Efficient signal reconstruction scheme for time-interleaved ADCs

Anu Kalidas Muralidharan Pillai and Håkan Johansson

Linköping University Post Print

N.B.: When citing this work, cite the original article.

©2012 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

Anu Kalidas Muralidharan Pillai and Håkan Johansson, Efficient signal reconstruction scheme for time-interleaved ADCs, 2012, Proc. IEEE 10th Int. New Circuits and Systems Conf. (NEWCAS), 357-360.

<http://dx.doi.org/10.1109/NEWCAS.2012.6329030>

Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-87582>

Efficient Signal Reconstruction Scheme for Time-Interleaved ADCs

Anu Kalidas Muralidharan Pillai and Håkan Johansson

Division of Electronics Systems, Department of Electrical Engineering, Linköping University, SE-581 83, Sweden

Email: {kalidas, hakanj}@isy.liu.se

Abstract—Time-interleaved analog-to-digital converters (ADCs) exhibit offset, gain, and time-skew errors due to channel mismatches. The time skews give rise to a nonuniformly sampled signal instead of the desired uniformly sampled signal. This introduces the need for a digital signal reconstructor that takes the “nonuniform samples” and generates the “uniform samples”. In the general case, the time skews are frequency dependent, in which case a generalization of nonuniform sampling applies. When the bandwidth of a digital reconstructor approaches the whole Nyquist band, the computational complexity may become prohibitive. This paper introduces a new scheme with reduced complexity. The idea stems from recent multirate-based efficient realizations of linear and time-invariant systems. However, a time-interleaved ADC (without correction) is a time-varying system which means that these multirate-based techniques cannot be used straightforwardly but need to be appropriately analyzed and extended for this context.

I. INTRODUCTION

A popular technique to increase the effective sampling rate of analog-to-digital converters (ADCs) is to have multiple ADCs in a time-interleaved fashion with each ADC operating at a lower sampling rate [1]. Sampling clocks to these time-interleaved ADCs (TI-ADCs) are provided in such a way that at any given sampling instant, only one ADC samples the input. Hence, in order to ensure that all the output samples are equally spaced, the sampling clocks should have uniform time skew. However in reality, timing mismatches between the ADCs create periodically nonuniformly spaced samples at the output of the TI-ADC. In an M -channel TI-ADC, such static time-skew errors lead to an M -periodic nonuniformly sampled signal at the output.

This paper considers reconstruction of nonuniformly sampled signals generated by two-channel TI-ADC, a popular TI-ADC configuration, in which two ADCs are used in parallel. Here static time-skew errors between the two channels result in two-periodic nonuniformly sampled signals as shown in Fig. 1(b). In periodic nonuniform sampling, the skew of the channels are expressed relative to the first channel. For example, in the two-periodic case shown in Fig. 1(b), the time-skew errors for the first and second channels will be 0 and ε_1 , respectively. This assumption means that the time skew in samples from the first channel needs no correction while skew in the samples from the second channel are corrected such that they become uniformly sampled with respect to samples from the first channel. In high-speed and high-resolution TI-ADCs, the overall performance may suffer from frequency response mismatches between the channels [2], [3], which will require special techniques to reconstruct the uniformly sampled sequence [4]–[7]. However, here it is assumed that the operating frequencies and resolution are not very high and it can be considered that the channel suffers only from static time-skew errors.

Reconstruction from periodically nonuniformly sampled signals using time-varying discrete-time FIR filters were considered in detail in [8]. Even though the method proposed in [8] gave the optimal filter order, whenever the time-skew error of a channel varies, all the filter coefficients have to be redesigned. This paper suggests a technique

Fig. 1. (a) Uniform sampling (b) Two-periodic nonuniform sampling.

which can be used to reduce the number of filter coefficients that need to be updated online when the time-skew error changes. For this purpose, the two-rate technique originally proposed in [9] is used to split the reconstruction filter into two parts: a regular linear-phase filter with symmetric coefficients and a simpler time-varying filter. The regular filter is designed in such a way that it needs no modification for a range of time-skew errors. Online redesign is needed only for the simpler time-varying filter with fewer multipliers.

An alternative is to perform reconstruction using polynomial impulse response FIR filters as proposed in [10], [11]. These reconstruction filters have the least number of variable multipliers and do not need any online redesign when the time-skew error changes, since the variable multipliers can be directly updated with the new time-skew. However they need a significantly larger number of fixed multipliers thereby increasing the total number of multipliers compared to the regular structure in [8].

In Section II, a review of the nonuniform sampling is provided followed by a discussion on periodic nonuniform sampling. Section III shows the basic two-rate approach. It explains how an equivalent single-rate structure, derived using the two-rate based approach, can be used for the reconstruction filter. It also outlines the offline and online design procedures to be followed to determine the coefficients of the subfilters in the single-rate structure. Section IV provides two design examples and Section V concludes the paper.

II. PERIODIC NONUNIFORM SAMPLING

With uniform sampling of a continuous-time signal, $x_a(t)$, we obtain the sequence

$$x(n) = x_a(nT) \quad (1)$$

for all integer values of n and where T is the sampling period. Nonuniform sampling of $x_a(t)$ results in a discrete-time sequence, $v(n)$, such that

$$v(n) = x_a(nT + \varepsilon_n T) \quad (2)$$

Fig. 2. (a) Time-varying reconstruction filter. (b) Equivalent two-channel two-rate filter-bank representation.

where ε_n is the deviation, in percentage, of the n th sample's actual sampling instant from the corresponding uniform sampling instant. In TI-ADCs, since the samples are formed by interleaving outputs from each ADC, these deviations are periodic. In an M -channel TI-ADC, these deviations or time-skew errors will be M -periodic as given by

$$\varepsilon_n = \varepsilon_{n+M}. \quad (3)$$

In the two-channel TI-ADC, the time-skew errors will be two-periodic with $\varepsilon_{2n} = \varepsilon_0$ and $\varepsilon_{2n+1} = \varepsilon_1$ for all values of n . Since we are interested in correcting the relative time-skew error between channels we assume that $\varepsilon_0 = 0$ and that ε_1 is the time-skew error of the second channel with respect to the first channel.

In order to reconstruct $x(n)$ from $v(n)$, a time-varying filter with impulse-response $h_n(k)$ is used such that its output $y(n)$, given by¹

$$y(n) = \sum_{k=-N}^N v(n-k)h_n(k), \quad (4)$$

approximates the signal $x(n)$ [8]. In this paper, the filter $h_n(k)$ is assumed to be noncausal to simplify the design and analysis, but can be easily converted to the causal filter by applying suitable delays. Also it should be noted that $h_n(k)$ is centered at the sample to be reconstructed.

Rather than designing a filter for the entire Nyquist band, we assume that the signals are oversampled such that they are bandlimited to $|\omega T| \leq \omega_0 T < \pi$. Such an assumption makes it feasible to actually implement the designed filters [8]. If $X(e^{j\omega T})$ is the Fourier transform of $x(n)$, and assuming that $x_a(t)$ is bandlimited to ω_0 , it can be shown that [8]

$$y(n) = \frac{1}{2\pi} \int_{-\omega_0 T}^{\omega_0 T} A_n(j\omega T) X(e^{j\omega T}) e^{j\omega T n} d(\omega T) \quad (5)$$

where

$$A_n(j\omega T) = \sum_{k=-N}^N h_n(k) e^{-j\omega T(k-\varepsilon_n-k)}. \quad (6)$$

As shown in [8], to obtain perfect reconstruction,

$$A_n(j\omega T) = 1, \quad \omega T \in [-\omega_0 T, \omega_0 T]. \quad (7)$$

Figure 2(b) shows the two-channel maximally decimated filter bank representation of the reconstruction filter in Fig. 2(a). Since $\varepsilon_0 = 0$, as shown in 2(b), the first channel should be passed to the output without any filtering while the time-skew error in the second channel should be corrected using the filter $H_1(z)$. The coefficients of $H_1(z)$ should be chosen such that it minimizes the error between $A_1(j\omega T)$ and 1 for a given bandwidth of $\omega_0 T$ and time-skew error ε_1 .

¹The order of the reconstruction filter is assumed to be even to simplify the derivations. With minor modifications, they can be applied to the odd-order case as well.

III. TWO-RATE BASED APPROACH

This section explains how the two-rate approach can be used to realize $H_1(z)$ such that its complexity compared to the regular reconstruction filter, shown in Fig. 3(a), is reduced. In the basic two-rate approach shown in Fig. 3(b), the input is upsampled by two and passed to a linear-phase half-band filter $F_1(z)$ whose output is then fed to $G_1(z)$. The output from $G_1(z)$ is downsampled by two to restore the original sampling rate. The filters $F_1(z)$ and $G_1(z)$ can be split into their polyphase components $F_{10}(z)$, $F_{11}(z)$ and $G_{10}(z)$, $G_{11}(z)$, respectively. Since a linear-phase half-band filter is symmetric with every second coefficient being zero, $F_{10}(z)$ is symmetric and $F_{11}(z)$ is equal to a delay $z^{-(D_{F_1}-1)/2}$ where D_{F_1} is the delay of $F_1(z)$. Using multirate theory [12], it can be shown that the overall structure in Fig. 3(b) corresponds to the zeroth-polyphase component of $F_1(z)G_1(z)$ as shown in Fig. 3(c). If $H_1(z)$ in Fig. 2(b) is to be represented using the structure in Fig. 3(c),

$$\begin{aligned} H_1(z) &= F_{10}(z)G_{10}(z) + z^{-1}F_{11}(z)G_{11}(z) \\ &= F_{10}(z)G_{10}(z) + z^{-(D_{F_1}+1)/2}G_{11}(z). \end{aligned} \quad (8)$$

Equation (8) shows that the $H_1(z)$ remains a single-rate structure. To compute the impulse response of $H_1(z)$, we have to determine the impulse responses of $F_{10}(z)$, $G_{10}(z)$, and $G_{11}(z)$. In order to reduce the number of impulse response coefficients that have to be updated online, $F_{10}(z)$ is designed such that it can be used for all values of time-skew errors between $\pm\varepsilon_1$. Hence, the reconstruction filter design problem can be split into two parts: offline design of $F_{10}(z)$ corresponding to the extreme time-skew errors, $-\varepsilon_1$ and ε_1 , and online redesign of $G_{10}(z)$ and $G_{11}(z)$ whenever the time-skew error varies between $\pm\varepsilon_1$.

A. Offline design of $F_{10}(z)$

To design $F_{10}(z)$, we follow a least-squares approach in which the filter design problem can be stated as:

Given the orders of the subfilters $F_{10}(z)$, $G_{10}(z)$, and $G_{11}(z)$, as well as ε_1 , determine the coefficients of these subfilters and a parameter δ , to minimize δ subject to

$$\frac{1}{2\pi} \int_{-\omega_0 T}^{\omega_0 T} |A_1(j\omega T) - 1|^2 d(\omega T) \leq \delta. \quad (9)$$

Usually, as part of the reconstruction filter specification, the maximum error that can be tolerated, δ_e , will be specified. This requirement will be satisfied if, δ satisfies the condition $\delta \leq \delta_e$ after optimization. Since $H_1(z)$ is obtained by cascading subfilters, (9) is a nonlinear optimization problem. Hence, in order to avoid a poor local optimum, it will be beneficial to have a good starting point for the subfilter coefficients used in the optimization.

The design steps to identify the coefficients of $F_1(z)$ which subsequently can be used for all time-skew errors within $\pm\varepsilon_1$ are as follows:

- 1) Determine the order, $\hat{N}_{F_1}$, of a standard half-band linear-phase FIR filter $F_1(z)$ such that its passband edge and stopband edges are $\Omega_c = \omega_0 T/2$ and $\Omega_s = \pi - \Omega_c$, respectively, with the maximum ripple in the passband and stopband being $\sqrt{\delta_e}$.
- 2) Determine the order, $\hat{N}_{G_1}$, of a filter $G_1(z)$ such that this filter approximates a regular reconstruction filter with the error $\sqrt{\delta_e}$ and bandwidth Ω_c .
- 3) For each combination of N_{F_1} and N_{G_1} around the values of $\hat{N}_{F_1}$ and $\hat{N}_{G_1}$:

Fig. 3. (a) Transfer function of the reconstructor for the second channel of a two-channel TI-ADC. (b) Basic two-rate approach where $F_1(z)$ is a half-band filter. (c) Equivalent single-rate realization of (b) using polyphase components of $F_1(z)$ and $G_1(z)$.

- (a) Design a regular half-band filter $F_1(z)$ and use polyphase decomposition to split it into $F_{10}(z)$ and the pure delay term $z^{-(D_{F_1}+1)/2}$.
- (b) Set the midtap of $G_1(z)$ to one and all other taps to zero and obtain the coefficients for $G_{10}(z)$ and $G_{11}(z)$ using polyphase decomposition.
- (c) Determine the filter coefficients by solving the optimization problem in (9). Use the coefficients for $F_{10}(z)$, $G_{10}(z)$, and $G_{11}(z)$ determined in Steps 3(a) and 3(b) as the initial values for the optimization. If δ obtained from the optimization routine is smaller than δ_e , save the results.

- 4) From all the results in Step 3(c) that satisfied the requirement, choose the one with lowest complexity as the final solution. If multiple results have the same total number of multipliers, choose the one that requires the least number of variable multipliers.

Once the coefficients for $F_{10}(z)$, $G_{10}(z)$, and $G_{11}(z)$, corresponding to the time-skew ε_1 , are identified, the filter to correct a time-skew of $-\varepsilon_1$, can be obtained by reversing the coefficients of $G_{10}(z)$ and $G_{11}(z)$ and using the same $F_{10}(z)$. It was shown in [8] that, as the magnitude of the time-skew error reduces, the less nonuniform the sampling pattern is, and a lower order reconstruction filter can be used to achieve the same reconstruction error. Therefore, when the time-skew error starts to decrease from the extremes $\pm\varepsilon_1$ and approaches 0, the reconstruction system becomes simpler. Hence $F_{10}(z)$ can still use the same coefficients and only $G_{10}(z)$ and $G_{11}(z)$ need to be redesigned for the new time-skew.

B. Online design of $G_{10}(z)$ and $G_{11}(z)$

With a fixed $F_1(z)$, online design of $G_{10}(z)$ and $G_{11}(z)$ is feasible by using a least-squares approach which can be implemented using matrix inversions.

Since $F_1(z)$ is a half-band filter of even order, the impulse response of $F_{10}(z)$ has a length of $2M$ and is denoted as

$$\mathbf{f}_{10} = [f_{10}(-M) \ f_{10}(-M+1) \ \dots \ f_{10}(M-1)] \quad (10)$$

while the delay term $z^{-(D_{F_1}+1)/2}$ in (8) can be considered as a sequence of length $2M$ and is denoted as

$$\mathbf{f}_{11} = [f_{11}(-M) \ f_{11}(-M+1) \ \dots \ f_{11}(M-1)]. \quad (11)$$

Since $\mathbf{f}_{11}$ represents a pure delay,

$$f_{11}(k) = \begin{cases} 1, & k = 1 \\ 0, & k \neq 1 \end{cases}. \quad (12)$$

Let $\mathbf{g}_{10}$ and $\mathbf{g}_{11}$ be the impulse response vectors of $G_{10}(z)$ and $G_{11}(z)$ respectively such that

$$\mathbf{g}_{10} = [g_{10}(-L) \ g_{10}(-L+1) \ \dots \ g_{10}(L)] \quad (13)$$

and

$$\mathbf{g}_{11} = [g_{11}(-L) \ g_{11}(-L+1) \ \dots \ g_{11}(L)] \quad (14)$$

where both $\mathbf{g}_{10}$ and $\mathbf{g}_{11}$ are assumed to have a length of $2L+1$ to simplify the calculations. However these results can be extended for any combination of impulse-response lengths of $\mathbf{g}_{10}$ and $\mathbf{g}_{11}$.

As was shown in [8], the error power function for the channel can be represented as

$$P_1 = \frac{1}{2\pi} \int_{-\omega_0 T}^{\omega_0 T} |A_1(j\omega T) - 1|^2 d(\omega T) \quad (15)$$

Using (6) in (15), followed by some algebraic manipulations, will yield

$$P_1 = \mathbf{g}_1^T \mathbf{F}_1^T \mathbf{S}_1 \mathbf{F}_1 \mathbf{g}_1 + \mathbf{g}_1^T \mathbf{F}_1^T \mathbf{b}_1 + C \quad (16)$$

where

$$\mathbf{F}_1 = [\mathbf{F}_{10}^T \ \mathbf{F}_{11}^T] \quad (17)$$

$$\mathbf{g}_1 = [\mathbf{g}_{10} \ \mathbf{g}_{11}]^T \quad (18)$$

$$\mathbf{S}_1 = [\hat{\mathbf{S}}_{1,kp}^T \ \hat{\mathbf{S}}_{1,kp}^T]^T \quad (19)$$

$$\hat{\mathbf{S}}_{1,kp} = \begin{cases} \frac{\omega_0 T}{\pi}, & p = k \\ \frac{\sin(\omega_0 T(p-k-\varepsilon_1-p+\varepsilon_1-k))}{\pi(p-k-\varepsilon_1-p+\varepsilon_1-k)}, & p \neq k \end{cases} \quad (20)$$

$$\mathbf{b}_1 = \begin{cases} -\frac{2\omega_0 T}{\pi}, & k = \varepsilon_1 - k \\ -\frac{2 \sin(\omega_0 T(k-\varepsilon_1-k))}{\pi(k-\varepsilon_1-k)}, & k \neq \varepsilon_1 - k \end{cases} \quad (21)$$

with $p, k = -R, -R+1, \dots, R-1$ and $2R = 2M + 2L$ is the length of $H_1(z)$. Also,

$$\mathbf{F}_{10} = [\mathbf{F}_{t10}^T \ \mathbf{Z}_{2M+2L,2L+1}^T] \quad (22)$$

$$\mathbf{F}_{11} = [\mathbf{Z}_{2M+2L,2L+1}^T \ \mathbf{F}_{t11}^T] \quad (23)$$

where $\mathbf{Z}_{r,q}$ is an $r \times q$ zero matrix and $\mathbf{F}_{t1r}$ is a $(2M+2L) \times (2L+1)$ Toeplitz matrix with first row $[f_{1r}(-M) \ \mathbf{Z}_{1,2L}]$ and first column $[\mathbf{f}_{1r} \ \mathbf{Z}_{1,2L}]^T$.

The value of $\mathbf{g}_1$ which minimizes the function P_1 is obtained by solving

$$\frac{\partial P_1}{\partial \mathbf{g}_1} = 0 \quad (24)$$

which gives

$$\mathbf{g}_1 = -0.5(\mathbf{F}_1^T \mathbf{S}_1 \mathbf{F}_1)^{-1} \mathbf{F}_1^T \mathbf{b}_1. \quad (25)$$

IV. DESIGN EXAMPLE

Example 1: Consider a reconstruction filter with the following specification: $\omega_0 T = 0.9\pi$, $\varepsilon_1 \in [-0.1, 0.1]$, with a maximum reconstruction error of $P_1 = -86$ dB. The first step is to identify the coefficients of the fixed filter $F_{10}(z)$. For this, the design steps mentioned in Section III-A are followed such that the final coefficients for $F_{10}(z)$, $G_{10}(z)$, and $G_{11}(z)$ provide the minimum number of multipliers when $\varepsilon_1 = \pm 0.1$. For the given specification, the number of coefficients in $F_{10}(z)$, $G_{10}(z)$, and $G_{11}(z)$ turned out to be 42, 4, and 3 respectively. Since $F_{10}(z)$ is symmetric only half the number of coefficients are needed and they can be implemented using fixed multipliers. $G_{10}(z)$ and $G_{11}(z)$ have to be implemented using variable multipliers since new coefficient values have to be identified when the time-skew changes. Table I shows

Fig. 4. Plot of $|A_1(j\omega T) - 1|$ versus bandwidth of the reconstruction filter in Example 1, after online redesign for $\varepsilon_1 = 0.1$.

Fig. 5. Plot of $|A_1(j\omega T) - 1|$ versus bandwidth of the reconstruction filter in Example 2, after online redesign for $\varepsilon_1 = 0.1$.

the number of multipliers required for $H_1(z)$ if implemented using the method mentioned in this paper as well as the methods in [8] and [10]. We used [10] instead of [11] since the design approach in [10] is optimized for the two-channel two-periodic case. The regular structure in [8] gives optimal order but all the coefficients should be implemented using variable multipliers. The error targeted in this example corresponds to -100 dB, if the error power measure specified in Example 3 of [10] is used. Even though the design approach in [10] provides the least number of variable multipliers, the total number of multipliers have increased. In the two-rate based approach, the overall filter order is somewhat higher compared to [8], but the overall number of multipliers is reduced. Also we need only a small number of variable multipliers compared to [8] (which means online coefficient updates can be done using simpler matrix inversions) and substantially fewer fixed multipliers compared to [10]. Figure 4 shows the magnitude of $|A_1(j\omega T) - 1|$ after online redesign for $\varepsilon_1 = 0.1$.

Example 2: Consider the following specification: $\omega_0 T = 0.9\pi$, $\varepsilon_1 \in [-0.1, 0.1]$, with an error power, P_1 , no larger than -60 dB. As seen from the results in Table II, as the requirements are relaxed, the complexity of the two-rate approach reduces even further. Figure 5 plots the magnitude of $|A_1(j\omega T) - 1|$ after online redesign for $\varepsilon_1 = 0.1$.

V. CONCLUSION

The reconstruction filter design method outlined in this paper provides a way to decrease the total number of multipliers with slightly larger overall reconstruction filter order. Compared to the regular structure, the proposed method provides a significant reduction in the number of variable multipliers and fewer total number of multipliers. When compared to the polynomial impulse response FIR filter implementation, the number of fixed multipliers is reduced significantly. Even though fewer variable multipliers helps in reducing the complexity of online redesign and lowers the power consumption

TABLE I
COMPARISON OF COMPLEXITY FOR EXAMPLE 1.

Design approach	Order	Multipliers	
		Fixed	Variable
Regular [8]	40	0	41
Polynomial based [10]	44	67	3
Two-rate based	44	21	7

TABLE II
COMPARISON OF COMPLEXITY FOR EXAMPLE 2.

Design approach	Order	Multipliers	
		Fixed	Variable
Regular [8]	30	0	31
Polynomial based [10]	32	33	2
Two-rate based	31	15	6

due to coefficient updates, a larger number of overall multipliers can increase the implementation complexity, chip area, and power consumption. It should be noted that the reconstruction filter always runs at the full rate while the rate at which the online redesign block runs depends on how often the reconstruction system is calibrated. One possible alternative to completely remove the need for online redesign of $G_1(z)$, is to use a polynomial impulse response based design proposed in [10] to implement $G_1(z)$ which however needs further investigation.

REFERENCES

- [1] W. C. Black and D. A. Hodges, "Time interleaved converter arrays," *IEEE J. Solid-State Circuits*, vol. 15, no. 6, pp. 1022–1029, 1980.
- [2] C. Vogel, "Modeling, identification, and compensation of channel mismatch errors in time-interleaved analog-to-digital converters," Ph.D. dissertation, Graz University, Graz, Austria, 2005.
- [3] T.-H. Tsai, P. J. Hurst, and S. H. Lewis, "Bandwidth mismatch and its correction in time-interleaved analog-to-digital converters," *IEEE Trans. Circuits Syst. II*, vol. 53, no. 10, pp. 1133–1137, Oct. 2006.
- [4] S. Saleem and C. Vogel, "Adaptive compensation of frequency response mismatches in high-resolution time-interleaved ADCs using a low-resolution ADC and a time-varying filter," in *Proc. IEEE Int. Symp. Circuits Syst.*, 2010, pp. 561–564.
- [5] H. Johansson and P. Löwenborg, "A least-squares filter design technique for the compensation of frequency response mismatch errors in time-interleaved A/D converters," *IEEE Trans. Circuits Syst. II*, vol. 55, no. 11, pp. 1154–1158, Nov. 2008.
- [6] M. Seo, M. J. W. Rodwell, and U. Madhoo, "Comprehensive digital correction of mismatch errors for a 400-M samples/s 80-dB SFDR time-interleaved analog-to-digital converter," *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 3, pp. 1072–1082, Mar. 2005.
- [7] C. Vogel and S. Mendel, "A flexible and scalable structure to compensate frequency response mismatches in time-interleaved adcs," *IEEE Trans. Circuits Syst. I*, vol. 56, no. 11, pp. 2463–2475, 2009.
- [8] H. Johansson and P. Löwenborg, "Reconstruction of nonuniformly sampled bandlimited signals by means of time-varying discrete-time FIR filters," *EURASIP J. Advances Signal Process.*, vol. 2006, 2006.
- [9] N. P. Murphy, A. Krukowski, and I. Kale, "Implementation of wideband integer and fractional delay element," *Electronics Letters*, vol. 30, no. 20, pp. 1658–1659, 1994.
- [10] H. Johansson, P. Löwenborg, and K. Vengattaramane, "Least-squares and minimax design of polynomial impulse response FIR filters for reconstruction of two-periodic nonuniformly sampled signals," *IEEE Trans. Circuits Syst. I*, vol. 54, no. 4, pp. 877–888, Apr. 2007.
- [11] S. Tertinek and C. Vogel, "Reconstruction of nonuniformly sampled bandlimited signals using a differentiator-multiplier cascade," *IEEE Trans. Circuits Syst. I*, vol. 55, no. 8, pp. 2273–2286, 2008.
- [12] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Prentice-Hall, Englewood Cliffs, NJ, USA, 1993.