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Capacity and Pricing Policies with Consumer Overflow Behavior

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Abstract

We analyze consumers' choice and overflow behavior between two potential market segments with different fares, capacity allocated, and then develop the optimal capacity and pricing polices assuming such consumers' strategic behavior can be observed. Every consumer prefers to choose a fare to obtain their utmost value surplus, and select the second if the first-best choice cannot be satisfied. Our study indicates that the effort of fencing the segments should be considered to cope properly with pricing and capacity decisions in order to direct the overflows. Disregarding overflows could create differences in decisions as well as economic consequences. The study results can be implemented, but not limited, to understand the flight seat allocation problem with strategic consumers.

Keywords: Flight seat allocation problem; Consumer overflow; Strategic choice; Market segmentation

1 Introduction

1.1 Research background

Revenue management started to gain significant research attention after the airline's deregulation on price control in USA in 1978 (Talluri and Van Ryzin, 2004, pp. 6-7). The seat allocation problem in a flight leg is an important issue in revenue management literature. So called "see-so-move" is a major problem during the early time of revenue management study. Market segmentation is then introduced as a technique which is followed by various models with an aim to price independently to different consumer groups and consequently maximize revenues. Later, the revenue management techniques were introduced to other industrial sectors such as restaurant, hotel and manufacturing to deal with problems for example booking limit, overbooking, pricing policy, auction, reselling, and others.

Understanding and describing consumers' demand behavior are important issues in revenue management. In literature, models often assume that consumers are already segmented and follow certain types of distributions. With regards to the economy and business class consumers, the assumptions of pre-defined and independent distributions are very appropriate in dealing with classic pricing and seat allocation problems. However, with appearing of many low-fare airlines such as Ryanair and EasyJet, the above assumptions need to be reinvestigated due to two reasons. First, such airline often aims at economy consumers who also likely have the same or similar value preference. Second, consumers also have strategic behavior, i.e. actively determining which product to buy and when to buy etc, based on the conditions and restrictions added by flight seat reservation and purchasing. Thus there are questions whether such airlines should enforce clear market segments for the distinguished products (services), and what the response of consumers will be? Such strategic consumers have been less investigated in revenue management literature (Shen and Su 2007). Further investigating consumers' choice behavior is thus important and it will definitely influence the revenue management decisions.

In this paper we therefore investigate the flight seat allocation problem in a single leg with "potentially" two market segments, in which consumers are willing to pay high and low fare tickets. Instead of assuming the demand of two classes are independent and exogenously defined (such as in Netessine and Shumsky, 2005), we let the consumers come from the same pool with a known distribution indicating their values. In addition, consumers have the opportunity to select seat classes based on ticket fares, perceived seat value and seat availability. The consumers' overflow pattern is analyzed with consideration of an airline's market segmentation policy. Furthermore, pricing and seat allocation policies are developed to understand the decisions in a flight leg including consumers' strategic behavior.

1.2 Related Literature

There are extensive studies focusing on how airlines can apply pricing and rationing policies to extract maximum revenue. Even before the airline's deregulation, Littlewood (1972) first describes revenue management principle in the airline industry, and afterwards numerous authors have expanded on Littlewood's work. For an overview on revenue management up to 1999 we refer to McGill and Van Ryzin (1999) whereas developments occurring afterwards are discussed in Hu, Caldentey and Vulcano (2010), Board and Skrzypacz (2011). For a comprehensive instruction of the methodology in revenue management, we refer to the book by Talluri and Van

Ryzin (2004).

The flight seat allocation problem in legs can be divided into static and dynamic models. In a static model, the booking period is regarded as a single interval. The tasks are setting a booking limit for every booking class at the start of the booking process and then making price decisions. Furthermore, the static models can be categorized into two types. The first type assumes that the distribution of the demand for different fare classes is known in advance. With an aim to maximize the expected revenue, this type of problem is often formulated as mathematical programming models (Haerian et al., 2006), or competitive game theoretic models (Netessine and Shumsky, 2005). The second type assumes that decision maker knows part of consumer behavior and demand information, and then it is solved by the dynamic programming approach, where the stages correspond to fare classes (Wright et al. 2010.).

In this paper, we carefully focus on consumers' overflow behavior and an airline’s pricing and booking policies in a static model. This paper connects three important streams of literature in revenue management, namely, consumers' valuation modeling, consumers' overflowing behavior, and an airline's pricing and capacity decisions. The literature review below will focus on these streams.

Some authors study revenue management with consideration of consumers' valuation. Shen and Su (2007) review exiting models, which consider customer behavior in revenue management. Dana et al. (2011) present a model of revenue management with strategic behavior, i.e. forward looking consumers. The consumers are heterogeneous in their valuations or willingness to pay. Using a mechanism design approach, the authors show that the optimal is a menu of expiring refund contracts. The authors also identify the conditions under which the manager can achieve the first-best solution, thereby extract the entire consumer surplus. With this optimal mechanism, contracting takes place after the consumers learn their types but before they learn their true valuations. Levin et al. (2008) present a pricing model for oligopolistic firms selling differentiated perishable goods to multiple finite segments of strategic consumers. They encompass strategic behavior of both firms and consumers into a unified stochastic dynamic game. The model provides insights about equilibrium price dynamics at different levels of competition, and multiple market segments with different properties. Ahmed and Abdelghany (2007) adopt a micro-simulation approach that replicates how prospective travelers select their travel itineraries that are provided through ticket distribution channels, and examine the trade-offs between two common types of ticket distribution channels: (i) one with high market penetration and high competition among subscribed flight legs and (ii) one with low market penetration and low flight legs competition. In the above literatures, the market segments are often clearly predefined. Consumer demand in the segments follows independent, externally defined distributions. Such assumptions are not true when we study low cost flight-legs (Ryanair and EasyJet), in which, consumers are likely coming from the same pool with similar valuation preference.

In the research domain of consumers' overflow behavior, Dumas and Soumis (2008) provide consumers flow estimation with given forecast data concerning (i) the demand distribution for each itinerary; (ii) the time distribution of booking requests for each itinerary; and (iii) the proportion of spill (from an itinerary) that is attracted to a given alternative itinerary. Zhang and

Bell (2010) present an approach to model demand leakage among different market segments and propose cost functions representing the effort devoted to fences. After establishing the connection between costs/revenue gains with market segmentation, they show how the optimal cost should be devoted to customer migration across segments. Little attention, however, has been given to show how consumers overflow behavior affects an airline's pricing and capacity decisions.

Another group of authors study revenue management considering airlines' pricing and capacity decisions. Jean (2009) discusses two nesting methods: net nesting and threshold nesting, and investigates the underlying assumptions. The findings indicate whether or not having stationary demand process is a key issue in such a problem, and an event study methodology has been suggested to reach the appropriate assumptions in practice. You (1999) considers a seat inventory control problem with multiple booking classes in both single- and multi- flight leg cases. Before the flight departure, the airline may face typical problems such as (i) what are the suitable prices for the opened booking classes, and (ii) when to close those opened booking classes. Chew et al. (2008) jointly determine the price and the inventory allocation for a perishable product with a predetermined lifetime and they develop a discrete time dynamic programming model to obtain the optimal prices. Dai et al. (2005) consider the pricing strategies of multiple firms providing the same service in competition for a common pool of customers in a revenue management context, each of which satisfies demand up to a given capacity limit.

As mentioned before, in this paper we aim at investigating consumers' overflow behavior. Instead of assuming consumers have already been segmented, we consider the case that consumers have similar demand pattern, however their tickets' selection depends on the airline’s effort of fencing the segments. This investigation should bring some insights about the interaction of consumers' choice behavior and an airline's seat allocation polices, such as pricing, capacity reservation and segment effort.

2 Problem Setting

In this section, we describe the background of problem settings. We describe the passenger types, present the decision of consumer's choice and then develop the expressions for demand function (volume).

2.1 Passenger Types

In the early studies, market segments are often assumed to be pre-defined and having independent distributions. These assumptions are true when we consider economy and business classes in commercial airlines, in particular with long distance flights. However, the low-fare airlines such as Ryanair and EasyJet, are becoming popular in practice. Among these airlines, consumers are more likely having the same or similar value preference. In this study, thus we assume that consumers have different values when buying the flight tickets, however this heterogeneity of consumers' value is uniformly distributed within a unit-length line $v \in U[0,1]$. We also normalize the number of consumers as one. Thus the maximum demand volume (market size) is one. We have to note this normalization does not change the conclusion of this paper, it only brings convenience in model development.

The above mentioned consumers' value refers to a normal booking condition of a flight leg. It can be interpreted as the consumers' utility of transferring the locations. If a consumer is offered a flight ticket with restrictive booking conditions or service limits, its value is discounted and then becomes θv. Hence we define the tickets with the normal booking condition as high fare, whereas the restrictive one as low fare. The value of θ can be interpreted in two ways: from consumers' aspect, it is considered as value depreciation; whereas from an airline's aspect, it is considered as an effort for differing the market segments, i.e. an airline provides different booking conditions (time restriction, cancellation policy) and services (food quality, seat space) for two fares. The above linear discount assumption for value depreciation could be over-simplified from practitioner's viewpoint, nevertheless we have to note that it links an airline's policy and consumers' response and thus provides an opportunity for further investigating consumer behavior. The value of θ could also be considered as a decision variable of an airline.

The following notations are used

C: flight's seat capacity;

B: seat capacity of low fare class;

$C-B$: seat capacity of high fare class;

v: value for buying high fare ticket;

θ: the customer value depreciation when receiving low fare ticket, or an airline's segment effort, $0 \le \theta \le 1$;

p_H: price for high fare ticket;

p_L: price for low fare ticket.

Since the consumers' value is assumed to be distributed within the interval [0, 1], the corresponding maximum price is one. Otherwise, there will be no consumer having positive surplus and the demand will be zero. Therefore the two prices p_H and p_L should also fall into the interval [0, 1]. Moreover, a flight leg can apply two different policies: first deciding capacity and booking limit, then pricing, which hereafter is called capacity-price policy, or alternatively opposite sequence called price-capacity policy. However, the two policies end with the same conclusion according to our study, thus we illustrate in this paper only the results of the price-capacity policy.

2.2 Consumer's Choice

A consumer buys a ticket only if its surplus is positive. If the consumer selects a high fare ticket at a price p_H and his value of the ticket is v, the resulting consumer surplus is $v - p_H$, correspondingly, a low fare ticket results $\theta v - p_L$. In case both fares contribute to positive surpluses, the consumer revolves the comparison of values of $(v - p_H)$ and $(\theta v - p_L)$: the consumer's first choice is the high fare and the second choice is the low fare if $v - p_H > \theta v - p_L > 0$, and vice verse. Because the consumer’s valuation is uniformly distributed, the demand of low and high fares corresponds to a piecewise-linear function, which depends on the interval of θ value. Since we have total number of consumers equals unit, the above demand volume can also be interpreted as the probability of purchasing for single consumer. For different intervals of the θ value, the consumer choice behavior and demand volume are developed and illustrated in Table1. For instance referring to the table, in the column with θ value at the interval $[p_L, \frac{p_L}{p_H})$, no consumer will choose only low fare, or with first choice of low fare ticket and then second choice high fare; whereas the demand volume for first choice of low fare ticket/second choice of high fare is $1 - \frac{p_L}{\theta}$, and that for only high fare $\frac{p_L}{\theta} - p_H$.

Table 1 indicates that, with a low value of θ (or alternatively, the airline has a great effort to segment the two fare classes), low fare seat leaves no surplus value and thus the demand occurs only for high fare (the first θ interval in Table 1). On the other hand, when the θ value is high (the airline provides less segment effort), a consumer prefers low fare seat (since its value has not been depreciated very much), whereas the consumer still consider the high fare when there is a surplus value and when low fare ticket is not available. However, in this case high fare will never be considered as a first choice (the last θ interval in Table 1). The proof of Table 1 is given in Appendix A.1.

Table 1. Consumer's choice behavior and demand volume with respect to different θ values, n/a=not applicable

Interval of $\theta \rightarrow$	$[0, p_L)$	$[p_L, \frac{p_L}{p_H})$	$[\frac{p_L}{p_H}, 1-p_H+p_L)$	$[1-p_H+p_L, 1]$
Choice alternatives ↓	**Demand volume ↓**			
Only low fare	n/a	n/a	$p_H - \frac{p_L}{\theta}$	$p_H - \frac{p_L}{\theta}$
First low fare-second high fare	n/a	n/a	$\frac{p_H - p_L}{1-\theta} - p_H$	$1-p_H$
First high fare-second low fare	n/a	$1-\frac{p_L}{\theta}$	$1-\frac{p_H - p_L}{1-\theta}$	n/a
Only high fare	$1-p_H$	$\frac{p_L}{\theta} - p_H$	n/a	n/a

According to Table 1, Corollaries 1 and 2 are obtained.

Corollary 1: The consumer demand is $Max\left\{(1-\frac{p_L}{\theta}),(1-p_H)\right\}$.

From Table 1, we can summarize the value in each column and easily see demand of consumers' buying one type of fares is $1-\frac{p_L}{\theta}$ if $\frac{p_L}{p_H}\le\theta\le 1$, and $1-p_H$ if $0\le\theta\le\frac{p_L}{p_H}$. Combining the two expressions together, we obtain the above corollary, which indicates that the airline's decision variables, p_L, p_H and θ should define the consumer's choice behavior.

From Table 1, we also note that a proportion of consumers have choice preference, i.e. taking the low fare as the first choices and high fare as the second one, or vice verse. This illustrates the consumer's overflow behavior in case there is no capacity left for the first choice. Thus we define here the overflow parameter as the ratio between the second choice demand to the sum of the first and second choice demand.

Corollary 2:

(i) The proportions of overflow from low fare class to high fare class are

$\lambda_1=\frac{1-p_H}{1-\frac{p_L}{\theta}}$, if $1-p_H+p_L\le\theta\le 1$;

$\lambda_2=\frac{\frac{p_H-p_L}{1-\theta}-p_H}{\frac{p_H-p_L}{1-\theta}-\frac{p_L}{\theta}}=\theta$, if $\frac{p_L}{p_H}\le\theta<1-p_H+p_L$;

$\lambda_3=0$, if $p_L\le\theta<\frac{p_L}{p_H}$;

$\lambda_4=0$, if $0\le\theta<p_L$.

(ii)The proportions of overflow from high fare class to low fare class are

$\gamma_1=0$, if $1-p_H+p_L\le\theta\le 1$;

$\gamma_2 = 1$, if $\frac{p_L}{p_H} \le \theta < 1 - p_H + p_L$;

$\gamma_3 = \frac{1 - \frac{p_L}{\theta}}{1 - p_H}$, if $p_L \le \theta < \frac{p_L}{p_H}$;

$\gamma_4 = 0$, if $0 \le \theta < p_L$.

Based on the consumer's choice behavior in Table 1 and overflow expressions in Corollary 2, we further develop the revenue function with given booking limit, capacity and prices,

$$\pi = \begin{cases} p_L Min[t_1, B] + p_H Min\left[\lambda_1 [t_1 - B]^+, C - B\right], \text{ if } 1 - p_H + p_L \le \theta \le 1 \\ p_L Min[t_2 + \gamma_2 [t_3 - C + B]^+, B] + p_H Min\left[t_3 + \lambda_2 [t_2 - B]^+, C - B\right], \text{if } \frac{p_L}{p_H} \le \theta < 1 - p_H + p_L \\ p_L Min[\gamma_3 [t_4 - C + B]^+, B] + p_H Min\left[t_4, C - B\right], \text{ if } p_L \le \theta < \frac{p_L}{p_H} \\ p_H Min\left[t_4, C - B\right], \text{ if } 0 \le \theta < p_L \end{cases} \tag{1}$$

in which $t_1 = 1 - \frac{p_L}{\theta}$, $t_2 = \frac{p_H - p_L}{1 - \theta} - \frac{p_L}{\theta}$, $t_3 = 1 - \frac{p_H - p_L}{1 - \theta}$, $t_4 = 1 - p_H$.

3 Price-Capacity Policy

In this section, we present the optimal decision, i. e. the airline has to decide pricing strategy first, and then capacity and booking limit. Eventually as we have investigated, the airline’s optimal decisions are the same from the capacity-price sequence.

3.1Capacity Decisions

First we investigate the capacity decision when prices are given. The following proposition is obtained.

Proposition 1: Given p_H and p_L the airline's optimal capacity decisions are

(i) $B = C = 1 - \frac{p_L}{\theta}$, $\pi = p_L(1 - \frac{p_L}{\theta})$ if $1 - p_H + p_L \le \theta \le 1$;

(ii) $B=0$, $C=1-p_H$, $\pi=p_H\left(1-p_H\right)$ if $0\le\theta<1-p_H+p_L$.

Proof is in Appendix A.2.

Proposition 1 shows that the optimal capacity decisions are of a bang-bang character with given p_H and p_L: either open the seats for all low fare by setting *B*= *C*, or have only high fare by setting *B*= 0. The intuitions of Proposition 1 are: (i) When the price level guarantees gains from selling high fare ticket, the airline makes those consumers with positive net value choose low fare tickets by setting $C-B=0$ to prevent consumer's overflow behavior. This can be realized by using a larger θ (i.e. less segment effort); (ii) Alternatively, the airline makes those consumer with positive net value choose high fare tickets by setting $B=0$. This is with a small value θ (high segment effort), or when the revenue is less in selling a low fare ticket than the high one. However, in this case the airline would loss consumers who only choose low fare tickets but gain consumers who have the first choice as low fare but overflow to the high fare. The first scenario explains the popularity of cheap flight legs. Consumers are fairly equally treated, even though there are some minor segment effort such as fast lane choice, extra luggage carriage.

3.2 Pricing Decisions

Now we investigate the pricing policy and develop the following proposition.

Proposition 2: Based on the intervals of θ corresponding to the ones in Proposition 1, the airline's optimal decisions are $B^*=0$, $C^*=\dfrac{1}{2}$, $p_H^*=\dfrac{1}{2}$ and $\dfrac{1}{2}>p_L^*>\theta-\dfrac{1}{2}$, the optimal profit is $\pi^*=\dfrac{1}{4}$.

Proof is in Appendix A.3.

First, we should note that the above optimal values of decisions and revenue are independent on

θ. As long as the low price p_L is less than 1/2 and larger than $\theta-\frac{1}{2}$, the above optimal profit will always be obtained. However, the setting of the lower price p_L and θ should direct the overflows, as shown in the following corollary.

According to the International Air Transport Association (IATA) , one airline industry trade group, the median fare ratio (p_H / p_L) among all 1500 markets is p_H / p_L =2.6, for more than 90% of the markets and this ratio has a value ranging from 1.3 to 4 usually (Netessine and Shumsky, 2005). Therefore Proposition 3 is obtained by assuming $p_H / p_L = a$.

Proposition 3: Given $p_H / p_L = a$ the airline's optimal revenues and decisions are

(i) $p_L^* = \frac{1}{2a}$, $B^* = 0$, $C^* = \frac{1}{2}$, $\pi^* = \frac{1}{4}$ if $0 \le \theta < \frac{a+1}{2a}$;

(ii) $p_L^* = \frac{1-\theta}{a-1}$, $B^* = 0$, $C^* = \frac{a\theta - 1}{a-1}$, $\pi^* = \frac{(a\theta-1)(1-\theta)a}{(a-1)^2\theta}$ if

$\frac{a+1}{2a} \le \theta \le \frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2}$;

(iii) $p_L^* = \frac{\theta}{2}$, $B^* = C^* = \frac{1}{2}$, $\pi^* = \frac{\theta}{4}$ if $\frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2} \le \theta \le 1$.

Proof of Proposition 3 is in Appendix A.4.

This proposition has the similar interpretation as Proposition 2. However, it provides an alternative way to examine the airline's policy based on the ratio of (p_H / p_L), which can be easily observed in practice.

Given the airlines optimal decisions in propositions 2 and 3 the consumer overflowing behavior is analyzed in Corollary 3.

Corollary 3: In the case of the airline's optimal choice, the behavior of consumer overflow is of the following character:

<u>Case one - without the constraint of $p_H / p_L = a$:</u>

(i)The volume of consumer's overflow from low fare class to high fare class is $\frac{\theta - 2p_L}{2(1-\theta)}$, if $\theta - \frac{1}{2} < p_L < \frac{\theta}{2}$.

<u>Case two - with the constraint of $p_H / p_L = a$:</u>

(i)The volume of consumer's overflow from low fare class to high fare class is $\frac{\theta - 2p_L}{2(1-\theta)}$, if $\frac{1}{a} \le \theta < \frac{a+1}{2a}$;

(ii)The volume of consumer's overflow from low fare class to high fare class is $\frac{a\theta - 1}{a-1}$, if $\frac{a+1}{2a} \le \theta < \frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2}$;

(iii)The volume of consumer's overflow from high fare class to low fare class is $\frac{1}{2}$ at the interval of $\frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2} \le \theta < \frac{1}{a}$ and $\frac{1-a}{2}$ if $\frac{1}{a} \le \theta < \frac{2}{a+1}$.

Proof of Corollary 3 is in Appendix A.5.

In the capacity-price policy, first, the airline decides the pricing policy based on the information of booking limit and the capacity; then the airline sets *B* and *C* to maximize its revenue by given the corresponding pricing decisions. The whole inference and reasoning processes are similar with those in the price-capacity policy. Finally, we get that the airline's optimal decisions in capacity-price policy are same with the optimal decisions in price-capacity policy.

4 Model Extension - Without Considering Overflows

In this section, we extend our model by investigating the case of ignoring the consumer's overflow behavior. Our intention is to see whether ignoring this behavior would, and to which extent will influence the airline's revenue.

The airline controls the booking limit, the capacity and the prices. Given the demand functions in Table 1, the airline's revenue function without considering overflow is,

$$\pi = \begin{cases} p_L Min[t_1, B] & \text{if } 1 - p_H + p_L \le \theta \le 1 \\ p_L Min[t_2, B] + p_H Min\left[t_3, C - B\right] & \text{if } \dfrac{p_L}{p_H} \le \theta < 1 - p_H + p_L \\ p_H Min\left[t_4, C - B\right] & \text{if } 0 \le \theta < \dfrac{p_L}{p_H} \end{cases}$$

Proposition 4: Given p_H and p_L and without considering overflows, the airline's optimal capacity decisions are

(i) $B = C = 1 - \dfrac{p_L}{\theta}$ if $1 - p_H + p_L \le \theta \le 1$;

(ii) $B = \dfrac{p_H - p_L}{1-\theta} - \dfrac{p_L}{\theta}$ and $C = 1 - \dfrac{p_L}{\theta}$ if $\dfrac{p_L}{p_H} \le \theta < 1 - p_H + p_L$;

(iii) $B = 0$ and $C = 1 - p_H$ if $0 \le \theta < \dfrac{p_L}{p_H}$.

Consequently, the optimal pricing decisions are

(i) $p_L = \dfrac{\theta}{2}$ if $1 - p_H \le \dfrac{\theta}{2}$;

(ii) $p_H = \dfrac{1}{2}$ if $0 \le \theta \le 2 p_L$.

Therefore Corollary 4 is obtained by assuming $p_H / p_L = a$.

Corollary 4: The optimal pricing decisions by assuming $p_H / p_L = a$ without considering overflows are

(i) $p_H^* = \dfrac{1}{2}$, $B^* = 0$ and $C^* = \dfrac{1}{2}$ if $0 \le \theta < \dfrac{1}{a}$;

(ii) $p_L^* = \dfrac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$, $B^* = \dfrac{a\left(a\theta-1\right)}{2\left(1-\theta+\theta(a-1)^2\right)}$ and $C^* = \dfrac{\left(1-\theta\right)\left(2-a\right)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$ if $\dfrac{1}{a} \le \theta < \dfrac{a+1}{2a}$;

(iii) $p_L^* = \dfrac{\theta}{2}$ and $B^* = C^* = \dfrac{1}{2}$ if $\dfrac{a+1}{2a} \le \theta \le 1$.

Proof is in Appendix A.6.

Given the optimal pricing decisions the corresponding profit functions can be obtained after the demand uncertainty is realized.

Proposition 5: without considering overflows

(i)at the interval $0 \le \theta < \dfrac{1}{a}$ the optimal profit is $\pi^* = \dfrac{1}{4}$;

(ii)at the interval of $\dfrac{1}{a} \le \theta < \dfrac{a+1}{2a}$

Case one: if $1 < a \le 1+\sqrt{2}$ the optimal profit is

$$\pi^* = \frac{(1-\theta)\theta\left(3a-a^2+\theta a^3-2a^2\theta-1\right)a^2}{4\left(1-\theta+\theta(a-1)^2\right)^2}$$

Case two: if $1+\sqrt{2} < a < 3$ the optimal profits are

$$\pi^* = \begin{cases} \dfrac{(1-\theta)\theta\left(3a-a^2+\theta a^3-2a^2\theta-1\right)a^2}{4\left(1-\theta+\theta(a-1)^2\right)^2} & \dfrac{2}{3a-a^2} \le \theta \le \dfrac{a+1}{2a} \\ \dfrac{(1-\theta)\theta\left(2-\left(4-6\theta\right)a+\left(4+2\theta\right)a^2-\left(1+3\theta\right)a^3+\theta a^4\right)a}{4\left(1-\theta+\theta(a-1)^2\right)^2} & \dfrac{1}{a} \le \theta < \dfrac{2}{3a-a^2} \end{cases}$$

Case three: if $3 \le a$ the optimal profit is

$$\pi^* = \frac{(1-\theta)\theta\left(2-\left(4-6\theta\right)a+\left(4+2\theta\right)a^2-\left(1+3\theta\right)a^3+\theta a^4\right)a}{4\left(1-\theta+\theta(a-1)^2\right)^2}$$

(iii) at the interval of $\dfrac{a+1}{2a} \le \theta \le 1$ the optimal profit is $\pi^* = \dfrac{\theta}{4}$.

Proof is in Appendix A.7.

Comparing the above results with Proposition 3 in which overflows are considered, we obtain the following table.

Table 2 Comparing the optimal decisions in cases with and without considering overflows with respect to the given value $a = p_H / p_L$ and $1+\sqrt{2} < a < 3$

Interval of θ	$[0,\frac{1}{a}]$	$(\frac{1}{a},\frac{a+1}{2a}]$	$(\frac{a+1}{2a},\Delta_a]$ *	$(\Delta_a,1]$
Decisions with overflow	$p_L^*=\frac{1}{2a}$ $B^*=0$ $C^*=\frac{1}{2}$		$p_L^*=\frac{1-\theta}{a-1}$ $B^*=0$ $C^*=\frac{a\theta-1}{a-1}$	$p_L^*=\frac{\theta}{2}$ $B^*=C^*=\frac{1}{2}$
Decisions without overflow	$p_H^*=\frac{1}{2}$ $B^*=0$ $C^*=\frac{1}{2}$	$p_L^*=\frac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$ $B^*=\frac{a\left(a\theta-1\right)}{2\left(1-\theta+\theta(a-1)^2\right)}$ $C^*=\frac{(1-\theta)\left(2-a\right)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$	$p_L^*=\frac{\theta}{2}$ $B^*=C^*=\frac{1}{2}$	
Revenue with overflow	$\pi=\frac{1}{4}$		$\pi^*=\frac{\left(a\theta-1\right)\left(1-\theta\right)a}{\left(a-1\right)^2\theta}$	$\pi^*=\frac{\theta}{4}$
Revenue without overflow	$\pi^*=\frac{1}{4}$	$\Delta\pi^*$	$\pi^*=\frac{\theta}{4}$	

Among of the Table 2 $\Delta_a = \dfrac{(6a+3-a^2)+\sqrt{(6a+3-a^2)^2-64a}}{8a}$ and

$$\Delta\pi^* = \begin{cases} \dfrac{(1-\theta)\theta\left(3a-a^2+\theta a^3-2a^2\theta-1\right)a^2}{4\left(1-\theta+\theta(a-1)^2\right)^2} & \dfrac{2}{3a-a^2} \le \theta \le \dfrac{a+1}{2a} \\ \dfrac{(1-\theta)\theta\left(2-\left(4-6\theta\right)a+\left(4+2\theta\right)a^2-\left(1+3\theta\right)a^3+\theta a^4\right)a}{4\left(1-\theta+\theta(a-1)^2\right)^2} & \dfrac{1}{a} \le \theta < \dfrac{2}{3a-a^2} \end{cases}.$$

From Table 2, first without considering overflows, the airline chooses to provide both high and low fare classes if $(\frac{1}{a},\frac{a+1}{2a}]$, this is also the case that capacity is no longer a bang-bang decision, but rather a proportion of capacity should be reserved for low fare, since the consumer surplus has not been explored entirely.

Second, we observe that, in case the overflows have been disregarded, the airline chooses to provide the high fare class if $(\frac{a+1}{2a},\Delta_a]$. Thus we can conclude that by considering consumers' overflow behavior, airline has more flexibility in pricing and the decision regarding the effort in θ. Furthermore, the maximum profit $\pi^* = \frac{1}{4}$ is obtained in the interval $0 \le \theta < \frac{a+1}{2a}$ with the consideration of overflow behavior, and it is achieved in the interval $0 \le \theta < \frac{1}{a}$ without that consideration (cf Proposition 5). Since $\frac{a+1}{2a} > \frac{1}{a}$ (recall a >1), the above results indicate that considering overflow behavior can increase the flexibility and enhance the chance of obtaining the maximum profit for the airline.

5 Conclusion

We have developed models to describe the consumer choice behavior and an airline's decision policy. The airline's optimal decisions with price-capacity sequence will be the same as in a capacity-price sequence, and in this paper we present only the former one. With the assumption that the consumer has the similar value distribution and they are allowed to overflow by choosing different fares, the airline still has a decision of bang-bang character with regard of capacity and booking limit, which should explain the popularity of cheap flight legs. Also we observe that the segment effort θ (which also affect consumer's choice value) should be carefully coordinated with the price level p_L in order to direct the overflow volume (Corollary 3). In addition, disregarding consumer overflow may still have a chance to maintain the best revenue, however, the choice of θ value becomes more restrictive.

The major contribution of this paper is its link with consumer choice behavior and airline's decision. The value of θ has been set in such a way that assuming a linear decreasing of the acceptance of the low fare tickets. This simple assumption provides convenience to observe and understand the changing pattern in current model. However, future research should include an investigation in order to understand precisely how θ varies with consumers and/or for various segments.

In addition, investigating different value distributions could also be important. For instance, what happens if the consumer value has a distribution of two peaks? In this case, the consumers have an internal tendency to segment because of their value differentiations. Would the externally factors, such as airlines' pricing and capacity decisions interfere with consumer's segment choice? Also, additional issues such as uncertainty demand, time preference can be introduced into the model. Current study should provide a framework for investigating the above topics, but the modeling complexity is expected to increase dramatically.

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Appendix

A.1 Proof of Demand volume and consumer's choice behavior in Table 1

The values of purchasing the high fare and the low fare are $v - p_H$ and $\theta v - p_L$ respectively,

$v-p_H > \theta v - p_L$ can be induced as $v > \frac{p_H - p_L}{1-\theta}$.

If $1-p_H+p_L \le \theta \le 1$ there are $v-p_H \le \theta v - p_L$ at the interval $[0,1]$, $v-p_H \ge 0$ at the interval $[p_H,1]$ and $\theta v - p_L \ge 0$ at the interval $[\frac{p_L}{\theta},1]$. Thus customer's first choice is the low fare ticket, and the second choice is high fare at the interval $[p_H,1]$; customer's unique choice is the low fare ticket at the interval $[\frac{p_L}{\theta}, p_H]$.

If $\frac{p_L}{p_H} \le \theta \le 1-p_H+p_L$ there are $v-p_H \ge \theta v - p_L$ at the interval $[\frac{p_H-p_L}{1-\theta},1]$, $v-p_H \le \theta v - p_L$ at the interval $[\frac{p_L}{\theta}, \frac{p_H-p_L}{1-\theta}]$; $v-p_H \ge 0$ at the interval $[p_H,1]$ and $\theta v - p_L \ge 0$ at the interval $[\frac{p_L}{\theta},1]$. Thus customer's first choice is the high fare ticket, and the second choice is low fare at the interval $[\frac{p_H-p_L}{1-\theta},1]$; customer's first choice is the low fare ticket, and the second choice is high fare at the interval $[p_H, \frac{p_H-p_L}{1-\theta}]$; customer's unique choice is the low fare ticket at the interval $[\frac{p_L}{\theta}, p_H]$.

If $p_L \le \theta \le \frac{p_L}{p_H}$ there are $v-p_H \ge \theta v - p_L$ at the interval $[p_H,1]$, $v-p_H \ge 0$ at the interval $[p_H,1]$ and $\theta v - p_L \ge 0$ at the interval $[\frac{p_L}{\theta},1]$. Thus customer's first choice is the high fare ticket, and the second choice is low fare at the interval $[\frac{p_L}{\theta},1]$; customer's unique choice is the high fare ticket at the interval $[\frac{p_L}{\theta}, p_H]$.

If $0 \le \theta \le p_L$ there are $v-p_H \ge \theta v - p_L$ at the interval $[p_H,1]$ and $v-p_H \ge 0$ at the interval $[p_H,1]$. Thus customer's unique choice is the high fare ticket at the interval $[\frac{p_L}{\theta}, p_H]$.

We can get Table 1 by summarizing the above analysis.

The end of proof□

A.2 Proof of Proposition 1.

Based on Table 1 the airline's profit function can be induced at the different interval of θ.

(i) If $1-p_H+p_L \le \theta \le 1$ then $\pi = p_L Min[t_1, B] + p_H Min\left[\lambda_1\left[t_1-B\right]^+, C-B\right]$.

The airline's best response are setting $t_1 - B \ge 0$ and $\lambda_1\left[t_1-B\right]^+ - C + B = 0$

Thus $\underset{B}{Max}\,\pi = p_L B + p_H \lambda_1\left(t_1-B\right)$

$C^* = B^* = 1-\dfrac{p_L}{\theta}$ are induced by the first order condition.

Then $\pi^* = p_L\left(1-\dfrac{p_L}{\theta}\right)$.

(ii)If $\dfrac{p_L}{p_H} \le \theta < 1-p_H+p_L$ then

$\pi = p_L Min[t_2+\gamma_2\left[t_3-C+B\right]^+, B] + p_H Min\left[t_3+\lambda_2\left[t_2-B\right]^+, C-B\right]$

$B^* = 0$ and $C^* = t_3+\lambda_2 t_2 = 1-p_H$ are induced due to $p_L \le \lambda_2 p_H$, $p_L\gamma_2 \le p_H$, thus

$\pi^* = p_H\left(t_3+\lambda_2 t_2\right) = p_H\left(1-p_H\right)$.

(iii) If $p_L \le \theta < \dfrac{p_L}{p_H}$ then $\pi = p_L Min[\gamma_3\left[t_4-C+B\right]^+, B] + p_H Min\left[t_4, C-B\right]$

The airline's best response is setting $t_4 \ge C-B$ and $\gamma_3\left[t_4-C+B\right]^+ = B$,

Thus $\underset{B}{Max}\,\pi = \gamma_3 p_L\left(t_4-C+B\right) + p_H\left(C-B\right) = p_L B + p_H\left(C-B\right)$

$B^* = 0$ and $C^* = t_4$ are induced by the first order condition.

Then $\pi^* = p_H(1-p_H)$.

(iv) If $0 \le \theta < p_L$ then $\pi = p_H Min\left[t_4, C-B\right]$

The airline's best response are $B^* = 0$ and $C^* = t_4$.

Thus $\pi^* = p_H t_4 = p_H(1-p_H)$.

The end of proof□

A.3 Proof of Proposition 2.

According to Proposition 1

(i) If $1-p_H+p_L \le \theta \le 1$ then $\pi = p_L(1-\frac{p_L}{\theta})$.

Thus $p_L=\frac{\theta}{2}$ and $\pi=\frac{\theta}{4}$ with the condition $1-\frac{\theta}{2} \le p_H$.

$B=C=\frac{1}{2}$ is obtained.

(ii)If $0 \le \theta < 1-p_H+p_L$ then $\pi = p_H\left(1-p_H\right)$

Thus $p_H=\frac{1}{2}$, $\pi=\frac{1}{4}$ with the condition $\frac{1}{2} > p_L > \theta-\frac{1}{2}$,

$B=0$ and $C=\frac{1}{2}$ are obtained.

Combining the above two cases together, we obtain the airline's best choices are $B^*=0$, $C^*=\frac{1}{2}$, $p_H^*=\frac{1}{2}$, $\pi^*=\frac{1}{4}$ and let $\frac{1}{2} > p_L^* > \theta-\frac{1}{2}$.

The end of proof□

A.4 Proof of Proposition 3

According to Proposition 1 there are two kinds of pricing strategies at the intervals $1-p_H+p_L \le \theta \le 1$ and $0 \le \theta < 1-p_H+p_L$ without the constraint of $p_H / p_L = a$.

Given $p_H / p_L = a$ there are $\pi_1 = p_{L1}(1-\frac{p_{L1}}{\theta})$ if $1-ap_{L1}+p_{L1} \le \theta \le 1$ and $\pi_2 = ap_{L2}\left(1-ap_{L2}\right)$ if $0 \le \theta < 1-ap_{L2}+p_{L2}$.

Thus the local optimal solutions are $p_{L1} = \max\left\{\frac{1-\theta}{a-1}, \frac{\theta}{2}\right\}$ with $\pi_1 = p_L(1-\frac{p_L}{\theta})$ and

$p_{L2} = \min\left\{\frac{1-\theta}{a-1}, \frac{1}{2a}\right\}$ with $\pi_2 = p_H\left(1-p_H\right)$.

There are the following three scenarios:

(i)If $0 \le \theta \le \frac{2}{a+1}$, there are $p_{L1} = \max\left\{\frac{1-\theta}{a-1}, \frac{\theta}{2}\right\} = \frac{1-\theta}{a-1}$, $p_{L2} = \min\left\{\frac{1-\theta}{a-1}, \frac{1}{2a}\right\} = \frac{1}{2a}$ and $\pi_1 - \pi_2 = \frac{(1-\theta)(a\theta-1)}{\theta(a-1)^2} - \frac{1}{4} < 0$, therefore the global optimal price is $p_L^* = \frac{1}{2a}$ and optimal profit is $\pi^* = \frac{1}{4}$.

(ii)If $\frac{2}{a+1} \le \theta \le \frac{a+1}{2a}$, there are $p_{L1} = \max\left\{\frac{1-\theta}{a-1}, \frac{\theta}{2}\right\} = \frac{\theta}{2}$, $p_{L2} = \min\left\{\frac{1-\theta}{a-1}, \frac{1}{2a}\right\} = \frac{1}{2a}$, $\pi_1 - \pi_2 = \frac{\theta}{4} - \frac{1}{4} < 0$. Therefore the global optimal price is $p_L^* = \frac{1}{2a}$ and optimal profit is $\pi^* = \frac{1}{4}$.

(iii)If $\frac{a+1}{2a} \le \theta \le 1$, there are $p_{L1} = \max\left\{\frac{1-\theta}{a-1}, \frac{\theta}{2}\right\} = \frac{\theta}{2}$ and $p_{L2} = \min\left\{\frac{1-\theta}{a-1}, \frac{1}{2a}\right\} = \frac{1-\theta}{a-1}$.

$\pi_1 - \pi_2 = \frac{\theta}{4} - \frac{(1-\theta)(a\theta-1)a}{(a-1)^2} < 0$ is induced as

$$\frac{a+1}{2a} \le \theta \le \frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2};$$

Therefore the global optimal price is $p_L^* = \frac{1-\theta}{a-1}$ and optimal profit is $\pi^* = \frac{(1-\theta)(a\theta-1)a}{(a-1)^2}$.

$\pi_1 - \pi_2 = \frac{\theta}{4} - \frac{(1-\theta)(a\theta-1)a}{(a-1)^2} \ge 0$ is induced as

$$\frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2} \le \theta \le 1.$$

Therefore the global optimal price is $p_L^* = \frac{\theta}{2}$ and optimal profit is $\pi^* = \frac{\theta}{4}$.

The proposition 3 is obtained by summarizing the above analysis.

The end of proof□

A.5 Proof of Corollary 3.

Case one: without the constraint of $p_H / p_L = a$

The airline provides the high fare class only according to Proposition 2.

There is not any kind of consumer overflow at the interval of $\theta < 2p_L$ according to Table 1.

The volume of consumer's overflow from low fare class to high fare class is $\frac{\theta - 2p_L}{2(1-\theta)}$ at the interval of $2p_L \le \theta < p_L + \frac{1}{2}$ according to Table 1.

Case two: with the constraint of $p_H / p_L = a$

(i)The optimal decisions are $p_L^* = \frac{1}{2a}$, $B^* = 0$ and $C^* = \frac{1}{2}$ at the interval of $0 \le \theta < \frac{a+1}{2a}$

The airline only provides the high fare class, there is not any consumer's overflow if $0 \le \theta < \frac{1}{a}$ and the volume of consumer's overflow from low fare class to high fare class is $\frac{\theta - 2p_L}{2(1-\theta)}$ if $\frac{1}{a} \le \theta < \frac{a+1}{2a}$ according to Table 1.

(ii)The optimal decisions are $p_L^* = \frac{1-\theta}{a-1}$, $B^* = 0$ and $C^* = \frac{a\theta - 1}{a-1}$ at the interval of $\frac{a+1}{2a} \le \theta < \frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2}$.

The airline only provides the high fare class, the volume of consumer's overflow from low fare class to high fare class is $\frac{a\theta - 1}{a-1}$ due to $\frac{1}{a} < \theta$ according to Table 1.

(iii)The optimal decisions are $p_L^* = \frac{\theta}{2}$ and $B^* = C^* = \frac{1}{2}$ at the interval $\frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2} \le \theta \le 1$.

The airline only provides the low fare class, the volume of consumer's overflow from high fare class to low fare class is $\frac{1}{2}$ at the interval of $\frac{(3a^2+6a-1)+\sqrt{(3a^2+6a-1)^2-64a^3}}{8a^2} \le \theta < \frac{1}{a}$

and $\frac{1-a}{2}$ at the interval of $\frac{1}{a} \le \theta < \frac{2}{a+1}$ according to Table 1.

The end of proof□

A.6 Proof of Corollary 4.

Given $p_H / p_L = a$ the airline's optimal capacity in Proposition 4 can be reformulated as:

(i) $B_1 = C_1 = 1 - \frac{p_{L1}}{\theta}$ if $\frac{1-\theta}{a-1} \le p_{L1}$;

(ii) $B_2 = p_{L2} \frac{a\theta - 1}{(1-\theta)\theta}$, $C_2 = 1 - \frac{p_{L2}}{\theta}$ if $\frac{1}{a} \le \theta \le 1 - ap_{L2} + p_{L2}$;

(iii) $B_3 = 0$, $C_3 = 1 - ap_{L3}$ if $0 \le \theta < \frac{1}{a}$.

Thus the local optimal prices are $p_{L1} = \max\left\{\frac{1-\theta}{a-1}, \frac{\theta}{2}\right\}$, $p_{L2} = \min\left\{\frac{1-\theta}{a-1}, \frac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}\right\}$

and $p_{L3} = \frac{1}{2a}$ respectively.

There are two scenarios according to the value of *a*.

Scenario one $1 < a < 3$

(i) If $0 \le \theta < \frac{1}{a}$

It is intuitive to induce that the local optimal profits are $\pi_1 = \frac{(a\theta - 1)(1-\theta)}{(a-1)^2 \theta}$,

$\pi_2 = \frac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$ and $\pi_3 = \frac{1}{4}$ for the airline.

Thus the global optimal profit is $\pi^* = \max\{\pi_1, \pi_2, \pi_3\} = \pi_3 = \frac{1}{4}$ and corresponding decisions are $p_H^* = \frac{1}{2}$, $B^* = 0$ and $C^* = \frac{1}{2}$.

(ii) If $\frac{1}{a} \le \theta < \frac{2}{a+1}$

The airline induces its local optimal profits are $\pi_1 = \frac{(a\theta - 1)(1-\theta)}{(a-1)^2 \theta}$ and

$\pi_2 = \dfrac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$.

Thus the global optimal profit is $\pi^* = \max\{\pi_1, \pi_2\} = \pi_2 = \dfrac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$ and corresponding decisions are $p_L^* = \dfrac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$, $B^* = \dfrac{a(a\theta-1)}{2\left(1-\theta+\theta(a-1)^2\right)}$ and $C^* = \dfrac{(1-\theta)(2-a)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$.

(iii) If $\dfrac{2}{a+1} \le \theta < \dfrac{2}{3a-a^2}$

The airline induces its local optimal profits are $\pi_1 = \dfrac{\theta}{4}$ and $\pi_2 = \dfrac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$.

Thus, the global optimal profit is $\pi^* = \dfrac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$ and corresponding decisions are $p_L^* = \dfrac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$, $B^* = \dfrac{a(a\theta-1)}{2\left(1-\theta+\theta(a-1)^2\right)}$ and $C^* = \dfrac{(1-\theta)(2-a)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$ if $\dfrac{2}{a+1} \le \theta < \dfrac{a+1}{2a}$; the global optimal profit is $\pi^* = \dfrac{\theta}{4}$ and corresponding decisions are $p_L^* = \dfrac{\theta}{2}$ and $B^* = C^* = \dfrac{1}{2}$ if $\dfrac{a+1}{2a} \le \theta < \dfrac{2}{3a-a^2}$.

(iv) If $\dfrac{2}{3a-a^2} \le \theta < 1$

The airline induces its local optimal profits are $\pi_1 = \dfrac{\theta}{4}$ and $\pi_2 = p_L^2 \dfrac{a\theta-1}{(1-\theta)\theta} + ap_L\left(1-\dfrac{a-1}{1-\theta}p_L\right) = \dfrac{(a\theta-1)(1-\theta)}{\theta(a-1)^2}$.

Thus the global optimal profit is $\pi^* = \max\{\pi_1, \pi_2\} = \pi_1 = \dfrac{\theta}{4}$ and corresponding decisions are $p_L^* = \dfrac{\theta}{2}$ and $B^* = C^* = \dfrac{1}{2}$.

Scenario two: $a \ge 3$

(i) If $0 \le \theta < \frac{1}{a}$

It is intuitive to induce that the local optimal profits are $\pi_1 = \frac{(a\theta - 1)(1-\theta)}{(a-1)^2 \theta}$, $\pi_2 = \frac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$ and $\pi_3 = \frac{1}{4}$.

Thus the global optimal profit is $\pi^* = \max\{\pi_1, \pi_2, \pi_3\} = \pi_3 = \frac{1}{4}$ and corresponding decisions are $p_H^* = \frac{1}{2}$, $B^* = 0$ and $C^* = \frac{1}{2}$.

(ii) If $\frac{1}{a} \le \theta < \frac{2}{a+1}$

The local optimal profits are $\pi_1 = \frac{(a\theta - 1)(1-\theta)}{(a-1)^2 \theta}$ and $\pi_2 = \frac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$

Thus the global optimal profit is $\pi^* = \max\{\pi_1, \pi_2\} = \pi_2 = \frac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$.

And the optimal decisions are $p_L^* = \frac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$, $B^* = \frac{a(a\theta-1)}{2\left(1-\theta+\theta(a-1)^2\right)}$ and $C^* = \frac{(1-\theta)(2-a)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$.

(iii) If $\frac{2}{a+1} \le \theta < 1$

The local optimal profits are $\pi_1 = \frac{\theta}{4}$ and $\pi_2 = \frac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$

Thus the global optimal profit is $\pi^* = \frac{a^2(1-\theta)\theta}{4\left(1-\theta+\theta(a-1)^2\right)}$ and the optimal decisions are $p_L^* = \frac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$, $B^* = \frac{a(a\theta-1)}{2\left(1-\theta+\theta(a-1)^2\right)}$ and $C^* = \frac{(1-\theta)(2-a)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$ if $\frac{2}{a+1} \le \theta < \frac{a+1}{2a}$; the global optimal profit is $\pi^* = \frac{\theta}{4}$ and the optimal decisions are $p_L^* = \frac{\theta}{2}$ and $B^* = C^* = \frac{1}{2}$ if $\frac{a+1}{2a} \le \theta < 1$.

We can obtain Corollary 4 by summarizing the above results and induce the corresponding optimal decisions.

The end of proof□

A.7 Proof of Proposition 5.

The airline's optimal decisions without considering overflow are given in Corollary 5. Recall equation (1) the realized airline's profit can be summarized as the followings at the different intervals of θ.

(i)At the interval $0 \le \theta < \frac{1}{a}$

The corresponding profit function is

$$\pi = p_L Min[\gamma_3 \left[t_4 - C + B\right]^+, B] + p_H Min\left[t_4, C - B\right]$$

With $p_H^* = \frac{1}{2}$, $B^* = 0$, $C^* = \frac{1}{2}$

Thus the optimal profit is $\pi^* = \frac{1}{4}$.

(ii) At the interval of $\frac{1}{a} \le \theta < \frac{a+1}{2a}$

The optimal decisions are $p_L^* = \frac{(1-\theta)\theta a}{2\left(1-\theta+\theta(a-1)^2\right)}$, $B^* = \frac{a\left(a\theta - 1\right)}{2\left(1-\theta+\theta(a-1)^2\right)}$ and $C^* = \frac{\left(1-\theta\right)\left(2-a\right)+\theta(a-1)^2}{2\left(1-\theta+\theta(a-1)^2\right)}$.

There are three cases according to different a.

Case one: if $1 < a \le 1+\sqrt{2}$ then $\frac{a+1}{2a} \le \frac{2}{3a-a^2}$

Given the optimal decisions the profit function is

$$\pi^* = p_L Min[t_2 + \gamma_2 [t_3 - C + B]^+, B] + p_H Min[t_3, C - B]$$
$$= p_L Min[B] + p_H Min[C - B] =$$
$$= \frac{(1-\theta)\theta\left(3a - a^2 + \theta a^3 - 2a^2\theta - 1\right)a^2}{4\left(1-\theta+\theta(a-1)^2\right)^2}$$

Case two: if $1+\sqrt{2} < a < 3$ then $\frac{a+1}{2a} > \frac{2}{3a-a^2}$

Given the optimal decisions the profit function is

$$\pi^* = \begin{cases} p_L Min[B] + p_H Min[C-B], \text{ if } \frac{2}{3a-a^2} \le \theta \le \frac{a+1}{2a} \\ p_L Min[t_2 + \gamma_2 [t_3 - C + B]] + p_H Min[C-B], \text{if } \frac{1}{a} \le \theta < \frac{2}{3a-a^2} \end{cases}$$
$$= \begin{cases} \frac{(1-\theta)\theta\left(3a - a^2 + \theta a^3 - 2a^2\theta - 1\right)a^2}{4\left(1-\theta+\theta(a-1)^2\right)^2}, \text{ if } \frac{2}{3a-a^2} \le \theta \le \frac{a+1}{2a} \\ \frac{(1-\theta)\theta\left(2-(4-6\theta)a+(4+2\theta)a^2-(1+3\theta)a^3+\theta a^4\right)a}{4\left(1-\theta+\theta(a-1)^2\right)^2}, \text{if } \frac{1}{a} \le \theta < \frac{2}{3a-a^2} \end{cases}$$

Case three: $3 \le a$

Given the optimal decisions the profit function is

$$\pi^* = p_L Min[t_2 + \gamma_2 [t_3 - C + B]] + p_H Min[C - B]$$
$$= \frac{(1-\theta)\theta\left(2-(4-6\theta)a+(4+2\theta)a^2-(1+3\theta)a^3+\theta a^4\right)a}{4\left(1-\theta+\theta(a-1)^2\right)^2}$$

(iii) At the interval of $\frac{a+1}{2a} \le \theta \le 1$

Given the optimal decisions the profit function is $\pi^* = \frac{\theta}{4}$

The end of proof□