A CALCULUS LECTURE: WHAT MADE IT GOOD?

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This paper reports from a study that investigates, from a student and a teacher perspective, the fact that the large group lecture format prevails and attracts students in undergraduate mathematics education despite the many arguments put forward against the educational value of lectures. The focus here is on students’ views on specific aspects of one calculus lecture they just attended.

INTRODUCTION

In beginning undergraduate mathematics education large group lectures is common in many courses worldwide, being an efficient model of teaching as an outcome of the massification of higher education. Since undergraduate mathematics courses are highly standardised and often based on textbooks, the purpose of the lecture format is questionable. Indeed, doubts about its value have been raised frequently (Bligh, 1972; Fritze & Nordkvelle, 2003; Holton, 2001). Students might rely on texts and spend more time in interactive study groups, with the option to ask questions after studying the texts. Still, students go to lectures. To study what reasons students might have for attending lectures, a case study was conducted with two lecture groups following the same calculus course at a Swedish university. The study included student questionnaires (given directly after a lecture) and interviews, as well as video recordings of the lectures and interviews with the lecturers. In Bergsten (2011), the results from the part of the questionnaire asking students about their general views of the lecture format in undergraduate mathematics education were presented. In this paper these results will be compared to the outcome of the part of the questionnaire dealing with how the students viewed the particular lecture they had just attended. Such comparison may provide some general insights about what attracts students to go to lectures.

LECTURES IN UNDERGRADUATE MATHEMATICS

In undergraduate education several teaching formats are commonly used (often in parallel), serving different functions. Fritze and Nordkvelle (2003) identify three different functions of a lecture: (i) it reflects scientific truth, by ways of argumentation and reflection; (ii) it reflects educational decisions in order to make this scientific content accessible to students; and (iii) it works as an “organization activity” since students regard lectures in a course as “a part of a socialization scheme” (ibid., p. 232). The study reported in this paper was set up to provide information concerning the first two of these functions.

Literature on tertiary mathematics education often discuss the quality of teaching and learning of subject content matter (Bradley, Sampson, & Royal, 2006). In the general context of tertiary education, Biggs (2003, p. 75) lists four aspects of
teaching/learning that from the literature seem to support quality learning of content matter: (i) a well-structured curriculum; (ii) an appropriate motivational context; (iii) learner activity, including interaction with others; and (iv) self-monitoring. The low extent to which the third of these categories applies to large group lectures has been used as an argument for the low learning potential of lectures (Fritze & Nordkvelle, 2003; Bligh, 1972). However, what “learner activity” here consists of remains unspecified. The educational value of large group lectures has also been questioned for several other reasons: students’ attention cannot be maintained during a whole lecture (Bligh, 1972); lectures are often not understood by the students (Rodd, 2003, p. 15); students do not learn much from lectures (Leron & Dubinsky, 1995); lectures are not effective in stimulating higher-order thinking (Bligh, 1972); lectures do not provide feedback and social interaction (Bligh, 1972); lectures show only what sometimes is called the “front” of mathematics and hiding the “back”, as they most often are linearly well ordered outlines of a ready made mathematical theory, not offering a view of mathematics as a human social activity, including creativity, struggles, and emotional aspects involved in mathematical activity (Alsina, 2001; Weber, 2004).

Despite such research results on lecturing, “the lecture survives, probably because it serves many functions not so well observed in the present research” (Fritze & Nordkvelle, 2003, p. 328). In this line Rodd (2003) argues that “university mathematics departments recognise the potential of lectures, not as information-delivery venues, but as a place where the ‘awe and wonder’ of mathematics can be experienced” (p. 20), and claims that ‘active participation’ and ‘identity and community’ can also be experienced by just being a ‘witness’, similar to the act of experiencing a theatre play. An important outcome from a good lecture may be inspiration (e.g. Alsina, 2001, pp. 3-6). Thus, the lecturer as a person, and humour, have been judged critical for how lectures are appreciated (Fritze & Nordkvelle, 2003). For this aspect, the notion of teacher immediacy (Frymier, 1994) has been used, referring to issues of closeness in classroom student-teacher interaction. Arguments for the importance of such personalisation in a mathematics education context can be found in several studies (Anthony, 1997; Anthony, Hubbard, & Swedosh, 2000; Forgasz & Swedosh, 1997), and the lecturer can be seen to provide a personalisation of the formal mathematical discourse (Anthony et al.; 2000; Wood & Smith, 2004). Also Wood, Joyce, Petocz and Rodd (2007) found that despite availability of textbooks and online material, for their learning students value lectures higher.

Weber (2004) identified three teaching styles used in undergraduate mathematics (small group) lecturing. In the logico-structural style a strictly formal way of working was used. While the procedural lecture style had the main focus on the technical work, the semantic style emphasized the intuitive meanings of the concepts. In large group lectures these styles can be mixed (e.g. Bergsten, 2007), as well as the linguistic modes used. Wood and Smith (2004) noted that lecturing
is a “mixed mode activity” (p. 3), employing both verbal and non-verbal means to organise students’ attention to “written language, mathematical notations, visual diagrams” (p. 3). These authors also noted that the lecturer often uses words like actually, fairly, and obviously to “personalise and introduce values and judgments into the presentation” (p. 7). They conclude that all these differences of modes and representational forms require a lot from the students.

The study by Anthony (1997) showed that the importance given to lectures was higher by students than by lecturers, both for success and failure, including “boring presentations of lectures” and “non attendance of lectures.” Students also “placed more importance than lecturers on active learning and note-taking” during lectures. Successful students found “the availability of worked examples in lectures and tutorials” and “clear presentation of lectures” more important than did non-successful students (pp. 60-61). Moreover, the study by Hubbard (1997) showed that students value the information about what is “important” provided by lectures but are often dissatisfied with the format of a lecture as well as lecturers’ ability to teach. This may well be due to a discrepancy in beliefs and perceptions about the role of lectures between lecturers and students (Anthony et al., 2000, p. 250).

A CASE STUDY INVOLVING CALCULUS LECTURES

Method

Since lectures make up a substantial component of beginning calculus courses in many university engineering and science programmes, it was an natural choice to study lectures within this context. As these lectures often are given to large groups of students they have a significant impact on undergraduate mathematics education. The case studied consists of two experienced lecturers running a calculus course with students enrolled in a five year engineering programme, one group of 141 engineering students studying physics and electric engineering (Group 1), and one group of 132 students specializing in industrial economy (Group 2). It represents a common situation for the phenomenon in focus.

All empirical data were collected by the author, who videotaped the lectures (one with each of the groups), distributed and collected a questionnaire immediately after the lectures, and interviewed a sample of the students who attended the lectures. In this paper data from one part of the questionnaire will be reported, while other data have been presented in Bergsten (2011) and will be referred to.

The questionnaire

Bergsten (2007) identified critical aspects that might influence students' appreciation and attendance of lectures. These aspects provided the basis for the questionnaire used in the study. To enhance validity, the questions were discussed with a research colleague and tried out in a small pilot study before the questionnaire was finalised. It used four level Likert items and was organised in
three parts; Part 1: four opening items concerning attendance, reading and note-taking; Part 2: items 1-16 on views about lectures in general; Part 3: items 17-34 on views about the particular lecture just attended. The answers could be commented and a final open question was asked. The results from Part 1 and Part 2 were presented in Bergsten (2011) and are summarised below. In this paper the analysis will focus on Part 3 of the questionnaire (items 17-34).

In addition to answer frequencies to all items, a simple correlation analysis was carried out to further elaborate the interpretation of the data. Not only the answers to Part 3 of the questionnaire but also to Part 1 and Part 2 were analysed also for each of the two lecture groups. There was no significant difference between the groups on Part 1 or Part 2 concerning the frequency distributions, or pair wise item correlations for Part 2. This homogeneity may be interpreted as an indication of reliability, as the groups are comparable in terms of previous mathematics studies. Before the presentation of the items and the new results, the outcome from Part 1 and Part 2 of the questionnaire will be shortly discussed.

Previous results

In the study by Bergsten (2007) of first year engineering students, it was noted how the complexity and richness of several educational aspects of a lecture in undergraduate mathematics can live within one and the same single large group lecture. With a TPA format (theorem-proof-application), the observed lecture was content-driven and rich in information delivery; employed a mixed mode of semantic-procedural teaching styles; displayed a strong inner coherence, was strong in rigour; showed a dominance of an algebraic mode over an imagistic; despite a frequent use of gestures and informal language it established mathematical norms for the course as well as for mathematics in general; created a relaxed atmosphere of doing mathematics as it seems together; and employed different semiotic means to objectify the knowledge. Aiming to structure this complexity, Bergsten (2007, pp. 69-70) outlined a systemic triangular model for characteristics of a mathematics lecture found critical for quality: mathematical exposition, teacher immediacy, and general quality criteria for mathematics teaching. Mathematical exposition here involves the dynamic interplay of the dimensions mathematical content, mathematical process, and institutionalisation.

The data presented in Bergsten (2011) showed that students in this study attend lectures almost always not only because they value its usefulness for the examination (as also found by Hubbard, 1997) but also because they find it easier to understand the course content by attending lectures than by only studying the textbook, and what mathematical proofs are and why they are needed. The students also strongly emphasise the value of both intuitive explanations (by way of diagrams, metaphors, gestures) and formal presentations (definitions, proof, algebraic calculations), as well as of examples. The need of seeing applications from outside mathematics was much less emphasised. The data also suggested
that the semantic mode of presentation is more important to attract students to lectures than the procedural or the logico-structural style (cf. Weber, 2004), though clarity and coherence were both seen as important. There was a strong agreement among the students about the key role of teacher immediacy: the lecturer should provide inspiration to study mathematics and use humour, show the strength and beauty of mathematics, and use everyday language, gestures and diagrams to illustrate the concepts and methods. In relation to the triangular model for a quality lecture (Bergsten, 2007), the significance of teacher immediacy and mathematical exposition thus found strong support in the data, while the general quality criteria were more implicitly expressed in aspects seen as important for a good lecture, such as coherence, clarity, pace, as well as student activity in terms of taking notes.

That lectures indeed have an impact on the students was evidenced by the great variety of open descriptions given to characterize a good lecture. This again points to the complexity and “richness” of the lecture format (Bergsten, 2007). It was evident from this study (Bergsten, 2011) that one main reason for the “success” of a lecture is given, by the students, to the lecturer as a person, being able to engage and inspire the students. Such emphasis on teacher immediacy has been noted by several authors (cf. above). The possibility to make illustrations adds non-symbolic and non-discursive elements to the semiotic objectification of knowledge, all which can serve students’ meaning-making processes. Some of the other aspects mentioned by many students in the questionnaire, such as clarity and coherence, a mix of examples and theory, an optimal pace, clear language and writing, can all be related more to lecturers’ pedagogical awareness (Nardi, Jaworski, & Hegedus, 2005) than to their personal charisma.

Part 3 of the questionnaire

All items of Part 3 are listed in Appendix. In Part 2, some of the items were designed to address the overall question Why do students go to lectures? more or less explicitly, and some more implicitly by asking on some specific issues that should be provided by lectures, or by asking if some aspects count as important for mathematics lectures. Part 3 included similar kinds of items but instead of addressing students’ views of mathematics lectures in general, the statements were referring to the particular lecture they had just attended. Some of the items in Part 3 were direct follow ups of items in Part 2 (cf. Bergsten, 2011).

Results

The results from items 17-34 of the questionnaire are displayed in Table 1. On most of the items the students in both groups fully agree or agree, while only four of the items (17, 18, 25 and 32) had a majority of responses in the categories partly agree or disagree. The response pattern is also very similar for the two groups on almost all of the items. Due to space limitations, I will comment on some of the items with reference to the observations of the video recording of
only one of the lectures, the one given for Group 2. This lecture presented some basic theorems about how the derivative can be used to analyse the behaviour of a function in terms of local extreme values.

**Responses to specific items**

Table 1 shows the frequencies of the response categories of the Part 3 items:

Table 1. Items 17-34 frequencies (%), mean (m) and number of responses (N); in each cell Group 1 = upper number and Group 2 = lower number.

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Some comments will here be given to each of these items:
17 - Much of the content presented in this lecture was well known from upper secondary school, though a formal outline of the theory is normally not included at school level.

18 and 21 - No extra-mathematical applications were observed in the video recorded lecture, though the introductory intuitive example may have given some students a sense of "application". This lecture was, by its introductory character, dominated by definitions and proofs, with only a few examples. A small majority of the students (54%) would have wanted more examples and applications.

19, 22 and 30 - The video clearly showed the links made between the different parts of the lecture as the students so strongly noted in their answer to item 19, where the lecturer's intuitively outlined introductions to the formal treatment, and the naturally floating language and pointing to relevant parts of the formulas and figures while presenting the definitions and proofs, may have contributed to that it was easy to follow and see structure. Also the short breaks often made after stating something (for students to reflect) may have mattered.

20 - References to previous lectures were made occasionally by words such as "you surely remember".

23 and 29 - During the proofs no "jumps" could be observed and the key ideas in the proofs (cf. Raman, 2003) were explicitly pointed out, explaining the students' opinions on these items.

24 - As examples of everyday language the metaphor "the top of this hill" was used to describe a local maximum on a graph, and the expression "alone ruling the roost" when talking about a strict local extreme value. Another example is the expression "a nasty corner here, it [the function] is not differentiable yet has a minimum". The lecturer also used gestures, such as hand movements in the air when saying that "an increasing function can be constant if you like but a strictly increasing function must increase all the time", or making an opening gesture with the arms to extend the whole number line when saying "all values".

25 and 26 - The video showed that there was extensive formalism employed in the lecture but algebraic calculations were kept at a very elementary level. As an additional explanation to the outcome, it was observed that things were generally first presented in words before writing down the algebraic formalism.

27 and 28 - No explicit comments from the lecturer on the issues formulated in these two items could be noted in the video, so the students may have experienced intuitively, possibly related to the clarity and coherence of the presentation, that what was explained in the lecture was important to learn.

31 - The clear agreement on this statement is difficult to relate to specific aspects of the presentation but may be explained by the teacher immediacy dimension.
32 - The response to this item may be explained by the fact that some of the content of the lecture was known through the upper secondary studies and that the results and examples were not very "surprising".

33 - The strong agreement on this item, though there was not much "jokes" with students laughing, may be explained by the teacher immediacy factor.

34 - The students' comments on this item varied, and the agreement will be discussed below in relation to the other items (correlations).

A matrix of the inter item correlations for both groups shows (not included here due to space) that many of the items were pair wise strongly correlated, similarly for both groups. Most notably, item 18 (on applications) is uncorrelated to almost all the other items, including item 34. Also in the students' views on mathematics lectures in general the importance of showing applications was not emphasised. Items 27, 28 and 31, and 32 for Group 1, display some low inter item correlations. Item 31, however, is one of the items that correlates most strongly with item 34. In fact, for Group 2 most of the items 17-33 show a correlation or a strong correlation to item 34, with more cases of low correlation for Group 1.

The observed positive correlations between the corresponding items in Part 2 and Part 3 of the questionnaire generally confirmed that the particular lectures studied here included many of those elements that students value, as expressed in Part 2 of the questionnaire.

CONCLUSION

The data in this and the previous study (Bergsten, 2011) have confirmed how the complexity and richness of different educational aspects present in the mathematics lecture as a teaching format all contribute to its "success" from the students' point of view, by way of a working balance between mathematical exposition, teacher immediacy and general quality criteria for teaching.

REFERENCES


**APPENDIX: STATEMENTS IN PART 3 OF THE QUESTIONNAIRE**

17. There was too much new material in this lecture
18. The lecture made links between the purely mathematical results and applications outside of mathematics
19. The different parts of the lecture were well connected
20. In the lecture references were made to results and methods from previous lectures
21. The lecture was too much about definitions and proofs instead of examples and applications
22. It was easy to follow the whole lecture
23. I felt that I understood the important steps of the proofs that were presented
24. The lecturer used everyday language, gestures and diagrams to illustrate the mathematical concepts and methods used
25. The lecture employed too much algebraic calculations
26. I understood how the algebraic calculations could be linked to the images and diagrams that were used
27. The lecturer made it clear what is important to know of what was presented
28. The lecturer made it clear what is required from a solution for it to be mathematically correct
29. It was easy to see how and on which ideas the proofs were built and how they began and ended
30. The lecturer gave a clear structure to the content and how different parts are connected
31. The lecture made me inspired to continue working with mathematics
32. The lecture made the strength and beauty of mathematics visible
33. The lecturer created a good atmosphere during the lecture
34. I got a lot out of this lecture (shortly explain why below)