Earthquake Rupture Dynamics in Complex Geometries using Coupled Summation-By-Parts High-order Finite Difference Methods and Node-Centered Finite Volume Methods

Ossian O’Reilly, Eric M. Dunham, Jeremy E. Kozdon and Jan Nordström

Linköping University Post Print

N.B.: When citing this work, cite the original article.

Original Publication:

Postprint available at: Linköping University Electronic Press
http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-81441
Abstract
We present a 2-D multi-block method for earthquake rupture dynamics in complex geometries using summation-by-parts (SBP) high-order finite differences on structured grids coupled to finite volume methods on unstructured meshes. The node-centered finite volume method is used on unstructured triangular meshes to resolve earthquake ruptures propagating along nonplanar faults with complex geometrical features. The unstructured meshes discretize the fault geometry only in the vicinity of the faults and have collocated nodes on the fault boundaries. Away from faults, where no complex geometry is present, the seismic waves emanating from the earthquake rupture are resolved using the high-order finite difference method on coarsened structured grids, improving the computational efficiency while maintaining the accuracy of the method.

In order for the SBP high-order finite difference method to communicate with the node-centered finite volume method in a stable manner, interface conditions are imposed using the simultaneous approximation term (SAT) penalty method. In the SAT method the interface conditions and boundary conditions are imposed in a weak manner. Fault interface conditions (rate-and-state friction) are also imposed in a provably stable manner using the SAT method. Another advantage of the SAT method is the ability to impose multiple boundary conditions at a single boundary node, e.g., at the triple junction of a branching fault. The accuracy and stability of the numerical implementation are verified using the method of manufactured solutions and against other numerical implementations. To demonstrate the potential of the method, we simulate an earthquake rupture propagating in a nonplanar fault geometry resolved with unstructured meshes in the fault zone and structured grids in the far-field.

1. Introduction
We demonstrate our numerical method motivated by the 2004 M6.0 Parkfield earthquake. Figure 1 shows a cross-section of the San Andreas fault (SAF) looking from the south to the north. The Pacific plate (PP) is moving into the page and the North-American plate (NAP) is moving out of the page. Observations indicate that surface fractures formed coseismically on the Southwest fracture zone (SWFZ) whereas surface fractures on the SAF were not recorded until 0.5 hours after the mainshock event. In our model, we consider that surface fractures formed coseismically on the Southwest fracture zone (SWFZ) whereas surface fractures formed coseismically on the Southwest fracture zone (SWFZ), whereas surface fractures formed coseismically on the Southwest fracture zone (SWFZ), whereas surface fractures formed coseismically on the Southwest fracture zone (SWFZ), whereas surface fractures formed coseismically on the Southwest fracture zone (SWFZ), whereas surface fractures formed coseismically on the Southwest fracture zone (SWFZ).

2. Hybrid method
Figure 2 shows the geometry in Figure 1 is tessellated into multiple blocks. The branches have been meshed using unstructured meshes and the remaining blocks have been meshed with structured grids. On the structured grids we solve the governing equations using the SBP high-order finite difference method and on the unstructured meshes using the finite volume method. Table 1 compares FVM in the whole domain vs the hybrid method with 4th-order accuracy outside the fault zone. We use a problem where the exact solution is known and remove points in the grids until the errors \( \| e \|_{L^2} \) and \( \| e \|_{L^\infty} \) closely match.

Table 1: Efficiency

<table>
<thead>
<tr>
<th>FVM</th>
<th>Points</th>
<th>( \log_{10} | e |_{L^2} ) Rate Points</th>
<th>( \log_{10} | e |_{L^\infty} ) Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,217</td>
<td>-1.39</td>
<td>1,421</td>
<td>-1.47</td>
</tr>
<tr>
<td>16,571</td>
<td>-2.01</td>
<td>2,073,094</td>
<td>-2.10</td>
</tr>
<tr>
<td>65,728</td>
<td>-2.64</td>
<td>2,09,418</td>
<td>-2.68</td>
</tr>
<tr>
<td>261,776</td>
<td>-3.24</td>
<td>2,022,380</td>
<td>-3.33</td>
</tr>
<tr>
<td>1,044,790</td>
<td>-3.83</td>
<td>1,941,054</td>
<td>-3.82</td>
</tr>
<tr>
<td>4,174,645</td>
<td>-4.37</td>
<td>1,815,353,169</td>
<td>-4.36</td>
</tr>
</tbody>
</table>

3. Finite volume method
Consider for example the momentum balance integrated over a small control volume \( \Omega \):

\[
\int_{\partial \Omega} \rho \frac{\partial \mathbf{v}}{\partial t} \cdot \mathbf{n} \, dA = \int_{\Omega} \rho \mathbf{g} \cdot \mathbf{v} \, dx + \int_{\Omega} \rho \mathbf{f} \cdot \mathbf{v} \, dx,
\]

where \( \mathbf{f} \) is the flux through the face with outward unit normal \( \mathbf{n} \), and

\[
\mathbf{f} = \frac{\partial \mathbf{u}}{\partial x} \cdot \mathbf{n}
\]

Therefore, the flux is approximated by a node-centered finite volume method on unstructured triangular meshes to resolve earthquake ruptures propagating along nonplanar faults with complex geometrical features. The unstructured meshes discretize the fault geometry only in the vicinity of the faults and have collocated nodes on the fault boundaries. Away from faults, where no complex geometry is present, the seismic waves emanating from the earthquake rupture are resolved using the high-order finite difference method on coarsened structured grids, improving the computational efficiency while maintaining the accuracy of the method.

For each node two source terms are imposed; one w.r.t the left boundary and one w.r.t the right boundary.

For example: At node 1 we have continuity between block B1 and B2, as well as fault friction between block B1 and B2. Fault friction at node 1:

\[ S_{north} = \Sigma (\sigma_{yy}(y = 0) - 0), \]

imposes the free-surface boundary condition in Figure 1. \( \Sigma \) is determined to yield a stable scheme. The weak boundary conditions converges to the continuous b.c. with mesh refinement.

5. Weak interface conditions

For example: At node 1 we have continuity between block B1 and B2, as well as fault friction between block B1 and B2. Fault friction at node 1:

\[ S_{north} = \Sigma (\sigma_{yy}(y = 0) - 0), \]

imposes the free-surface boundary condition in Figure 1. \( \Sigma \) is determined to yield a stable scheme. The weak boundary conditions converges to the continuous b.c. with mesh refinement.

6. Simulation

Figure 6: Simulation results

* In reality we use characteristic boundary and interface conditions. The characteristic form of \( F \) leads to a nonlinear equation that we solve for \( V \) with a bracketed secant method.