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Nonlinear propagation of coherent electromagnetic waves in a dense magnetized plasma

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We present an investigation of the nonlinear propagation of high-frequency coherent electromagnetic waves in a uniform quantum magnetoplasma. Specifically, we consider nonlinear couplings of right-hand circularly polarized electromagnetic-electron-cyclotron (CPEM-EC) waves with dispersive shear Alfvén (DSA) and dispersive compressional Alfvén (DCA) perturbations in plasmas composed of degenerate electron fluids and non-degenerate ion fluids. Such interactions lead to amplitude modulation of the CPEM-EC wave packets, the dynamics of which is governed by a three-dimensional nonlinear Schrödinger equation (NLSE) with the frequency shift arising from the relativistic electron mass increase in the CPEM-EC fields and density perturbations associated with the DSA and DCA perturbations. Accounting for the electromagnetic and quantum forces, we derive the evolution equation for the DSA and DCA waves in the presence of the magnetic field-aligned ponderomotive force of the CPEM-EC waves. The NLSE and the driven DSA and DCA equations are then used to investigate the modulational instability. The relevance of our investigation to laser-plasma interaction experiments and the cores of white dwarf stars is pointed out. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4731733>]

I. INTRODUCTION

The field of quantum plasma physics is vibrant and evolving rapidly.^{1–8} It is concerned with the collective behavior of charged particles in dense plasmas in which the electrons are degenerate and the ions non-degenerate. Such dense plasmas are found in astrophysical settings^{9–11} (e.g., in the cores of white dwarf stars and magnetars) and in warm dense matter,¹² in planetary systems¹³ (e.g., in the core of Jupiter), in intense laser-solid compressed density plasma experiments for inertial confined fusion (ICF),¹⁴ and in quantum free-electron-laser (Q-FEL) systems^{15,16} for producing coherent x-rays, as well as in metallic thin films/nanostructures¹⁸ and semiconductor devices.¹⁷ In dense quantum plasmas, the degenerate electrons are Fermions and their equilibrium distribution is governed by the Fermi-Dirac statistics. Furthermore, at high plasma number densities the inter-electron separation could be of the order of atomic dimensions (the Bohr radius $a_B = 5.29 \times 10^{-9}$ cm) and nano-scales, which could be comparable with the De-Broglie thermal wavelength of the charged particles. Here, the quantum mechanics comes into the picture due to (1) overlapping of electron wave functions owing to the Heisenberg uncertainty principle, leading to electron tunneling through the quantum Bohm potential (also referred to as the quantum recoil effect⁶), and (2) electron exchange and electron correlations because of the electron-one-half spin effect. Thus, there are new equations of state^{19,20}

(the degenerate electron pressure that relates the Fermi electron temperature and the electron number density), new quantum forces involving the electron exchange and electron correlation potentials,²¹ as well as the Bohm potential^{1,5,7,8} and electron-one-half spin effects.^{22,23} Inclusion of these forces in the collective behavior of the dense quantum plasma plays a significant role, since the physical phenomena appear on the atomic and nanoscales. For example, recent laboratory studies^{24,25} have conclusively demonstrated the existence of quantum electron plasma oscillations with a frequency spectrum $\omega = [\Omega_p^2 + 3k^2V_F^2(1 + 0.088n_0\lambda_B^3) + \Omega_q^2]^{1/2}$, where $\Omega_p = (4\pi n_0 e^2/m_0)^{1/2}$ is the electron plasma frequency, n_0 is the unperturbed number density, m_0 the rest mass of the electrons, e the magnitude of the electron charge, and $\Omega_q = \hbar k^2/2m_0$, which has a signature of quantum dispersion effects through the electron Fermi speed $V_F = (k_B T_F/m_0)^{1/2}$ and the quantum recoil effect involving the frequency Ω_q . Here, k_B is the Boltzmann constant, \hbar is the Planck constant divided by 2π , and T_F the electron Fermi temperature (in the zero-temperature limit, one has $k_B T_F = (\hbar^2/5m_0)(3\pi^2)^{2/3} n_0^{2/3}$, when the plasma number densities are typically $(1.5 - 4.5) \times 10^{23}$ cm⁻³ in a low-temperature ($T_e < 25$ eV) dense plasma that is unmagnetized). We note that at such high densities, we have $\Omega_p \sim 2 \times 10^{17}$ rad/s, which is in the x-ray regime. Recently, Shukla and Eliasson²⁶ discovered novel physical attraction between ions that are shielded by degenerate electrons. Henceforth, ions can be brought closer in order for fusion to occur at nanoscales, a fusion scenario for ICF schemes.

Intense high-frequency electromagnetic (HF-EM) waves are emitted from the cores of white dwarf stars, in addition

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to being used for heating high density plasmas in ICF schemes. Furthermore, since there exist huge magnetic fields^{27–29} in the cores of massive white dwarf stars and in ICF schemes, it is of practical interest to examine the consequences of ambient magnetic fields on the transport of electrons and the nonlinear propagation of large amplitude electromagnetic waves in a dense magnetoplasma. In this paper, we thus extend our previous study³⁰ on stimulated scattering instabilities of coherent electromagnetic waves in quantum plasmas to include the external magnetic field effect. The latter has a profound influence on the HF-EM wave^{31–39} and also on the low-frequency electromagnetic perturbations^{40–42} that are reinforced by the HF-EM wave pressure. Accordingly, as shown below, the coupling coefficient of stimulated scattering instabilities of HF-EM waves is significantly modified by the presence of an external magnetic field in a dense quantum magnetoplasma.

The manuscript is organized in the following fashion. In Sec. II, we present the governing equations for nonlinearly coupled right-hand circularly polarized electromagnetic-electron-cyclotron (CPEM-EC) and driven (by the low-frequency ponderomotive force of the high-frequency CPEM-EC waves) dispersive shear and compressional Alfvén waves in a dense quantum magnetoplasma. For the high-frequency CPEM-EC wave envelopes, we have our three-dimensional nonlinear Schrödinger equation (NLSE),³⁶ while driven (by the low-frequency ponderomotive force of the CPEM-EC envelopes) equations for the low-frequency dispersive shear Alfvén (DSA) and dispersive compressional Alfvén (DCA) waves are derived by using the two-fluid quantum magnetohydrodynamic equations⁷ and Faraday and Ampère's law. The nonlinearly coupled mode equations are then used in Sec. III to derive the nonlinear dispersion relations for studying the modulational instabilities and associated growth rates. Section IV contains a summary and conclusion of our investigation.

II. FORMULATION

We consider the nonlinear propagation of a large amplitude right-hand CPEM-EC wave along an external magnetic field $\hat{z}B_0$ in a dense quantum magnetoplasma composed of degenerate electron and non-degenerate ion fluids, where \hat{z} is the unit vector along the z -axis in a Cartesian coordinate system and B_0 the strength of the ambient magnetic field. The electric field of the right-hand CPEM-EC is of the form $\mathbf{E} = E_\perp(\hat{x} + i\hat{y})\exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r}) + \text{complex conjugate}$, where the frequency and the wave vector are denoted by ω and \mathbf{k} , respectively, and \hat{x} (\hat{y}) is the unit vector along the x (y) axis. For the magnetic field-aligned right-hand CPEM-EC wave, which carries zero density perturbations, the wave frequency and the wave number k_z are related by the dispersion relation³⁵

$$\omega^2 = k_z^2 c^2 + \frac{\omega_p^2 \omega}{(\omega - \omega_c)}, \quad (1)$$

where c is the speed of light in vacuum, $\omega_p = (4\pi n_e e^2 / m_e)^{1/2}$, $\omega_c = eB_0 / m_e c$, $m_e = m_0 \gamma$, $\gamma = 1 / (1 - u^2 / c^2)^{1/2}$ the relativistic gamma factor, and u the electron quiver velocity in the CPEM wave fields.

The nonlinear interaction between the magnetic field-aligned right-hand CPEM-EC wave with multi-dimensional DSA perturbations in a dense magnetoplasma gives rise to a slowly varying 3D right-hand CPEM-EC wave envelope, which is governed by the NLSE (Ref. 37)

$$i(\partial_t + V_g \partial_z)E_\perp + \frac{S}{2} \partial_z^2 E_\perp + \frac{T}{2} \nabla_\perp^2 E_\perp - \Delta E_\perp = 0, \quad (2)$$

and where the parallel (to \hat{z}) group velocity, parallel group dispersion, and perpendicular (to \hat{z}) group dispersion are given by, respectively,

$$V_g = \frac{k_z c^2}{\omega + \Omega_p^2 \Omega_e / 2(\omega - \Omega_e)^2}, \quad (3)$$

$$S = \left[1 - 4k_z^2 c^2 (\omega - \Omega_e) \frac{(\omega - \Omega_e)^3 - \Omega_e \Omega_p^2}{[2\omega(\omega - \Omega_e)^2 + \Omega_p^2 \Omega_e]^2} \right] \frac{V_g}{k_z} \quad (4)$$

and

$$T = \left[1 - \frac{\Omega_p^2 \Omega_e}{2(\omega - \Omega_e)(\omega^2 - \Omega_p^2)} \right] \frac{V_g}{k_z}, \quad (5)$$

where $\Omega_e = eB_0 / m_0 c$ is the gyrofrequency defined in terms of the rest mass of the electrons. The nonlinear frequency shift is denoted by Δ , which we present below in the context of the equations governing the dynamics of low-frequency electromagnetic perturbations in the presence of the ponderomotive force of the right-hand CPEM-EC waves.

A. The driven DSA perturbations

The DSA perturbations in a magnetoplasma are the low-frequency (in comparison with the electron gyrofrequency Ω_e) mixed modes with finite density fluctuations n_{e1} , the electric field $-\nabla\phi - (1/c)\hat{z}\partial_t A_z$, and the magnetic field $\nabla A_z \times \hat{z}$, where ϕ and A_z are the scalar and parallel (to \hat{z}) components of the vector potentials, respectively. There is no compressional magnetic field perturbation B_z associated with the DSA waves.

The nonlinear frequency shift Δ arising from the relativistic electron mass increase and the ponderomotive force driven density n_{e1} and magnetic field-aligned electron velocity V_z perturbations associated with the DSA waves, for our purposes, reads

$$\Delta = \frac{V_g \omega \Omega_p^2}{2k_z c^2 (\omega - \Omega_e)} \left[N + \frac{k_z V_z \Omega_e}{\omega(\omega - \Omega_e)} - \frac{\omega e^2 |E_\perp|^2}{m_0^2 c^2 (\omega - \Omega_e)^3} \right], \quad (6)$$

where $N = n_{e1} / n_0 \ll 1$. The third term in the right-hand side of Eq. (6) comes from the relativistic electron mass increase in the right-hand CPEM-EC fields. The slowly varying electron density and velocity perturbations are related by

$$\partial_t N + \frac{c}{B_0 \Omega_e} \partial_t \nabla_\perp^2 (\phi - \phi_p) + \partial_z V_z = 0, \quad (7)$$

where the parallel electron velocity perturbation is obtained from Ampère's law

$$V_z = \frac{c}{4\pi n_0} \nabla_{\perp}^2 A_z. \quad (8)$$

In writing the electron continuity equation (7), we have used the perpendicular (to $\hat{\mathbf{z}}$) component of the electron fluid velocity perturbation

$$\begin{aligned} \mathbf{V}_{\perp} = & \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla(\phi - \phi_p) + \frac{c}{B_0 \Omega_e} \partial_t \nabla_{\perp}(\phi - \phi_p) \\ & + \mathbf{V}_D + \mathbf{V}_x + \mathbf{V}_q, \end{aligned} \quad (9)$$

where $\phi_p = e|E_{\perp}|^2/m_0(\omega - \Omega_e)^2$ the perpendicular component of the CPEM-EC wave envelope ponderomotive potential, $\mathbf{V}_D = -(c/B_0 n_e) \hat{\mathbf{z}} \times \nabla P_e$, $\mathbf{V}_x = (c/B_0) \hat{\mathbf{z}} \times \nabla U_x$, and $\mathbf{V}_q = (c/B_0) \hat{\mathbf{z}} \times \nabla V_B$, with $P_e = (n_0 m_0 V_*^2/5)(n_e/n_0)^{5/3}$, $V_* = \hbar(3\pi^2)^{1/3}/m_0 r_0$, $r_0 = n_0^{-1/3}$,²¹ $U_x = 0.985 n_e^{1/3} e^2 [1 + (0.034/a_B n^{1/3}) \ln(1 + 18.37 a_B n^{1/3})]$, where $a_B = \hbar^2/m_0 e^2$ the Bohr radius, and $V_B = (\hbar^2/2m_0 \sqrt{n_e}) \nabla^2 \sqrt{n_e}$. Furthermore, in Eq. (8), we have neglected the magnetic field-aligned ion velocity perturbation, since the parallel ion current is small and the magnetic field-aligned current is primarily carried by the electrons in the DSA electromagnetic fields. Also, the displacement current has been neglected because the DSA wave phase velocity is much smaller than c .

1. DSA perturbations in plasmas with immobile ions

When the frequencies of the DSA perturbations are much larger than the ion plasma and ion gyrofrequencies, one can neglect the ion dynamics and write Poisson's equation as

$$\nabla^2 \phi = 4\pi n_{e1}. \quad (10)$$

Combining Eqs. (7)–(10), we have

$$\begin{aligned} \left(\nabla^2 + \frac{\Omega_p^2}{\Omega_e^2} \nabla_{\perp}^2 \right) \partial_t N + \partial_z \nabla^2 V_z - \frac{e^2}{m_0^2 \Omega_e^2 (\omega - \Omega_e)^2} \\ \times \nabla_{\perp}^2 \nabla^2 \partial_t |E_{\perp}|^2 = 0, \end{aligned} \quad (11)$$

where V_z is given by Eq. (8).

We now relate N and A_z through the parallel component of the electron momentum equation, which includes the magnetic field-aligned ponderomotive force³² of the CPEM-EC waves, the quantum statistical electron pressure perturbation P_1 , the force involving the electron-exchange and electron correlations potential perturbation V_{x1} , and the first order quantum force arising from the quantum recoil effect caused by overlapping of electron wave functions due to the Heisenberg uncertainty principle. We have

$$\begin{aligned} \frac{e^2}{m_0^2 \omega (\omega - \Omega_e)} \left[\partial_z - \frac{k_z \Omega_e}{\omega (\omega - \Omega_e)} \partial_t \right] \nabla^2 |E_{\perp}|^2 \\ = \frac{e}{m_0 c} \partial_t (1 - \lambda_e^2 \nabla^2) \nabla^2 A_z + \Omega_p^2 \partial_z N \end{aligned}$$

$$-U_*^2 \partial_z \nabla^2 N + \frac{\hbar^2}{4m_0^2} \partial_z \nabla^4 N, \quad (12)$$

where $\lambda_e = c/\Omega_p$ is the electron skin depth. In Eq. (12), we have denoted $U_*^2 = (3\hbar^2/m_0^2 r_0^2) + (0.328e^2/m_0 r_0) [1 + 0.62/(1 + 18.36 a_B/r_0)]$. The first and second terms in the definition of U_*^2 are due to the quantum electron pressure and perturbations of the electron exchange and electron correlations potentials,²⁶ respectively. The fourth term in the right-hand side of Eq. (12) is the quantum recoil effect associated with the perturbation of the quantum Bohm potential.

Combining Eq. (8) with Eq. (11) and the resultant equation with Eq. (12), we obtain the desired equation governing the dynamics of the driven DSA waves in a dense magnetoplasma with immobile ions. The result is

$$\begin{aligned} \mathcal{L}_0 N = & \frac{e^2 (1 - \lambda_e^2 \nabla_{\perp}^2)}{m_0^2 \Omega_e^2 (\omega - \Omega_e)^2} \\ & \times \left[\partial_t^2 - \frac{c^2 \Omega_e^2 (\omega - \Omega_e)}{\Omega_p^2 \omega} \left(\partial_z^2 - \frac{k_z \Omega_e}{\omega (\omega - \Omega_e)} \partial_{zt}^2 \right) \right] \\ & \times \nabla_{\perp}^2 \nabla^2 |E_{\perp}|^2, \end{aligned} \quad (13)$$

where $\mathcal{L}_0 = [\nabla^2 + (\Omega_p^2/\Omega_e^2) \nabla_{\perp}^2] \partial_t^2 + (1 - \lambda_e^2 \nabla_{\perp}^2) [c^2 - U_*^2 \lambda_e^2 \nabla^2 + (c^2/4) \lambda_e^2 \lambda_c^2 \nabla^4] \nabla^2 \partial_z^2$ and $\lambda_c = \hbar/m_0 c$ is the Compton length. Equations (11) and (13) shall be used in Eq. (6) to calculate Δ .

2. DSA perturbations in plasmas with mobile ions

Here, we turn our attention on the low-frequency (in comparison with the ion gyrofrequency $\Omega_i = eB_0/m_i c$, where m_i is the ion mass) DSA perturbations in a dense magnetoplasma including ion dynamics. The perpendicular (to $\hat{\mathbf{z}}$) component of the ion fluid velocity perturbation in the DSA electric field is

$$\mathbf{U}_{\perp} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla \phi + \frac{ck_B T_i}{eB_0 n_0} \hat{\mathbf{z}} \times \nabla n_{i1} - \frac{c}{B_0 \Omega_i} \partial_t \nabla_{\perp} \phi, \quad (14)$$

where T_i the ion temperature and $n_{i1} (\ll n_0)$ is the ion density perturbation. Since the ponderomotive force of the right-hand CPEM-EC waves acting on non-degenerate ion fluids is much smaller than that on the degenerate electron fluids, we have neglected it here.

Inserting Eq. (14) into the linearized ion continuity equation, we have

$$n_{i1} = \frac{n_0 c}{B_0 \Omega_i} \nabla_{\perp}^2 \phi, \quad (15)$$

which shows that the ion density perturbation is caused by the ion polarization drift.

By using the quasi-neutrality condition $n_{e1} = n_{i1}$, we obtain from Eqs. (7) and (15)

$$\partial_t N + \partial_z V_z - \frac{e^2}{m_0^2 \Omega_e^2 (\omega - \Omega_e)^2} \partial_t \nabla_{\perp}^2 |E_{\perp}|^2 = 0. \quad (16)$$

Furthermore, for the present purposes, we write the parallel component of the electron equation of motion as

$$\begin{aligned} & \frac{e^2}{m_0^2 \omega(\omega - \Omega_e)} \left[\partial_z - \frac{k_z \Omega_e}{\omega(\omega - \Omega_e)} \partial_t \right] \nabla_\perp^2 |E_\perp|^2 \\ &= \frac{e}{c} \partial_t (1 - \lambda_e^2 \nabla_\perp^2) \nabla_\perp^2 A_z + \Omega_L^2 \partial_z N \\ & \quad - U_*^2 \nabla_\perp^2 \partial_z N + \frac{\hbar^2}{4m_0^2} \nabla_\perp^2 \nabla^2 \partial_z N, \end{aligned} \quad (17)$$

where we have used (15) to eliminate $\nabla_\perp^2 \phi$, and denoted $\Omega_L = (\Omega_e \Omega_i)^{1/2}$.

Let us now eliminate A_z from Eq. (17) by using Eqs. (8) and (16), obtaining the driven DSA wave equation in a dense magnetoplasma with mobile ions

$$\mathcal{L}N = -\frac{e^2 \lambda_e^2}{m_0^2 \omega(\omega - \Omega_e)} \left[\partial_z^2 - \frac{k_z \Omega_e}{\omega(\omega - \Omega_e)} \partial_{zt} \right] \nabla_\perp^2 |E_\perp|^2, \quad (18)$$

where $\mathcal{L} = (1 - \lambda_e^2 \nabla_\perp^2) \partial_t^2 - V_A^2 [1 - (\rho_q^2 - \rho_h^4 \nabla_\perp^2) \nabla_\perp^2] \partial_z^2$, $\rho_q = U_*/\Omega_L$, $\Omega_L = (\Omega_e \Omega_i)^{1/2}$ are the lower-hybrid resonance frequency, $\rho_h = (a_c^2 \lambda_B^2/4)^{1/4}$, $a_c = \hbar/m_0 c$ the Compton length, and $\lambda_B = c/\Omega_L$. In Eq. (17), we have neglected terms proportional to ω/Ω_e . Equations (17) and (19) have to be inserted in Eq. (6) to determine Δ .

3. DCA perturbations in electron-ion plasmas

The low-frequency (in comparison with Ω_i) DCA perturbations are characterized by the magnetic field perturbation $\mathbf{B}_c = \mathbf{B}_{c\perp} + \hat{\mathbf{z}} B_{cz}$. They carry finite electron and ion number density fluctuations and magnetic field-aligned electron and ion velocity perturbations. Thus, the frequency shift associated with the DCA perturbations is³⁷

$$\begin{aligned} \Delta &= \frac{V_g \omega \Omega_p^2}{2k_z c^2 (\omega - \Omega_e)} \\ & \times \left[N + \frac{k_z V_z \Omega_e}{\omega(\omega - \Omega_e)} + \frac{\Omega_e \mathcal{B}}{(\omega - \Omega_e)} - \frac{\omega e^2 |E_\perp|^2}{m_0^2 c^2 (\omega - \Omega_e)^3} \right], \end{aligned} \quad (19)$$

where $\mathcal{B} = B_{cz}/B_0$.

The dynamics of the DCA perturbations is governed by the quantum magnetohydrodynamic equations that are composed of the ion continuity equation

$$\partial_t N + \nabla \cdot \mathbf{u}_i = 0, \quad (20)$$

the ion momentum equation

$$\begin{aligned} \partial_t \mathbf{u}_i &= \frac{B_0}{4\pi n_0 m_i} (\hat{\mathbf{z}} \cdot \nabla) \mathbf{B} - \frac{\nabla \mathbf{B}^2}{8\pi n_0 m_i} - 3V_{Ti}^2 \nabla N \\ & \quad - \frac{m_0 U_*^2}{m_i} \nabla N + \frac{\hbar^2}{4m_0 m_i} \nabla \nabla^2 N + \frac{\mathbf{F}_p}{m_i}, \end{aligned} \quad (21)$$

and Faraday's law

$$\partial_t \mathbf{B}_c = B_0 \nabla \times (\mathbf{u}_i \times \hat{\mathbf{z}}), \quad (22)$$

where $\mathbf{u}_i = \mathbf{u}_\perp + \hat{\mathbf{z}} U_z$, $u_z \approx V_z$, $V_{Ti} = (k_B T_i/m_i)^{1/2}$, $\nabla \cdot \mathbf{B}_c = 0$, and $\mathbf{F}_p = \mathbf{F}_\perp + \hat{\mathbf{z}} F_z$, with

$$\mathbf{F}_\perp = \frac{e^2}{m_0 (\omega - \Omega_e)^2} \nabla_\perp |E_\perp|^2, \quad (23)$$

and

$$F_z = \frac{e^2}{m_0 \omega (\omega - \Omega_e)} \left[\partial_z - \frac{k_z \Omega_e}{\omega(\omega - \Omega_e)} \partial_t \right] |E_\perp|^2. \quad (24)$$

We note that in deducing Eq. (21) we have used the electric field vector from the inertialess electron equation of motion to eliminate the electric force from the inertial ion momentum equation, while in Eq. (22) the electric field vector is eliminated by using the ion momentum equation in the low-frequency approximation, viz. $|\partial_t| \ll \Omega_i$.

The perpendicular and parallel components of Eq. (21) are, respectively,

$$\begin{aligned} \partial_t \mathbf{u}_\perp &= \frac{V_A^2}{B_0} \partial_z \mathbf{B}_{c\perp} - \frac{V_A^2}{B_0} \nabla_\perp B_{cz} - C_s^2 \nabla_\perp N \\ & \quad + \frac{\hbar^2}{4m_0 m_i} \nabla_\perp \nabla^2 N + \frac{\mathbf{F}_\perp}{m_i} \end{aligned} \quad (25)$$

and

$$\partial_t u_z = -C_s^2 \partial_z N + \frac{\hbar^2}{4m_0 m_i} \partial_z \nabla^2 N + \frac{F_z}{m_i}, \quad (26)$$

where we have denoted $C_s = [3V_{Ti}^2 + (m_0/m_i)U_*^2]^{1/2}$. In Eq. (25), we shall replace $\nabla_\perp \cdot \mathbf{B}_{c\perp} = -\partial_z B_{cz}$, where the compressional magnetic field perturbation B_{cz} is obtained from

$$\partial_t B_{cz} = -B_0 \nabla \cdot \mathbf{u}_\perp. \quad (27)$$

From Eqs. (20), (25), and (26), we readily obtain

$$\begin{aligned} & \left[\partial_t^2 - \left(C_s^2 - \frac{\hbar^2 \nabla^2}{4m_0 m_i} \right) \partial_z^2 \right] N - \partial_t^2 \mathcal{B} \\ &= -\frac{e^2}{m_0 m_i \omega (\omega - \Omega_e)} \left[\partial_z^2 + \frac{k_z \Omega_e}{\omega(\omega - \Omega_e)} \partial_{zt} \right] |E_\perp|^2. \end{aligned} \quad (28)$$

Furthermore, we combine Eqs. (25) and (27) to obtain

$$\begin{aligned} & (\partial_t^2 - V_A^2 \nabla^2) \mathcal{B} - \left(C_s^2 - \frac{\hbar^2 \nabla^2}{4m_0 m_i} \right) \nabla_\perp^2 N \\ &= \frac{e^2}{m_0 m_i (\omega - \Omega_e)^2} \nabla_\perp^2 |E_\perp|^2. \end{aligned} \quad (29)$$

Eliminating \mathcal{B} from Eq. (29) by using (28) we have

$$\mathcal{L}_2 N = \frac{e^2}{m_0 m_i (\omega - \Omega_e)^2} \left[\nabla_{\perp}^2 \partial_t^2 + \frac{(\Omega - \omega)}{\omega} (\partial_t^2 - \nabla^2) \right. \\ \left. \times \left(\partial_z^2 + \frac{k_z \Omega_e}{\omega(\omega - \Omega_e)} \partial_{zt}^2 \right) \right] |E_{\perp}|^2, \quad (30)$$

where $\mathcal{L}_2 = (\partial_t^2 - V_A^2 \nabla^2) [\partial_t^2 - (C_s^2 - \hbar^2 \nabla^2 / 4m_0 m_i) \partial_z^2] - (C_s^2 - \hbar^2 \nabla^2 / 4m_0 m_i) \nabla_{\perp}^2 \partial_t^2$. Equations (26), (28), and (29), shall be used to determine Δ through Eq. (19).

III. NONLINEAR DISPERSION RELATIONS

We now Fourier transform Eqs. (13), (18), and (30) by assuming that N and $|E_{\perp}|^2$ are proportional to $\exp(-i\Omega t + i\mathbf{K} \cdot \mathbf{r})$. The results are

$$D_C N = G_C |E_{\perp}|^2, \quad (31)$$

where

$$D_C = (K^2 + \Omega_p^2 K_{\perp}^2 / \Omega_e^2) \Omega^2 - (1 + K_{\perp}^2 \lambda_e^2) \\ + K^2 K_z^2 [c^2 + U_*^2 K^2 \lambda_e^2 + (c^2/4) K^4 \lambda_e^2 \lambda_c^2]$$

and

$$G_C = -\frac{e^2 (1 + K_{\perp}^2) K^2 K_{\perp}^2}{m_0^2 \Omega_e^2 (\omega - \Omega_e)^2} \\ \times \left[\Omega^2 - \frac{c^2 \Omega_e^2 (\omega - \Omega_e)}{\Omega_p^2 \omega} \left(K_z^2 + \frac{k_z K_z \Omega_e \Omega}{\omega(\omega - \Omega_e)} \right) \right]. \quad (32)$$

$$D_A N = G_A |E_{\perp}|^2, \quad (33)$$

where $D_A = \Omega^2 - \Omega_A^2$, $\Omega_A = K_z V_A (1 + K_{\perp}^2 \rho_q^2 + K_{\perp}^2 K^2 \rho_h^4)^{1/2} / (1 + K_{\perp}^2 \lambda_e^2)^{1/2}$, and

$$G_A = \frac{K_{\perp}^2 c^2 K_z^2 e^2}{m_0^2 \Omega_p^2 \omega (\omega - \Omega_e) (1 + K_{\perp}^2 \lambda_e^2)} \left(1 + \frac{k_z \Omega_e \Omega}{K_z \omega (\omega - \Omega_e)} \right). \quad (34)$$

$$D_M N = C_M |E_{\perp}|^2, \quad (35)$$

where $D_M = (\Omega^2 - K^2 V_A^2) [\Omega^2 - K_z^2 C_s^2 (1 + d)] - \Omega^2 K_{\perp}^2 C_s^2 (1 + d)$, $d = \hbar^2 K^2 / 4m_0 m_i C_s^2$, and

$$C_M = \frac{e^2}{m_0 m_i (\omega - \Omega_e)^2} \left[K_{\perp}^2 \Omega^2 + \frac{(\Omega_e - \omega)}{\omega} (\Omega^2 - K^2 V_A^2) \right. \\ \left. \times \left(K_z^2 + \frac{k_z K_z \Omega_e \Omega}{\omega(\omega - \Omega_e)} \right) \right], \quad (36)$$

where \mathbf{K}_{\perp} and K_z are the components of the wave vector \mathbf{K} across and along \hat{z} .

From Eqs. (11) and (17), we also have $V_z \approx (\Omega / K_z) N$, while from Eqs. (17) and (29) we have $u_z = V_z \approx N K_z C_s^2 (1 + d) / \Omega$, and $\mathcal{B} = N K_{\perp}^2 C_s^2 (1 + d) / (\Omega^2 - K^2 V_A^2)$,

which are needed for calculating nonlinear frequency shifts. The latter corresponding to Eqs. (31), (33), and (35) are, respectively,

$$\Delta_C = C_c(\omega, \Omega, \mathbf{k}, \mathbf{K}) |E_{\perp}|^2 \quad (37)$$

with

$$C_c = \frac{V_g \omega \Omega_p^2}{2k_z c^2 (\omega - \Omega_e)} \\ \times \left[\left(1 + \frac{k_z \Omega_e \Omega}{K_z \omega (\omega - \Omega_e)} \right) \frac{G_C}{D_C} - \frac{\omega e^2}{m_0^2 c^2 (\omega - \Omega_e)^3} \right] |E_{\perp}|^2. \quad (38)$$

$$\Delta_a = C_a(\omega, \Omega, \mathbf{k}, \mathbf{K}) |E_{\perp}|^2 \quad (39)$$

with

$$C_a = \frac{V_g \omega \Omega_p^2 e^2}{2k_z c^4 (\omega - \Omega_e)^2 m_0^2} \left[\left(1 + \frac{k_z \Omega_e \Omega}{K_z \omega (\omega - \Omega_e)} \right)^2 \right. \\ \left. \times \frac{K_{\perp}^2 K_z^2 c^4}{\Omega_e^2 \omega (1 + K_{\perp}^2 \lambda_e^2) D_A} - \frac{\omega}{(\omega - \Omega_e)^2} \right] \quad (40)$$

and

$$\Delta_m = C_m(\omega, \Omega, \mathbf{k}, \mathbf{K}) |E_{\perp}|^2 \quad (41)$$

with

$$C_m = \frac{V_g \omega \Omega_p^2}{2k_z c^2 (\omega - \Omega_e)} \left[\left(1 + \frac{k_z K_z C_s^2 (1 + d) \Omega_e}{\omega (\omega - \Omega_e)} \right) \right. \\ \left. + \frac{\Omega_e K_{\perp}^2 C_s^2 (1 + d)}{(\Omega^2 - K^2 V_A^2)} \right] \frac{C_M}{D_M} - \frac{\omega e^2}{m_0^2 c^2 (\omega - \Omega_e)^3}, \quad (42)$$

where $d = \hbar^2 K^2 / 4m_0 m_i C_s^2$.

Following the general method for treating wave-wave interactions,^{32,36,43–46} we derive the general dispersion relation for the modulational instability of a constant amplitude CPEM-EC pump wave $E_0 \exp(-i\omega_0 t + i\mathbf{k}_0 \cdot \mathbf{r})$, where E_0 is the electric field amplitude, and $\omega_0(\mathbf{k}_0)$ the frequency and (wave vector) of the pump wave. Thus, we decompose the electric field as $E_{\perp} = [E_0 + E_1(\xi)] \exp(-i\Delta_0 t)$, where $\Delta_0 = C_A(\omega_0, \Omega, \mathbf{k}_0, \mathbf{K}) |E_0|^2$, and E_1 is determined from

$$i(\partial_t + V_g \partial_z) E_1 + \frac{S}{2} \partial_z^2 E_1 + \frac{T}{2} \nabla_{\perp}^2 E_1 - \Delta_0 (E_1 + E_1^*) = 0, \quad (43)$$

where the asterisk denotes the complex conjugate, and Δ_0 represents Δ_c , Δ_a , and Δ_m .

Letting $E_1 = R + iI$ in Eq. (43) and separating the real and imaginary parts, Fourier transforming the resultant equations and combining them, we obtain the nonlinear dispersion relation

$$\begin{aligned}
& (\Omega - k_z V_g)^2 - \frac{1}{4}(SK_z^2 + TK_\perp^2)^2 \\
& = C_{j0}(\omega_0, \Omega, \mathbf{k}_0, \mathbf{K})(SK_z^2 + TK_\perp^2)|E_0|^2, \quad (44)
\end{aligned}$$

where $C_{j0} = C_j(\omega_0, \Omega = K_z V_g, \mathbf{k}_0, \mathbf{K})$, with j equals c , a , and m .

We now put $\Omega = K_z V_g + i\Omega_i$ in Eq. (44), where $\Omega_i \ll KV_g$, to obtain the growth rate

$$\Omega_i = \left[-\frac{1}{4}(SK_z^2 + TK_\perp^2)^2 - C_{j0}(SK_z^2 + TK_\perp^2)|E_0|^2 \right]^{1/2}. \quad (45)$$

Instability arises provided that

$$C_{j0}(SK_z^2 + TK_\perp^2) < 0. \quad (46)$$

We have thus found the growth rates and threshold criteria for the modulational instabilities of coherent right-hand CPEM-EC waves against three types of low-frequency electromagnetic perturbations in dense quantum magnetoplasmas.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have investigated the nonlinear propagation of right-hand CPEM-EC waves in a dense quantum magnetoplasma that is composed of degenerate electron fluids and non-degenerate ion fluids. Specifically, we have considered nonlinear interactions between coherent CPEM-EC waves and non-resonant DSA and DCA perturbations in a dense quantum magnetoplasma. Both DSA and DCA perturbations are accompanied with finite density fluctuations and thus there appear modifications of their frequency spectra due to the quantum effects, viz., the quantum statistical electron pressure perturbation, perturbations of the electron-exchange and electron correlation potentials, and the quantum Bohm potential perturbation. It is shown that nonlinear wave interactions are governed by a three-dimensional NLSE for the CPEM-EC wave envelope and equations for the DSA and DCA perturbations that are reinforced by the ponderomotive force of the right-hand CPEM-EC waves. Since the quantum mechanical effects modify the frequency spectra of the DSA and DCA perturbations and the associated coupling constants, we have new nonlinear dispersion relations for the study of the parametric instabilities of a constant amplitude CPEM-EC pump wave against low-frequency electromagnetic perturbations in dense magnetized plasmas. An analysis of the nonlinear dispersion relation reveals that the quantum effects significantly enhance the growth rates when the real parts of the amplitude modulation frequencies are close to $K_z V_g$. This is attributed to a reduction of the obliquely propagating low-frequency electromagnetic disturbances that are participating in the modulational interactions in solid density plasmas with sufficiently high electron number densities. Our three-dimensional (3D) NLSE and the governing driven equations for 3D DSA and DCA perturbations can be numerically investigated to depict the intrinsic localization of amplitude modulated 3D right-hand CPEM-EC pulses in a dense

magnetoplasma. Furthermore, the present description of 3D driven DCA waves should also be generalized to include the electron-spin magnetization⁴⁷ in a magnetized quantum plasma. In conclusion, we stress that the results of the present investigation are relevant for understanding the salient features of stimulated scattering processes associated with high-frequency electromagnetic waves in the next generation of intense laser-solid density plasma interaction experiments in which huge magnetic fields (of the order of several gigagauss) can be created by return electron currents in high-energy density plasmas (electron number densities of the order of 10^{23} cm^{-3} and beyond) that are of ICF interest. One might also exploit the present idea of stimulated scatterings of coherent CPEM-EC waves for understanding the origin of intense x-ray pulses from the cores of massive white dwarf stars, where the plasma number densities are significantly high ($\sim 10^{30} \text{ cm}^{-3}$), and the magnetic fields are rather strong (of the order of 300 megagauss).

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