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Non-Rigid Diffeomorphic Image Registration of Medical Images using Polynomial Expansion

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Abstract. The use of polynomial expansion in image registration has previously been shown to be beneficial due to fast convergence and high accuracy. However, earlier work has only briefly out-lined how non-rigid image registration is handled, e.g. not discussing issues like regularization of the displacement field or how to accumulate the displacement field. In this work, it is shown how non-rigid image registration based upon polynomial expansion can be integrated into a generic framework for non-rigid image registration achieving diffeomorphic displacement fields. The proposed non-rigid image registration algorithm using diffeomorphic field accumulation is evaluated on both synthetically deformed data and real image data and compared to additive field accumulation. The results clearly demonstrate the power of the diffeomorphic field accumulation.

1 Introduction

Image registration is a well-known concept, widely applied in a number of different areas, for instance geophysics, robotics and medicine. The use of image registration within the medical image domain is vast and includes various tasks, such as; surgical planning, radiotherapy planning, image-guided surgery, disease progression monitoring and image fusion. The basic idea of image registration is to find a displacement field \mathbf{d} that geometrically aligns a moving image, I_m , with a fixed image, I_f . This can be more strictly defined as an optimization problem, where the aim is to find a displacement field that maximizes the similarity between the moving and the fixed images while maintaining a certain control (smoothness) of the displacement field.

A commonly stated requirement for medical non-rigid image registration is to have diffeomorphic displacement fields, i.e. displacement fields that are invertible, differentiable and where its inverse also is differentiable. This is considered important, since it allows compression and deformation of organs but prevents non-invertible spatial transforms. Diffeomorphism is considered to be a necessary condition for having physically plausible displacement fields [1].

Polynomial expansion was introduced by Farnebäck [3] as a method to locally approximate a signal with a polynomial. In a work by Farnebäck and Westin [4] it was shown how polynomial expansion can be used to perform both linear (e.g. translation and affine) and non-rigid image registration. This idea was further

developed by Wang *et al.* [8]. Both Farneback and Westin [4] and Wang *et al.* [8] showed that image registration using polynomial expansions has some valuable qualities. Since it is based on an analytical solution, the convergence rate is fast, typically only needing a few iterations per scale. Also the accuracy of the registration has been shown to be similar or even better than the accuracy of the well-known demons algorithm. However, thus far, previous works have not satisfactorily dealt with questions related to the regularization of the displacement field and accumulation of the displacement field.

The contribution of this paper is to present how non-rigid image registration based upon polynomial expansion can be integrated into a generic framework for non-rigid registration, which enforces diffeomorphic displacement fields. The framework also holds the possibility of including other types of regularizers. The proposed non-rigid image registration algorithm using diffeomorphic field accumulation is evaluated on both synthetically deformed data and real image data and compared to additive field accumulation.

2 Background

2.1 Polynomial Expansion

The basic idea of polynomial expansion is to locally approximate each signal value with a polynomial. In case of a quadratic polynomial, this approximation can be expressed as $f(\mathbf{x}) \sim \mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c$, where \mathbf{A} is a symmetric matrix, \mathbf{b} a vector and c a scalar. In the linear case, the approximation reduces to $f(\mathbf{x}) \sim \mathbf{b}^T \mathbf{x} + c$. The coefficients are determined by a weighted least squares fit to the local signal. The weighting depends on two factors, certainty and applicability. These terms are the same as in normalized convolution, see [3, 6], which forms the basis for polynomial expansion.

2.2 Image Registration Using Polynomial Expansion

Global Translation Let both the fixed and the moving images be locally approximated with a linear polynomial expansion and assume that the moving image is a globally translated version of the fixed image, thus,

$$I_f(\mathbf{x}) = \mathbf{b}_f^T \mathbf{x} + c_f, \quad (1)$$

$$I_m(\mathbf{x}) = \mathbf{b}_m^T \mathbf{x} + c_m = I_f(\mathbf{x} - \mathbf{d}) = \mathbf{b}_f^T (\mathbf{x} - \mathbf{d}) + c_f, \quad (2)$$

which gives $\mathbf{b}_m = \mathbf{b}_f$ and $c_m = c_f - \mathbf{b}_f^T \mathbf{d}$, where the latter is sufficient to find the translation \mathbf{d} .

$$\mathbf{d} = (\mathbf{b}_f \mathbf{b}_f^T)^{-1} \mathbf{b}_f (c_f - c_m). \quad (3)$$

In practice, a point-wise polynomial expansion is estimated, let $\mathbf{b}_f(\mathbf{x})$, $c_f(\mathbf{x})$, $\mathbf{b}_m(\mathbf{x})$, and $c_m(\mathbf{x})$ be the polynomial expansion coefficients for the fixed and the moving images. Since it cannot be expected that $\mathbf{b}_f(\mathbf{x}) = \mathbf{b}_m(\mathbf{x})$ holds, they

are replaced with the average $\mathbf{b}(\mathbf{x}) = (\mathbf{b}_f(\mathbf{x}) + \mathbf{b}_m(\mathbf{x}))/2$, also set $\Delta c(\mathbf{x}) = c_f(\mathbf{x}) - c_m(\mathbf{x})$. Thus, the primary constraint is given by $\mathbf{b}(\mathbf{x})^T \mathbf{d} = \Delta c(\mathbf{x})$, where \mathbf{d} is computed by minimizing the squared error in the constraint over the whole image,

$$\epsilon^2 = \sum_{\mathbf{x}} \|\mathbf{b}(\mathbf{x})\mathbf{d} - \Delta c(\mathbf{x})\|^2, \quad (4)$$

with the least squares solution given as

$$\mathbf{G} = \sum_{\mathbf{x}} \mathbf{b}(\mathbf{x})\mathbf{b}(\mathbf{x})^T, \quad (5)$$

$$\mathbf{h} = \sum_{\mathbf{x}} \mathbf{b}(\mathbf{x})\Delta c(\mathbf{x}), \quad (6)$$

$$\mathbf{d} = \mathbf{G}^{-1}\mathbf{h}. \quad (7)$$

Non-Rigid Registration A non-rigid registration algorithm can be achieved if the assumption about a global translation is relaxed and we instead sum over a neighborhood around each pixel in (4), thereby obtaining an estimate of a local translation for each pixel. More precisely (4) is changed to

$$\epsilon^2(\mathbf{x}) = \sum_{\mathbf{y}} w(\mathbf{y})(\|\mathbf{b}(\mathbf{x} - \mathbf{y})^T \mathbf{d}(\mathbf{x}) - \Delta c(\mathbf{x} - \mathbf{y})\|^2), \quad (8)$$

where w weights the points in the neighborhood around each pixel. This weight can be any low-pass function, but here it is assumed to be Gaussian. Clearly this equation can be interpreted as a convolution of the point-wise contributions to the squared error in (4) with the low-pass filter w . The solution is given as

$$\mathbf{G}(\mathbf{x}) = \mathbf{b}(\mathbf{x})\mathbf{b}(\mathbf{x})^T, \quad (9)$$

$$\mathbf{h}(\mathbf{x}) = \mathbf{b}(\mathbf{x})\Delta c(\mathbf{x}), \quad (10)$$

$$\mathbf{G}_{\text{avg}}(\mathbf{x}) = (\mathbf{G} * w)(\mathbf{x}), \quad (11)$$

$$\mathbf{h}_{\text{avg}}(\mathbf{x}) = (\mathbf{h} * w)(\mathbf{x}), \quad (12)$$

$$\mathbf{d}(\mathbf{x}) = \mathbf{G}_{\text{avg}}(\mathbf{x})^{-1}\mathbf{h}_{\text{avg}}(\mathbf{x}). \quad (13)$$

Note that in this work, we have only used a linear polynomial expansion, but as shown in [4, 8] a quadratic polynomial expansion along with similar derivations can also be used for image registration. In fact, it is possible to combine both in order to obtain a more robust solution.

3 Generic Framework For Non-Rigid Image Registration

An often encountered benefit of non-rigid image registration algorithms, is that they allow for a decoupling of the optimization of the similarity measure and the regularization of the displacement field, for instance as shown in [7] for the demons algorithm. Janssens *et al.* [5] utilizes this to present a generic framework

for non-rigid image registration, where different components can easily be exchanged and, thus, efficiently evaluated. In their work, the framework consists of the three main components *field computation*, *field accumulation* and *field regularization*.

In this section and also in the rest of this work, the focus is on the last two components, field accumulation and field regularization, since the actual field computation is what has been covered by earlier work and also in Section 2.2.

3.1 Field Accumulation

Field accumulation is most often implemented as additive accumulation, i.e.

$$\mathbf{d}_a = \mathbf{d}_a + \mathbf{d}_u. \quad (14)$$

Here an iterative process is assumed and \mathbf{d}_u refers to the update field and \mathbf{d}_a to the accumulated field. However, considering this iterative process, it turns out that compositive field accumulation is more correct to use. If first defining the "deformation" operation of \mathbf{d} on I_m as

$$I_d(\mathbf{x}) = I_m(\mathbf{x}) \diamond \mathbf{d}(\mathbf{x}) = I_m(\mathbf{x} + \mathbf{d}(\mathbf{x})), \quad (15)$$

then compositive field accumulation can be defined as

$$\mathbf{d}_a \oplus \mathbf{d}_u = \mathbf{d}_u + \mathbf{d}_a \diamond \mathbf{d}_u. \quad (16)$$

This means, that the two displacements fields are accumulated by first deforming \mathbf{d}_a according to \mathbf{d}_u and then adding \mathbf{d}_u . Since, \mathbf{d}_u is estimated from $I_m \diamond \mathbf{d}_a$ it is obvious that $\mathbf{d}_a + \mathbf{d}_u$ is not valid but rather that $\mathbf{d}_a \oplus \mathbf{d}_u$ is consistent with the applied spatial transformation. If the two displacements fields are diffeomorphic, than their composition is also diffeomorphic [2].

Assuming a smooth vector field \mathbf{d} and a point \mathbf{x} , the diffeomorphic flow $\phi_{\mathbf{d}}(\mathbf{x}, t)$ is the solution $\mathbf{u}(t)$ of

$$\frac{d}{dt} \mathbf{u}(t) = \mathbf{d}(\mathbf{u}), \quad (17)$$

$$\mathbf{u}(0) = \mathbf{x}. \quad (18)$$

Vercauteren *et al.* [7] showed that the exponential of \mathbf{d} is the deformation obtained by the flow of \mathbf{d} at time $t = 1$ and therefore the exponential mapping of the vector field (\mathbf{d}) can be used as an operation for obtaining a diffeomorphic displacement field. In [7] it is further shown that a scaling and squaring approach can be used as an efficient approximation, i.e.

$$\exp(\mathbf{d}) \approx \exp(2^{-k} \mathbf{d})^{2^k}, \quad (19)$$

which is implemented with the following three steps:

- Scale \mathbf{d} with a factor 2^{-k} to ensure that $2^{-k} \mathbf{d}$ is small enough, for example, $\max \|2^{-k} \mathbf{d}\| < 0.5$.

- Compute a first-order integration of the flow: $\phi_{\mathbf{d}}(\mathbf{x}, 2^{-k}) \approx \text{Id}(\mathbf{x}) + 2^{-k} \mathbf{d}(\mathbf{x})$
- Perform k squarings of the flow in order to obtain the flow a time 1. This is implemented as k recursive compositive field accumulations of the flow $\phi_{\mathbf{d}}(\mathbf{x}, 2^{-k})$.

Using the compositive field accumulation and the exponentiation of the displacement field, a diffeomorphic field accumulation can be achieved using

$$\mathbf{d}_a = \mathbf{d}_a \oplus \exp(\mathbf{d}_u) \quad (20)$$

3.2 Field Regularization

Regularization of the displacement field is often used to ensure the smoothness of the displacement field, typically requiring a very strong Gaussian smoothing, which often will lead to a reduction of the registration accuracy. However, using a diffeomorphic field accumulation, a regularization is already in place and the need for extra regularization is not as strong as before. Regularization is not only used for smoothing of the displacement field in general, but can also be used for enforcing various characteristics on the displacement field. Common examples are modeling of physical processes, for instance, fluid-like or diffusion-like regularization. Fluid-like regularization is obtained by smoothing of the update field

$$\mathbf{d}_u = \mathbf{d}_u * g, \quad (21)$$

whereas diffusion-like regularization is implemented as smoothing of the accumulated displacement field

$$\mathbf{d}_a = \mathbf{d}_a * g. \quad (22)$$

4 Results

Three different experiments were performed to evaluate how the diffeomorphic field accumulation affects the end result for non-rigid image registration based on polynomial expansion, when compared to additive field accumulation. In all experiments the field computation described in (9)-(13) was used along with both applying fluid-like and diffusion-like regularization ($\sigma_{fluid} = \sigma_{diff} = 1.0$) and a multi-scale strategy. Also the number of iterations per scale was fixed, 20 iterations on the finest scale and 10 iterations on the coarser scales.

The first experiment involved the registration of a filled O with a C, see left column of Fig. 1. These images are often used to analyze how an algorithm handles very large deformations and is here only evaluated qualitatively. In the remaining columns of Fig. 1, we can observe the results of the registration using the two different accumulation methods, where the images clearly show that a similar registration accuracy is achieved, whereas the additive field accumulation fails to provide an invertible displacement field.

In the second experiment, a 3D dimensional T1-weighted MRI data set of the brain (size 144x144x144 and spatial resolution 1.66x1.66x1.7 mm³) has been

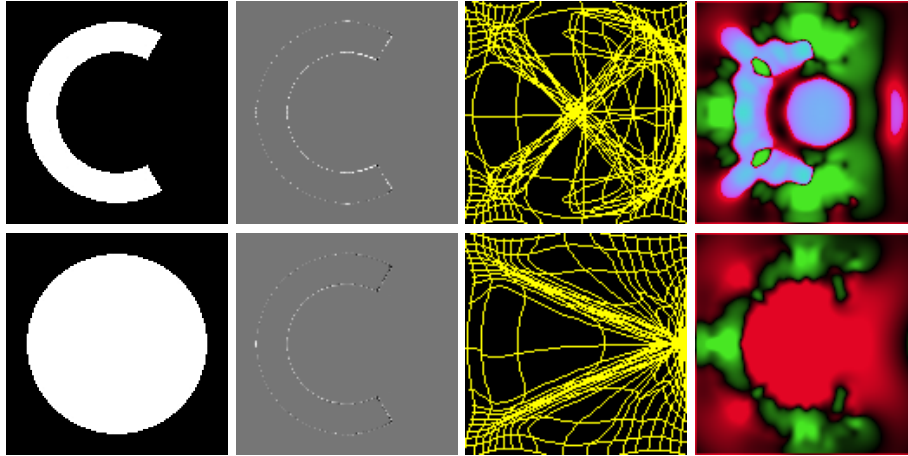


Fig. 1: The results from the first experiment, where a filled O is registered to a C. The registration is done using two different field accumulation methods. **Top row:** Additive accumulation **Bottom row** Diffeomorphic accumulation. **First column:** The filled O to be registered to the C. **Second column:** The difference image between the C and the registered O. **Third column:** The grid used to resample the filled O. **Fourth column:** The Jacobian determinant of the displacement field, where green corresponds to expansion, red contraction and purple to folding.

deformed using 20 randomly created diffeomorphic displacement fields. The original data set is then registered to the deformed data sets. Example images from a registration in this experiment are given in Fig. 2. Since the true displacement fields are known, the average displacement error (ADE) along with the mean square error (MSE) can be used to measure the accuracy. The smoothness of the displacement field is given by the harmonic energy (HE) and the minimum value of the Jacobian determinant of the displacement field ($\min |\text{Jac}|$). The HE refers to the mean Frobenius norm of the Jacobian of the displacement field. Boxplots of the results are given in Fig. 3 and the results show that even though the accuracy of diffeomorphic accumulation surpasses additive accumulation, the obtained displacement fields are still smoother and invertible.

In the third experiment, ten data sets of the same type as in the second experiment but from ten different subjects have been used. In this experiment, one data set acted as fixed image and the remaining nine data sets were registered to the fixed. The same metrics as in the second experiment, apart from ADE, have been used to analyze the obtained results. Also the number of voxels with a negative Jacobian determinant was used (voxels $|\text{Jac}| < 0$). The results are given in Fig. 4 and show that the two methods for field accumulation provide similar registration accuracy while diffeomorphic field accumulation produces smoother displacement fields.

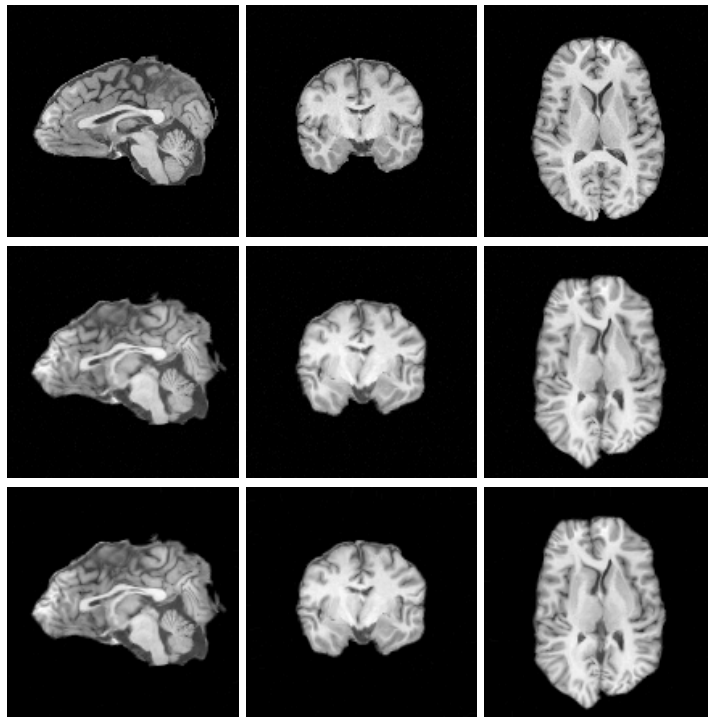


Fig. 2: Example images from the second experiment showing the center sagittal, coronary and axial planes. **Top row** Original data set used as moving image. **Middle row** Synthetically deformed data set used as fixed image. **Bottom row** Result data set from registering the moving to the fixed image.

5 Discussion

In this work, we have shown how non-rigid image registration based upon polynomial expansion can be integrated into a generic framework for non-rigid image registration, where the different components of the framework allow diffeomorphic field accumulation and various regularization methods to be used. The focus of the evaluation in this work has been on the effects of diffeomorphic field accumulation when compared to additive field accumulation.

The results from the three different experiments clearly show the benefit of the diffeomorphic field accumulation for obtaining diffeomorphic displacement fields, while at the same time maintaining the registration accuracy or even providing superior registration accuracy. The results are clearly along the line of previous works utilizing a similar approach for obtaining diffeomorphic displacement field, for example [1] and [5]. However, a difference is that in the third experiment, we obtained some displacement fields that contained a negative Jacobian determinant, whereas other works have no report of this.

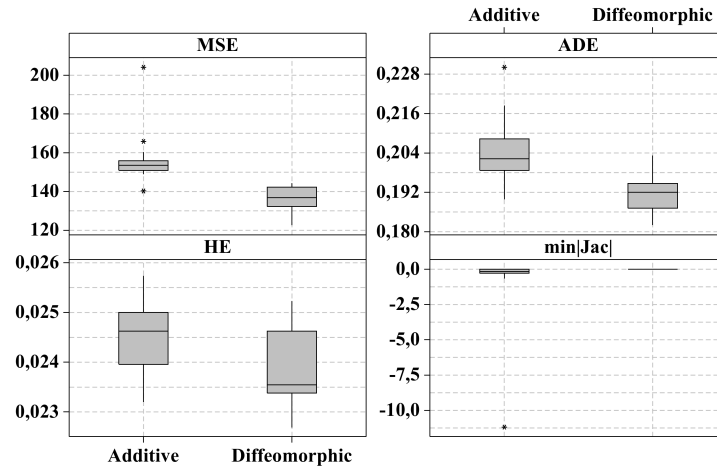


Fig. 3: Boxplots from the second experiment, where a data set is registered to a synthetically deformed data set. For each method 20 randomly created diffeomorphic displacement fields were used.

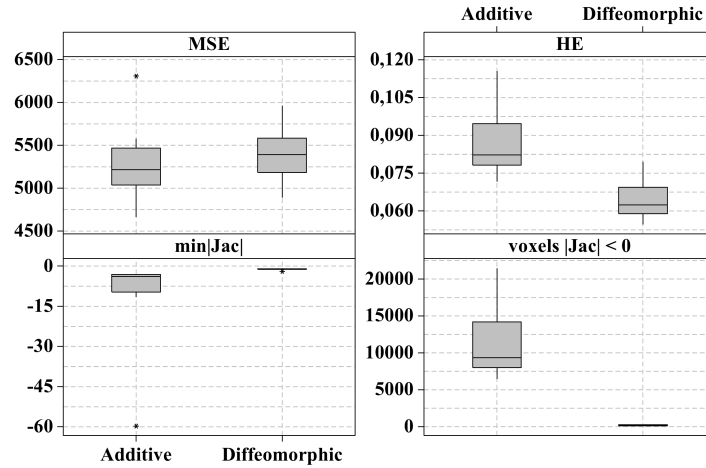


Fig. 4: Results from the third experiment, where nine subjects were registered to a tenth subject.

The diffeomorphic field accumulation comes of course with a cost, the cost of squaring the displacement fields, which in this case refers to recursive composite field accumulations, where the most computationally demanding part refers to the deformation operation, i.e. interpolation. Instead of just performing a single deformation per iteration when using additive field accumulation, roughly 10–16

deformations had to be performed on average per iteration in our experiments. For example, the run-time for a registration in the second or the third experiment was approximately 110 seconds for additive field accumulation and 180 seconds for diffeomorphic field accumulation, when using a MATLAB implementation.

An interesting aspect for future work, which was not evaluated in our work, is the difference between compositive field accumulation and diffeomorphic field accumulation. Some initial experiments showed no sign of difference in the smoothness of the obtained displacement fields using the two different methods, and this would also be the expected result if the maximum step length per iteration is small enough. If this is the case, then the extra computational burden associated with diffeomorphic field accumulation is uncalled for.

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