Simulation Platform for Resource Allocation in Multi-Cellular Wireless Networks

Examensarbete utfört i Kommunikationssystem vid Tekniska högskolan i Linköping
av

Tony Khosravi Dehkourdi

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The goal of this Master’s thesis was to solve resource allocation problems in wireless networks through the implementation of a lightweight simulation platform. The spectrum and power resources of wireless networks have to be efficiently used to accommodate the growing number of wireless terminals and the massive increase of data transferred by their applications. The major problem that needs to be tackled is interference, which significantly limits the performance of wireless systems. In this thesis, the resource allocation of interest was the joint problem of scheduling and power control with Quality of Service (QoS) constraints. The Signal-to-Interference-plus-Noise Ratio (SINR) was used to quantify QoS. This thesis studied the recently proposed mixed-integer linear programming (MILP) formulation of the problem. Due to the scheduling component, the problem is inherently combinatorial and NP-hard, therefore computationally expensive and difficult to solve in tractable time. A simulation platform was implemented in order to automate and facilitate the solving process.

As a starting point, wireless channels and channel modeling issues were studied. Then, the platform was implemented to simulate random instances of multi-cellular wireless networks, with several mobile stations per cell, and generate the corresponding channels. Finally, the platform was extended to use the GNU Linear Programming Kit (GLPK) API in order to optimally solve the aforementioned formulated problem for various inputs of generated channels.

Tests of the simulation platform were performed to check the consistency of the results. Indeed, the output results satisfied the initial expectations regarding the SINR constraints and the formulation. Moreover, they were produced in reasonable time. An analysis of the output results was presented.

This thesis resulted in a configurable and lightweight simulation platform which is able to solve the MILP-formulated resource allocation problem. The simulation platform is basic and does not cover all the aspects of multi-cellular wireless networks and wireless channels. Due to its modularity, it can be extended in a future project.
Abstract

The goal of this Master's thesis was to solve resource allocation problems in wireless networks through the implementation of a lightweight simulation platform. The spectrum and power resources of wireless networks have to be efficiently used to accommodate the growing number of wireless terminals and the massive increase of data transferred by their applications. The major problem that needs to be tackled is interference, which significantly limits the performance of wireless systems. In this thesis, the resource allocation of interest was the joint problem of scheduling and power control with Quality of Service (QoS) constraints. The Signal-to-Interference-plus-Noise Ratio (SINR) was used to quantify QoS. This thesis studied the recently proposed mixed-integer linear programming (MILP) formulation of the problem. Due to the scheduling component, the problem is inherently combinatorial and NP-hard, therefore computationally expensive and difficult to solve in tractable time. A simulation platform was implemented in order to automate and facilitate the solving process.

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List of Symbols

**DoF**  Degree of Freedom.

**GLPK**  GNU Linear Programming Kit.

**GMPL**  GNU Mathematical Programming Language.

**MILP**  Mixed-Integer Linear Programming.

**OFDM**  Orthogonal Frequency Division Multiplexing.

**QoS**  Quality of Service.

**SINR**  Signal-to-Interference-plus-Noise Ratio.

**SIR**  Signal-to-Interference Ratio.

**SNR**  Signal-to-Noise Ratio.
Chapter 1

Introduction

This chapter gives the motivations that led to this thesis work. It presents the main goal, the method used during the work and the report outline.

1.1 Background

Wireless systems have been subject to a tremendous expansion for some years. In addition, they are very challenging since many constraints are faced in order to ensure a sufficient quality of communication. One of these constraints is the growing demand for transmission of wireless data which results in an increasing consumption of wireless resources which are limited. Thus, resource allocation in wireless networks is a struggling problem. Let us consider a scenario with many transmitters and receivers in the system sharing the same system resources. The performance of this system is limited by interference, which should be considered in the resource allocation approaches. There are some frameworks that can be used to solve many joint power control and scheduling problems in wireless systems e.g. the Mixed-Integer Linear Programming (MILP) (an example of such a formulation can be found in [21]). Due to the scheduling component, the joint problem is combinatorial and NP-hard ("at least as hard as the hardest problems in NP" [9]). Such problems are computationally expensive and known to be difficult to solve in tractable time.

1.2 Simulation platform

The main goal of this project was to develop a suitable simulation platform to simulate a multi-cellular wireless network in the first place. The simulation consisted in generating a number of base stations and mobile stations given various parameters in order to obtain their respective channels. Then, an existing solver, i.e. GLPK API [10], was used for solving the resource allocation problems in an efficient way. In previous experiments (as in [21]), simulations are run through other tools such as MATLAB [41] in order to create instances of networks, which
are then used in order to find some solutions to the given problem. Such task is very tedious since MATLAB is quite slow to perform the former mentioned task (it is called as a script which is slower than a compiled program) and solvers cannot be used to perform the latter mentioned task at a great scale. A dedicated simulation platform was thus suitable.

The simulation platform had to be implemented so it would be very easy and simple to use. It focused on the MILP at first, but possible extensions might be added later. In addition, there was a need to run the simulation over different channel models such as Hata Model [1] or Spatial Channel Model [34]. Making the simulation platform generic or extendable was therefore desirable. The simulation platform had to exploit available computational resources, i.e., to run under Linux and in parallel over many cores. The possibility of a multi-threaded application had to be considered.

1.3 Working method

As a student in computer science, my knowledge in telecommunication and in wireless networks particularly was limited to computer network and internetworking. The theoretical approach of this field, especially resource allocation, was totally unfamiliar to me. Therefore, I had first to look for information about this topic to understand the problem statement very well, so that I would be able to formulate and design a suitable simulation platform. A first draft of the latter was proposed and tested first, to establish a basic design. Once this first step was achieved, a more advanced design was proposed and implemented to offer more features until reaching the implementation of a simulation platform which was satisfying for a first version. Thus, the work was performed step-by-step in order to understand what was implemented so that it is well-implemented.

1.4 Report outline

The content in this report is outlined as follow:

Chapter 2 gives some key elements to understand cellular wireless networks and wireless channels.

Chapter 3 describes channel modeling.

Chapter 4 states the resource allocation problem in cellular wireless networks.

Chapter 5 gives some key elements to understand MILP and describes the MILP formulation for the problem statement.

Chapter 6 describes the design and the implementation of the simulation platform.

Chapter 7 presents the results of the simulation platform.
Chapter 8 discusses the results and possible future work.

Appendices contain additional documents: user manual and code examples.
Chapter 2

Wireless communications

This chapter describes the specifications and the challenges of wireless communications, especially cellular wireless networks. It also explains the factors that characterize a wireless channel.

2.1 Wireless systems

Wireless communications have become a tremendous challenge and very attractive segment for communications industry. Indeed, with the great increase of the market of cellular phones and laptops, the needs of efficient wireless technologies have exponentially grown too [12, 20, 43]. This part does not describe in details how wireless systems work. Instead, it gives some technical issues one faces when one deals with wireless systems. Finally, we give a deeper description of the wireless system we focused on during this thesis: cellular wireless networks.

2.1.1 Challenges and issues

Wireless communications is one of the most successful technologies of the last 25 years in terms of scientific innovations, market size and impact on society, and will surely still be during the next few years. The needs of more efficient wireless technology increase every year since the demand increases as well [20]. This growing demand implies many challenges to be addressed [25, 28].

Wireless terminals add more features and incorporate multiple modes of operation to support the different applications and media. There is a need to process various kinds of data: voice, image, text and video data. However, unlike computers, it needs to be done through a cheap lightweight, handheld device. The challenge faced here is to be able to have equipments that can perform transmission and signal processing with a minimum power consumption and in the same time having multimedia applications and networking functions supported by signal processing [12, Ch. 1.3][28].

The aforementioned issue is accompanied by an additional lack of resources for mobile broadband. Presently, this lack of resources is not noticeable but is
expected within the next decade with the expansion of the use of wireless data. The growing popularity of wireless mobile broadband provokes the available spectrum to be less and less available. Besides, the spectrum of mobile broadband is limited in range (from 300 to 3500 MHz). Out of this range, radio waves are less efficient. In addition, mobile broadband has to "fight" against over-the-air TV broadcast spectrum allocation to obtain licenses for the bands to use. This is partly due to the fast evolution of technologies which resulted in the growing use of wireless terminals and the slow improvement of frequency allocation dictated by government and/or companies [25].

The nature of the wireless channel makes the design of wireless networks to be different than the design of wired networks. Indeed, the wireless channel represents an unpredictable and hence a difficult medium for communication. Through this channel, signal propagation is subject to random fluctuations (see Section 2.2) [28]. Allocating the radio spectrum becomes more and more complicated due to the diversity of applications and systems so spectrum need to be controlled by regulatory bodies (regionally and globally)[12, 32]. The issue is that the spectrum of high frequency is very crowded [25].

Wireless networking is another challenge. Since links in wired networks do not move whereas mobile terminals move (sometimes at high speed like in vehicular systems), routing data to users in those conditions may be difficult since channels are varying fast. Resources of the network are limited. Therefore, they must be allocated in a fair and efficient way regarding user demands and locations. Indeed, both change over time. Besides, since wired and wireless networks differ in performance capabilities, interfacing them is a difficult problem [12, 28].

Finally, one last technical challenge in wireless network design is a revision of the design process itself. The basic approach of networking is based on the layered approach. This approach applies for wired networks, but also for wireless networks. Each layer of the system operation has an associated protocol ensuring less complexity and more modularity. It adds standardization as well, making systems reusable and flexible. On the other hand, it causes a lack of global design optimization. This is not a big issue in wired networks since equipments are reliable whereas it is in wireless networks. Wireless links can show very poor performance, and this performance changes over time along with user connectivity and network topology. Thus, there is a need of optimizing wireless network design by revisiting the global existing approach [12, 32].

2.1.2 Cellular systems

This section focuses on cellular wireless systems since it is the topic of our study, without giving too much details though. However, it is important to note that today, many different wireless systems exist, and it is still interesting to mention them. Thus, beside cellular wireless networks, one can encounter other wireless technologies: cordless phones, wireless LANs, wide area wireless data services, broadband wireless access, paging systems, satellite networks, low-cost low-power radios such as Bluetooth and Zigbee, ultrawideband radios, etc [12, 28, 32]. Many wireless systems imply a broad range of products. However, nowadays, cellular
2.1 Wireless systems

Wireless systems prevail among all those forms of wireless communications [12].

Cellular telephone systems ignited the wireless revolution. It is the economically most important form of wireless communications [28]. Cellular systems provide two-way voice and data communication with different coverages: regional, national and international [32]. Today these systems have evolved to support lightweight handheld mobile terminals operating inside and outside buildings [12].

An important part of these systems is frequency reuse. Spectrum is limited so there is a need of reusing it for different wireless connections in different locations. Since signal power falls off with distance, the same frequency can be reused on different spatially-separated locations. The area of the cellular system is divided into non-overlapping cells. A set of frequencies is assigned for each cell so that the same set of frequencies is reused in another cell some distance away. The reason why adjacent cells use different sets of frequencies is interference. Users in different cells operating on the same set of frequencies cause inter-cell interference (see Section 4.2.2). Cells using the same set of frequencies are separated by a reuse distance. On one hand, this reuse distance should be as small as possible so that frequencies are reused as often as possible to maximize spectral efficiency [36]. On the other hand, the smaller the reuse distance is, the more the system is subject to inter-cell interference because of the smaller propagation distance between interfering cells. For acceptable performance, inter-cell interference must remain below a given threshold. Thus, reuse distance cannot be reduced under some minimum value [12, 20, 28, 32]. Efficient cellular networks are interference-limited, i.e. the interference dominates the noise floor. Thus, techniques reducing interference in cellular systems increase system capacity and performance [12, 20, 28]. Several methods for interference reduction (in modern and emerging systems) exist: cell sectorization, smart antennas, multiuser detection, dynamic resource allocation, etc (see Section 4.4).

Initially, cellular systems were based on large cells called macrocells. Base stations (each cell has a base station on its center) were placed on tall buildings or mountains, and transmitted at very high power with big cell coverage areas. Signal power is radiated uniformly in all directions from the base station. This circular shape of constant power results in a hexagonal shape for each cell of a system (see Figure 2.1) [12, 20].

Nowadays, cellular systems in urban areas are composed by smaller cells (microwells, picocells). Base stations are close to street level and transmit at much less power. Advantages are higher capacity, less transmission power, local interference only and robustness. Disadvantages are that infrastructure is needed, handover is needed since mobiles traverse small cells faster than large cells, and frequency planning [20]. In addition, propagation models are difficult to develop since base stations cannot be placed anywhere in an urban areas and the signal propagation depends on surrounding objects or buildings so a hexagonal cell is generally not a good approximation to signal propagation in microcells [12]. Microcellular systems are often designed using square or triangular cell shapes [16].
Figure 2.1. Cellular system

2.2 Wireless channel

The wireless channel is the link between a transmitter and a receiver. Downlink refers to the link from the base station from the mobile station and uplink refers to the link from the mobile station to the base station. We studied the former. More precisely, the term channel refers to the medium between the transmitting antenna and the receiving antenna [19, 20]. More precisely, The characteristics of a wireless signal change as it goes from the transmitting antenna to the receiving antenna. The characteristics depend on different factors: distance between the antennas of the transmitter and the receiver, the path(s) taken by the signal, and the environment the path goes through. Furthermore, the strength of the wireless channel varies over time and frequency so it poses a serious challenge for high-speed communication [12, 28]. The profile of a received signal can be obtained if we have a model of the medium between the two antennas. This model of the medium is called channel model [19] (see Chapter 3).

The wireless channel is susceptible to noise, interference and other channel impediments. In addition, the user moves, making those impediments to change over time in an unpredictable way. Those variations affect the channel strength over time and frequency. They are basically called fading [12], but they are divided into two categories: large-scale fading and small-scale fading [42].
2.2 Wireless channel

2.2.1 Large-scale fading

Large-scale fading (or large-scale propagation effects) are due to *path loss* and *shadowing*. This occurs over relatively large distance (of the order of the cell size), and is typically frequency independent [12, 42].

**Path loss**

The effects of the dissipation of the power radiated by the transmitter is referred as *path loss*. In general, models for path loss assume path loss is the same along the distance between the transmitter and the receiver. Path loss provokes variations over very large distances (100-1000 meters) [12, 19, 28]. Figure 2.2 shows how the distance affects the signal power (with the propagation attenuation model, see Section 3.1.1).

![Figure 2.2](image)

**Figure 2.2.** Ratio (in dB) between received power and transmitted power over the distance ($d$ is in metres) due to path loss effect

**Shadowing**

Large obstacles between the transmitter and the receiver cause signal power attenuation. This is referred as *shadowing* (or shadow fading) (see Figure 2.3) [12]. Effects of shadowing are caused by absorption, reflection, scattering and diffraction [20]. The signal can be blocked due to these effects if attenuation is too strong. Variations due to shadowing effects occur depending on the environment: over distances up to 10-100 meters in outdoor environments and less in indoor environments. These distances are proportional to the length of the obstructing object [12].
2.2.2 Small-scale fading

Small-scale fading (or small-scale propagation effects) is caused by constructive and destructive addition of self-interference of the different signal paths between the transmitter and the receiver. It occurs at the spatial scale of the order of the carrier wavelength. In addition, it is frequency dependent [42]. The general term fading is usually used for all the different types of small-scale fading but we focused on the multipath fading. Multiple versions of the transmitted signal cause interference since those versions arrive at different times (before or after). This is referred as delay spread (typical values are $3 \, \mu s$ in cities up to $12 \, \mu s$) [20]. These waves are called multipath waves. They are received as a combined wave at the receiving antenna. It results in a signal which varies in amplitude and phase because the reflected signal takes different paths. The paths depend on the distribution of the intensity and relative propagation time of the waves and the bandwidth of the transmitted signal [32]. Small-scale fading is caused by several physical factors: multipath propagation, speed of the mobile, speed of surrounding objects and the transmission bandwidth of the signal [32].

Flat fading and frequency-selective fading

The coherence bandwidth (see Definition 2.1) of the channel relatively to the bandwidth of the signal may cause two kind of fading. In flat fading, the bandwidth of the transmitted signal is much smaller than the coherence bandwidth. Thus, the fading across the signal bandwidth is highly correlated. In frequency-selective fading, the signal bandwidth is larger than the coherence bandwidth. Thus, the channel can be decomposed to many parallel and independent channels which are frequency separated by more than the coherence bandwidth. Otherwise, communications through frequency-selective fading channel suffer inter-symbol interference (ISI) (interference between multipath components) [12, 42].
Definition 2.1 (Coherence bandwidth) The coherence bandwidth is the range of frequencies defining whether the channel can be considered flat or not \([12, 28]\). The coherence bandwidth \(B_c\) is related to the delay spread \(S\) as \(B_c = \frac{1}{S}\). Thus, when the delay spread is large, the coherence bandwidth is smaller and the channel becomes frequency-selective.

### 2.2.3 Thermal Noise

Noise is an important factor for the analysis of communications systems. There are different sources of noise: thermal noise, shot noise or flicker noise. In wireless communications, the major source of noise generation is temperature. Thermal noise always alters a transmitted signal through a wireless channel \([37, 23]\). An object at a temperature above absolute zero provokes the transmitted charges to be excited, creating a thermal noise. The latter is very random and difficult to remove from the transmitted signal. Thus, objects around the receiving antenna cause thermal noise altering the received signal \([7]\). Thermal noise is generally assumed to be an additive white Gaussian noise \([42]\).

### 2.3 Multiuser communications

Multiplexing is a way for several users to share a medium with minimum or no interference (not only in wireless communication systems). In wireless communications, four dimensions can be used for multiplexing \([20, 12]\):

- **space** i.e. **Space Division Multiple Access (SDMA)** by using multiple antennas
time i.e. Time Division Multiple Access (TDMA) by assigning different time-slots to different users

different frequencies to different users

code i.e. Code Division Multiple Access (CDMA) by assigning different codes to different users

The idea is to assign space, time, frequency and code to each wireless channel with a minimum of interference and a maximum use of the medium. The channel can be split into \( N \) divisions with minimum interference between them. Those \( N \) divisions can be assimilated to \( N \) Degrees of Freedom [42].

--- Example 2.1: Orthogonal Frequency Division Multiplexing ---

Orthogonal Frequency Division Multiplexing (OFDM) is a way of exploiting Degree of Freedom (DoF). Besides, it can operate in a frequency-selective channel.

OFDM is a modulation scheme and is suited for high-data-rate transmission. The mechanism is as follow: a high-rate data stream is split into a number of low-rate data streams. These streams are transmitted over parallel, narrowband channels. Each channel corresponds to a subcarrier. Subcarriers are orthogonal in order for the receiver to be able to separate signal carried by these subcarriers [28]. Each subcarrier is narrow band, therefore each of them experience flat-fading only [33, 47].
Chapter 3

Channel modeling

In general, mobile communication systems operate in complex propagation environments. For designing, simulating and planning wireless systems, models for the propagation channels are needed because environments are too complex to describe accurately. Empirical and statistical models have been developed based on measurements taken in different real environments. These models are used for most simulation studies. In this section, only the models used for the simulation platform are described.

3.1 Path loss modeling

3.1.1 Propagation attenuation model

The free space propagation model is a model that permits to predict the received signal strength when there is an unobstructed Line-of-Sight (LoS)\(^1\) path between the transmitter and the receiver [32]. A very simple model can be used to model the effect of the path loss. Indeed, the first factor to take into consideration during a signal propagation is the distance. The larger the distance is, the more important the propagation loss is (see Figure 2.2). Thus, we can simply model the propagation loss as a factor of the distance. However, given the environment, the propagation loss, i.e. the path loss exponent \(\alpha\), varies. In fact, it can vary from 2 in an ideal free space environment to 6 in a very obstructed building. A typical value for outdoor environment is 3 [32, 35]. Here is a simple formulation for this model:

\[
PL = 10\alpha \log_{10}(d)
\]

where, \(PL\) is the path loss in dB, \(d\) is the distance between the transmitter and the receiver and \(\alpha\) is the exponent (note that \(\alpha\) is the slope of Figure 2.2).

This model is very simple since the environment factor is only represented by one parameter (the exponent \(\alpha\)). That is why other models, more complex, may be used.

\(^1\)Type of propagation which can ensure communication between the transmitter and the receiver when they are in view of each other without any obstacles between them.
3.1.2 Hata models

Hata model

The Hata model is an empirical formulation based on the graphical path loss data provided by Okumura. It is valid over the range of 150-1500 MHz. Under the Hata model, the standard formulation for empirical path loss in urban areas is

\[
PL_{urban}(d) = 69.55 + 26.16 \log_{10} f - 13.82 \log_{10} h_t - a(h_r)
+ (44.9 - 6.55 \log_{10} h_t) \log_{10} d \tag{3.2}
\]

with PL the path loss in dB, \( f \) is the frequency in MHz, \( d \) is the distance between the transmitter and receiver antennas in km, \( h_t \) is the antenna height above ground level in metres and \( h_r \) is the receiver antenna height above ground level in metres.

The parameter \( a(h_r) \) is defined for urban environments as

\[
a(h_r) = 3.2(\log_{10}(11.75h_r))^2 - 4.97, \text{ for } f > 300 \text{ MHz} \tag{3.3}
\]

and for suburban or rural environments

\[
a(h_r) = (1.1 \log_{10} f - 0.7)h_r - (1.56 \log_{10} f - 0.8) \tag{3.4}
\]

Corrections to the urban model are made for suburban and rural propagation, so that these models are

\[
PL_{suburban}(d) = PL_{urban}(d) - 2 \left( \log_{10} \left( \frac{f}{28} \right) \right)^2 - 5.4 \tag{3.5}
\]

and

\[
PL_{rural}(d) = PL_{urban}(d) - 4.78(\log_{10} f)^2 + 18.33 \log_{10} f - K, \tag{3.6}
\]

where \( K \) ranges from 35.94 (countryside) to 40.94 (desert). The Hata model well-approximates the Okumura model for distances \( d > 1 \text{ km} \) which means it is a good model for first generation cellular systems, but not for current cellular systems with smaller cell sizes and higher frequencies [28].

COST-231 Hata model

An extension to the Hata model is the COST-231 Hata model. This model is widely used for predicting path loss. It is designed to be used over the range 500-2000 MHz [1, 12, 28].

The basic equation for path loss in dB is

\[
PL = 46.3 + 33.9 \log_{10}(f) - 13.82 \log_{10} h_t - a(h_r)
+ (44.9 - 6.55 \log_{10} h_t) \log_{10} d + c_m \tag{3.7}
\]

where, \( PL \) is the path loss in dB, \( f \) is the frequency in MHz, \( d \) is the distance between the transmitter and receiver antennas in km, \( h_t \) is the antenna height above ground level in metres and \( h_r \) is the receiver antenna height above ground level in metres. The parameter \( c_m \) is defined as 0 dB for suburban or open environments and 3 dB for urban environments.

As the Hata model, this model only is a good model for first generation cellular systems and meant for \( 1 \text{ km} < d < 20 \text{ km} \) [12].
3.2 Shadowing modeling

3.2.1 Log-normal shadowing model

A model for the random attenuation due to shadowing effects is also needed. Since the location and the size of the blocking objects or the changes in reflecting surfaces and scattering objects that cause the random attenuation are generally unknown, statistical models are used to characterize this attenuation. The most common model is the log-normal shadowing model. In this model the ratio of transmit-to-receive power is random with a log-normal distribution [12].

The shadowing can be directly combined with the path loss to give a more complete path loss model, or both models can be just superimposed [12]. We used the latter. In linear unit, it consists in adding a zero-mean Gaussian distributed random variable. The log-normal shadowing is then modeled as a random variable $L$ which has Gaussian distribution with standard deviation $\sigma$ in $dB$. Thus, in linear unit, we obtain $10 \frac{L}{10}$ which is log-normal.

3.3 Multipath modeling

3.3.1 Rician and Rayleigh model

Since in practice deterministic channel models are rarely available, we must characterize multipath channels statistically. One frequently used model is the Rician Model in which the LoS path is large and has a known magnitude, and there are also a large number of independent paths. In that case, the channel is modeled as

$$h_f = \sqrt{\frac{K}{K+1}} \sigma e^{j\theta} + \sqrt{\frac{1}{K+1}} \bar{h}_f$$

with $\bar{h}_f \sim \mathcal{CN}(0, \sigma^2)$ (3.8)

where $K$ is the Rician K-factor, $\sigma$ the standard deviation, $\theta$ the uniform phase of the path and $\bar{h}_f$ follows a circular symmetric complex normal distribution (see [8, Ch. 3.8] for more details about complex normal distribution) with mean zero and variance $\sigma^2$ [42].

The first term corresponds to the LoS path and the second term corresponds to the aggregation of the large number of reflected and scattered paths, independent of $\theta$. The $K$-factor is the ratio of the energy in the LoS path to the energy in the scattered paths. If $K = 0$, then this model corresponds to the Rayleigh model with the following form [42]

$$h_f \sim \mathcal{CN}(0, \sigma^2)$$

(3.9)

3.4 Combining different models

We have seen that there are many models which model a channel given different parameters and variations. They model different behaviours of a channel on different scales. It would be interesting to combine those different models to take into account the fading and the loss together.
Combining those variations gives the following formulation for the channel

$$ h = \frac{h_f}{\sqrt{L}} $$

(3.10)

where $h_f$ is the small-scale fading factor and $L$ the loss factor\footnote{The loss factor is here expressed in linear scale unlike in previous section where it was expressed in dB}, which is a combination of the path loss and the shadowing. This is done by doing the product $L_{PL} \times L_S$ with $L_{PL}$ the path loss and $L_S$ the shadowing loss.
Chapter 4

Resource allocation in wireless networks

This chapter describes the resource allocation problem encountered in wireless networks, which is the main interest of this thesis work.

In wireless communications, resource management is an important issue. We saw in Chapter 2.2 that the channel has different kinds of variation. Hence, the signal also suffers different phenomena. At the receiver side, the received signal is not the same as the emitted signal due to distance and fading indeed, but also due to interference and noise. The Quality of Service (QoS) is very important for the performance of a cellular network to ensure that the quality of the received signal is satisfying for the particular application [28].

4.1 Quality of service

There is no real common definition of QoS but several definitions given by books or communication systems organisms [14]. We can give a definition from [18] which refers to QoS as

Definition 4.1 A set of quality requirements on the collective behavior of one or more objects. […] Quality of Service is concerned with such characteristics as the rate of information transfer, the latency, the probability of a communication being disrupted, the probability of system failure, the probability of storage failure, etc.

[46] gives a more general definition of QoS for applications that must communicate in real-time

Definition 4.2 The set of those quantitative and qualitative characteristics of a distributed multimedia system, which are necessary in order to achieve the required functionality of an application.

In order to achieve a certain QoS for a system, one needs to consider the phenomena a wireless system is subject to, that is noise and interference.
4.2 Noise and interference in wireless systems

4.2.1 Noise-limited systems

Providing a certain minimum transmission quality is a requirement of wireless systems. Considering only the noise, the transmission quality can be quantified with the Signal-to-Noise Ratio (SNR). The minimum quality is ensured by a minimum SNR at the receiver side. In this situation, only two factors determine the performance of the system: the strength of the signal and the noise. The further a mobile station moves away, the more the received signal power decreases, and at some distance, the SNR does not achieve the required threshold for reliable communication. We call the range of the system noise limited or signal power limited [28, 32].

4.2.2 Interference-limited systems

Unlike wired networks, the channel in wireless networks is subject to interference. In a situation where the interference is so strong that noise can be neglected, the interference dominates the performance of the system. The transmission quality is quantified by the Signal-to-Interference Ratio (SIR) [12]. Wireless systems are mostly limited by interference. There are two major types of cellular interference: co-channel interference and adjacent channel interference [32]. Co-channel interference (or inter-cell interference) is caused by the power of neighboring base stations. Adjacent channel interference (or intra-cell interference) is caused by the power used by the base station that serves other mobile stations within the same cell. One difference between interference and noise is that interference suffers from fading, while the noise power is typically constant [28].

4.2.3 Signal-to-interference-plus-noise ratio

Instead of having the SNR and the SIR separately, there is a figure which takes into account both: the Signal-to-Interference-plus-Noise Ratio (SINR). This figure is a way to measure the quality of wireless connections and is the key figure of our simulation platform. Thus, we have the following formulation for the SINR [12, 28]:

\[
SINR = \frac{S}{I + N}
\]  

(4.1)

where \( S \) is the received intended power, \( I \) is the interference received power of other transmissions at the same time and \( N \) is the noise power.

Ideally, to have an optimized system, the SINR of all mobile stations should be above a given threshold which is sufficient enough to satisfy QoS requirements. This is discussed in more details in Chapter 5.2 and Section 7.2.
4.3 Quality of service and SINR

In general, we talk about bandwidth, delay and error rates to specify QoS requirements \[28\]. Since a wireless link is subject to interference and noise, QoS need to be ensured in order to have a good quality of communication. Thus, we obtain the rate \( R \) of a stream \[42\]:

\[
R \leq \log_2(1 + \text{SINR})
\]

We can thus ensure the QoS of the stream with the SINR by ensuring that the latter is greater than the threshold \( T \), that is:

\[
\text{SINR} \geq T \triangleq 2^R - 1
\]

4.4 Reducing interference

4.4.1 Joint power control and link scheduling

Power control

In order to have reliable communication, the SINR should be above a given threshold corresponding to a particular communication rate. In a wireless system, as aforementioned in Section 2.2, large-scale fading and small-scale fading cause signal attenuation. The latter is accompanied by interference. For one active link between a base station and a mobile station, interference is caused by the power used by other base stations to serve other mobile stations and influence the SINR for each receiver. Thus, there is a need of power control in order to maintain a target SINR. Power control is done for each base station in order to use minimum power to serve mobile stations. Minimizing the power used for each link reduces interference on every other link \[42, 38\]. The power control formulation is discussed in Section 5.2.1.

Scheduling

In addition to power control, the achieved QoS of a mobile station depends on the allocated Degrees of Freedom. In a system with only one DoF available, active links can only be scheduled for this one. The power is controlled over this only one. In a situation where the system has more than one DoF available, active links can be scheduled over each of them, and the power control can be done over each of them as well. Indeed, in case of many subcarriers or time slots, a base station could schedule some links for a given DoF and some other links for a different one in order to satisfy the target SINR for every receiver. Thus, each DoF is used in a most efficient way that is in order to as many active links (in total) as possible \[32\].

4.4.2 Interference reduction techniques

We have seen that it is possible to optimize the SINR with joint power control and link scheduling. However, there are other techniques to reduce interference...
in cellular systems. These techniques include cell splitting, sectorization, smart antennas, interference averaging, multiuser detection, and interference precancellation. In this section, we describe few of them.

**Sectorization**

Sectorization uses directional antennas at the base station to divide up the cell into sectors. The sectorization is done by assigning different sets of frequencies per sector. For given mobile station, intra-cell interference comes from its sector only (instead of coming from the whole cell) and overall interference is reduced. This feature is commonly used in cellular systems, typically by dividing each cell into three sectors [12]. Figure 4.1 shows an example of sectorization (with frequency reuse).

![Figure 4.1. An example of sectorization: three cell cluster with three sector antennas. It shows a combination of sectorization and frequency reuse (see Section 2.1.2). Frequency f, g and h are reused over the whole network (frequency reuse). Each set of frequencies is divided in three set of frequency (for example, $f_1$, $f_2$ and $f_3$) for each cell.](image)

**Smart antennas**

Smart antennas (also referred as beamforming) is a technique that use antenna arrays in order to provide directional gain. By controlling the phase of each antenna element in the array, the angle of antenna beams can change in order to direct the reception or the transmission (see Figure 4.2) [12, 28, 39].

---

1The simulation platform is implemented in order to be extended to those techniques.
Figure 4.2. An example of beamforming with 4 antennas
Chapter 5

Resource allocation problem as mixed-integer linear programming problem

We have seen that the resource allocation problem is a struggling issue in wireless networks.

This chapter explains general concepts of MILP and how it is used to help solve the resource allocation problem.\(^1\)

5.1 Definition of mixed-integer linear programming

A mixed-integer linear program is the minimization or maximization of a linear function subject to linear constraints. The "mixed-integer" term refers to the fact that only some of the variables are required to be integers. More explicitly, a mixed-integer linear program with \(n\) variables and \(m\) constraints has the form

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad A_1 x = b_1 \\
& \quad A_2 x \leq b_2 \\
& \quad l_i \leq x_i \leq u_i \quad \text{for } i = 1 \ldots n \\
& \quad x_j \in \mathbb{Z} \quad \forall j \in D \subset \{1 \ldots n\},
\end{align*}
\]

where \(A_1\) is a \(m_1 \times n\) matrix, \(A_2\) is a \(m_2 \times n\) matrix, \(m_1 + m_2 = m\).

If all the variables can be real, the problem is called linear programming problem (solvable in polynomial time). When some of the variables must be integer

\(^1\)This way of solving resource allocation problem is not the only one. Hence, it is not general (there are other ways of solving it), nor practical (it is difficult to use this model in practice)
5.2 MILP formulations of resource allocation

The problem stated in [21] is how to jointly allocate DoF and power optimally in a generic model of wireless networks with \( K \) transmitter-receiver pairs, considering the downlink channel. The problem is based on the received SINR and how to adjust the latter to have maximum active links and minimum power over a network. In [21], the problem is stated for only one receiver per transmitter. In order to provide a more generic formulation, we refined the latter to include more than one receiver per transmitter.

We present the refinements of the formulation step-by-step. We first present the formulation to its most basic form, that is one receiver per transmitter and one DoF. Then, we extend it to have more than one receiver per transmitter. Finally, we extend it to handle more than one DoF.

5.2.1 Problem formulation: 1 receiver per transmitter, 1 degree of freedom

In this case, we have a system of \( K \) transmitters, 1 receiver per transmitter and 1 DoF. Thus, we have \( K \) transmitter-receiver pairs (or links).

SINR formulation

The SINR is the key figure of the resource allocation problem, and of the simulation platform (see Section 4.2.3). The equation (4.1) is a very basic form for the SINR. In more details, for the \( k^{th} \) link, the SINR is formulated as following:

\[
SINR_k = \frac{G_{kk}p_k}{\sum_{l \neq k} G_{lk}p_l + \sigma_k^2}
\]  

(5.6)

Referring to the equation (4.1), the power \( S \) is expressed with the actual transmit power \( p_k \) used on the \( k^{th} \) link and the gain of the channel \( G_{kk} \), and the interference \( I \) is expressed with the summation of channel gain \( G_{lk} \) between the \( l^{th} \) transmitter (that is all other transmitters) with the \( k^{th} \) receiver and the transmit power of the \( l^{th} \) transmitter. The noise variance \( \sigma_k^2 \) is also added.

Relatively to (3.10), the gain \( G_{lk} \) between the \( l^{th} \) transmitter and the \( k^{th} \) is expressed as \( G_{lk} = |h_{lk}|^2 \) with \( h_{lk} \) the channel between the \( l^{th} \) transmitter and the \( k^{th} \).

Power control with SINR constraints

The SINR determines the quality of the \( k^{th} \) link. Therefore, the QoS is dictated by the SINR. Thus, given a predetermined threshold \( T_k \), the QoS of the \( k^{th} \) link is satisfied, when the SINR is larger or equal to \( T_k \).
The problem is that if there is too much interference, the SINR of the link drops. In this case, one would just increase the power $p_k$. However, by increasing the power of the $k^{th}$ link, interference on the other links is also increased. So the issue is to control the transmit power of every link in order to satisfy the QoS on each link (see Section 4.4.1). Beside the power control, scheduling some links might also be needed to satisfy the QoS (see Section 4.4.1) This leads to two sets of variables we need to optimize: the transmit power $p_k$ and the scheduling variable $s_k$. The aim of the problem is to (i) maximize the number of scheduled links to have as many links served as possible (ii) minimize the power on each link in order to limit interference on every link.

According to [21], the problem is formulated as follow

$$\max \left\{ \left\{ p_k \in [1, P] \right\} \right\} \sum_{k=1}^{K} s_k - W \sum_{k=1}^{K} p_k$$ (5.7)

subject to

$$G_{kk}p_k + M(1 - s_k) \geq T_k \quad \forall k \in K,$$ (5.8)

$$p_k - Ps_k \leq 0 \quad \forall k \in K.$$ (5.9)

The objective function (5.7) is the sum of two terms. The objective of the first term is to serve as many users as possible (by counting the number of links served). The objective of the second term is to minimize the power spent (by subtracting the total power spent to the first term) and is scaled with a weight parameter $W \geq 0$ (explained later).

As aforementioned, the variables of the optimization problem (5.7)–(5.9) are:

- $p_k$ the power used on the $k^{th}$ link
- $s_k$ a binary variable for modeling the scheduling question for the link $k$ (equals to 1 if the $k^{th}$ link is active, 0 otherwise)

Through (5.7), the problem tries to maximize the number of scheduled links as well as minimizing the allocated power so long as the constraint (5.8) is respected. The parameters of (5.7)–(5.9) are:

- $K$ the number of links of the network
- $P$ the maximum power per base station
- $W$ a weight parameter to scale the second term of the objective function defined as $0 < W < 1/KP$. It is tuned so that the objective of serving many users has higher priority than the objective of saving power
- $M$ a scalar parameter. This parameter is chosen so that all $K$ inequalities (5.8) are fulfilled when $s_k = 0$, without taking into account the values for $p_k$. When considering the worst-case scenario, all the interfering transmitters use full power, whereas the transmitter in the direct link is silent. Setting $\{p_l =
Resource allocation problem as mixed-integer linear programming problem

Equation (5.8) and $p_k = 0$ in each inequality, selecting the maximum resulting lower bound we have

$$M \geq \max_k \{ T_k \sum_{l \neq k} G_{lk} P + T_k \sigma_k^2 \}$$  (5.10)

MILP formulation

In [21], the problem is finally formulated as a MILP problem. We need to recast our previous problem as a MILP representation as well.

The constraints (5.8) are actually linear inequalities. Since the denominator of the fraction is positive, (5.8) can be equivalently rewritten as

$$G_{kk} p_k + M (1 - s_k) \geq T_k \sum_{l \neq k} G_{lk} p_l + T_k \sigma_k^2 \Leftrightarrow$$

$$T_k \sum_{l \neq k} G_{lk} p_l - G_{kk} p_k + M s_k \leq M - T_k \sigma_k^2 \Leftrightarrow$$

$$\sum_{l=1}^{K} A_{lk} p_l + M s_k \leq B_k,$$  (5.11)

where we have defined $B_k \triangleq M - T_k \sigma_k^2$ and

$$A_{lk} \triangleq \begin{cases} -G_{kk} & \text{if } l = k, \\ T_k G_{lk} & \text{if } l \neq k, \end{cases}$$

Thus, the problem can be equivalently written as:

$$\max_{\{ p_k \in [1, P] \}, s_k \in \{0,1\} : k \in K} \sum_{k=1}^{K} s_k - W \sum_{k=1}^{K} p_k$$

subject to

$$\sum_{l=1}^{K} A_{lk} p_l + M s_k \leq B_k \quad \forall k \in K,$$  (5.13)

$$p_k - P s_k \leq 0 \quad \forall k \in K,$$  (5.14)

5.2.2 Problem formulation: $R$ receivers per transmitter, 1 degree of freedom

The next step to extend the optimization problem is to have $R$ receivers per transmitter. Indeed, in practice, more than one receiver may be served by one transmitter. Having more than one receiver per transmitter makes the SINR formulation altered. In fact, it increases the interference for one receiver. For one transmitter-receiver pair, the SINR is subject to more interference caused by the power from the other links linked to the same transmitter (intra-cell interference). In addition, the power transmitted by one transmitter has to be distributed among all the receivers.
We thus have the following optimization problem:

\[
\max_{\{ p_{kr}, s_{kr} \in [0,1] \}} \sum_{k=1}^{K} \sum_{r=1}^{R} s_{kr} - W \sum_{k=1}^{K} \sum_{r=1}^{R} p_{kr} \quad (5.15)
\]

subject to

\[
G_{kkr} p_{kr} + M (1 - s_{kr}) \geq T_{kr} \quad \forall k \in K, \forall r \in R, \quad (5.16)
\]

\[
\sum_{v \neq r} G_{kkr} p_{kv} + \sum_{l \neq k} \sum_{v=1}^{R} G_{lkr} p_{lv} + \sigma_{kr}^2 \geq T_{kr} \quad \forall k \in K, \forall r \in R, \quad (5.17)
\]

\[
\sum_{r=1}^{R} p_{kr} - P s_{kr} \leq 0 \quad \forall k \in K, \forall r \in R, \quad (5.18)
\]

Constraint (5.16) takes into account two types of interference (see Section 4.2.2).

- **Intra-cell interference** is represented by \( \sum_{v \neq r} G_{kkr} p_{kv} \). It is calculated using the power used by the \( k^{th} \) base station for the other \( v^{th} \) receivers of the cell (so that \( v \neq r \) for the \( k^{th} \) link) and the channel gain of the \( r^{th} \) receiver of the \( k^{th} \) base station.

- **Inter-cell interference** is represented by \( \sum_{l \neq k} \sum_{v=1}^{R} G_{lkr} p_{lv} \). It is calculated using the total power used by the other \( l^{th} \) base stations (so that \( l \neq k \)) and the channel gain between those base stations and the \( r^{th} \) receiver of the \( k^{th} \) base station.

Taking into account \( R \) receivers per transmitter, we have \( RK \) links. Thus, we have \( K \) transmitters and \( RK \) receivers in total. The previous case can be assimilated to this case with \( R = 1 \). The number of receivers per transmitter is represented with \( r \in R \triangleq \{1, \ldots, R\} \). Then, the channel gain between the \( l^{th} \) transmitter and the \( r^{th} \) receiver of the \( k^{th} \) transmitter is represented with \( G_{kr} \).

Thus, we have:

- \( p_{kr} \) the power used by the \( k^{th} \) base station for its \( r^{th} \) receiver
- \( s_{kr} \) binary variable for modeling the scheduling question of the \( k^{th} \) base station for its \( r^{th} \) receiver
- \( K \) the number of base stations
- \( R \) the number of receivers per base station i.e. per cell

Note that the \( M \) parameter has to take into account intra-cell interference for the maximum lower bound. In the worst-case scenario, intra-cell interference use full power whereas the direct link is silent

\[
M \geq \max_{k,r} \{ T_k G_{kkr} P + T_k \sum_{l \neq k} G_{lkr} P + T_k \sigma_{kr}^2 \} \quad (5.19)
\]
Resource allocation problem as mixed-integer linear programming problem

The distribution of the transmit power among \( R \) receivers for one transmitter is done with (5.18).

MILP formulation

As in Section 5.2.1, the problem has to be recast as a MILP representation.

\[
G_{kkr}p_{kr} + M(1 - s_{kr}) \geq T_{kr} \sum_{v \neq r} G_{kkv}p_{kv} + T_{kr} \sum_{l \neq k} \sum_{v=1}^{R} G_{lkr}p_{lv} + T_{kr}\sigma_{kr}^2 \Leftrightarrow \\
T_{kr} \sum_{v \neq r} G_{kkv}p_{kv} + T_{kr} \sum_{l \neq k} \sum_{v=1}^{R} G_{lkr}p_{lv} - G_{kkr}p_{kr} + M s_{kr} \leq M - T_{kr}\sigma_{kr}^2 \Leftrightarrow \\
\sum_{l=1}^{K} \sum_{v=1}^{R} A_{lkv}p_{lv} + M s_{kr} \leq B_{kr},
\]

(5.20)

where we have defined \( B_{kr} \triangleq M - T_{kr}\sigma_{kr}^2 \) and

\[
A_{lkv} \triangleq \begin{cases} 
-G_{kkr} & \text{if } l = k \text{ and } v = r, \\
T_{kr}G_{lkv} & \text{if } l = k \text{ and } v = r, \text{ or } l \neq k,
\end{cases}
\]

This leads to the following formulation

\[
\max \left\{ \sum_{k=1}^{K} \sum_{r=1}^{R} s_{kr} - W \sum_{k=1}^{K} \sum_{r=1}^{R} p_{kr} \right\} \text{ subject to }
\sum_{l=1}^{K} \sum_{v=1}^{R} A_{lkv}p_{lv} + M s_{kr} \leq B_{kr},
\]

(5.22)

\[
p_{kr} - P s_{kr} \leq 0 \quad \forall k \in \mathcal{K}, \forall r \in \mathcal{R},
\]

(5.23)

\[
\sum_{r=1}^{R} p_{kr} - P \leq 0 \quad \forall k \in \mathcal{K}.
\]

(5.24)

5.2.3 Problem formulation: \( R \) receivers per transmitter, \( N \) degrees of freedom

Ideally, the final formulation of the problem should include DoF. For each transmitter, we have \( N \) DoF. Given the previous formulation, we have the following
5.2 MILP formulations of resource allocation

optimization problem:

\[
\begin{align*}
\max & \\\\\\\\\\\\{p^n_{kr} \in [0, P]\} \quad s^n_{kr} \in \{0, 1\} \quad k \in K \quad r \in R \\
\text{subject to} & \quad \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{n=1}^{N} s^n_{kr} - W \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{n=1}^{N} p^n_{kr} \\
& \quad \sum_{v \neq r} G^n_{kr} p^n_{kv} + \sum_{l \neq k} \sum_{l=1}^{L} G^n_{lkr} p^n_{lv} + \sigma^2_{kr} \geq T_{kr} \quad \forall k \in K, \forall r \in R, \forall n \in N, \\
& \quad p^n_{kr} - Ps^n_{kr} \leq 0 \quad \forall k \in K, \forall r \in R, \forall n \in N, \\
& \quad \sum_{r=1}^{R} p^n_{kr} - P \leq 0 \quad \forall k \in K, \forall n \in N, \\
& \quad \sum_{n=1}^{N} s^n_{kr} \leq 1 \quad k \in K, \forall r \in R.
\end{align*}
\]

Here we have:

- \(p^n_{kr}\) the power used by the \(k\)th base station for its \(r\)th receiver for the \(n\)th DoF
- \(s^n_{kr}\) binary variable for modeling the scheduling question of the \(k\)th base station for its \(r\)th receiver for the \(n\)th DoF
- \(K\) the number of base stations
- \(R\) the number of receivers per base station i.e. per cell
- \(N\) the number of DoF
- \(G^n_{kr}\) the channel gain between the \(l\)th base station and the \(r\)th receiver of the \(k\)th cell for the \(n\)th DoF

The scalar parameter \(M\) has to include the DoF parameter

\[
M \geq \max_{k,r,n} \{T_k G^n_{kr} P + T_k \sum_{l \neq k} G^n_{lkr} P + T_k \sigma^2_{kr}\}
\]

The last constraint (5.29) ensures that for the \(k\)th link, only one DoF is assigned.

Note that with (5.28), the power used per base station (to serve its active links) is limited by \(P\) for each DoF so the total power used by one base station is limited by \(NP\). In order to limit the total power used per base station by \(P\), the equation would be

\[
\sum_{n=1}^{N} \sum_{r=1}^{R} p^n_{kr} - P \leq 0 \quad \forall k \in K
\]
MILP formulation

As in Section 5.2.1, the problem has to be recast as a MILP representation.

\[
G_{kkr}^n p_{kr}^n + M(1 - s_{kr}^n) \geq T_{kr} \sum_{v \neq r} G_{kkv}^n p_{kv}^n + T_{kr} \sum_{l \neq k} \sum_{v=1}^{R} G_{lkr}^n p_{lv}^n + T_{kr} \sigma_{kr}^2 \iff
\]

\[
T_{kr} \sum_{v \neq r} G_{kkv}^n p_{kv}^n + T_{kr} \sum_{l \neq k} \sum_{v=1}^{R} G_{lkr}^n p_{lv}^n - G_{kkv}^n p_{kr}^n + M s_{kr}^n \leq M - T_{kr} \sigma_{kr}^2 \iff
\]

\[
\sum_{l=1}^{K} \sum_{v=1}^{R} A_{lkv}^n p_{lv}^n + M s_{kr}^n \leq B_{kr},
\]

where we have defined \( B_{kr} \triangleq M - T_{kr} \sigma_{kr}^2 \) and

\[
A_{lkv}^n \triangleq \begin{cases} 
-G_{kkv}^n & \text{if } l = k \text{ and } v = r, \\
T_{kr} G_{kkv}^n & \text{if } l = k \text{ and } v \neq r, \\
T_{kr} G_{lkr}^n & \text{if } l \neq k, 
\end{cases}
\]

Thus, the optimization problem (5.25)–(5.29) can be equivalently written as:

\[
\max \left\{ \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{n=1}^{N} s_{kr}^n - W \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{n=1}^{N} p_{kr}^n \right\}
\]

subject to

\[
\sum_{l=1}^{K} \sum_{v=1}^{R} A_{lkv}^n p_{lv}^n + M s_{kr}^n \leq B_{kr} \quad \forall k \in K, \forall r \in R, \forall n \in N, \tag{5.34}
\]

\[
p_{kr}^n - P s_{kr}^n \leq 0 \quad \forall k \in K, \forall r \in R, \forall n \in N, \tag{5.35}
\]

\[
\sum_{r=1}^{R} p_{kr}^n - P \leq 0 \quad \forall k \in K, \forall n \in N, \tag{5.36}
\]

\[
\sum_{n=1}^{N} s_{kr}^n \leq 1 \quad k \in K, \forall r \in R. \tag{5.37}
\]

Now that we have this optimization problem, what is the relationship with the simulation platform? In [21], the MILP formulation was used through a simulation run with the GNU Linear Programming Kit (GLPK) [10] and a set of data generated (that is channel gains) with MATLAB. Using this method is quite tedious when one wants to run the solver through many sets of data. In addition, it is tedious to generate those data as well. The simulation platform ought to generate the data at large scale and in a simple way. Then, this formulation (written as a GMPL model, see Appendix B) is used to find optimal solutions.

This leads to our next part, the design and implementation of the simulation platform.
Chapter 6

Simulation platform

Simulation platforms for cellular networks already exist. For example, OpenWNS offers a way to simulate networks. It provides a graphical interface to plot data, different possibilities of scenario and the loading of data from database [4]. The ns-3 project offers also a wide range of simulations and can interact with real systems [29]. These simulation tools offer a wide range of possibilities to simulate and measure performance in cellular networks. However, they are very often complicated and do not fit the purpose of our work. Thus, the implementation of a simulation platform is suitable for the aim of our work.

This chapter describes the way the simulation platform was designed and implemented.

6.1 Description

As partly aforementioned in Section 1.2, the simulation platform should meet the following criteria:

- **a parametrized cellular network** the simulation platform must permit to simulate a multi-cellular network. The size (that is the number of cells) should be given as a parameter so that the size of the network may vary as well as the coordinates of the base stations

- **different channel models** instead of choosing only one channel model, the platform should give the possibility to choose different models for the generation of the data

- **several receivers** in order to have realistic simulations, the simulation platform should provide the possibility to generate more than one receiver for each transmitter

- **simple and flexible** one of the reason for the development of the simulation platform is that generating data is quite tedious with existing tools (MATLAB). Providing a very simple and extendable tool is one main aspect of this platform
**consistency and stability** the data generated should be consistent for each simulation, no matter the number of simulations and the size of the network. In addition, the simulation platform has to be stable and not limited by bugs or errors. A simulation consists in building a cellular network with base stations and mobile stations and generating channels and channel gains (see 6.2)

**multi-core capable** if possible, the simulation platform must use all the available resources in order to perform simulations and computations as fast as possible

### 6.2 Design and implementation

The simulation platform is in fact divided into two modules:

- a *simulator* module which takes care of simulating and generating data (network topologies and channel gains)

- a *solver* module which takes care of solving the MILP problem given the data generated by the simulation module

In addition, some useful functions were developed in order to ease some processes (grouped as a *util* module).

Figure 6.1 describes how the simulation platform is structured.

![Diagram](main.cpp

Solver module

Simulator module

Util module

**Figure 6.1.** Diagram describing the modules structure

The principle of the simulation platform basically follows the Monte Carlo method [2, 48]. The aim is to generate many simulations in a random way to finally have enough data to study the average performance (which would be the variables of the MILP problem (5.25)–(5.29)).

The entry point of the application consists in a main program which allows the user to perform several actions. These actions are shown in Figure 6.2 as a use-case diagram.
6.2 Design and implementation

6.2.1 Simulator module

Figure 6.3 shows the simulator diagram generated by Doxygen (see Section 6.3.3).

Cellular Network

The simulation of a cellular network consists in building hexagonal cells by placing base stations. Base stations are identified with their coordinates over the network given a central cell. Even if in practice base stations are not placed uniformly and regularly, keeping the simulation simple obliged to have a simple way to place them. Thus, given the central cell, cells are placed according to one parameter: the intersite distance. Indeed, in theory, cells have regular hexagonal shapes and are thus positioned given this shape. The radius of the hexagon is given by the distance between adjacent base stations. The structure of the cellular network itself follows the principle of having rings around the central cell (see Figure 6.4). Thus, the parameter to take into consideration is the number of rings to build.

Snapshots

Once the cellular network is built, receivers need to be generated. Since we may need to generate many receivers for the same network structure, it is useless to
Figure 6.3. Simulator diagram generated by Doxygen
generate the cellular network as many times as we need to generate receivers. Thus, we generate the cellular network only once, and we generate 'snapshots' of receivers over it. A snapshot is basically a set of mobile stations which are only identified by coordinates over the network. The way mobile stations are positioned is described in Example 6.1.

Figure 6.5 shows an example of snapshot.

---

**Example 6.1: Mobile station positioning**

Mobile stations are basically positioned given three parameters. The coordinates of the base station \( p_{bs} \) (with \( p_{bs} = x_{bs} + i \cdot y_{bs} \)) i.e. the center of the cell, the radius of the cell \( d_r \), therefore the apothem \( a_r \) and the inner-zone radius \( d_i \) which corresponds to the radius of the near area around the base station. Indeed, receivers cannot be placed too close to the base station location for practical reasons. The first step is to draw the \( X \) and \( Y \) coordinates of each mobile station. The coordinates are randomly drawn according to a *uniform distribution* defined on \([x_{bs} - d_r; x_{bs} + d_r]\) for \( X \) and \([y_{bs} - a_r; y_{bs} + a_r]\) for \( Y \). If the generated point is out of the cell i.e. out of the hexagon or inside the inner-zone radius, then the point is dropped and another point is generated until a point inside the cell and outside the inner-zone radius is found. Figure 6.6 shows where mobile stations can be dropped and where they cannot.
Figure 6.5. An example of a snapshot

Figure 6.6. Dropping zone of mobile station
Wrap-around

In our basic model for the simulation we have a finite number of base stations. When mobile stations are dropped in the cellular network to produce what we have called snapshots, they experience interference from base stations all around (interference calculated when computing the SINR). However, in this case, the state and figures with regard to the interference is not equivalent for all the cells, especially those which are located on the edge of the cellular network. Indeed, at the boundary of the cellular network, resulting interference is biased since some is caused by base stations located far from the mobile station. Figure 6.7 shows that for a mobile station located in a bordering cell, interference are coming only from one side of the cell since no interference are coming from the void.

![Figure 6.7. Cellular network without wrap-around technique](image)

The wrap-around technique is used to solve the aforementioned problem due to cellular networks with a finite number of base stations. The basic concept is to simulate an infinite number of base stations taking into account the resource-limited factor. Indeed, having an infinite number of base stations is not possible. The wrap-around technique is similar to the principle of frequency reuse in cellular networks. A theoretical overview can be found in [16] based on the tiling principle. It describes how to use tiles to have a network that is wrapped around so that we would have an infinite number of base stations with a kind of a loop all over the base stations. In our case, we have used the one described in [15]. Basically, the technique used is having a central cluster (like the basic cellular network) which contains the mobile stations, and six copies of the latter which cover the boundaries of the central cluster. Base stations of a copy have the same properties as the corresponding base stations of the central cluster. The only difference is the coordinates of the base stations. Figure 6.8 shows the structure of the wrap-around cellular network. We see that each cell has its corresponding translated copy in each copy cluster.
Channel Profile

Choosing different channel models is an important feature of the simulation platform. Once the cellular network and several snapshots are generated, channels are calculated according to the chosen channel model. The channel model is referred as the channel profile of the simulation platform. In Chapter 3, we have seen different ways and models for channel modeling. Beside choosing one channel model for the channel profile, it is also possible to combine different channel models (given the three phenomena that affect the channel) with the equation (3.10).

Output

The simulator module generates different kinds of data which are used by the solver module.

First, a generated cellular network and its corresponding snapshots can be stored so that they can be visualized by users. Originally, the format chosen was the Gnuplot data file format [22] because it can be visualized by Gnuplot and MATLAB. However, for reason of flexibility, we chose the IT++ (see Section 6.3.2) format to store them as matrices readable by MATLAB. In addition, channels and channels gains (given a channel profile) can be stored so they can be used to generate GNU Mathematical Programming Language (GMPL) files [11, 26] or analyzed for other purpose. The format used is the IT++ file format as well.
Finally, GMPL data files are generated to be used by the solver module (see Section 6.2.2). An example of such a file is attached in Appendix C.

### 6.2.2 Solver module

The solver module is the part which solves MILP problems. Provided with data files and a model file, solutions of the problem are generated and stored.

Figure 6.9 shows the solver diagram generated by Doxygen (see Section 6.3.3).

![Solver diagram generated by Doxygen](image)

**Figure 6.9.** Solver diagram generated by Doxygen

#### Model Files

GMPL model files are used by the solver module. In our case, the model file consists of a MILP problem based on the final formulation in Section 5.2.3. The MILP formulation is based on the formulation in [21] and results in the formulation in Section 5.2.3. The corresponding GMPL file can be found in Appendix B.

#### Data Files

Model files may include data but in order to be able to load many data files in the same time, data are put in separated GMPL data files. Those data files are generated with the channel gains produced by the simulator. Some parameters need to be calculated given those data, $M$ and $W$ in particular (see Section 5.2.1, 5.2.2 or 5.2.3). $M$ is calculated given its lower bound, and $W$ its upper bound. The solver module can load either one data file or several data files. Moreover, it can load an entire folder containing data files (which are identified by the `.dat` extension). An example of such a data file can be found in Appendix C.
Output

Two formats have been chosen for the output. The first format is the one used by GLPK to output data as a single file for each data file analyzed. In addition, the solver module outputs some data to be analyzed as an IT++ file so it can be read with MATLAB. The data output corresponds to the optimized solution found and the optimized values of the variables. In our case, the optimal solution corresponds to the $p_k$ and $s_k$ variables mentioned in Chapter 5.

6.3 Implementation

6.3.1 C++

The implementation of the simulation platform has been made in C++. The reason for choosing this language was performance. Indeed, at first sight, implementing a simulation platform that runs many simulations and has to perform many computations (especially when solving MILP problems) may require an important use of the resources. Thus, a language such as Java would have been unwise. In addition, useful libraries for the purpose of the simulation platform are available and easy to integrate in C++ such as those after-mentioned. In addition, C++ is an advanced language which permits object-oriented programming.

6.3.2 External libraries

IT++

IT++ is a very useful C++ library of mathematical, signal processing and communication classes and functions [30]. For the simulation platform, it provides useful functions for generating random number. In addition, useful C++ objects are available in this library such as matrices and vectors (and operations on these objects). Finally, the IT++ file format is a format readable by MATLAB.

GLPK

The GNU Linear Programming Kit (GLPK) package is intended for solving different kind of problems at large-scale: linear programming (LP), mixed integer programming (MIP), and other related problems. It provides a set of routines written in ANSI C and organized in the form of a callable library but may be called as a stand-alone application. The GLPK package is used by the solver module to solve MILP problems. It offers many functions that are easily callable in C++ [10]. With GLPK, GMPL model files can be read and solved, and solutions can be retrieved and output.

6.3.3 Documentation

Documentation is very important especially when the project has to be resumed later on by other persons. Moreover, since the intended users are not computer
scientists, documentation and a user manual are needed. The documentation is generated by Doxygen [44] which permits to generated \LaTeX\ files or HTML files. The documentation is written directly into the source code as comments and then analyzed by Doxygen. In addition, a user manual has been written to explain to future users how to use the simulation platform. It has been written as a \LaTeX\ document. The latter can be found in Appendix A.

6.3.4 Testing

The testing is a part that should not be neglected. It is needed to be sure the simulation platform does what it is supposed to do. There are two categories of testing.

There is the proper testing part which is testing the consistency of the results. This is basically done by launching the program and check whether the results are consistent or not. This is done by computation or by visualizing some results through graphs (such graphs are generated by Gnuplot or GNU Octave and can be seen in the next part).

Unit testing tests basic features of the simulation platform. This part should be done through an automatic process. Such a program is a redundant task, therefore we looked for some unit testing framework. Since this work is not about finding the ideal unit testing frameworks, the one chosen had to meet few but essential requirements: C++ tests, Linux-platform compatible, simple. Hence, we chose the Google C++ Testing Framework [13]. In addition, we used a debugger in order to find and correct errors in the code. The debugger we used was GDB, the GNU Project Debugger [6].
Chapter 7

Results

This part explains and describes the test and results of the simulation platform.

7.1 Test of simulator module

7.1.1 Representation of cellular network

The first feature of the simulator module is generating a multi-cellular wireless network. As written in Section 6.2.1, generated cellular networks are stored in IT++ files. With the generated data files, it is possible to generate a representation of a cellular network. This permits to check that cellular networks generated by the simulation platform are well-generated. Figure 7.1 shows an example of a generated cellular network rendered by GNU Octave. Here, we can see a cellular network with 19 base stations (that is a 2-ring cellular network) which is similar to the example shown in the Figure 6.4.

7.1.2 Representation of snapshots

The second feature of the simulator module is generating mobile stations across the cellular network. This was referred as snapshots (see Section 6.2.1). Snapshots are generated as IT++ files as well. They can be stored in the same file as the one used for the cellular network in separate variables or in a different file. Figure 7.1 shows the previously-mentioned cellular network with one snapshot superimposed. The simulation platform is able to generate many snapshots, and this randomly. Visualizing those snapshots gives the possibility to check that they are indeed generated through a random process so that we do not have redundant data (and respect the Monte Carlo Method).

7.1.3 Performance of simulator

Performance is an important matter since the aim of the simulation platform is to release users from tedious tasks as simulating cellular networks. Thus, we
Figure 7.1. Cellular network with one snapshot generated by GNU Octave (× denotes base stations, · denotes mobile stations)
measured performance of the simulator given the time needed in order to generate data. The following tables show the elapsed time after generating snapshots and channels given the variation of different parameters.

<table>
<thead>
<tr>
<th>Number of snapshots</th>
<th>Snapshots</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.00342</td>
<td>0.451</td>
</tr>
<tr>
<td>5000</td>
<td>0.0261</td>
<td>2.2773</td>
</tr>
<tr>
<td>10000</td>
<td>0.0358</td>
<td>4.59842</td>
</tr>
</tbody>
</table>

Table 7.1. Duration (in seconds) of the snapshots and channels generation given the number of snapshots (with 7 base stations and 1 receiver per base station)

<table>
<thead>
<tr>
<th>Number of receivers per base station</th>
<th>Snapshots</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.035897</td>
<td>4.59842</td>
</tr>
<tr>
<td>2</td>
<td>0.0616651</td>
<td>10.5937</td>
</tr>
<tr>
<td>3</td>
<td>0.089319</td>
<td>17.7256</td>
</tr>
<tr>
<td>4</td>
<td>0.117353</td>
<td>26.1559</td>
</tr>
</tbody>
</table>

Table 7.2. Duration (in seconds) of the snapshots and channels generation given the number of receivers per base station (with 10000 snapshots and 7 base stations)

Table 7.1 and table 7.2 show that increasing the number of receivers per base station or the number of snapshots to generate has no incidence on the performance of the simulator module: the complexity is linear.

<table>
<thead>
<tr>
<th>Number of base stations</th>
<th>Snapshots</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.035897</td>
<td>4.59842</td>
</tr>
<tr>
<td>19</td>
<td>0.084955</td>
<td>32.9952</td>
</tr>
<tr>
<td>37</td>
<td>0.158123</td>
<td>131.798</td>
</tr>
<tr>
<td>51</td>
<td>0.228507</td>
<td>377.508</td>
</tr>
</tbody>
</table>

Table 7.3. Duration (in seconds) of the snapshots and channels generation given the number of base stations (with 10000 snapshots and 1 receiver per base station)

Table 7.3 shows that increasing the number of base station has a bigger impact on the time needed to generate data. We observe that the time to generate channels increases quadratically. Thus, the complexity is \( O(KKR) \) with \( K \) the number of transmitters and \( R \) the number of receivers. \( KR \) represents the total number of receivers.

The previous results give data generation following the wrap-around technique (see Section 6.2.1). Results shown in Table 7.4 come from the simulator run without using the wrap-around technique. We observe that without wrap-around, generating channels takes less time. The time needed to generate the cellular network is the same since the copies of the central cluster are generated anyway.

Experiments were run on a simple laptop with 3.8 GB of memory and a dual-core processor 2×1.30 GHz.
<table>
<thead>
<tr>
<th>Number of base stations</th>
<th>Snapshots</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.041632</td>
<td>4.38848</td>
</tr>
<tr>
<td>19</td>
<td>0.0873709</td>
<td>30.8442</td>
</tr>
<tr>
<td>37</td>
<td>0.150223</td>
<td>117.642</td>
</tr>
<tr>
<td>51</td>
<td>0.226918</td>
<td>333.782</td>
</tr>
</tbody>
</table>

Table 7.4. Duration (in seconds) of the snapshots and channels generation given the number of base station (with 10000 snapshots and 1 receiver per base station, no wrap-around)

7.2 Results of solver

Beside the simulation of a cellular network with its base stations and mobile stations, the second purpose of the simulation platform consists in solving the MILP formulation with data generated by simulations.

In order to test the consistency of the data, the MILP formulation and the efficiency of the solver, some tests were run under different parameters. Results of those tests are supposed to acknowledge certain assumptions (concerning the SINR) and confirm the good use of the solver.

Tests were run varying different kinds of parameter: intersite distance, number of rings for each cluster, number of receivers, SINR threshold, etc. Since the variables of the MILP formulation are \( s \) (binary variables for the scheduling of a link) and \( p \) (assigned power for a link), the latter are the values we focus on.

Figures were generated with GNU Octave and Gnuplot.

7.2.1 Consistency of results

Input data

Some input data is needed in order to run the simulation. This data need to be close to reality in order to have consistent and useful results.

**number of snapshots** in order to have sufficient variation, experiments need to generate many snapshots during each simulation.

**intersite distance** the distance between two neighboring base stations can be parametrized. This distance may vary if the base station is located in rural area, suburban area or urban area. It models the density of the network. We based our experiments on the data given in [36, 45] and [28, p. 18].

**inner-zone radius** the radius of the very near area around the base station which is impractical for a mobile station to be. We based our experiments on the data given in [3, p. 12] and [27, p. 12].

**channel model** different channel models have been implemented. Experiments were run in order to have realistic results that is including path loss, shadowing and fading (see Section 3.4).
SINR threshold given the application, the SINR threshold may change. Requirements on the rate and QoS are different for video call or voice call for instance. The QoS of an application is guaranteed by this threshold. Thus, the SINR threshold can be chosen and the resource allocation formulation (see equation (5.26)) tries to determine the best scheduling and power control decisions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of snapshots</td>
<td>1000</td>
</tr>
<tr>
<td>intersite distance (km)</td>
<td>1 (urban), 2 (suburban/rural)</td>
</tr>
<tr>
<td>inner-zone radius (m)</td>
<td>100</td>
</tr>
<tr>
<td>channel model</td>
<td>propagation attenuation (with $\alpha = 3$) with log-normal shadowing (with $\sigma = 4$) and rician fading (with $\sigma^2 = 1$)</td>
</tr>
<tr>
<td>$SINR$ threshold (dB)</td>
<td>$[-10; 20]$</td>
</tr>
<tr>
<td>noise variance $V$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>maximum power</td>
<td>1</td>
</tr>
<tr>
<td>wrap-around technique</td>
<td>yes</td>
</tr>
</tbody>
</table>

| Table 7.5. Parameter values used as input for the experiments |

Wrap-around vs no wrap-around

First of all, we show that the results of the simulation run with the wrap-around technique (see Section 6.2.1) are different than those that run without. It might be interesting to analyze how the proportion of scheduled link and the allocated power per scheduled link may evolve given a non-wrap-around network structure, as shown in Figure 7.2.

For the wrap-around technique, we expect overall more interference in the system. Indeed, mobile stations located in outer-ring cells are subject to more interference than in a non-wrap-around network (see Section 6.2.1). Figure 7.2 shows that the number of scheduled links in slightly lower for the wrap-around technique. In addition, less power is used. Even though we would expect more power used for the wrap-around technique (in order to compensate interference), the fact that there are less scheduled links may cause the allocated power to be lower than the non-wrap-around technique. Another reason would be that there is no significant difference due to the choice of parameters (e.g. small network, one receiver).

Density of cellular network

The density of a cellular network is referred as the percentage of base stations given an area. This corresponds to the deployment of base stations over a network (assuming there is only one type of base station). The larger the distance between
Figure 7.2. Impact of the wrap-around technique on the proportion of scheduled links and the average allocated power per scheduled link (number of base station = 7, number of receiver per cell = 1, intersite distance = 1 km, DoF = 1)
adjacent base stations of a network is, the less dense the network is. In different environments, the intersite distance is different. Indeed, in rural environment, a dense network is not needed since few users need to be allocated, unlike in urban environment. Figure 7.3 shows the impact of the environment on scheduled links and allocated power (given data mentioned in Table 7.5).

As expected, we can see that in an urban environment, the proportion of scheduled links is higher than in a rural environment. This is due to the power needed to serve links. Indeed, on one hand mobile stations in rural environment can be located a lot farther than in urban environment, the path loss effect is more important so more power is needed to serve links. This increase of power causes more interference on other channels as well. Thus, the percentage of scheduled links appears to be lower in rural environment. On the other hand, we observe that in urban environment, more links are scheduled since less interference is caused by a lower needed power to allocate the formers. Note that due to infrastructure, the densification in urban environment is costly.

**Size of network**

The size of the network also dictates the amount of interference the channels are subject to, that is the number of base stations that cannot be neglected when interference needs to be calculated. This number basically corresponds to the number of base stations in a cluster as defined in Figure 6.8. Figure 7.4 shows the impact of the size of the network on scheduled links and allocated power. We observe that the proportion of scheduled link is lower and decreases a little faster when the network is larger. Likewise, the allocated power is higher and increases faster when the network is larger. The difference is quite small though. This is due to more important interference among the larger network which overcomes path loss effect. Indeed, the larger a network is, the more important interference is, i.e. more cells create more inter-cell interference. In order to have a sufficient number of scheduled links, the power is increased a lot in order to ensure the QoS therefore causing more interference in the network.

**Density of a cell**

In Section 7.2.1, we mentioned the density of the network. In this section, we refer to density as the density of a cell, i.e. the number of direct links for one base station. A big number of links for one base station means a big number of mobile stations within a cell therefore potentially served by the base station.

Figure 7.5 and Figure 7.6 show the impact of cell density on scheduled links and allocated power.\(^1\) We observe that the more there are mobile stations per base station, the lower the proportion of scheduled link is. Note that we study the proportion for the first graph on the top. At the middle, we observe that the number of scheduled links is bigger since there are more receivers per base station. The probability to have good links to serve is higher with more receivers.

---

\(^1\) All the mobile stations were generated at the first place, so the curves are correlated since all the original mobile stations were kept when adding one more receiver per base station.
Figure 7.3. Comparison of the proportion of scheduled links and the average allocated power per scheduled link given the density of the network (number of base station = 7, number of receiver per cell = 1, DoF = 1)
7.2 Results of solver

Figure 7.4. Comparison of the proportion of scheduled links and the average allocated power per scheduled link given the size of the network (number of receiver per cell = 1, intersite distance = 1 km, DoF = 1)
Figure 7.5. Comparison of the proportion of scheduled links, the absolute number of scheduled links and the average allocated power per scheduled link given the density of a cell i.e. number of receivers per base station (number of base station = 7, intersite distance = 1 km, DoF = 1)
Figure 7.6. Comparison of the proportion of scheduled links, the absolute number of scheduled links and the average allocated power per scheduled link given the density of a cell i.e. number of receivers per base station (number of base station = 7, intersite distance = 2 km, DoF = 1)
Results

in a cell therefore it increases opportunistic scheduling of links (multiuser diversity [31, 40, 42]). We observe though that the number of scheduled users is reaching a common floor when the SINR threshold get bigger.

Likewise, the more there are mobile stations per base station, the lower the allocated power per scheduled link is in spite of a close value for the two curves when the SINR threshold is low. While in Section 7.2.1 we referred to inter-cell interference, having many receivers per base station causes intra-cell interference. Indeed, on one hand the whole power of one base station need to be distributed over all the mobile stations in a cell, on the other hand one channel is subject to interference due to the power used to serve other mobile stations within the same cell. Distributing the power and keeping a low allocated power is possible with a low SINR threshold that is with a low QoS, but increasing this threshold imposes first the allocated power to be lower since intra-cell interference have a bigger impact than inter-cell interference and then a fewer percentage of scheduled links (but more links are served). Over one cell, the user with the best instantaneous channel quality communicates with the base station. This is also part of multiuser diversity.

Degrees of freedom

So far, DoF were not used to influence with results. We used only 1 DoF, so the effectively used MILP constraints were only (5.26), (5.27) and (5.28). However, in order to have a more realistic representation of cellular network therefore more realistic results for the optimisation, DoF need to vary (see Section 2.3).

Figure 7.7 and Figure 7.8 show the impact of the number of DoF on scheduled links and allocated power.\(^2\) We observe a big difference for the percentage of scheduled links for \(\text{DoF} = [1, 2, 3]\). We can see that with more DoF, the proportion of scheduled link is larger since the scheduling is distributed within the DoF. With a low SINR threshold, this percentage is still higher for a larger number of DoF. In addition, the allocated power keeps being lower with a larger number of DoF before reaching the same asymptotic value when the SINR threshold is high. Thus, we can actually see the impact of link scheduling on power control. Indeed, channels are subject to interference. Intra-cell interference has a greater impact on the QoS of the links than inter-cell interference. The density of the network is also an important factor since within a less dense network, the power needed is more important therefore interference is more important (at the inter-cell scale). Finally, we observe that increasing the number of DoF permits to have on one hand a better percentage of scheduled links, on the other hand less power used per scheduled link, at the cost of using more bandwidth though.\(^3\)

\(^2\)All the channels were generated at the first place, so the curves are correlated since all the original channels were kept when adding one DoF per channel per link.

\(^3\)A bigger difference would be expected between 1 DoF and 3 DoF. This small difference might be due to path loss effects which dominate over interference.
Figure 7.7. Comparison of the proportion of scheduled links and the average allocated power per scheduled link given the DoF (number of base station = 7, number of receiver per cell = 1, intersite distance = 1 km)
Figure 7.8. Comparison of the proportion of scheduled links and the average allocated power per scheduled link given the DoF (number of base station = 7, number of receiver per cell = 1, intersite distance = 2 km)
Results of solver 59

<table>
<thead>
<tr>
<th>Link</th>
<th>Allocated Power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SINR = 0</td>
</tr>
<tr>
<td>1</td>
<td>0.1192</td>
</tr>
<tr>
<td>2</td>
<td>0.0297</td>
</tr>
<tr>
<td>3</td>
<td>0.1457</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.3279</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.5188</td>
</tr>
<tr>
<td>8</td>
<td>0.9376</td>
</tr>
<tr>
<td>9</td>
<td>0.1516</td>
</tr>
<tr>
<td>10</td>
<td>0.3057</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0.1642</td>
</tr>
<tr>
<td>13</td>
<td>0.0257</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>0.1945</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>0.1293</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
</tr>
</tbody>
</table>

| Scheduled links | 12 | 7 | 2 |

Table 7.6. Distribution of the allocated power over a network for one snapshot given the SINR threshold (intersite distance = 1 km, DoF = 1)

Analysis of one snapshot

Figure 7.1 shows an example of a snapshot over a 19-base station network. Results of the solver were extracted in order to analyze them and are presented in Table 7.6. We observe that some receivers are never served no matter the value of the SINR threshold (cell 4, 6, 11, 14, 16, 18 and 19). All of them are located quite far from the base station but the one in cell 4. We can see that six of them are located close to or on the edge of their respective cell (cell 6, 11, 14, 16, 18 and 19). Serving them would obviously cause a lot of interference since they would need a lot of power to be allocated due to path loss. However, some receivers which are located quite far from their base station are served (depending on the SINR threshold). On the contrary, some receivers are always served (cell 2 and 13) since the configuration of the network and the snapshot does not change within the variation of the SINR threshold. We can see that for them, the allocated power is already very low with a low SINR threshold. In addition, for the last column (with SINR threshold = 10 dB), the allocated power is still very low for the active links. This is explained by the fact that given the formulation we have in Section 5.2.1, 5.2.2 or 5.2.3 and the distances of those mobile stations which are quite close. We

---

4Every time a ring is added, the first cell to be added is the upper north one and the next ones are added clockwisely.
Figure 7.9. Distribution of the allocated power over a network for one snapshot given the SINR threshold equal respectively to 1, 7 and 10 dB (intersite distance = 1 km, DoF = 1). Active cells i.e. links are in gray.
7.2 Results of solver

It can be noticed that in cell 4, despite the close location of the mobile station from the base station, the link is not active. This might be caused by an important shadowing and/or fading. Figure 7.9 permits to visualize the geographical distribution of active links.

7.2.2 Performance of solver

Like the simulation platform, the performance of the solver is an important matter. Users do not want to wait over and over every time the solver is launched. However, performance of the solver depends a lot on GLPK since the latter is used to solve the optimization problem. Unfortunately, GLPK does not support multi-threading. Indeed, it is not possible to call multiple instances of the solver, and a call cannot be interrupted which causes multi-threading not viable. The following figures show the performance of the solver relatively in term of duration of execution.\(^5\)

![Figure 7.10. Duration (in seconds) of the solver optimisation relatively to the number of base stations (number of DoF = 1, number of receivers per base station = 1, intersite distance = 1 km, number of snapshots = 100)](image)

Figure 7.10 shows that increasing the size of the network increases the duration of the optimization process exponentially (which was expected). Figure 7.11 shows that increasing the number of receivers per base station increases the duration of the optimization process. Moreover, this increase appears to be even more significant when the number of base stations is increased. Figure 7.12 shows

\(^5\)Duration is given for the execution of whole set of snapshots, not on average
Figure 7.11. Duration of the solver optimisation relatively to the number of receivers per base station (number of base stations = 7, number of DoF = 1, intersite distance = 1 km, number of snapshots = 100)

Figure 7.12. Duration of the solver optimisation relatively to the number of DoF (number of base stations = 7, number of receivers per base station = 1, intersite distance = 1 km, number of snapshots = 1000)
that increasing the number of DoF per base station increases the duration of the optimization process the most significantly.

Another interesting measure of performance is the distribution of the duration over a set of snapshots shown with the histogram of Figure 7.13. First, we can

Figure 7.13. Proportion of snapshots relatively to the duration of the solver optimization for 1 & 2 receivers per base station (number of DoF = 1, number of base stations = 19, intersite distance = 1 km, number of snapshots = 100)
see that the range of duration for solving a problem is quite wide even though we observe that a big proportion of these problems is optimized quite fast. The variation of duration for optimizing snapshots is due to the fact that the majority of the problems are easy (therefore fast) to optimize whereas a few need more time to be optimized.

Second, we see that increasing the number of receivers per base station increases the duration of solving each snapshots. Besides, we observe that increasing the number of receivers per base station actually increases the complexity of some problems. We see that the number of problems solved 'quickly' decreases as well.

Experiments were run on a simple laptop with 3.8 GB of memory and a dual-core processor $2 \times 1.30$ GHz.
Chapter 8

Discussion and future work

This chapter analyzes the simulation platform comparing with the initial expectations and the future work to be done in order to improve the accomplished work.

8.1 Discussion and conclusions

The simulation platform consists more of a set of classes and functions (like a library) than a real application. The main program provides just a simple access to the main functions of the simulation platform.

Overall, the simulation platform provides a tool which respects initial expectations (see Section 6.1):

- various parameters can be set for the cellular network (intersite distance, inner-zone radius, size), the snapshots (number of snapshots, number of receivers), the channel models (exponent, multi-antenna) and the solver (QoS, SINR threshold).

- the simulation platform is stable enough to generate up to 10000 simulations in less than 10 seconds.

- the program uses very few external C++ libraries. IT++ and GLPK are the only external libraries used. This choice is motivated by the fact that the program is about to be used by non-programmer user, so using external libraries may cause troubles if problems come from them.

However, some weaknesses may be pointed out:

- the designed program is not the most-user friendly since it is launched and run within the terminal. Thus, even if the simulation platform can be parametrized, doing it through a terminal can be tedious. That is the reason why a configuration file can be used instead.

- the program has been started from scratch. Therefore since the design has been made by only one person which has programming skills, there may be some flaws in it.
• the program uses few external libraries. However, the use of external libraries may increase performance of the simulation platform since this is one of the points we focused on.

• the resources of the machine the simulation platform runs on are not entirely used. It is not a problem for the generation of snapshots since it is rather fast, but it may have been useful for the solver given the performance of the solver (see Section 7.2.2). However, this has not been possible since GLPK does not support multi-threading yet.

• some aspects of channels have not been implemented or not entirely. First, beamforming (see Section 4.4.2) has not been implemented but the implementation gives a starting point for the use of smart antennas. One might need to read [39] to have good understanding of the topic and [49] to have a first approach of beamforming used with the SINR. Indeed, each base station has a given number of antennas. Second, multiple channels for one link are used but consists only in a parameter given by the user. It means that DoF generated consists just in uncorrelated vector of channel rather that correlated one (see Section 2.2.2). Those aspects should be treated in a further version of the simulation platform.

8.1.1 Conclusions

Overall, the work done during this thesis has been achieved regarding initial expectations. A simulation platform has been implemented and has proved to provide consistent data. Its use reveals to be quite simple and many parameters can be taken into consideration. In addition, performance is satisfying regarding the fact that GLPK does not allow multi-threading. Results can be produced in a satisfying amount of time. Furthermore, the use of widely used output format makes data produced by the simulation platform easy to reuse for later studies.

8.2 Future work

The simulation platform is far from being complete. Here are some suggestions for the future work to be made on it.

a Graphical User Interface as aforementioned, the program provided is not very user-friendly. An interesting improvement would be to add a Graphical User Interface for instance with the visualization of the cellular network and generated snapshots. First, the use of the simulation platform would be easier. It would provide a better way to analyze the position of the mobile stations and the base stations than having generated graphs with Gnuplot or GNU Octave. Second, it would lead to other improvements for the simulation platform such as moving dynamically mobile stations over the network or changing the parameters dynamically and observing the change. Different libraries exist: Qt, Gtk+, WxWidgets... However, it may be useful to point out that the implementation of a GUI may have an negative impact
on performance since GUI usually consume lots of resources when running. Running the solver is particularly resource consuming and in order to speed up the optimization process, resources should not be overloaded. A GUI which would be used after simulations are run would be suitable though.

different network topologies in our simulation platform, we only simulate cellular network as regular hexagonal cellular network. In reality, the topology depends on the environment (urban, rural). It could be interesting to have different topologies for the generation of the network or to be able to put the base station wherever we want (and then move them if we want to) even though hexagonal cellular networks are very common in simulation studies. In addition, it might be useful to have other ways of placing mobile stations since in urban environment in particular, the location of base station would depend on the location of buildings or other external factors (with different random distributions for example).

different size of cells in the network the network simulated by the simulation platform is only composed by one type of cell. Given the environment, cells can have various sizes. Moreover, some cells can be split up in smaller cells which is the case with micro-cells which may belong to (bigger) macro-cells.

overlapping network the simulation platform consists only in one network. Despite the notion of cluster implemented, there is no overlapping between them. The possibility to have overlapping network (like with different operators) would be an interesting feature.

sectorization sectorization was mentioned in Section 4.4.2 and could offer other possibilities for the simulation platform by splitting cell into sectors and reducing interference.

directional antennas directional antennas could be used for both sectorization and smart antennas.

pre-calculation with the generated data when a simulation is run, some receivers may obviously not meet the QoS constraint (channel gain too low). It would be interesting to discard them in the optimization process to find a better and faster solution. Thus, a pre-calculation on the generated data in order to do so would be needed before the solving process.

exceptions handling the simulation platform uses very few exceptions handling mechanisms. Some exception are handled with assertions mechanisms, but they don’t provide any way of troubleshooting. If inputs do not have the good format, the program crashes. A big improvement would be to add exceptions handling to the program.

design improvement since the simulation platform has been made from scratch, a work of analysis might be needed by someone from out of the box. In addition, a work of expertise might be needed. Indeed, it might help to have
Discussion and future work

more telecommunication skills in the design of the platform (in the same way as it has been done for the IT++ library) since I don’t have those skills (at least not at the same level). The design part should be a 2-people work.

**make the solver more flexible** the solver does not analyze any data. It might be interesting to have a solver which recognizes the variables (in our case they were $s[k]$ and $p[k]$) and provides a way of analyzing them.

**other optimizers** so far, only the GLPK library was used in order to optimize the MILP formulation. A suggestion would be to use another optimizer and see the results. It might be interesting to see if the results differ or if another solver optimizes the MILP formulation faster. CPLEX [17] is a commercial optimizer that might be worth to look at.

**utilize full resources** GLPK does not support multi-threading, the simulation platform is not a multi-threaded application. With another optimizer, it might be possible to use multi-threading therefore increase the performance.

**optimize the code** as aforementioned, external libraries have been used only when needed (IT++, GLPK). The use of external libraries may slightly improve performance though. Libraries such as Boost [5] provide C++ objects which may improve performance or make the code more generic. This is not an essential part but it could be interesting to have a better, cleaner and reusable code thanks to external libraries.

**other channel models** the simulation platform allows the user to choose different channel models. However, many studies about different channel models have been done and it might be interesting to have a large number of channel models in order to test channels on various environments. There are many path loss, shadowing and small-scale fading models which might be interesting to use. Many are given in [12, 28, 32].

This simulation platform might be a starting point for a bigger project heading up to a complete program to simulate cellular network and optimizes different kind of problems.
Bibliography


Appendix A

User Manual
A.1 Introduction

This document is the user manual of the simulation platform. It consists in describing the way the sources are compiled to have a proper executable (through a Makefile), the commands the simulation platform can execute and its behavior during the execution.

A.2 Compilation

The compilation is done with the *make* mechanism. The program is organized in modules and called through the main program. Each module has its own Makefile in order to compile the source files. Makefile of each module are called by a "Master" Makefile which is located in the same directory of the main program.

The command *make* compiles the whole program and produces the executable. Here are the different options:

- **all** equivalent to *make*
- **clean** remove all the files created by *make*
- **create-dir** create the bin directory
- **simulation_make** execute the sub Makefile of the simulator module
- **solver_make** execute the sub Makefile of the solver module
- **util_make** execute the sub Makefile of the util module

The compilation creates a *bin/* directory where the executable is stored. In order to execute it, one needs to move to this directory.

A.3 Execution

The basic execution of the program is simply done with:

```
./simulationplatform
```

like a usual program.

The first step of the program consists in configuring all the variables that will be used by the program such as the number of simulation. All the variables are needed at the beginning of the program to ease the execution later. That is why the use of a configuration file is more comfortable (see Section A.3.1). Variables which consist in path to file or directory should be set according to the location of the running executable.

Then, the program prompts a menu with different actions to perform. Here is a description of the entries of the menu:

1: run a complete simulation + optimization
2: **run a simulation and generate channels** build an entire cellular network and generate several snapshots. Then, generate channels and channel gains for each snapshot

3: **generate Gmpl data files from existing itpp data files** use the existing itpp data files which contains channel gains, and build Gmpl data files from them

4: **generate one Gmpl data file from one variable of an existing itpp data file** extract the channel gain of one variable from an existing itpp file and build the corresponding Gmpl data file

5: **run the solver on generated data files** run the solver over several data files and store the results in an itpp file

6: **run the solver on one generated data file** run the solver on only one data file and store the result in an itpp file

7: **reload the configuration file** reload the configuration file by replacing all the previous parameters by the new ones

8: **quit** quit the program

### A.3.1 Configuration File

Since entering all those parameters is tedious, it is possible to use a configuration file. The path of the configuration file should be given with the `-c` option as follow:

```
./simulationplatform -c path/to/configuration_file
```

With this configuration file, the program will not ask any questions for the different parameters.

### A.3.2 Menu Entry Option

It may be tedious as well to enter the entry of an action if one already knows what action to perform. The use of the `-e` option may be used to directly the action with the entry that follow the `-e` option. For example, the following command

```
./simulationplatform -e 1
```

will execute the entry 1 of the menu

### A.3.3 Read output IT++ files

The simulation platform output files in various formats: Gnuplot, Gmpl and IT++. The latter can be read by MATLAB or Gnu Octave but some commands are needed in order to load the IT++ module. In order to load a file, one has to enter the following command

```
p = path; path(p, "/usr/local/share/itpp");
```

Then one loads a file with the command `itload('path/to/file')`. 
Appendix B

GMPL Model file

param K;
param R;
param N;

set TRANS := {1..K};
set RCV := {1..R};
set CHAN := {1..N};

param e {TRANS, TRANS, RCV, CHAN};
param gamma {TRANS, RCV};
param pbar;
param eps;
param sigma;
param M;

param a {k in TRANS, l in TRANS, r in RCV, n in CHAN} :=
    if k = l then (e[k,l,r,n]*-1)/sigma else (e[k,l,r,n]*gamma[k, r]/sigma);
param b {k in TRANS, r in RCV, n in CHAN} := e[k,k,r,n]*gamma[k, r]/sigma;
param MS := M/sigma;

var s {k in TRANS, r in RCV, n in CHAN} >= 0, binary;
var p {k in TRANS, r in RCV, n in CHAN} >= 0, <= pbar;

maximize cost:
    sum {k in TRANS, r in RCV, n in CHAN} s[k, r, n] -
    sum {k in TRANS, r in RCV, n in CHAN} eps * p[k, r, n];

subject to c1 {k in TRANS, r in RCV, n in CHAN}:
\[ b[k,r,n]*\sum\{v \in RCV \text{ diff } r..r\} \ p[k,v,n] + \sum\{l \in \text{TRANS} \text{ diff } k..k, v \in RCV\} \ a[k,l,r,n]*p[l,v,n] + a[k,k,r,n]*p[k,r,n] + MS*s[k,r,n] \leq MS - \gamma[k,r]; \]

subject to c2 \{k in TRANS, r in RCV, n in CHAN\}: \[ p[k,r,n] - \bar{p} * s[k,r,n] \leq 0; \]

subject to c3 \{k in TRANS, n in CHAN\}: \[ \sum\{r \in RCV\} p[k,r,n] - \bar{p} \leq 0; \]

subject to c4 \{k in TRANS, r in RCV\}: \[ \sum\{n \in CHAN\} s[k,r,n] \leq 1; \]

end;
Appendix C

GMPL Data file

data;
param K := 7;
param R := 1;
param N := 1;

param eps := 0.142857;
param pbar := 1;
param M := 0.00297228;
param sigma := 1e−08;

param gamma :
   1 :=
   1 10
   2 10
   3 10
   4 10
   5 10
   6 10
   7 10;

param e :=
   [*,*,1,1]:
   1  2  3  4  5
   6  7 :=
   1  0.0002805 4.32811e−08 5.60984e−08 2.30374e−07 5.57489e−07
   e−07 4.33834e−06 1.14926e−05
   2  1.58774e−06 0.000281892 1.63165e−07 1.36857e−06 9.36065e−07
   e−07 1.34216e−06 9.29846e−07
   3  2.45256e−06 1.3003e−06 3.12114e−06 1.27763
end;

end;}