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Reconfigurable Two-Stage Nyquist Filters Utilizing the Farrow Structure

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Abstract—This paper introduces reconfigurable two-stage finite-length impulse response (FIR) Nyquist filters. In both stages, the Farrow structure realizes reconfigurable lowpass linear-phase FIR Nyquist filters. By adjusting the variable multipliers of the Farrow structure, various FIR Nyquist filters and integer interpolation/decimation structures are obtained, online. However, the filter design problem is solved only once, offline. Design examples illustrate the method.

Index Terms—Nyquist Filter, Farrow Structure.

I. INTRODUCTION

Nyquist (Lth-band) filters find applications in, e.g., filter banks, spectrum sensing, pulse shaping, and timing/carrier recovery [1]–[3]. Interpolation/decimation is composed of anti-imaging/anti-aliasing filters and upsamplers/downsamplers. With Nyquist filters, we can exactly recover the input signal [4]. Efficient design of Nyquist filters is thus a necessity in communication systems.

In addition, communication engineers aim to design reconfigurable systems for multistandard communications. This leads to supporting different bandwidths and sampling rate conversion (SRC) structures which can, in principle, be handled using dedicated blocks for each standard. However, this would then require to either (i) design a large set of filters offline, or (ii) design the filters online. This is not desirable because of the resulting high complexity in both design and realization. We hence need structures which dynamically perform SRC at a low cost. This necessitates reconfigurable filters which also require a low arithmetic complexity.

With a noncausal Nyquist filter \( H(z) \) of order \( N \), we have

\[
T(z) = \sum_{l=0}^{L-1} H(zW_L^l) = 1, \quad W_L = e^{-j\frac{2\pi}{N}}. \tag{1}
\]

In the time domain,

\[
h(n) = \begin{cases} 
\frac{1}{T} & n = 0 \\
0 & n = mL \\
\text{arbitrary} & n \neq mL.
\end{cases} \tag{2}
\]

A classical solution to \( h(n) \) is the root-raised cosine pulse [5] but solutions based on optimization are generally more efficient [2], [6].

The SRC can be performed in single or multiple stages [4]. In multi-stage realizations, the overall SRC ratio is factorized into multiple ratios thereby reducing the arithmetic complexity [7]. However, the constraints on the overall anti-imaging/anti-aliasing filters do not change and the analysis method can hence be extended from the single-stage to the multi-stage case.

This paper proposes reconfigurable structures for two-stage realization [7] of finite-length impulse response (FIR) Nyquist filters. The Farrow structure is used in both stages so as to obtain reconfigurable Nyquist filters [8]. The zeroth polyphase component of the filters, in each stage, is a pure delay. The remaining polyphase components are realized by the Farrow structure. Therefore, both stages can be reconfigured, online. By combining these reconfigurable stages, we can perform reconfigurable two-stage SRC with a low arithmetic complexity. This reconfigurability does not need filter redesign. This paper only discusses the structures for interpolation because a decimator can be obtained by transposing the corresponding interpolator.

Section II discusses the single-stage SRC, two-stage SRC, and the Farrow structure. The reconfigurable two-stage Nyquist filters are outlined in Section III. In Section IV, the filter design is treated and some design examples are provided. Some discussion about the arithmetic complexity is given in Section V with the concluding remarks outlined in Section VI.

II. PREREQUISITES

As seen in Fig. 1(a), interpolation by \( L \) requires an upsampled and a lowpass anti-imaging filter \( H(z) \) so that \( Y(z) = X(z^L)H(z) \). The filter \( H(z) \) has a lowpass characteristic with a roll-off of \( 0 \leq \rho \leq 1 \).

The passband and stopband edges are

\[
\omega_c T = \frac{1 - \rho}{L}, \quad \omega_s T = \frac{1 + \rho}{L}. \tag{3}
\]

Figure 1(a) is the single-stage equivalent of Fig. 1(b) where \( L = L_1L_2 \) [4]. Using the noble identities, the equivalent lowpass filter is

\[
H(z) = H_1(zL^2)L_2(z). \tag{4}
\]

Like (1), if \( H(z) \) is a noncausal Nyquist filter, we have

\[
T(z) = \sum_{l=0}^{L-1} H_1(zL^2W_L^l)H_2(zW_L^l) = 1. \tag{5}
\]

A. Farrow Structure

The Farrow structure, shown in Fig. 2, is composed of fixed linear-phase FIR subfilters \( S_k(z), k = 0, 1, \ldots, L_F \), and it can approximate reconfigurable fractional delay (FD) filters. If \( \mu \) is the FD value, the transfer function is [9]

\[
\begin{align*}
S_k(z) &= \sum_{l=0}^{L_F} a_l z^{-l} H_1(zL_F^l)H_2(zW_L^l) \\
&= \sum_{l=0}^{L_F} a_l z^{-l} H_1(zL^l)H_2(zW_L^l). \\
&= \sum_{l=0}^{L_F} a_l z^{-l} H(zL^l) \\
&= \frac{1}{z^\mu} H(z). \\
&= H(zL^{-\mu}).
\end{align*}
\]

Fig. 1. Interpolation by \( L \) using single-stage and two-stage structures.

Fig. 2. Farrow structure with fixed subfilters \( S_k(z) \) and variable FD \( \mu \).
\[ F(z, \mu) = \sum_{k=0}^{L_p} S_k(z)\mu^k, \quad |\mu| \leq 0.5. \]  

The \( S_k(z) \) are designed so that \( F(z, \mu) = z^{-\mu} \) [10]. For simplicity, the rest of the paper uses \( F(z) \) instead of \( F(z, \mu) \).

### III. Reconfigurable Two-Stage SRC

This section considers \( H_1(z) \) and \( H_2(z) \) to have orders \( N_1 \) and \( N_2 \), respectively. The Type I polyphase decomposition of \( H_1(z) \) is

\[ H_1(z) = \sum_{m=0}^{L_1-1} z^{-m} H_{1,m}(z^{L_1}). \]  

If \( H_1(z) \) is an ideal causal lowpass filter of order \( N_1 \), we have

\[ H_1(z) = \begin{cases} 
    z^{\frac{N_1}{2}} & \text{in the passband} \\
    0 & \text{in the stopband.}
\end{cases} \]  

From (7) and (8), we get

\[ H_{1,m}(z) = \begin{cases} 
    z^{\frac{N_1}{2}} & \text{in the passband} \\
    0 & \text{in the stopband.}
\end{cases} \]  

A general \( N_1 \)-th order causal Nyquist filter can thus be designed if

\[ H_{1,0}(z) = \frac{N_1}{L_1} \]  

and by utilizing the Farrow structure to realize \( H_{1,m}(z) \), \( m = 1, 2, \ldots, L_1 - 1 \), of odd\(^1\) order \( N_{F_1} \) as [8], [11], [12]

\[ N_{1,0} = \frac{N_1}{L_1} = N_{F_1} + 1. \]  

Then, we can use (6) to obtain

\[ H_{1,m}(z) = \sum_{k=0}^{L_{F_1}} S_{1,k}(z)\mu_1^k, \]  

with

\[ \mu_{1,m} = \frac{-m}{L_{F_1}} + \frac{1}{2} \Rightarrow \mu_{1,m} = -\mu_{1,L_1-m}. \]  

From (7), (10), and (12), we have

\[ H_1(z) = z^{\frac{L_1 N_{1,0}}{2}} \sum_{m=0}^{L_1-1} z^{-m} \sum_{k=0}^{L_{F_1}} S_{1,k}(z^{L_1})\mu_1^k. \]  

The same principle, as in (7)-(14), can be applied so that

\[ H_2(z) = z^{\frac{L_2 N_{2,0}}{2}} + \sum_{m=1}^{L_2-1} z^{-m} \sum_{k=0}^{L_{F_2}} S_{2,k}(z^{L_2})\mu_2^k, \]  

where

\[ N_{2,0} = \frac{N_2}{L_2} = N_{F_2} + 1 \]  

and

\[ \mu_{2,m} = \frac{-m}{L_{F_2}} + \frac{1}{2} \Rightarrow \mu_{2,m} = -\mu_{2,L_2-m}. \]  

With (4), (5), (14), and (15), some manipulations give (18) and (19) on the next page. The filter \( H(z) \), in (4), is a Type I linear-phase FIR filter of order

\[ N = \frac{L_2 N_1 + N_2}{2} = \frac{L_2}{2}(N_{F_1} + 1) + \frac{L_2}{2}(N_{F_2} + 1). \]  

Here, each filter \( H_u(z), u = 1, 2 \), is a Nyquist (\( L_u \)-th-band) filter, as in [7], leading to

\[ H_{u,0}(z) = z^{\frac{N_{u,0}}{2}}. \]

\(^1\)With proper modifications, even-order filters can also be designed [8].

Fig. 3. Efficient interpolation by variable integer ratio \( L_1 \) using fixed subfilters, variable multipliers, and commutators.

Considering (13) and (17), we have [8], [11], [12]

\[ H_{1,m}(z) = \Phi_{1,m}(z) + \Psi_{1,m}(z), \quad H_{1,L_1-m}(z) = \Phi_{1,m}(z) - \Psi_{1,m}(z), \]  

where

\[ \Phi_{1,m}(z) = \sum_{k=0}^{L_{F_1} - 1} S_{1,2k}(z)\mu_{1,m}^{2k}, \]  

\[ \Psi_{1,m}(z) = \sum_{k=0}^{L_{F_1} - 1} S_{1,2k-1}(z)\mu_{1,m}^{2k-1}. \]

As in Fig. 3, reconfigurable SRC by \( L_1 \) requires fixed filters \( S_{1,k}(z) \) and \( H_{1,0}(z) \), variable multipliers \( \mu_{1,m} \), and commutators [8], [11], [12]. Consequently, reconfigurable two-stage interpolation by \( L = L_1 L_2 \) can be realized according to Fig. 4.

### IV. Filter Design

With noncausal ideal filters, we have

\[ H(z) = \begin{cases} 
    1 & \text{in the passband} \\
    0 & \text{in the stopband.}
\end{cases} \]  

With nonideal filters, we can approximate (25). This section treats the minimax design problem as

\[ \min \delta \quad \text{subject to} \quad |H(e^{j\omega T})| \leq \delta, \quad \omega T \in \Omega \]  

and we consider two cases where

Case 1: \( \Omega = [\omega_s T, \pi] \)

Case 2: \( \Omega = \bigcup_{p=1}^{L/2} \left[ \frac{2(p-1)+\pi}{L}, \frac{2(p+1)-\pi}{L} \right] \cup \left[ \frac{2N_f+\pi}{L}, \pi \right] \)

Here, Case 1 covers the whole stopband from \( \omega_s T \) up to \( \pi \) but Case 2 excludes the don’t-care bands centered on \( \left\{ \frac{2(p+1)-\pi}{L} \right\} \) for \( L > 2 \). This is admissible in some applications [7]. Note that in a Nyquist filter, the passband and stopband ripples are related to each other [7], [13].

In (26), the free optimization parameters are the coefficients of \( S_{u,k}(z) \) and \( H_{u,0}(z) \) with \( u = 1, 2 \), and \( k = 1, 2, \ldots, L_{F_u} \). During the filter design, the values of \( \mu_{u,m}^{1,2}, L_u, N_{F_u} \), and \( L_{F_u} \) are pre-determined. After solving (26) only once, we can realize reconfigurable FIR Nyquist filters. For this realization, the impulse responses of \( S_{u,k}(z) \) and \( H_{u,0}(z) \) as well as the values of \( N_{F_u} \) and \( L_{F_u} \) are fixed. However, only the values of \( L_{u} \) need to be adjusted, online.

Here, the design problem for each stage is a convex optimization problem [8] where, ideally, \( S_{u,k}(e^{j\omega T}) = \frac{(-\pi/\omega T)^{k}}{\pi} \) [8]. This can be used to find the initial solutions, using the linprog routine of MATLAB. These initial solutions can then be used to solve the general nonlinear problem of (26), using the fminimax routine of MATLAB.
Nyquist filters obtained from (26) where $A$. Design Examples

$L$ can exactly meet (1). Further, we can decrease the stopband ripple are, respectively,

\[ \delta_{L} \]

In Figs. 5(b) and 7(b), the magnitude response, in don't-care bands,

\[ |H(e^{\mu_{f}T})| = 10^{-\frac{20}{3}x10^0}, \mu_{f} \]

Fig. 5. Characteristics of $|H(e^{\mu_{f}T})|$ for reconfigurable Nyquist filters.

A. Design Examples

Figures 5–8 show the characteristics of some reconfigurable Nyquist filters obtained from (26) where $\rho = 0.2$. As can be seen, we can exactly meet (1). Further, we can decrease the stopband ripple by increasing $L_{F_1}, L_{F_2}, N_{F_1}$, and $N_{F_2}$. Also, allowing don’t-care bands helps further decrease $\delta$, in (26), without increasing the values of $L_{F_1}, L_{F_2}, N_{F_1}$, and $N_{F_2}$. In Figs. 5(a) and 5(b), the values of $\delta$ are, respectively, $7.7981 \times 10^{-2}$ and $6.7892 \times 10^{-2}$. As can be seen from Figs. 5(b) and 7(b), the magnitude response, in don't-care bands, is exceeding the corresponding $\delta$. For Figs. 7(a) and 7(b), the values of $\delta$ are, respectively, $4.7369 \times 10^{-3}$ and $3.8423 \times 10^{-3}$.

V. ARITHMETIC COMPLEXITY

For interpolation by $L_u$, $u = 1, 2$, using a Nyquist filter $H_u(z)$ which is realized with a Farrow structure having $L_{F_2}$ subfilters of orders $N_{F_2}$, we need [8]

\[ C_u = \begin{cases} \frac{(L_{F_2} + 1)(N_{F_2} + 1)}{2} + \frac{L_{F_2}(L_{F_2} - 1)}{2} & \text{odd } L_u \\ \frac{(L_{F_2} + 1)(N_{F_2} + 1)}{2} + \frac{L_{F_2}(L_{F_2} - 2)}{2} & \text{even } L_u \end{cases} \]  

\[ (28) \]

distinct multiplications per input sample. Therefore, the total number of distinct multiplications, for reconfigurable two-stage SRC by $L = L_1L_2$, becomes

\[ C_T = (C_1 + L_1C_2)\].  

(29)

In the conventional fixed two-stage case [7], one can utilize fixed $L_u$th-band filters $H_u(z)$, $u = 1, 2$, so as to obtain fixed Nyquist filters $H(z)$. Then, the total number of distinct multiplications becomes

\[ C_{ct} = \left( \left[ \frac{N_1}{2} \right] - \left[ \frac{N_u - 1}{L_1} \right] \right) + L_1 \left( \frac{N_2}{2} - \left[ \frac{N_u - 1}{L_2} \right] \right) \]  

\[ (30) \]

where $k_u = \frac{N_u}{L_u}$ mod $L_u$ with $\frac{N_u}{L_u}$ mod $L_u$ being the remainder of $\frac{N_u}{L_u}$. Also, $\left[ . \right]$ represents the floor operation. In (30), the terms inside parentheses refer to the number of distinct nonzero coefficients in a general $L_u$th-band filter of order $N_u$.

Tables I and II summarize the results of the designs in Fig. 7(a) using, respectively, reconfigurable and the conventional fixed two-stage Nyquist filters. In Table I, we need a maximum of 141 distinct multiplications to simultaneously realize $L$th-band filters with $L = \{4, 6, 8, 9, 10, 12, 15, 16, 20\}$. On the other hand, Table II will require $\sum C_{ct} = 375$ distinct multiplications to realize all of these Nyquist filters. This shows a 62% reduction of the arithmetic
In comparison to the conventional fixed two-stage Nyquist filters, the filter design and through adjusting (i) the number of polyphase complexity while obtaining a reconfigurability. Note also that Table I only requires to save the coefficients of $S_{u,k}(z)$ and $H_{u,0}(z)$ but Table II needs to save all of the coefficients for $H_1(z)$ and $H_2(z)$, i.e., $\sum N_1 + N_2$. This also means that the reconfigurable two-stage design has fewer optimization parameters.

VI. CONCLUSION

Reconfigurable two-stage Nyquist filters, using the Farrow structure, were outlined. These Nyquist filters are obtained by one offline filter design and through adjusting (i) the number of polyphase components, and (ii) the variable multipliers of the Farrow structure. In comparison to the conventional fixed two-stage Nyquist filters, the arithmetic complexity (in terms of the number of distinct multiplications) is reduced by 62%.

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