encouraging me in whatever course I have taken in life. I dedicate the dissertation to my wife Hanna and my children Elliot, Alicia and, at the time of writing, expected third child. Thank you for your endless support, understanding and love!

Linköping, July 2012
Erik Axell

Contents

Abstract **v**

Populärvetenskaplig Sammanfattning (in Swedish) **vii**

Acknowledgments **ix**

I Introduction **1**

1 Background and Problem Formulation **3**

2 Fundamentals of Signal Detection **7**
2.1 Unknown Parameters 8
2.2 Constant False-Alarm Rate (CFAR) Detectors 9
2.3 Energy Detection 9
2.4 Fundamental Limits for Sensing: SNR Wall 11

3 Feature Detection Based on Second-Order Statistics **13**
3.1 Detectors Based on Autocorrelation Properties 14
3.2 Detectors Based on Cyclostationarity 18
3.3 Detectors that Rely on a Specific Structure of the Sample Covariance Matrix 19
3.4 Blind Detection 22

4 Cooperative Spectrum Sensing **25**
4.1 Soft Combining 25
4.2 Hard Combining 26

5 Contributions of the Dissertation **29**
5.1 Included Papers 29
5.2 Not Included Papers 32

References **35**

II Included Papers **39**

A Optimal and Sub-Optimal Spectrum Sensing of OFDM Signals in Known and Unknown Noise Variance **41**
1 Introduction . . . 44
1.1 Background . . . 44
1.2 Contributions . . . 45
2 Model . . . 46
3 Optimal Neyman-Pearson Detector . . . 47
3.1 Known σ_n^2 and σ_s^2 . . . 49
3.2 Special Cases . . . 51
3.3 Unknown σ_n^2 and σ_s^2 . . . 53
4 Detection Based on Second-Order Statistics . . . 54
4.1 GLRT-Approach for Unknown σ_n^2 and σ_s^2 . . . 54
4.2 Special Case: Known σ_n^2 and Unknown σ_s^2 . . . 59
4.3 Special Case: Known σ_n^2 and σ_s^2 . . . 59
4.4 Benchmarks . . . 60
4.5 Detector Complexity . . . 62
5 Comparisons . . . 62
5.1 Optimal Neyman-Pearson Detector . . . 63
5.2 Detectors Based on Second-Order Statistics . . . 65
6 Concluding Remarks . . . 69
A Efficient Computation of $\mathbf{x}^H \left(\mathbf{Q}_\tau^{-1} - \frac{1}{\sigma_n^2}\mathbf{I}\right)\mathbf{x}$ . . . 71
B Efficient Computation of $\det(\mathbf{Q}_\tau)$ . . . 72
C Moments of $\overline{R}_i$ . . . 73
D Moments of $\widetilde{R}_i$ . . . 75
E Derivation of $\mathrm{Cov}\left[\overline{R}_i, \widetilde{R}_i\right]$ . . . 77
References . . . 79

B A Unified Framework for GLRT-Based Spectrum Sensing of Signals with Covariance Matrices with Known Eigenvalue Multiplicities **81**
1 Introduction . . . 84
2 Model and Problem Formulation . . . 85
3 Detection . . . 85
3.1 ML-Estimation of the Covariance Matrices . . . 86
3.2 Generalized Likelihood-Ratio Test . . . 87
4 Spectrum Sensing Applications . . . 88
4.1 Multiple Receive Antennas (SIMO) . . . 88
4.2 Orthogonal Space-Time Block Codes . . . 89
4.3 Signal with Unknown Correlation Structure . . . 90
4.4 OFDM . . . 91
5 Monte-Carlo Simulations . . . 92
5.1 Benchmarks . . . 92

5.2 Numerical Results 92
6 Concluding Remarks 93
References 95

C Multiantenna Spectrum Sensing of a Second-Order Cyclostationary Signal 97
1 Introduction 100
2 Model and Problem Formulation 101
3 Synchronized Detection 101
3.1 GLRT Exploiting Known Eigenvalue Multiplicities 102
4 Unsynchronized Detection 103
5 Spectrum Sensing of an OFDM Signal 105
6 Numerical Results 105
7 Concluding Remarks 108
References 111

D Eigenvalue-Based Spectrum Sensing of Orthogonal Space-Time Block Coded Signals 113
1 Introduction 116
2 Synchronized Detection 117
2.1 Model and Problem Formulation 117
2.2 Optimal Genie Detection for known $\mathbf{H}$, σ^2 and γ^2 120
2.3 Detection with Unknown Parameters – GLRT 120
2.4 Theoretical Asymptotic Performance 123
3 Unsynchronized Detection 127
3.1 Carrier Frequency Offset (CFO) 127
3.2 Timing Synchronization Error 128
4 Numerical Results 129
4.1 Benchmarks 129
4.2 Simulations 130
5 Concluding Remarks 134
A Proof of Equivalent GLRT 135
B Asymptotic Performance of the GLRT 136
References 139

E Spectrum Sensing of Signals with Structured Covariance Matrices Using Covariance Matching Estimation Techniques 143
1 Introduction 146
2 Optimal Detection – Known Covariance 147
2.1 Optimal Detector Based on the Sample Covariance Matrix 148
3 Detection – Unknown Covariance 148
3.1 Covariance Matching Estimation Techniques (COMET) . . 149
4 Detector Based on a Gaussian Approximation of the Sample Covariance Matrix 151
4.1 Relation to COMET 152

5 Spectrum Sensing Applications 153
5.1 OFDM Signal Detection 153
6 Numerical Results 154
7 Concluding Remarks 156
References 157

F A Bayesian Approach to Spectrum Sensing, Denoising and Anomaly Detection 159
1 Introduction 162
2 Problem Formulation 162
3 Optimal Detector 164
4 Detector for Unknown ρ, σ 165
4.1 Estimation of ρ, σ Using Prior Knowledge 165
4.2 Elimination of σ Via Marginalization 166
5 Detector Approximations 166
6 Numerical Results 167
7 Concluding Remarks 170
References 173

Part I

Introduction

Chapter 1

Background and Problem Formulation

The ever increasing demand for higher data rates in wireless communications in the face of limited or under-utilized spectral resources has motivated the introduction of cognitive radio. Traditionally, licensed spectrum is allocated over relatively long time periods, and is intended to be used only by licensees. The usage of frequency bands, or spectrum, is strictly regulated and allocated to specific communication techniques. The vast majority of frequency bands are allocated to licensed users, which are also steered by standards. There are a number of organizations working on standards for frequency allocation, for example the International Telecommunication Union (ITU), the European Telecommunications Standards Institute (ETSI) and the European Conference of Postal and Telecommunications Administrations (CEPT). Various measurements of spectrum utilization have shown substantial unused resources in frequency, time and space [1, 2]. The concept behind cognitive radio is to exploit these under-utilized spectral resources by reusing unused spectrum in an opportunistic manner [3, 4]. The phrase "cognitive radio" is usually attributed to Mitola [4], but the idea of using learning and sensing machines to probe the radio spectrum was envisioned several decades earlier (cf. [5]).

Cognitive radio systems typically involve primary users of the spectrum, who are incumbent licensees, and secondary users who seek to opportunistically use the spectrum when or where the primary users are idle. The unused resources are often referred to as spectrum holes or white spaces. There might, for example, be geographical positions where some frequency bands are allocated to a primary user system, but not currently used. These geographical spectrum holes could be employed by secondary users as shown in Figure 1.1. There might also be

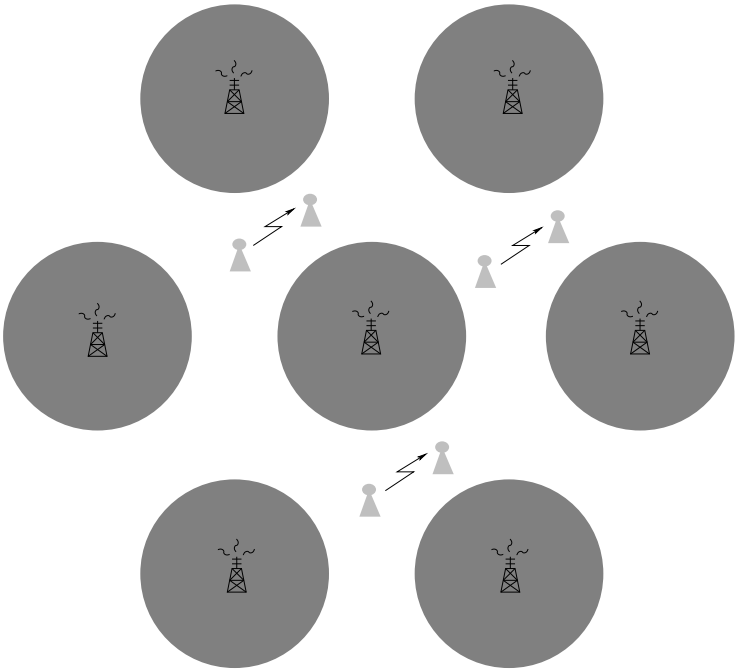

Figure 1.1: Example of geographical spectrum holes.

certain time intervals for which the primary system does not use the licensed spectrum, as shown in Figure 1.2. These time domain spectrum holes could also potentially be employed by secondary users.

The introduction of cognitive radios inevitably creates increased interference and thus can degrade the quality-of-service of the primary system. The impact on the primary system, for example in terms of increased interference, must be kept at a minimal level. Therefore, cognitive radios must sense the spectrum to detect whether it is available or not, and must be able to detect very weak primary user signals [6, 7]. Thus spectrum sensing is one of the most essential components of cognitive radio. Note that here we are describing and addressing so-called "interweave" cognitive radio systems, meaning that the secondary users exploit only those resources which are not used by the primary users. Other methods of spectrum sharing have also been envisioned. These include overlay and underlay systems, where the secondary and primary users coexist and techniques such as spread-spectrum or dirty-paper coding are used to avoid excessive interference. Such systems are not addressed here except to the extent that they may also rely on spectrum sensing.

The problem of spectrum sensing is to decide whether a particular slice of

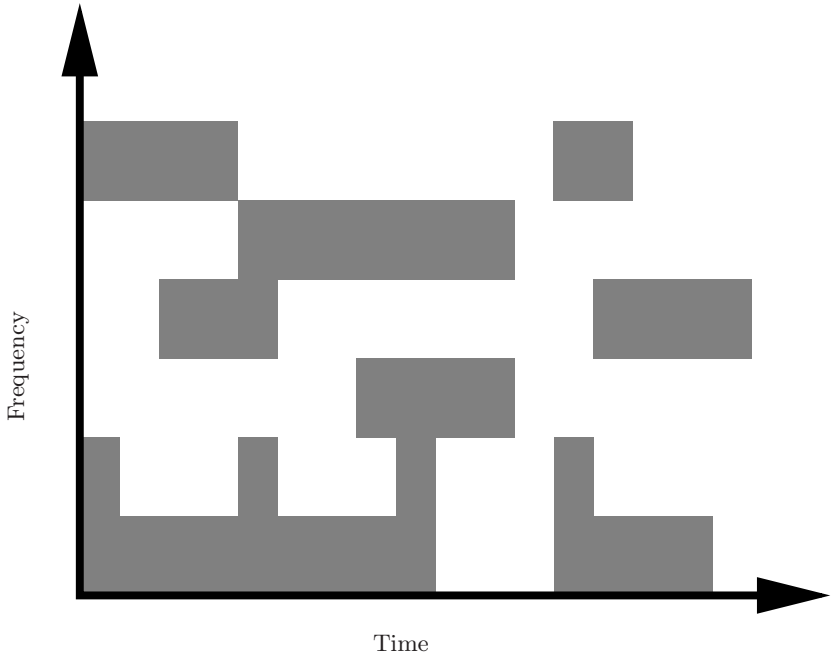

Figure 1.2: Example of time domain spectrum holes.

the spectrum is "available" or not. That is, in its simplest form we want to discriminate between the two hypotheses

$$\begin{aligned} H_0 :\ & \mathbf{y}[n] = \mathbf{w}[n], \quad n = 1, \ldots, N \\ H_1 :\ & \mathbf{y}[n] = \mathbf{x}[n] + \mathbf{w}[n], \quad n = 1, \ldots, N, \end{aligned} \tag{1.1}$$

where $\mathbf{x}[n]$ represents a primary user's signal, $\mathbf{w}[n]$ is noise and n represents time. The received signal $\mathbf{y}[n]$ is vectorial, of length L. Each element of the vector $\mathbf{y}[n]$ could represent, for example, the received signal at a different antenna at the time instant n. Note that (1.1) is a classical detection problem, which is treated in numerous detection theory textbooks. Detection of very weak signals $\mathbf{x}[n]$, in the setting of (1.1) is also a traditional topic, dealt with in depth in [8, Ch. II-III], for example. The novel aspect of the spectrum sensing when related to the long-established detection theory literature is that the signal $\mathbf{x}[n]$ has a specific structure that stems from the use of modern modulation and coding techniques in contemporary wireless systems. Clearly, since such a structure may not be trivial to represent, this has resulted in substantial research efforts. At the same time, this structure offers the opportunity to design very efficient spectrum sensing algorithms.

In the sequel, we will use bold-face lowercase letters to denote column vectors and bold-face capital letters to denote matrices. A discrete-time index is denoted with square brackets and the mth user is denoted with a subscript. That is, $\mathbf{y}_m[n]$ is the vectorial observation for user m at time n. When considering a single user, we will omit the subscript for simplicity. Moreover, if the sequence is scalar, we use the convention $y[n]$ for the time sequence. The lth scalar element of a vector is denoted by $y_l[n]$, not to be confused with the vectorial observation $\mathbf{y}_m[n]$ for user m.

For simplicity of notation, let the vector $\mathbf{y} \triangleq [\mathbf{y}[1]^T,\ \mathbf{y}[2]^T,\ \ldots,\ \mathbf{y}[N]^T]^T$ of length LN contain all observations concatenated in one column vector. In the same way, denote the total concatenated signal by $\mathbf{x}$ and the noise by $\mathbf{w}$. The hypothesis test (1.1) can then be rewritten as

$$\begin{aligned} H_0 &: \mathbf{y} = \mathbf{w}, \\ H_1 &: \mathbf{y} = \mathbf{x} + \mathbf{w}. \end{aligned} \tag{1.2}$$

A standard assumption in the literature, which we also make throughout this introduction, is that the additive noise $\mathbf{w}$ is zero-mean, white, and circularly symmetric complex Gaussian. We write this as $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$, where σ^2 is the noise variance.

Chapter 2

Fundamentals of Signal Detection

In signal detection, the task of interest is to decide whether the observation $\mathbf{y}$ was generated under H_0 or H_1. Typically, this is accomplished by first forming a test statistic $\Lambda(\mathbf{y})$ from the received data $\mathbf{y}$, and then comparing $\Lambda(\mathbf{y})$ with a predetermined threshold η:

$$\Lambda(\mathbf{y}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{2.1}$$

The performance of a detector is quantified in terms of its *receiver operating characteristics* (ROC), which gives the probability of detection $P_{\text{D}} = \Pr(\Lambda(\mathbf{y}) > \eta | H_1)$ as a function of the probability of false alarm $P_{\text{FA}} = \Pr(\Lambda(\mathbf{y}) > \eta | H_0)$. By varying the threshold η, the operating point of a detector can be chosen anywhere along its ROC curve. There is always a tradeoff between the probabilities of false alarm and detection. The choice of the operating point along the ROC curve depends on the application. On the one hand, from a primary user's perspective it is desirable that the probability of detection is large. The secondary users must detect the primary user's signal with high probability in order not to interfere with the primary user. On the other hand, from a secondary user's perspective it is desirable to detect the available spectrum holes with high probability to be able to use as much of the spectrum as is possible. In that case we wish to detect a spectrum hole with high probability $\Pr(\Lambda(\mathbf{y}) \leq \eta | H_0) = 1 - P_{\text{FA}}$. That is, we wish to keep P_{FA} small. To conclude this reasoning, in the cognitive radio context there is a tradeoff between the secondary users' spectrum usage and the impact of interference to the primary users, and this tradeoff is embodied in a tradeoff between P_{FA} and P_{D}.

Clearly, the fundamental problem of detector design is to choose the test statistic $\Lambda(\mathbf{y})$ and to set the decision threshold η in order to achieve good detection performance. These matters are treated in detail in many books on detection theory (e.g. [8]). Detection algorithms are either designed in the framework of classical statistics or in the framework of Bayesian statistics. In the classical (also known as deterministic) framework, either H_0 or H_1 is deterministically true and the objective is to choose $\Lambda(\mathbf{y})$ and η so as to maximize P_{D} subject to a constraint on P_{FA}: $P_{\text{FA}} \leq \alpha$. In the Bayesian framework, by contrast, it is assumed that the source selects the true hypothesis at random according to some *a priori* probabilities $\Pr(H_0)$ and $\Pr(H_1)$. The objective in this framework is to minimize the so-called *Bayesian cost*. Interestingly, although the difference in philosophy between these two approaches is substantial, both result in a test of the form (2.1) where the test statistic is the likelihood-ratio [8, Ch. II]

$$\Lambda(\mathbf{y}) = \frac{p(\mathbf{y}|H_1)}{p(\mathbf{y}|H_0)}. \tag{2.2}$$

2.1 Unknown Parameters

To compute the likelihood ratio $\Lambda(\mathbf{y})$ in (2.2), the probability distribution of the observation $\mathbf{y}$ must be perfectly known under both hypotheses. This means that one must know all parameters, such as noise variance, signal variance and channel coefficients. If the signal to be detected, $\mathbf{x}$, is perfectly known, then[1], $\mathbf{y} \sim \mathcal{N}(\mathbf{x}, \sigma^2\mathbf{I})$ under H_1, and it is easy to show that the optimal test statistic is the output of a matched filter [8, Sec. III.B]:

$$\text{Re}(\mathbf{x}^H\mathbf{y}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta,$$

where $\mathbf{x}^H$ denotes the Hermitian (conjugate) transpose of $\mathbf{x}$. In practice, the signal and noise parameters are not known. In the following, we will discuss two standard techniques that are used to deal with unknown parameters in hypothesis testing problems.

In the Bayesian framework, the optimal strategy is to marginalize the likelihood function to eliminate the unknown parameters. More precisely, if the vector $\boldsymbol{\theta}$ contains the unknown parameters, then one computes

$$p(\mathbf{y}|H_i) = \int p(\mathbf{y}|H_i, \boldsymbol{\theta})p(\boldsymbol{\theta}|H_i)d\boldsymbol{\theta},$$

where $p(\mathbf{y}|H_i, \boldsymbol{\theta})$ denotes the conditional PDF of $\mathbf{y}$ under H_i and conditioned on $\boldsymbol{\theta}$, and $p(\boldsymbol{\theta}|H_i)$ denotes the *a priori* probability density of the parameter

[1] Recall that we assume circularly symmetric Gaussian noise throughout.

vector given hypothesis H_i. In practice, the actual *a priori* parameter density $p(\boldsymbol{\theta}|H_i)$ often is not perfectly known, but rather is chosen to provide a meaningful result. How to make such a choice, is far from clear in many cases. One alternative is to choose a non-informative distribution in order to model a lack of *a priori* knowledge of the parameters. One example of a non-informative prior is the gamma distribution, which is used in Paper F to model an unknown noise power. Another option is to choose the prior distribution via the so-called *maximum entropy principle*. According to this principle, the prior distribution of the unknown parameters that maximizes the entropy given some statistical constraints (e.g. limited expected power or second-order moment) should be chosen. The maximum entropy principle has been used in the context of spectrum sensing for cognitive radio in [9].

In the classical hypothesis testing framework, the unknown parameters must be estimated somehow. A standard technique is to use maximum-likelihood (ML) estimates of the unknown parameters, which gives rise to the well-known generalized likelihood-ratio test (GLRT):

$$\frac{\max_{\boldsymbol{\theta}} p(\mathbf{y}|H_1, \boldsymbol{\theta})}{\max_{\boldsymbol{\theta}} p(\mathbf{y}|H_0, \boldsymbol{\theta})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

This is a technique that usually works quite well, although it does not necessarily guarantee optimality. Other estimates than the ML estimate may also be used.

2.2 Constant False-Alarm Rate (CFAR) Detectors

A detector is said to have the property of constant-false alarm rate (CFAR), if its false alarm probability is independent of parameters such as noise or signal powers. In particular, the CFAR property means that the decision threshold can be set to achieve a pre-specified P_{FA} without knowing the noise power. The CFAR property is normally revealed by the equations that define the test (2.1): if the test statistic $\Lambda(\mathbf{y})$ and the optimal threshold are unaffected by a scaling of the problem (such as multiplying the received data by a constant), then the detector is CFAR. CFAR is a very desired property in many applications, especially when one has to deal with noise of unknown power, as we will see later.

2.3 Energy Detection

As an example of a very basic detection technique, we present the well known energy detector, also known as the radiometer [10]. The energy detector mea-

sures the received energy during a finite time interval, and compares it to a predetermined threshold. It should be noted that the energy detector works well also for other cases than the one we will present, although it might not be optimal.

To derive this detector, assume that the signal to be detected does not have any known structure that could be exploited, and model it via a zero-mean circularly symmetric complex Gaussian variable $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \gamma^2\mathbf{I})$. Then, $\mathbf{y}|H_0 \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ and $\mathbf{y}|H_1 \sim \mathcal{N}(\mathbf{0}, (\sigma^2+\gamma^2)\mathbf{I})$. After removing irrelevant constants, the optimal (Neyman-Pearson) test can be written as

$$\Lambda(\mathbf{y}) = \frac{\|\mathbf{y}\|^2}{\sigma^2} = \frac{1}{\sigma^2}\sum_{i=1}^{LN} |y_i|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{2.3}$$

The operational meaning of (2.3) is to compare the energy of the received signal against a threshold and this is why (2.3) is called the energy detector. Its performance is well known, cf. [8, Sec. III.C], and is given by

$$P_{\text{FA}} = \Pr(\Lambda(\mathbf{y}) > \eta | H_0) = 1 - F_{\chi^2_{2NL}}\left(\frac{2\eta}{\sigma^2}\right)$$

and

$$\begin{aligned} P_{\text{D}} &= \Pr\left(\Lambda(\mathbf{y}) > \eta | H_1\right) = 1 - F_{\chi^2_{2NL}}\left(\frac{2\eta}{\sigma^2+\gamma^2}\right) \\ &= 1 - F_{\chi^2_{2NL}}\left(\frac{F^{-1}_{\chi^2_{2NL}}(1-P_{\text{FA}})}{1+\frac{\gamma^2}{\sigma^2}}\right), \end{aligned}$$

where $F_{\chi^2_{2NL}}(\cdot)$ denotes the cumulative distribution function of a χ^2-distributed random variable with $2NL$ degrees of freedom. Clearly, P_{D} is a function of P_{FA}, NL and the SNR $\triangleq \gamma^2/\sigma^2$. Note that for a fixed P_{FA}, $P_{\text{D}} \to 1$ as $NL \to \infty$ at any SNR. That is, ideally any pair $(P_{\text{D}}, P_{\text{FA}})$ can be achieved if sensing can be done for an arbitrarily long time. This is typically not the case in practice, as we will see in the following section.

It has been argued that for several models, and if the probability density functions under both hypotheses are perfectly known, energy detection performs close to the optimal detector. For example, it was shown in [7] that the performance of the energy detector is asymptotically equivalent, at low SNR, to that of the optimal detector when the signal is modulated with a zero-mean finite signal constellation, assuming that the symbols are independent of each other and that all probability distributions are perfectly known. Similar results have been shown numerically in other works, for example in Paper A for the detection of an orthogonal frequency-division multiplexing (OFDM) signal. These results hold if all probability density functions, including that of the noise, are

perfectly known. By contrast, if for example the noise variance is unknown, the energy detector cannot be used because knowledge of σ^2 is needed to set the threshold. If an incorrect ("estimated") value of σ^2 is used in (2.3) then the resulting detector may perform rather poorly. We discuss this matter in more depth in the following section.

2.4 Fundamental Limits for Sensing: SNR Wall

Cognitive radios must be able to detect very weak primary user signals [6, 7]. This is difficult, because there are fundamental limits on detection at low SNR. Specifically, due to uncertainties in the model assumptions, accurate detection is impossible below a certain SNR level, known as the *SNR wall* [11, 12]. The reason is that to compute the likelihood ratio $\Lambda(\mathbf{y})$, the probability distribution of the observation $\mathbf{y}$ must be perfectly known under both hypotheses. In any case, the signal and noise in (1.2) must be modeled with some known distributions. Of course, a model is always a simplification of reality, and the true probability distributions are never perfectly known. Even if the model would be perfectly consistent with reality, there will be some parameters that are unknown such as the noise power, the signal power and the channel coefficients, as noted above.

To exemplify the SNR wall phenomenon, consider the energy detector. To set its decision threshold, the noise variance σ^2 must be known. If the knowledge of the noise variance is imperfect, the threshold cannot be correctly set. Setting the threshold based on an incorrect noise variance will not result in the desired value of false-alarm probability. In fact, the performance of the energy detector quickly deteriorates if the noise variance is imperfectly known [7, 11]. Let σ^2 denote the imperfect estimate of the noise variance, and let σ_t^2 be the true noise variance. Assume that the estimated noise variance is known only to lie in a given interval, such that $\sigma^2 \in [\frac{1}{\rho}\sigma_t^2, \rho\sigma_t^2]$ for some $\rho > 1$. To guarantee that the probability of false alarm is always below a required level, the threshold must be set to fulfill the requirement in the worst case. That is, we need to make sure that

$$\max_{\sigma^2 \in [\frac{1}{\rho}\sigma_t^2, \rho\sigma_t^2]} P_{\text{FA}}$$

is below the required level. The worst case occurs when the noise power is at the upper end of the interval, that is when $\sigma^2 = \rho\sigma_t^2$. It was shown in [12] that under this model, the number of samples LN that are required to meet a P_{D} requirement, tends to infinity as the SNR $= \gamma^2/\sigma_t^2 \searrow (\rho^2 - 1)/\rho$. That is, even with an infinite measurement duration, it would be impossible to meet the P_{D} requirement when the SNR is below the SNR wall $(\rho^2 - 1)/\rho$. This effect occurs only because of the uncertainty in the noise level. The effect of the SNR wall for energy detection is shown in Figure 2.1. The figure shows the number of

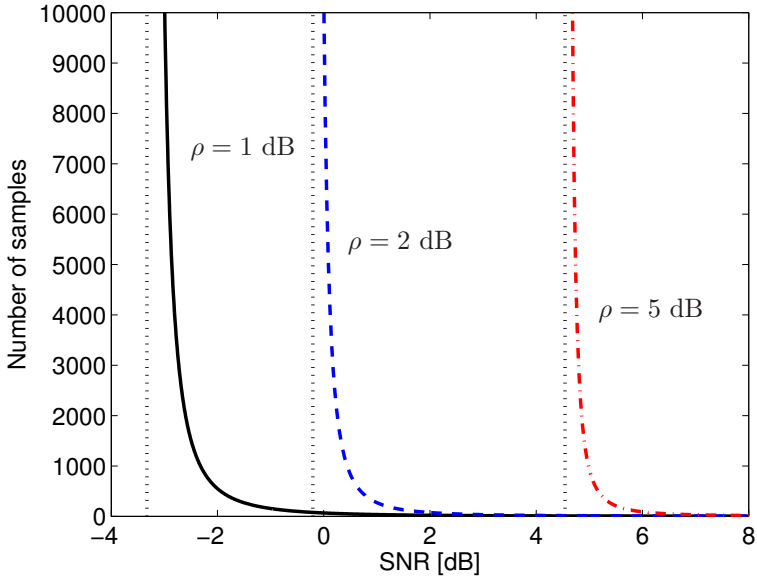

Figure 2.1: The number of samples required to meet $P_{\text{FA}} = 0.05$ and $P_{\text{D}} = 0.9$ using energy detection under noise uncertainty.

samples that are needed to meet the requirements $P_{\text{FA}} = 0.05$ and $P_{\text{D}} = 0.9$ for different levels of the noise uncertainty.

It was shown in [12] that errors in the noise power assumption introduce SNR walls to any moment-based detector, not only to the energy detector. This result was further extended in [12] to any model uncertainties, such as color and stationarity of the noise, simplified fading models, ideality of filters and quantization errors introduced by finite-precision analog-to-digital (A/D) converters. It is possible to mitigate the problem of SNR walls by taking the imperfections into account, in the sense that the SNR wall can be moved to a lower SNR level. For example, it was shown in [12] that noise calibration can improve the detector robustness. Exploiting known features of the signal to be detected can also improve the detector performance and robustness. Known features can be exploited to deal with unknown parameters using marginalization or estimation as discussed before. It is also known that fast fading effects can somewhat alleviate the requirement of accurately knowing the noise variance in some cases [13]. Note also that a CFAR detector is not exposed to the SNR wall phenomenon, since the decision threshold is set independently of any potentially unknown signal and noise power parameters.

Chapter 3

Feature Detection Based on Second-Order Statistics

Information theory teaches us that communication signals with maximal information content (entropy) are statistically white and Gaussian and hence, we would expect signals used in communication systems to be nearly white Gaussian. If this was the case, then no spectrum sensing algorithm could do better than the energy detector. However, signals used in practical communication systems always contain distinctive features that can be exploited for detection and that enable us to achieve a detection performance that substantially surpasses that of the energy detector. Perhaps even more importantly, known signal features can be exploited to estimate unknown parameters such as the noise power. Therefore, making use of known signal features effectively can circumvent the problem of SNR walls discussed in the previous section. The specific properties that originate from modern modulation and coding techniques have aided in the design of efficient spectrum sensing algorithms.

The term feature detection is commonly used in the context of spectrum sensing and usually refers to exploitation of known statistical properties of the signal. The signal features referred to may be manifested both in time and space. Features of the transmitted signal are the result of redundancy added by coding, and of the modulation and burst formatting schemes used at the transmitter. For example, OFDM modulation adds a cyclic prefix which manifests itself through a linear relationship between the transmitted samples. Also, most communication systems multiplex known pilots into the transmitted data stream or superimpose pilots on top of the transmitted signals, and doing so results in very distinctive signal features. A further example is given by space-time coded signals, in which the space-time code correlates the transmitted signals. The

received signals may also have specific features that occur due to characteristics of the propagation channel. For example, in a MIMO (multiantenna) system, if the receiver array has more antennas than the transmitter array, then samples taken by the receiver array at any given point in time must necessarily be correlated.

In this section, we will review a number of state-of-the-art detectors that exploit signal features and which are suitable for spectrum sensing applications. This will serve as a background to the techniques proposed in this dissertation. Most of the presented methods are very recent advances in spectrum sensing, and there is still much ongoing research in these areas.

3.1 Detectors Based on Autocorrelation Properties

A very popular and useful approach to feature detection is to estimate the second-order statistics of the received signals and make decisions based on these estimates. Clearly, in this way we may distinguish a perfectly white signal from a colored one. This basic observation is important, because typically, the redundancy added to transmitted signals in a communication system results in its samples becoming correlated. The correlation structure incurred this way does not necessarily have to be stationary; in fact, typically it is not as we shall see. Since $\text{cov}(\mathbf{A}\mathbf{x}) = \mathbf{A}\text{cov}(\mathbf{x})\mathbf{A}^H$ for any deterministic matrix $\mathbf{A}$ and stochastic vector $\mathbf{x}$, the correlation structure incurred by the addition of redundancy at the transmitter is usually straightforward to analyze if the transmit processing consists of a linear operation. Moreover, we know that the distribution of a Gaussian signal is fully determined by its first and second-order moments. Therefore, provided that the communication signals in question are sufficiently near to Gaussian and that enough samples are collected, we expect that estimated first and second-order moments are sufficient statistics to within practical accuracy. Since communication signals are almost always of zero-mean (in order to minimize the power spent at the transmitter), just looking at the second-order moment is adequate. Taken together, these arguments tell us that in many cases we can design near-optimal spectrum sensing algorithms by estimating second-order statistics from the data, and making decisions based on these estimates.

We explain detection based on second-order-statistics using OFDM signals as an example. Further details on the detectors presented in this introductory section are given in Paper A. OFDM signals have a very explicit correlation structure imposed by the insertion of a cyclic prefix (CP) at the transmitter. Moreover, OFDM is a popular modulation method in modern wireless standards. Consequently a sequence of papers have proposed detectors that exploit

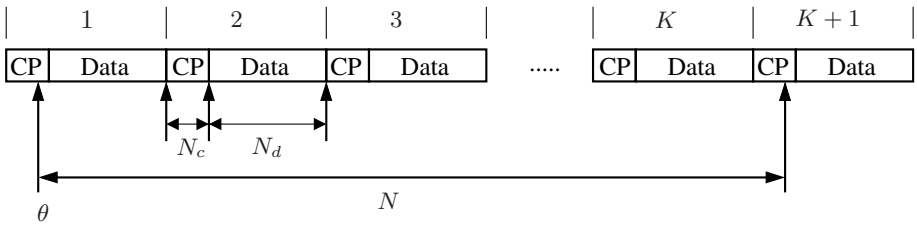

Figure 3.1: Model for the N samples of a received OFDM signal.

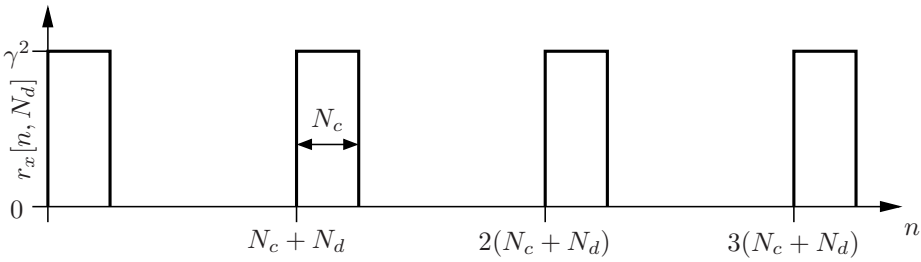

Figure 3.2: Example of a periodic autocorrelation function for an OFDM signal with a cyclic prefix.

the correlation structure of OFDM signals [14–16] and Paper A. We will briefly describe those detectors in the following. These detectors can be used for any signal with a CP structure, for example single-carrier transmission with a CP and repeated training or so-called known symbol padding, but in what follows we assume that we deal with a conventional OFDM signal.

Consider an OFDM signal with a CP, as shown in Figure 3.1. Let N_d be the number of data symbols, that is, the block size of the inverse fast Fourier transform (IFFT) used at the transmitter or equivalently the number of subcarriers. The CP has length N_c, and it is a repetition of the last N_c samples of the data. Assume that the transmitted data symbols are independent and identically distributed (i.i.d.), zero-mean and have variance γ^2, and consider the autocorrelation function (ACF)

$$r_x[n, \tau] \triangleq E\left[x[n]x^*[n+\tau]\right]. \tag{3.1}$$

Owing to the insertion of the CP, the OFDM signal is nonstationary and therefore the ACF $r_x[n, \tau]$ in (3.1) is time-varying. In particular, it is non-zero at time lag $\tau = N_d$ for some time instances n, and zero for others. This is illustrated in Figure 3.2. The non-zero values of the ACF occur due to the repetition of symbols in the CP. This non-stationarity property of the ACF can be exploited

in different ways by the detectors, as we will see in what follows. Of course, the more knowledge we have of the parameters that determine the shape of the ACF (N_c and N_d specifically, and σ^2 and γ^2), the better performance we can obtain.

For simplicity of notation, assume that the receiver has observed $K+1$ consecutive OFDM signals out of an endless stream of OFDM modulated data, so that the received signal $y[n]$ contains $N = K(N_c + N_d) + N_d$ samples. Furthermore, for simplicity we consider an additive white Gaussian noise (AWGN) channel. The quantitative second-order statistics will be the same in a multipath fading channel, but the exact ACF may be smeared out due to the time dispersion. However, averaging the second-order statistics over multiple OFDM symbols mitigates the impact of multipath fading, and the detection performance is close to the performance in an AWGN channel in many cases (cf. [14]). We are interested in estimating $r_x[n, N_d]$, and we form the following estimate of it:

$$\widehat{r}[n] \triangleq y[n]y^*[n + N_d], \quad n = 1, \ldots, K(N_c + N_d).$$

Note that $r_w[n, \tau] = 0$ for any $\tau \neq 0$, since the noise is white and zero-mean. Here $r_w[n, \tau]$ and $r_y[n, \tau]$ are defined based on $w[n]$ and $y[n]$ similarly to (3.1). Hence, $r_y[n, N_d] = r_x[n, N_d]$ whenever $N_d \neq 0$. By construction $E[\widehat{r}[n]] = r_y[n, N_d] = r_x[n, N_d]$ is the ACF of the OFDM signal at time lag N_d for $N_d \neq 0$. We know from the above discussion (see Figure 3.2) that $\widehat{r}[n]$ and $\widehat{r}[n + k(N_c + N_d)]$ have identical statistics and that they are independent. Therefore, it is useful to define

$$\widehat{R}[n] \triangleq \frac{1}{K} \sum_{k=0}^{K-1} \widehat{r}[n + k(N_c + N_d)], \; n = 1, \ldots, N_c + N_d.$$

What is the best way of making decisions on signal presence versus absence based on $\widehat{r}[n]$? We know that the mean of $\widehat{r}[n]$ is nonzero for some n and zero for others and this is the basic observation that we would like to exploit. It is clear that the design of an optimal detector would involve an accurate analysis of the statistical distribution of $\widehat{r}[n]$. This is a nontrivial matter, since $\widehat{r}[n]$ is a nonlinear function of $y[n]$. Moreover, this analysis is difficult if there are unknown parameters such as the noise power.

The recent literature has proposed several ways forward.

- One of the first papers on the topic was [15], in which the following statistical test was proposed:

$$\max_{\theta} \left| \sum_{n=\theta+1}^{\theta+N_c} \widehat{r}[n] \right| \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{3.2}$$

The test in (3.2) exploits the non-stationarity of the OFDM signal. The variable θ in (3.2) has the interpretation of synchronization mismatch. The intuition behind this detection is therefore to catch the "optimal" value of θ and then measure, for that θ, how large is the correlation between values of $y[n]$ spaced N_d samples apart. For this to work, the detector must know N_c and N_d. Perhaps more importantly, in order to set the threshold one also needs to know σ^2 and hence the detector in (3.2) is susceptible to the SNR wall phenomenon. This is so for the same reasons as previously discussed for the energy detector: the test statistic in (3.2) is not dimensionless and hence the test is not CFAR.

The original test in [15] looks only at one received OFDM symbol but it can be extended in a straightforward manner to use all K symbols. The resulting statistic then sums the variables $\widehat{R}[n]$ instead of $\widehat{r}[n]$ and we have

$$\max_{\theta\in\{0,\ldots,N_c+N_d-1\}} \left| \sum_{n\in S_\theta} \widehat{R}[n] \right| \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{3.3}$$

where $S_\theta \subset \{1, 2, \ldots, N_c + N_d\}$ denotes the set of N_c (cyclic) consecutive indices for which $E[\widehat{R}[n]] \neq 0$, given the synchronization error θ.

- A different path was taken in [14]. The detector proposed therein uses the empirical mean of the autocorrelation normalized by the received power, as the test statistic. More precisely, the test is

$$\frac{\sum_{n=1}^{N-N_d} \mathrm{Re}(\widehat{r}[n])}{\sum_{n=1}^{N} |y[n]|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{3.4}$$

  The advantage of (3.4) is that in order to use this test, one needs to know only N_d, but not N_c. This is useful if N_c is unknown, or if there is substantial uncertainty regarding N_c; think for example, of a system that alternates between CPs of different lengths or that uses different CPs on different component carriers. On the other hand, a potential disadvantage of (3.4) is that it does not exploit the fact that the OFDM signal is non-stationary. This is evident from (3.4) as all samples of $\widehat{r}[n]$ are weighed equally when forming the test statistic; hence, the time-variation of the ACF is not reflected in the detection criterion. Not surprisingly, one can obtain better performance if this time-variation is exploited.

  By construction, (3.4) is a CFAR test. Hence, it requires no knowledge of the noise power σ^2. We note in passing that a detector similar to [14], but without the power normalization, was proposed in [16].

- In Paper A we propose another detector that exploits the non-stationarity of an OFDM signal. The detector of Paper A is essentially an approximation of the GLRT, treating the synchronization mismatch between the

transmitter and the receiver, and the signal and noise variances, as unknown parameters. It is also a CFAR detector and needs no knowledge of σ_n^2. It differs from the detectors in [14] and [16] in that it explicitly takes the non-stationarity of $x[n]$ into account. Further details of the proposed detector are given in Paper A.

3.2 Detectors Based on Cyclostationarity

In many cases, the ACF of the signal is not only non-stationary, but is also periodic. Most man-made signals show periodic patterns related to symbol rate, chip rate, channel code or cyclic prefix. Such second-order periodic signals can be appropriately modeled as second-order cyclostationary random processes [17]. As an example, consider again the OFDM signal shown in Figure 3.1. The autocorrelation function of this OFDM signal, shown in Figure 3.2, is periodic. The fundamental period is the length of the OFDM symbol, $N_c + N_d$. Knowing some of the cyclic characteristics of a signal, one can construct detectors that exploit the cyclostationarity [18, 19] and benefit from the spectral correlation.

A discrete-time zero-mean stochastic process $y[n]$ is said to be *second-order cyclostationary* if its time-varying ACF $r_y[n,\tau] = E\left[y[n]y^*[n+\tau]\right]$ is periodic in n [17, 18]. Hence, $r_y[n,\tau]$ can be expressed by a Fourier series

$$r_y[n,\tau] = \sum_{\alpha} R_y(\alpha,\tau)e^{j\alpha n},$$

where the sum is over integer multiples of fundamental frequencies and their sums and differences. The Fourier coefficients depend on the time lag τ and are given by

$$R_y(\alpha,\tau) = \lim_{T\to\infty} \frac{1}{T}\sum_{n=0}^{T-1} r_y[n,\tau]e^{-j\alpha n}.$$

The Fourier coefficients $R_y(\alpha,\tau)$ are also known as the *cyclic autocorrelation* at cyclic frequency α. The process $y[n]$ is second-order cyclostationary when there exists an $\alpha \neq 0$ such that $R_y(\alpha,\tau) > 0$, because $r_y[n,\tau]$ is periodic in n precisely in this case. The *cyclic spectrum* of the signal $y[n]$ is the Fourier coefficient

$$S_y(\alpha,\omega) = \sum_{\tau} R_y(\alpha,\tau)e^{-j\omega\tau}.$$

The cyclic spectrum represents the density of correlation for the cyclic frequency α.

Knowing some of the cyclic characteristics of a signal, one can construct detectors that exploit the cyclostationarity and thus benefit from the spectral

correlation (see, e.g., [18–20]). Note that the inherent cyclostationarity property appears both in the cyclic ACF $R_y(\alpha, \tau)$ and in the cyclic spectral density function $S_y(\alpha, \omega)$. Thus, detection of the cyclostationarity can be performed both in the time domain, and in the frequency domain. The paper [18] proposed detectors that exploit cyclostationarity based on one cyclic frequency, either from estimates of the cyclic autocorrelation or of the cyclic spectrum. The detector of [18] based on cyclic autocorrelation was extended in [19] to use multiple cyclic frequencies. The cyclic autocorrelation is estimated in [18] and [19] by

$$\hat{R}_y(\alpha, \tau) \triangleq \frac{1}{N - N_\mathrm{d}} \sum_{n=0}^{N-N_\mathrm{d}-1} y[n]y^*[n+\tau]e^{-j\alpha n}.$$

The cyclic autocorrelation $\hat{R}_y(\alpha_i, \tau_{i,N_i})$ can be estimated for the cyclic frequencies of interest α_i, $i = 1, \ldots, p$, at time lags $\tau_{i,1}, \ldots, \tau_{i,N_i}$. The detectors of [18] and [19] are then based on the limiting probability distribution of $\hat{R}_y(\alpha_i, \tau_{i,N_i})$, $i = 1, \ldots, p$.

In practice only one or a few cyclic frequencies are used for detection, and this is usually sufficient to achieve a good detection performance. Note, however, that this is an approximation. For example, a perfect Fourier series representation of the signal shown in Figure 3.2 requires infinitely many Fourier coefficients. The autocorrelation-based detector of [14] and the cyclostationarity detector of [19] are compared in [21], for detection of an OFDM signal in AWGN. The results show that the cyclostationarity detector using two cyclic frequencies outperforms the autocorrelation detector, but that the autocorrelation detector is superior when only one cyclic frequency is used.

3.3 Detectors that Rely on a Specific Structure of the Sample Covariance Matrix

Signal structure, or correlation, is also inherent in the covariance matrix of the received signal. Some communication signals impart a specific known structure to the covariance matrix. This is the case, for example, when the signal is received by multiple antennas [22–24] (single-input/multiple-output - SIMO), [9] (multiple-input/multiple-output - MIMO). In many cases, the covariance matrix has eigenvalues with known multiplicities owing to the reduncandy added to a communication signal. In Paper B, we show how the knowledge of arbitrary eigenvalue multiplicities can be exploited to obtain CFAR detectors. In Papers C and D the result of Paper B is used as a building block to propose detectors for spectrum sensing of a second-order cyclostationary signal (such as

OFDM) received at multiple antennas and when the signal is encoded with an orthogonal space-time block code (OSTBC) respectively.

Consider again the vectorial discrete-time representation (1.1). For better understanding we will start with the example of a single symbol received by multiple antennas (SIMO). This case was dealt with, for example, in [9, 22, 23] and [24]. Suppose that there are $L > 1$ receive antennas at the detector. Then, under H_1, the received signal can be written as

$$\mathbf{y}[n] = \mathbf{h}s[n] + \mathbf{w}[n],\ n = 1, \ldots, N, \tag{3.5}$$

where $\mathbf{h}$ is the $L \times 1$ channel vector and $s[n]$ is the transmitted symbol sequence. Assume further that the signal is zero-mean Gaussian, i.e. $s[n] \sim \mathcal{N}(0, \gamma^2)$, and as before $\mathbf{w}[n] \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$. Then, the covariance matrix under H_1 is $\mathbf{\Psi} \triangleq E[\mathbf{y}[n]\mathbf{y}[n]^H|H_1] = \gamma^2\mathbf{h}\mathbf{h}^H + \sigma^2\mathbf{I}$. Let $\lambda_1, \lambda_2, \ldots, \lambda_L$ be the eigenvalues of $\mathbf{\Psi}$ sorted in descending order. Since $\mathbf{h}\mathbf{h}^H$ has rank one, then $\lambda_1 = \gamma^2 \|\mathbf{h}\|^2 + \sigma^2$ and $\lambda_2 = \ldots = \lambda_L = \sigma^2$. In other words, $\mathbf{\Psi}$ has two distinct eigenvalues with multiplicities one and $L - 1$ respectively. Denote the sample covariance matrix by

$$\hat{\mathbf{R}} \triangleq \frac{1}{N}\sum_{n=1}^{N} \mathbf{y}[n]\mathbf{y}[n]^H.$$

Moreover, let $\nu_1, \nu_2, \ldots, \nu_L$ denote the eigenvalues of $\hat{\mathbf{R}}$ sorted in descending order. An example of the eigenvalues $\nu_1, \nu_2, \ldots, \nu_L$ in this case, with four receive antennas, $N = 1000$ and SNR $= 10\log_{10}(\gamma^2/\sigma^2) = 0$ dB, is shown at the top of Figure 3.3. It is clear that there is one dominant eigenvalue under H_1 due to the rank-one channel vector. It can be shown (cf. [22] and [23]) that the GLRT when the channel $\mathbf{h}$ and the powers σ^2 and γ^2 are unknown is given by

$$\frac{\nu_1}{\mathrm{tr}(\hat{\mathbf{R}})} = \frac{\nu_1}{\sum_{i=1}^{L}\nu_i} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{3.6}$$

Here, we have considered independent observations $\mathbf{y}[n]$ at multiple antennas. A similar covariance structure could of course also occur for a time-series. Then, we could construct the sample covariance matrix by considering a scalar time-series $y[n]$, $n = 1, 2, \ldots, N$ as in [25] and [26], and letting $\mathbf{y}[n] = [y[n], y[n+1], \ldots, y[n+L-1]]^T$ for some integer $L > 0$. This can be seen as a windowing of the sequence $y[n]$ with a rectangular window of length L. The choice of the window length L will of course affect the performance of the detectors. The reader is referred to the original papers [25] and [26] for discussions of this issue.

Now, consider more generally that the received signal under H_1 can be written as

$$\mathbf{y}[n] = \mathbf{G}\mathbf{s}[n] + \mathbf{w}[n],\ n = 1, \ldots, N, \tag{3.7}$$

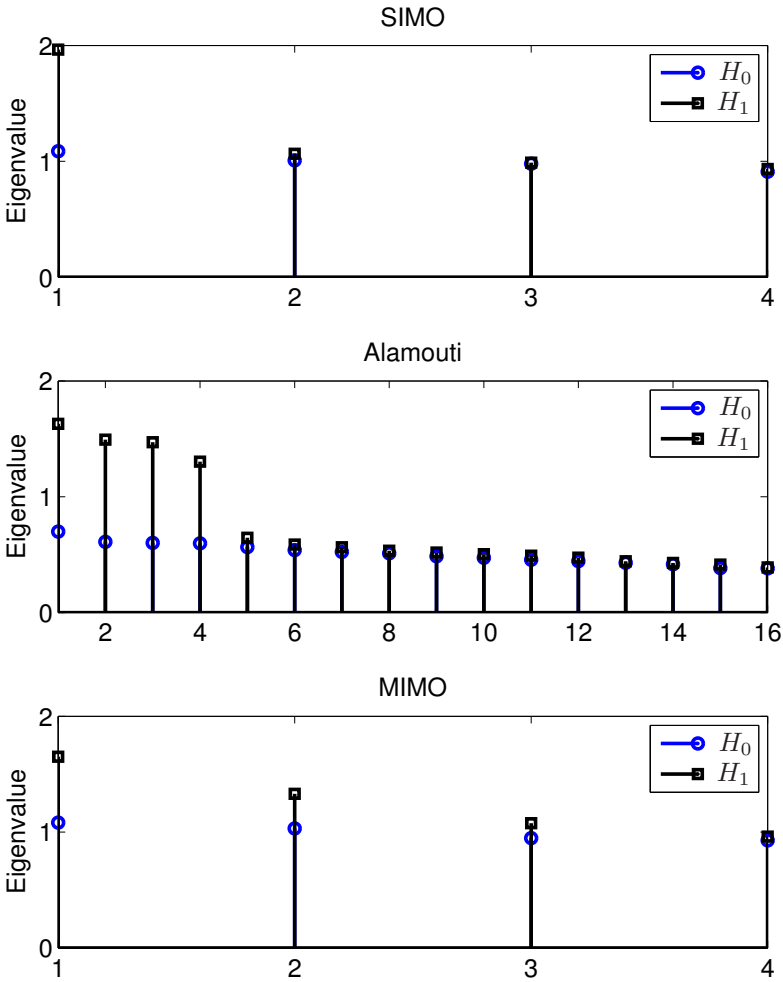

Figure 3.3: Example of the sorted eigenvalues of the sample covariance matrix $\hat{\mathbf{R}}$ with four receive antennas, for $N = 1000$ and $\mathrm{SNR} = 10\log_{10}(\gamma^2/\sigma^2) = 0$ dB. The Alamouti scheme codes two complex symbols over two time intervals and two antennas.

where $\mathbf{G}$ is a low rank matrix, and $\mathbf{s}[n] \sim \mathcal{N}(\mathbf{0}, \gamma^2\mathbf{I})$ is an i.i.d. sequence. Then, the covariance matrix is $\mathbf{\Psi} = \gamma^2\mathbf{G}\mathbf{G}^H + \sigma^2\mathbf{I}$ under H_1, which is a "low-rank-plus-identity" structure. Suppose that $\mathbf{\Psi}$ has d distinct eigenvalues with multiplicities $q_1, q_2, \ldots, q_d$ respectively. This can happen if the signal has some specific structure, for example in a multiple antenna (MIMO) system [9], when the signal is encoded with an orthogonal space-time block code (Paper D), or if the signal is an OFDM signal (Paper C). Examples of the sorted eigenvalues of $\hat{\mathbf{R}}$ for an orthogonal space-time block code (Alamouti), and for a general MIMO system [9], with two transmit and four receive antennas, are shown in Figure 3.3 (middle and bottom respectively). The reason that the number of eigenvalues for the Alamouti case is four times higher than for the general MIMO system is that the space-time code is coded over two time intervals, and the observation is divided into real and imaginary parts (see Paper D for details). For the Alamouti code, the four largest eigenvalues are significantly larger than the others. In fact, the expected values of the four largest eigenvalues are equal, due to the orthogonality of the code. For the general MIMO case, we note that two of the eigenvalues are larger than the others, because the channel matrix has rank two (there are two transmit antennas). In this case, however, the expectations of the two largest eigenvalues are different in general. Define the set of indices $\mathcal{S}_i \triangleq \{(\sum_{j=1}^{i-1} q_j)+1, \ldots, (\sum_{j=1}^{i-1} q_j)+q_i\}$, $i = 1, 2, \ldots, d$. For example, if there are two distinct eigenvalues with multiplicities q_1 and q_2 $(= L - q_1)$ respectively, then $\mathcal{S}_1 = \{1, \ldots, q_1\}$ and $\mathcal{S}_2 = \{q_1 + 1, \ldots, L\}$. It is shown in Paper B that the GLRT when the eigenvalues are unknown, but have known multiplicities and order, is

$$\frac{\left(\frac{1}{L}\mathrm{tr}(\hat{\mathbf{R}})\right)^L}{\prod_{i=1}^{d}\left(\frac{1}{q_i}\sum_{j\in\mathcal{S}_i}\nu_j\right)^{q_i}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{3.8}$$

It can be shown that in the special case when $q_1 = 1$ and $q_2 = L - 1$, this test is equivalent to the test (3.6). Further details on these matters are given in Paper B.

Properties of the covariance matrix are also exploited for detection in [25] and [26], without knowing the structure. Detection without any knowledge of the transmitted signal is usually referred to as *blind detection*, and will be discussed further in the following section.

3.4 Blind Detection

Even though a primary user's signal is correlated or has some other structure, this structure might not be perfectly known. An example of this is shown at the

bottom of Figure 3.3. This eigenvalue structure occurs in a general MIMO system, when the number of receive antennas is larger than the number of transmit antennas. In general, the number of antennas and the coding scheme used at the transmitter might not be known. The transmit antennas could of course also belong to an (unknown) number of users that transmit simultaneously [27, 28]. If the transmitted signals have a completely unknown structure, we must consider *blind detectors*. Blind detectors are blind in the sense that they exploit structure of the signal without any knowledge of signal parameters. We saw in the previous section that the eigenvalues of the covariance matrix behave differently under H_0 and H_1 if the signal is correlated. This is still true, even if the exact structure of the eigenvalues is not known. Blind eigenvalue-based tests, similar to those described in the previous section, have been proposed recently in [25] and [26].

We will begin by describing the blind detectors of [25] and [26] based on the eigenvalues of the sample covariance matrix. The presentation here will be slightly different from the ones in [25] and [26], in order to include complex-valued data and be consistent with the notation used above. The paper [25] proposes two detectors based on the eigenvalues of $\hat{\mathbf{R}}$, similar to the detectors of the previous section. The detectors proposed in [25] are

$$\frac{\nu_1}{\nu_L} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \quad \text{and} \quad \frac{\mathrm{tr}(\hat{\mathbf{R}})}{\nu_L} \underset{H_0}{\overset{H_1}{\gtrless}} \eta,$$

where ν_i, $i = 1, 2, \ldots, L$ are the sorted eigenvalues of $\hat{\mathbf{R}}$, as before. Thus, ν_1 is the maximum eigenvalue and ν_L is the minimum eigenvalue. The motivation for these tests is based on properties similar to those discussed in Section 3.3. If the received sequence contains a (correlated) signal, the expectation of the largest eigenvalues will be larger than if there is only noise, but the expectation of the smallest eigenvalues will be the same in both cases.

Blind detectors are commonly also based on information theoretic criteria, such as Akaike's Information Criterion (AIC) or the Minimum Description Length (MDL) [27–30]. These information theoretic criteria typically result in eigenvalue tests similar to those of the previous section. The aims of [27] and [28] are not only to decide whether a signal has been transmitted or not, but rather also to estimate the number of signals transmitted. Assume as in the previous section that the received signal under H_1 is $\mathbf{y}[n] = \mathbf{G}\mathbf{s}[n] + \mathbf{w}[n]$. The number of uncorrelated transmitters is the rank d of the matrix $\mathbf{G}$. The problem under consideration in [27] and [28] is then to determine the rank of $\mathbf{G}$ by minimizing the AIC or MDL, which are functions of d (written as AIC(d) and MDL(d)). The result of [27] is applied in [29] and [30] to the problem of spectrum sensing. More specifically, the estimator of [27] is used in [29] to determine whether the number of signals transmitted is zero or non-zero. This idea is further simplified in [30] to that of using only the difference AIC(0) − AIC(1) as a test statistic.

Note that these detectors are very similar to the detectors of the previous section and to the detectors described in the beginning of this section. They all exploit properties of the eigenvalues of the sample covariance matrix, and use functions of the eigenvalues as test statistics. The detectors of this section use only the assumption that the received signal is correlated. They are all blind detectors, in the sense that they do not require any more knowledge.

Chapter 4

Cooperative Spectrum Sensing

Spectrum sensing using a single cognitive radio has a number of limitations. First of all, the sensitivity of a single sensing device might be limited because of energy constraints. Furthermore, the cognitive radio might be located in a deep fade of the primary user signal, and as such might miss the detection of this primary user. Moreover, although the cognitive radio might be blocked from the primary user's transmitter, this does not mean it is also blocked from the primary user's receiver, an effect that is known as the hidden terminal problem. As a result, the primary user is not detected but the secondary transmission could still significantly interfere at the primary user's receiver. To improve the sensitivity of cognitive radio spectrum sensing and to make it more robust against fading and the hidden terminal problem, cooperative sensing can be used. The concept of cooperative sensing is to use multiple sensors and combine their measurements into one common decision. In this section, we will consider this approach, including both soft combining and hard combining, where for the latter we will also look at the influence of fading of the reporting channels to the fusion center.

4.1 Soft Combining

Assume that there are M sensors. Then, the hypothesis test (1.2) becomes

$$\begin{aligned} H_0 &: \mathbf{y}_m = \mathbf{w}_m,\ m = 1, \ldots, M, \\ H_1 &: \mathbf{y}_m = \mathbf{x}_m + \mathbf{w}_m,\ m = 1, \ldots, M. \end{aligned}$$

Suppose that the received signals at different sensors are independent of one another, and let $\mathbf{y} = \left[\mathbf{y}_1^T,\ \mathbf{y}_2^T, \ldots, \mathbf{y}_M^T\right]^T$. Then, the log-likelihood ratio is

$$\log\left(\frac{p(\mathbf{y}|H_1)}{p(\mathbf{y}|H_0)}\right) = \log\left(\prod_{m=1}^{M}\frac{p(\mathbf{y}_m|H_1)}{p(\mathbf{y}_m|H_0)}\right) = \sum_{m=1}^{M}\log\left(\frac{p(\mathbf{y}_m|H_1)}{p(\mathbf{y}_m|H_0)}\right) = \sum_{m=1}^{M}\Lambda^{(m)}, \tag{4.1}$$

where $\Lambda^{(m)} = \log\left(\frac{p(\mathbf{y}_m|H_1)}{p(\mathbf{y}_m|H_0)}\right)$ is the log-likelihood ratio for the mth sensor. That is, if the received signals for all sensors are independent, the optimal fusion rule is to sum the local log-likelihood ratios.

Consider the case in which the noise vectors $\mathbf{w}_m$ are independent $\mathbf{w}_m \sim \mathcal{N}(\mathbf{0}, \sigma_m^2\mathbf{I})$, and the signal vectors $\mathbf{x}_m$ are independent $\mathbf{x}_m \sim \mathcal{N}(\mathbf{0}, \gamma_m^2\mathbf{I})$. After removal of irrelevant constants, the log-likelihood ratio (4.1) can be written as

$$\sum_{m=1}^{M}\frac{\|\mathbf{y}_m\|^2}{\sigma_m^2}\frac{\gamma_m^2}{(\sigma_m^2+\gamma_m^2)}. \tag{4.2}$$

The statistic $\frac{\|\mathbf{y}_m\|^2}{\sigma_m^2}$ is the soft decision from an energy detector at the mth sensor, as shown in (2.3). Thus, the optimal cooperative detection scheme is to use energy detection for the individual sensors, and combine the soft decisions by the weighted sum (4.2). This result is also shown in [31], for the case when $\sigma_m^2 = 1$, and thus γ_m^2 is equivalent to the SNR experienced by the mth sensor.

The main source of correlation between users is shadow fading. Multipath fading is uncorrelated at very small distances, on the scale of half a wavelength, and thus this kind of correlation can easily be avoided. Hence the correlation is mainly distance-dependent, and the cooperation gains are limited by the physical separation of the cognitive users. From a detection perspective, a large separation between users is desired. However, if cognitive users are to be able to communicate without disturbing the primary system they must be sufficiently near to one another. Thus, there is a distance trade-off between detection performance and cognitive communication. The cooperative gain and the effect of untrusted users, under the assumption that the noise and signal powers are equal for all sensors, are analyzed in [32]. It is shown in [32] that correlation between the sensors severely decreases the cooperation gain and that if one out of M sensors is untrustworthy, then the sensitivity of each individual sensor must be as good as that achieved with M trusted users.

4.2 Hard Combining

So far we have considered optimal cooperative detection. That is, all users transmit soft decisions to a fusion center, which combines the soft values to

one common decision. This is equivalent to the case in which the fusion center has access to the received data for all sensors, and performs optimal detection based on all data. This potentially requires a very large amount of data to be transmitted to the fusion center. The other extreme case of cooperative detection is that each sensor makes its own individual decision, and transmits only a binary value to the fusion center. Then, the fusion center combines the hard decisions into one common decision, for instance using a voting rule (c.f. [33]).

Suppose that the individual statistics $\Lambda^{(m)}$ are quantized to one bit, such that $\Lambda^{(m)} \in \{0, 1\}$ is the hard decision from the mth sensor. Here, 1 means that a signal is detected and 0 means that the channel is deemed to be available. The voting rule then decides that a signal is present if at least C of the M sensors have detected a signal, for $C \in [1, M]$. The test decides on H_1 if

$$\sum_{m=1}^{M} \Lambda^{(m)} \geq C.$$

A majority decision is a special case of the voting rule when $C = M/2$, whereas the AND-logic and OR-logic are obtained for $C = M$ and $C = 1$, respectively. Paper F contains a numerical comparison of cooperative sensing with independent sensors using optimal soft combining and hard combining with a majority decision, for the proposed detector in a multiband scenario. Let P_{md} and P_{fa} indicate the local probabilities of missed detection and false alarm, respectively, and denote their global representatives as P_{MD} and P_{FA}. In [34], hard combining is studied for energy detection with equal SNR for all cognitive radios. In particular, the optimal voting rule, optimal local decision threshold and minimal number of cognitive radios are derived, where optimality is defined in terms of the (unweighted) global probability of error $P_{\text{FA}} + P_{\text{MD}}$ (note that this is different from the true global probability of error). It turns out that when the local probability of false alarm P_{fa} and missed detection P_{md} are of the same order, the majority rule is optimal, whereas the optimal voting rule leans towards the OR rule if $P_{\text{fa}} \ll P_{\text{md}}$ and to the AND rule if $P_{\text{fa}} \gg P_{\text{md}}$.

Chapter 5

Contributions of the Dissertation

The dissertation is comprised of six included publications, all in different ways related to spectrum sensing based on second-order statistics. All papers consider exploitation of partial knowledge of the received signal to circumvent the problem of unknown parameters as discussed in Chapter 2. In Papers A-D, the proposed detectors are based on a GLRT approach, whereas in Paper E we use so called covariance matching estimation techniques (COMET) instead of maximum-likelihood estimation. Papers A-E exploit known second-order signal features as mentioned in Chapter 3, where Paper A exploits autocorrelation properties of an OFDM signal as presented briefly in Section 3.1, and Papers B-E all consider different structures of the covariance matrix as mentioned in Section 3.3. In Paper F the problem of unknown noise and signal powers are dealt with by marginalization in a Bayesian framework, and also by estimation using knowledge of the primary user activity in a multiband scenario. Paper F also evaluates the gain of using independent cooperative sensors as discussed in Chapter 4.

5.1 Included Papers

The following publications are included in the dissertation. All publications are authored by Erik Axell and Erik G. Larsson. Brief summaries of the included papers are given in the following.

Paper A: Optimal and Sub-Optimal Spectrum Sensing of OFDM Signals in Known and Unknown Noise Variance

Published in IEEE Journal on Selected Areas in Communications 2011.

This is an evolved version of the paper
E. Axell and E. G. Larsson, "Optimal and Near-Optimal Spectrum Sensing of OFDM Signals in AWGN Channels", in *Proc. 2nd International Workshop on Cognitive Information Processing (CIP)*, Elba Island, Italy, June 14-16 2010, pp. 128-133.

In this work, we consider spectrum sensing of OFDM signals in an AWGN channel. For the case of perfectly known noise and signal powers, we set up a vector-matrix model for an OFDM signal with a cyclic prefix and derive the optimal Neyman-Pearson detector from first principles. The optimal detector exploits the inherent correlation of the OFDM signal incurred by the repetition of data in the cyclic prefix, using knowledge of the length of the cyclic prefix and the length of the OFDM symbol. We compare the optimal detector to the energy detector numerically, and show that the energy detector is near-optimal (within 1 dB SNR) when the noise variance is known. Thus, when the noise power is known, no substantial gain can be achieved by using any other detector than the energy detector.

For the case of completely unknown noise and signal powers, we derive a GLRT based on empirical second-order statistics of the received data with a few approximations. The proposed GLRT detector exploits the time varying correlation structure of the OFDM signal and does not require any knowledge of the noise or the signal power. The GLRT detector is compared to state-of-the-art OFDM signal detectors, and shown to improve the detection performance with 5 dB SNR in relevant cases.

Paper B: A Unified Framework for GLRT-Based Spectrum Sensing of Signals with Covariance Matrices with Known Eigenvalue Multiplicities

Published in the proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP) 2011.

In this paper, we create a unified framework for spectrum sensing of signals which have covariance matrices with known eigenvalue multiplicities. We derive the GLRT for this problem, with arbitrary eigenvalue multiplicities under both hypotheses. We also show a number of applications to spectrum sensing for cognitive radio and show that the GLRT for these applications, of which some are already known, are special cases of the general result.

Paper C: Multiantenna Spectrum Sensing of a Second-Order Cyclostationary Signal

Published in the proceedings of IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP) 2011.

We consider spectrum sensing of a second-order cyclostationary signal received at multiple antennas. The proposed detector exploits both the spatial and the temporal correlation of the received signal, from knowledge of the fundamental period of the cyclostationary signal and the eigenvalue multiplicities of the temporal covariance matrix. All other parameters, such as the channel gains or the noise power, are assumed to be unknown. The proposed detector is shown numerically to outperform state-of-the-art detectors for spectrum sensing of an OFDM signal, both when using a single antenna and with multiple antennas.

Paper D: Eigenvalue-Based Spectrum Sensing of Orthogonal Space-Time Block Coded Signals

This paper is a reworked combination, with some additions, of the papers
E. Axell and E. G. Larsson, "Eigenvalue-Based Spectrum Sensing of Orthogonal Space-Time Block Coded Signals", submitted to *IEEE Transactions on Signal Processing*
and
E. Axell and E. G. Larsson, "Spectrum Sensing of Orthogonal Space-Time Block Coded Signals with Multiple Receive Antennas", in *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Process. (ICASSP)*, Dallas, Texas, March 14-19 2010, pp. 3110-3113.

We consider spectrum sensing of signals encoded with an orthogonal space-time block code (OSTBC). Such signals are shown to have a specific covariance structure which is exploited for detection. We consider unknown noise and signal powers as well as channel coefficients, and derive the GLRT based on knowledge of the eigenvalue multiplicities of the covariance matrix, that are inherent owing to the OSTBC. We derive theoretical bounds for the performance of the proposed detector. In addition, we consider unsynchronized detection. We show that the proposed detector is robust to carrier frequency offset, and propose a detector that deals with timing synchronization using the detector for the synchronized case as a building block. The proposed detectors are shown numerically to perform very well.

Paper E: Spectrum Sensing of Signals with Structured Covariance Matrices Using Covariance Matching Estimation Techniques

Published in the proceedings of IEEE Global Communications Conference (GLOBECOM) 2011.

In this work, we consider spectrum sensing of Gaussian signals with structured covariance matrices. We show that the optimal detector based on the probability distribution of the sample covariance matrix is equivalent to the optimal detector based on the raw data, if the covariance matrices are known. However, the covariance matrices are unknown in general. Therefore, we propose to estimate the unknown parameters using covariance matching estimation techniques (COMET). We also derive the optimal detector based on a Gaussian approximation of the sample covariance matrix, and show that this is closely connected to COMET.

Paper F: A Bayesian Approach to Spectrum Sensing, Denoising and Anomaly Detection

Published in the proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP) 2009.

In this paper, we deal with the problem of discriminating samples that contain only noise from samples that contain a signal embedded in noise. The focus is on the case when the variance of the noise is unknown. We derive the optimal soft decision detector using a Bayesian approach. The complexity of this optimal detector grows exponentially with the number of observations and as a remedy, we propose a number of approximations to it. The problem under study is a fundamental one and it has applications in signal denoising, anomaly detection, and spectrum sensing for cognitive radio. We illustrate the results in the context of the latter.

5.2 Not Included Papers

The following publications by the author are not included in the dissertation either because they do not fit within the main scope of the dissertation, or another person is the lead author. The first paper in the list has served as a foundation for the introductory part of the dissertation.

E. Axell, G. Leus, E. G. Larsson and H. V. Poor, "Spectrum Sensing for Cognitive Radio: State-of-the-Art and Recent Advances", *IEEE Signal Processing Magazine*, vol. 29, no. 3, pp. 101-116, May 2012.

A. Blad, E. Axell and E. G. Larsson, "Spectrum Sensing of OFDM Signals in the Presence of CFO: New Algorithms and Empirical Evaluation Using USRP", in *Proc. IEEE Workshop on Signal Processing Adv. in Wireless Commun. (SPAWC)*, Çeşme, Turkey, June 17-20 2012.

E. Axell and E. G. Larsson, "Comments on "Multiple Antenna Spectrum Sensing in Cognitive Radios" ", *IEEE Transactions on Wireless Communications*, vol. 10, no. 5, pp. 1678-1680, May 2011.

D. Danev, E. Axell, and E. G. Larsson, "Spectrum Sensing Methods for Detection of DVB-T Signals in AWGN and Fading Channels", in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Istanbul, Turkey, Sep. 26-30 2010, pp. 2721-2726.

E. Axell, G. Leus and E. G. Larsson, "Overview of Spectrum Sensing for Cognitive Radio", in *Proc. 2nd International Workshop on Cognitive Information Processing (CIP)*, Elba Island, Italy, June 14-16 2010, pp. 322-327.

E. Axell, E. G. Larsson and J-Å Larsson, "On the Optimal K-term Approximation of a Sparse Parameter Vector MMSE Estimate", in *Proc. IEEE Workshop on Statistical Signal Processing (SSP)*, Cardiff, Wales, UK, Aug.31-Sept. 3 2009, pp. 245-248.

E. Axell, E. G. Larsson and D. Danev, "Capacity Considerations for Uncoordinated Communication in Geographical Spectrum Holes", *Physical Communication Journal (ELSEVIER)*, vol. 2, no. 1-2, pp. 3-9, 2009.

References

[1] FCC, "Spectrum policy task force report," Federal Communications Commission, Tech. Rep. 02-135, Nov. 2002, available: http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-228542A1.pdf.

[2] M. McHenry, "NSF spectrum occupancy measurements project summary," SSC, Tech. Rep., Aug. 2005, available: http://www.sharedspectrum.com/.

[3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, pp. 201–220, Feb. 2005.

[4] I. J. Mitola, "Software radios: Survey, critical evaluation and future directions," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 8, pp. 25–36, Apr. 1993.

[5] C. E. Shannon, "Computers and automata," *Proc. IRE*, vol. 41, pp. 1234–1241, Oct. 1953.

[6] E. G. Larsson and M. Skoglund, "Cognitive radio in a frequency-planned environment: some basic limits," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 4800–4806, Dec. 2008.

[7] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Proc. 42nd Annual Allerton Conf. on Commun., Cont., and Comp.*, Monticello, Illinois, USA, Oct. 2004, pp. 1662–1671.

[8] H. V. Poor, *An Introduction to Signal Detection and Estimation.* Springer Verlag, New York, 1994.

[9] R. Couillet and M. Debbah, "A Bayesian framework for collaborative multisource signal sensing," *IEEE Trans. Signal Process.*, vol. 58, pp. 5186–5195, Oct. 2010.

[10] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proc. IEEE*, vol. 55, pp. 523–531, Apr. 1967.

[11] A. Sonnenschein and P. Fishman, "Radiometric detection of spread-spectrum signals in noise of uncertain power," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 3, pp. 654–660, Jul. 1992.

[12] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE J. Sel. Topics Signal Process.*, vol. 2, pp. 4–17, Feb. 2008.

[13] E. G. Larsson and G. Regnoli, "Primary system detection for cognitive radio: Does small-scale fading help?" *IEEE Commun. Lett.*, vol. 11, no. 10, pp. 799–801, 2007.

[14] S. Chaudhari, V. Koivunen, and H. V. Poor, "Autocorrelation-based decentralized sequential detection of OFDM signals in cognitive radios," *IEEE Trans. Signal Process.*, vol. 57, pp. 2690–2700, Jul. 2009.

[15] Huawei Technologies and UESTC, "Sensing scheme for DVB-T," Jul. 2006, IEEE Std.802.22-06/0127r1.

[16] Z. Lei and F. Chin, "OFDM signal sensing for cognitive radios," in *PIMRC*, Sep. 2008, pp. 1–5.

[17] W. A. Gardner, "Cyclostationarity: Half a century of research," *Signal Processing*, vol. 86, pp. 639–697, Apr. 2006.

[18] A. V. Dandawaté and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Trans. Signal Process.*, vol. 42, pp. 2355–2369, Sep. 1994.

[19] J. Lundén, V. Koivunen, A. Huttunen, and H. V. Poor, "Collaborative cyclostationary spectrum sensing for cognitive radio systems," *IEEE Trans. Signal Process.*, vol. 57, pp. 4182–4195, Nov. 2009.

[20] S. Enserink and D. Cochran, "A cyclostationary feature detector," in *Proc. IEEE Asilomar Conf. Signals, Systems, and Computers*, Pacific Grove, CA, Oct-Nov 1994, pp. 806–810.

[21] S. Chaudhari, J. Lundén, and V. Koivunen, "Collaborative autocorrelation-based spectrum sensing of OFDM signals in cognitive radios," in *Proc. Annual Conf. on Information Sciences and Systems (CISS)*, Princeton, NJ, Mar. 2008, pp. 191–196.

[22] P. Bianchi, M. Debbah, M. Maida, and J. Najim, "Performance of statistical tests for single source detection using random matrix theory," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 2400–2419, Apr 2011.

[23] P. Wang, J. Fang, N. Han, and H. Li, "Multiantenna-assisted spectrum sensing for cognitive radio," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 1791–1800, May 2010.

[24] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 814–823, Feb. 2010.

[25] Y. Zeng and Y.-C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE Trans. Commun.*, vol. 57, pp. 1784–1793, Jun. 2009.

[26] ——, "Spectrum-sensing algorithms for cognitive radio based on statistical covariances," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 1804–1815, May 2009.

[27] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoustics, Speech, and Signal Process.*, vol. 33, no. 2, pp. 387–392, Apr. 1985.

[28] M. Chiani and M. Win, "Estimating the number of signals observed by multiple sensors," in *Proc. IEEE Int. Workshop on Cognitive Information Process. (CIP)*, Elba Island, Italy, Jun. 2010, pp. 156–161.

[29] M. Haddad, A. M. Hayar, M. Debbah, and H. M. Fetoui, "Cognitive radio sensing information-theoretic criteria based," in *Proc. Int. Conf. on Cognitive Radio Oriented Wireless Networks and Commun. (CrownCom)*, Orlando, FL, USA, Aug. 2007, pp. 241–244.

[30] R. Wang and M. Tao, "Blind spectrum sensing by information theoretic criteria for cognitive radios," *IEEE Trans. Veh. Technol.*, vol. 59, no. 8, pp. 3806–3817, Oct. 2010.

[31] J. Ma, G. Zhao, and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 4502–4507, Nov. 2009.

[32] S. M. Mishra, A. Sahai, and R. W. Brodersen, "Cooperative sensing among cognitive radios," in *Proc. IEEE Int. Conf. on Commun. (ICC)*, Istanbul, Turkey, Jun. 2006, pp. 1658–1663.

[33] Z. Quan, S. Cui, H. V. Poor, and A. Sayed, "Collaborative wideband sensing for cognitive radios," *IEEE Signal Process. Mag.*, vol. 25, pp. 60–73, Nov. 2008.

[34] W. Zhang, R. Mallik, and K. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 5761–5766, Dec. 2009.