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Method to Estimate the Position and Orientation of a Triaxial Accelerometer Mounted to an Industrial Manipulator

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Abstract

A novel method to find the orientation and position of a triaxial accelerometer mounted on a six degrees-of-freedom industrial robot is proposed and evaluated on experimental data. The method consists of two consecutive steps, where the first is to estimate the orientation of the sensor data from static experiments. In the second step the sensor position relative to the robot base is identified using sensor readings when the sensor moves in a circular path and where the sensor orientation is kept constant in a path fixed coordinate system. Once the accelerometer position and orientation are identified it is possible to use the sensor in robot model parameter identification and in advanced control solutions. Compared to previous methods, the sensor position estimation is completely new, whereas the orientation is found using an analytical solution to the optimisation problem. Previous methods use a parameterisation where the optimisation uses an iterative solver.

Keywords: Robotics, Accelerometer, Estimation

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***Abstract*— A novel method to find the orientation and position of a triaxial accelerometer mounted on a six degrees-of-freedom industrial robot is proposed and evaluated on experimental data. The method consists of two consecutive steps, where the first is to estimate the orientation of the sensor data from static experiments. In the second step the sensor position relative to the robot base is identified using sensor readings when the sensor moves in a circular path and where the sensor orientation is kept constant in a path fixed coordinate system. Once the accelerometer position and orientation are identified it is possible to use the sensor in robot model parameter identification and in advanced control solutions. Compared to previous methods, the sensor position estimation is completely new, whereas the orientation is found using an analytical solution to the optimisation problem. Previous methods use a parameterisation where the optimisation uses an iterative solver.**

I. INTRODUCTION

A novel method to estimate the position and orientation of a triaxial accelerometer mounted on an industrial robot is presented. The estimation method uses a two step procedure where the first step is to identify the orientation of the sensor using a number of static experiments. It is assumed that the sensor is mounted in such a way that it can be arbitrarily oriented using the six *degrees-of-freedom* (DOF) robot arm. The desired orientation of the sensor is hence known while the actual orientation is unknown. In [1] and [2] the accelerometer calibration is considered and internal parameters of the accelerometer, such as sensitivity and bias, but also alignment of each one of the three accelerometer measurement channels, are identified. The main differences between the approach presented in the present paper, and [1], [2], are that the orientation, sensitivity, and bias are found using an iterative optimisation approach in [1], [2] while in the approach presented in this paper the solution can be found in closed form. In addition, the present method also uses the dynamics of the process to identify the position of the accelerometer. In [1], [2] it is assumed that the accelerometer is moved in such a way that only gravity affects the measurements. In contrast, to identify the position it is necessary to excite the dynamic acceleration, and it is presented how this can be achieved by doing a number of measurements using the motion capabilities of the robot while keeping the accelerometer in different orientations with respect to the path coordinate system. Finally, the proposed method is evaluated on experimental data.

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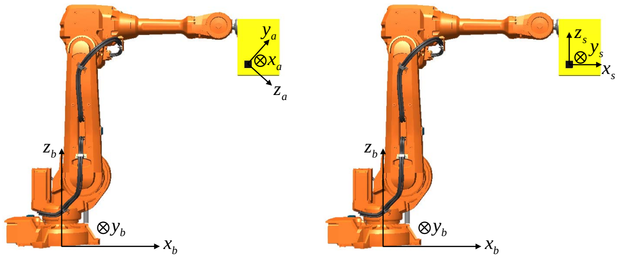
The estimation problem is formulated in Section II. In Section III, the method to find the orientation of the sensor is described, and the method to estimate the mounting position is described in Section IV. The orientation and position estimation is evaluated on experimental data in Section V and Section VI concludes the results.

II. PROBLEM FORMULATION

Assume that the accelerometer is mounted on the robot according to Figure 1(a) where the sensor is assumed to be rigidly attached to the robot tool. Given a definition of the tool coordinate system the estimation method presented in this paper finds the relative orientation and position of the triaxial sensor. The orientation of the desired coordinate system can be seen in Figure 1(b). Let ρ_a be an accelerometer measurement vector in the sensor coordinate system $Ox_ay_az_a$ of the accelerometer and ρ_s an acceleration vector in the desired coordinate system $Ox_sy_sz_s$, describing the acceleration in $\mathrm{m/s^2}$. The relation between ρ_a and ρ_s is given by,

$$\rho_s = \kappa R \rho_a + \rho_0, \tag{1}$$

where R is the rotation matrix R from $Ox_ay_az_a$ to $Ox_sy_sz_s$, κ is the accelerometer sensitivity and ρ_0 the bias. It is assumed that the same sensitivity value κ can be used for all three sensors in the triaxial accelerometer. The sensitivity and bias is chosen such that the units in $Ox_sy_sz_s$ are $\mathrm{m/s^2}$. When the unknown parameters in (1) have been found the position of the accelerometer is identified, expressed relative to the tool coordinate system. To solve for the unknown parameters ρ_a is measured while ρ_s is computed from a model. In the static case ρ_s is simply the gravity vector, while in the dynamic case when the sensor is moved the acceleration will depend on the speed and orientation of the sensor. To be able to divide the estimation problem in two distinct problems the orientation is estimated using static measurements only while the position of the sensor is found by moving the accelerometer along a known path with known speed. Using the known orientation of the accelerometer it is possible to numerically cancel the effect of gravity and only measure the dynamic acceleration, with constant speed in a circular path, perpendicular to the gravity field. The orientation of the accelerometer is kept fixed with respect to the path coordinates during the motion. This means that the acceleration originating from the movement can be isolated from the gravity component. A special case is when $Ox_sy_sz_s$ is rotated such that the coordinate system of the accelerometer is directed to give gravity measurements along one



(a) The accelerometer and its actual coordinate system $Ox_ay_az_a$.

(b) The accelerometer and the desired coordinate system $Ox_sy_sz_s$.

Fig. 1. The accelerometer mounted on the robot. The yellow rectangle represents the tool or a weight and the black square on the yellow rectangle is the accelerometer. The base coordinate system $Ox_by_bz_b$ of the robot is also shown.

coordinate axis only. The two other axes of the accelerometer directly gives the dynamic acceleration component which can be used to estimate the position.

III. Identification of Orientation, Sensitivity and Bias

To solve for the parameters R, κ and ρ_0 in (1), first define the residual

$$e_k = \rho_{s,k} - \kappa R \rho_{a,k} - \rho_0, \tag{2}$$

where k indicates the sample number. Next, minimise the sum of the squared norm of the residuals,

$$\begin{array}{ll} \text{minimise} & \sum_{k=1}^{N} ||e_k||^2 \\ \text{subject to} & \det(R) = 1 \\ & R^T = R^{-1} \end{array} \tag{3}$$

where the constrains guarantee that R is an orthonormal matrix. There exists a closed form solution to this optimisation problem [3],

$$\kappa = \sqrt{\sum_{k=1}^{N} ||\rho'_{s,k}||^2 \Big/ \sum_{k=1}^{N} ||\rho'_{a,k}||^2}, \tag{4a}$$

$$R = M\left(M^T M\right)^{-1/2}, \tag{4b}$$

$$\rho_0 = \bar{\rho}_s - \kappa R \bar{\rho}_a, \tag{4c}$$

where

$$\bar{\rho}_s = \frac{1}{N}\sum_{k=1}^{N} \rho_{s,k}, \tag{5a}$$

$$\bar{\rho}_a = \frac{1}{N}\sum_{k=1}^{N} \rho_{a,k}, \tag{5b}$$

are the centroids for the measurements in $Ox_ay_az_a$ and $Ox_sy_sz_s$.

$$\rho'_{s,i} = \rho_{s,i} - \bar{\rho}_s, \tag{6a}$$

$$\rho'_{a,i} = \rho_{a,i} - \bar{\rho}_a, \tag{6b}$$

denote new coordinates and

$$M = \sum_{k=1}^{N} \rho'_{s,k}(\rho'_{a,k})^T. \tag{7}$$

N is the total number of measurements and it has to be assumed that $N \geq 3$. In addition a condition of sufficient excitement has to be fulfilled, such that $M^T M$ has full rank. As an alternative to the formulation above where the rotation is parameterised by the orthonormal matrix R it is also possible to find a closed-form solution to (1) using unit quaternions, see e.g. [4]. Considering the number of operations the matrix formulation is, however, computationally more efficient.

As indicated in Section II the orientation and the sensor parameters are found using static measurements, i.e., moving the tool into a number, N_C, of different configurations. The gravity vector is measured by the accelerometer in each of the N_C configurations, which gives $N_{M,j}$, $j = 1, \ldots, N_C$ measurements for each configuration. Let

$$\{\rho_a\} = \left\{\{\rho^1_{a,i}\}_{i=1}^{N_{M,1}}, \ldots, \{\rho^{N_C}_{a,i}\}_{i=1}^{N_{M,N_C}}\right\} \tag{8}$$

denote the set of all the $N = \sum_{j=1}^{N_C} N_{M,j}$ measurements in all N_C configurations, and let

$$\{\rho_s\} = \left\{\{\rho^1_{s}\}_{i=1}^{N_{M,1}}, \ldots, \{\rho^{N_C}_{s}\}_{i=1}^{N_{M,N_C}}\right\} \tag{9}$$

be the gravity vector from the model in the desired coordinate system $Ox_sy_sz_s$ for each configuration, where ρ^j_s, $j = 1, \ldots, N_C$ is a constant. Using the measured accelerations and the model values to solve the optimisation problem in (4) to (7) the transformation parameters can be computed.

The N_C different configurations can be chosen arbitrary but here we suggest six different configurations according to Figure 2, which give

$$\begin{array}{ll} \rho^1_s = \begin{pmatrix} 0 & 0 & g \end{pmatrix}^T & , \rho^2_s = \begin{pmatrix} 0 & g & 0 \end{pmatrix}^T, \\ \rho^3_s = \begin{pmatrix} 0 & 0 & -g \end{pmatrix}^T & , \rho^4_s = \begin{pmatrix} 0 & -g & 0 \end{pmatrix}^T, \\ \rho^5_s = \begin{pmatrix} -g & 0 & 0 \end{pmatrix}^T & , \rho^6_s = \begin{pmatrix} g & 0 & 0 \end{pmatrix}^T, \end{array} \tag{10}$$

where $g = 9.81\,\mathrm{m/s^2}$. The sign of g in (10) is opposite the gravity vector in Figure 2. The explanation for this is that an accelerometer measures the normal force which is opposite the gravity vector.

The six configurations in Figure 2 are straightforward to obtain for a six degree of freedom industrial manipulator [5]. The procedure to estimate the triaxial accelerometer sensor parameters is summarised in Algorithm 1.

Algorithm 1 *Estimation of the sensor parameters*

1) *Measure the acceleration for the different configurations in Figure 2 to obtain* $\{\rho_a\}$ *according to* (8).
2) *Construct* $\{\rho_s\}$ *in* (9) *from* (10).
3) *Calculate* R, κ *and* ρ_0 *from* (4) *to* (7).

It is possible to use other configurations than the one in Figure 2 in Algorithm 1 as long as $M^T M$ has full rank[1].

[1]The matrix $M^T M$ has always full rank if none of the two sets $\{\rho_a\}$ and $\{\rho_s\}$ are coplanar.

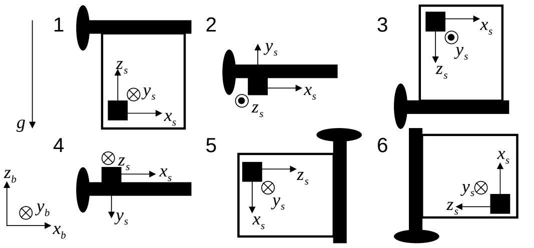


Fig. 2. Six different configurations of the robot tool used in Algorithm 1. The orientation of the desired coordinate system $Ox_sy_sz_s$ is shown for each configuration. The base coordinate system $Ox_by_bz_b$ and the gravity vector are also shown.

IV. Estimation of the Position of the Accelerometer

Using a mathematical model of the robot motion it is possible to compute the acceleration, parameterised in some unknown parameters. In the second step of the proposed orientation and position estimation process a method for the position estimation is explained for the accelerometer's coordinate system $Ox_sy_sz_s$, expressed in a coordinate system $Ox_{bf}y_{bf}z_{bf}$ fixed to the robot. From Section III the orientation and sensor parameters are known, hence the acceleration measured by the accelerometer has a known orientation.

To simplify the mathematical model for the acceleration and to make it possible to parameterise the unknown parameters , consider the case when the robot is in the configuration shown in Figure 3. The figure shows the vector r_s, the two coordinate systems $Ox_{bf}y_{bf}z_{bf}$ and $Ox_sy_sz_s$, a world fixed coordinate system $Ox_by_bz_b$ attached to the base of the robot, a coordinate system $Ox_wy_wz_w$ fixed to the end of the robot arm, a vector $a_s \triangleq \frac{d^2}{dt^2}(r_s)$ describing the acceleration of $Ox_sy_sz_s$, which we want to find an expression for. The figure also shows a parameter θ describing the rotation between $Ox_{bf}y_{bf}z_{bf}$ and $Ox_by_bz_b$, two known parameters L_1 and L_2 describing the arm lengths and three unknown parameters l_i, $i = 1, 2, 3$ describing the vector $r_{s/w}$ in $Ox_wy_wz_w$.

All the calculations are done in the world fixed coordinate system in order to obtain an expression for $\frac{d^2}{dt^2}(r_s)$. In a body fixed coordinate system $Ox_{bf}y_{bf}z_{bf}$ $\frac{d^2}{dt^2}(r_s) = 0$. The notation $[r_s]_i$ is used to emphasise that r_s is expressed in coordinate system i.

In Figure (3) we see that r_s can be written as a sum of two vectors,

$$[r_s]_{bf} = [r_w]_{bf} + [r_{s/w}]_{bf}, \tag{11}$$

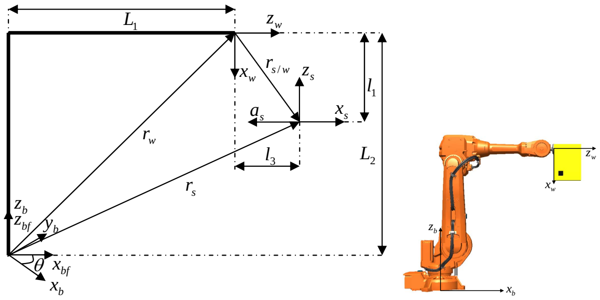
where

$$[r_{s/w}]_{bf} = \begin{pmatrix} l_3 & -l_2 & -l_1 \end{pmatrix}^T, \tag{12}$$

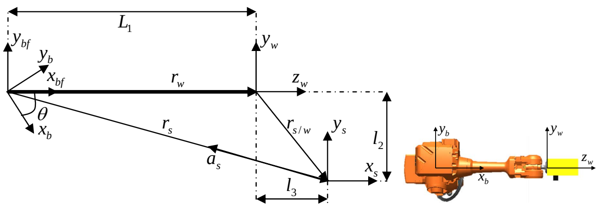
$$[r_w]_{bf} = \begin{pmatrix} L_1 & 0 & L_2 \end{pmatrix}^T. \tag{13}$$

The transformation of r_s from $Ox_{bf}y_{bf}z_{bf}$ to $Ox_by_bz_b$ can be expressed as

$$[r_s]_b = [Q_{bf/b}]_b \left([r_w]_{bf} + [r_{s/w}]_{bf}\right), \tag{14}$$



(a) From the side.



(b) From above.

Fig. 3. The first robot configuration for estimation of the mounting position. The black cube on the yellow box indicates the sensor, i.e., the origin of $Ox_sy_sz_s$. The yellow box is attached to the robot in the point $\begin{pmatrix} L_1 & 0 & L_2 \end{pmatrix}^T$ expressed in $Ox_{bf}y_{bf}z_{bf}$.

where

$$[Q_{bf/b}]_b = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{15}$$

is the rotation matrix that relates the coordinate system $Ox_{bf}y_{bf}z_{bf}$ to $Ox_by_bz_b$. $\theta = \theta(t)$ is the angle relating $Ox_by_bz_b$ and $Ox_{bf}y_{bf}z_{bf}$ according to Figure 3. Taking the derivative of $[r_s]_b$ with respect to time gives

$$\frac{d}{dt}\left([r_s]_b\right) = \frac{d}{dt}\left([Q_{bf/b}]_b\right)\left([r_w]_{bf} + [r_{s/w}]_{bf}\right). \tag{16}$$

From [6] we have that

$$\frac{d}{dt}\left([Q_{bf/b}]_b\right) = S(\omega)[Q_{bf/b}]_b, \tag{17}$$

where $\omega = \begin{pmatrix} 0 & 0 & \dot{\theta} \end{pmatrix}^T$ and

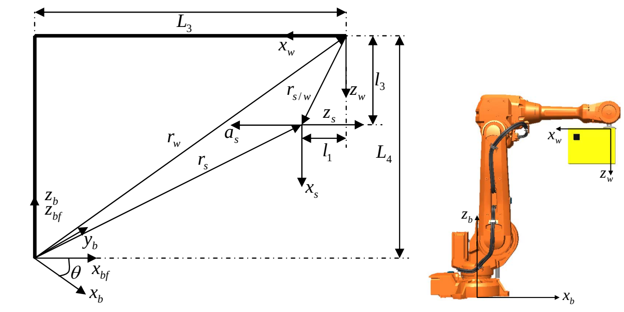
$$S(\omega) = \begin{pmatrix} 0 & -\dot{\theta} & 0 \\ \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{18}$$

is a skew symmetric matrix. Hence, the time derivative of $[r_s]_b$ can be written

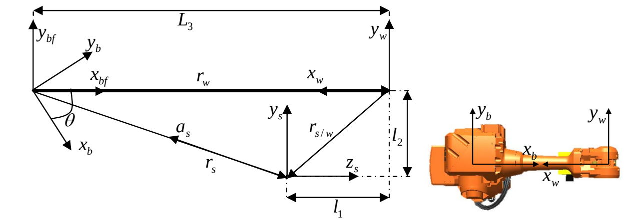
$$\frac{d}{dt}\left([r_s]_b\right) = S(\omega)[Q_{bf/b}]_b\left([r_w]_{bf} + [r_{s/w}]_{bf}\right). \tag{19}$$

The second time derivative of $[r_s]_b$ becomes

$$\begin{aligned}
[a_s]_b =& \frac{d^2}{dt^2}\left([r_s]_b\right) = \frac{d}{dt}\left(S(\omega)\right)[Q_{bf/b}]_b\left([r_w]_{bf} + [r_{s/w}]_{bf}\right) \\
&+ S(\omega)\frac{d}{dt}\left([Q_{bf/b}]_b\right)\left([r_w]_{bf} + [r_{s/w}]_{bf}\right) \\
=& S(\dot{\omega})[Q_{bf/b}]_b\left([r_w]_{bf} + [r_{s/w}]_{bf}\right) \\
&+ S(\omega)S(\omega)[Q_{bf/b}]_b\left([r_w]_{bf} + [r_{s/w}]_{bf}\right) \\
=& S(\omega)S(\omega)[Q_{bf/b}]_b\left([r_w]_{bf} + [r_{s/w}]_{bf}\right),
\end{aligned} \tag{20}$$

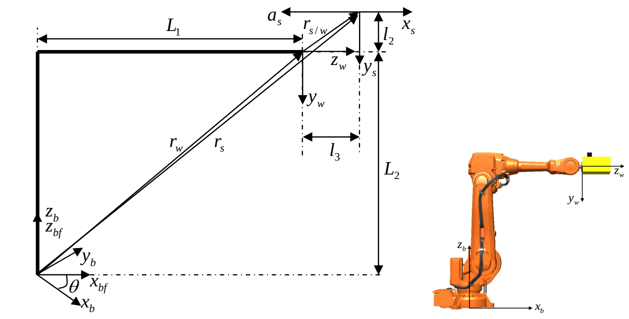


(a) From the side.

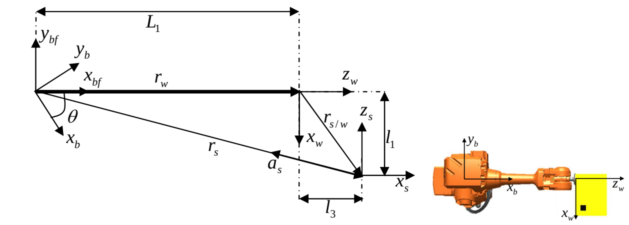


(b) From above.

Fig. 4. The second robot configuration for estimation of the mounting position. The black cube on the yellow box indicates the sensor, i.e., the origin of $Ox_sy_sz_s$. The yellow box is attached to the robot in the point $\begin{pmatrix} L_3 & 0 & L_4 \end{pmatrix}^T$ expressed in $Ox_{bf}y_{bf}z_{bf}$.



(a) From the side.



(b) From above.

Fig. 5. The third robot configuration for estimation of the mounting position. The black cube on the yellow box indicates the sensor, i.e., the origin of $Ox_sy_sz_s$. The yellow box is attached to the robot in the point $\begin{pmatrix} L_1 & 0 & L_2 \end{pmatrix}^T$ expressed in $Ox_{bf}y_{bf}z_{bf}$.

where $\dot{\omega} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$ follows from the assumption of constant angular velocity.

It now remains to transform the measured acceleration a_s^M from $Ox_sy_sz_s$ to $Ox_by_bz_b$. From Figure 3 we see directly that

$$[a_s^M]_{bf} = \begin{pmatrix} a_{s,x}^M & a_{s,y}^M & 0 \end{pmatrix}^T, \tag{21}$$

hence

$$[a_s^M]_b = [Q_{bf/b}]_b [a_s^M]_{bf}. \tag{22}$$

Equations (20) and (22) give

$$\begin{gathered} [Q_{bf/b}]_b [a_s^M]_{bf} = S(\omega)S(\omega)[Q_{bf/b}]_b \left([r_w]_{bf} + [r_{s/w}]_{bf}\right) \\ \Leftrightarrow \\ [a_s^M]_{bf} = [Q_{bf/b}]_b^T S(\omega)S(\omega)[Q_{bf/b}]_b \left([r_w]_{bf} + [r_{s/w}]_{bf}\right) \end{gathered} \tag{23}$$

since $[Q_{bf/b}]_b^T = [Q_{bf/b}]_b^{-1}$. Carrying out the matrix multiplication in the right hand side expression of (23) gives

$$[a_s^M]_{bf} = \begin{pmatrix} -\dot{\theta}^2(L_1 + l_3) \\ \dot{\theta}^2 l_2 \\ 0 \end{pmatrix}, \tag{24}$$

where (12), (13), (15) and (18) have been used. Equations (21) and (24) can now be written as a system of equations where l_2 and l_3 are unknown,

$$\begin{pmatrix} 0 & -\dot{\theta}^2 \\ \dot{\theta}^2 & 0 \end{pmatrix} \begin{pmatrix} l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} a_{s,x}^M + \dot{\theta}^2 L_1 \\ a_{s,y}^M \end{pmatrix}. \tag{25}$$

It is thus possible to find l_2 and l_3 from (25) but unfortunately not l_1. To find l_1, rotate the sensor according to Figure 4 and do the same kind of movement. The same calculations as before with

$$[r_{s/w}]_{bf} = \begin{pmatrix} -l_1 & -l_2 & -l_3 \end{pmatrix}^T, \tag{26}$$

$$[r_w]_{bf} = \begin{pmatrix} L_3 & 0 & L_4 \end{pmatrix}^T, \tag{27}$$

$$[a_s^M]_{bf} = \begin{pmatrix} a_{s,z}^M & a_{s,y}^M & 0 \end{pmatrix}^T, \tag{28}$$

see Figure 4, give

$$\begin{pmatrix} \dot{\theta}^2 & 0 \\ 0 & \dot{\theta}^2 \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} = \begin{pmatrix} a_{s,z}^M + \dot{\theta}^2 L_3 \\ a_{s,y}^M \end{pmatrix}. \tag{29}$$

Equations (25) and (29) can now be used to estimate the unknown parameters. The estimation of l_i, $i = 1, 2, 3$ will be more accurate if more data are used with different configurations. Therefore, one more robot configuration is used according to Figure 5, which gives

$$[r_{s/w}]_{bf} = \begin{pmatrix} l_3 & -l_1 & l_2 \end{pmatrix}^T, \tag{30}$$

$$[r_w]_{bf} = \begin{pmatrix} L_1 & 0 & L_2 \end{pmatrix}^T, \tag{31}$$

$$[a_s^M]_{bf} = \begin{pmatrix} a_{s,x}^M & a_{s,z}^M & 0 \end{pmatrix}^T. \tag{32}$$

From (23) we now get

$$\begin{pmatrix} 0 & -\dot{\theta}^2 \\ \dot{\theta}^2 & 0 \end{pmatrix} \begin{pmatrix} l_1 \\ l_3 \end{pmatrix} = \begin{pmatrix} a_{s,x}^M + \dot{\theta}^2 L_1 \\ a_{s,z}^M \end{pmatrix}. \tag{33}$$

Equations (25), (29) and (33) can now be written as one system of equations according to

$$\underbrace{\begin{pmatrix} 0 & 0 & -\dot{\theta}_{c1}^2 \\ 0 & \dot{\theta}_{c1}^2 & 0 \\ \dot{\theta}_{c2}^2 & 0 & 0 \\ 0 & \dot{\theta}_{c2}^2 & 0 \\ 0 & 0 & -\dot{\theta}_{c3}^2 \\ \dot{\theta}_{c3}^2 & 0 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}}_{l} = \underbrace{\begin{pmatrix} a_{s,x,c1}^M + \dot{\theta}_{c1}^2 L_1 \\ a_{s,y,c1}^M \\ a_{s,z,c2}^M + \dot{\theta}_{c2}^2 L_3 \\ a_{s,y,c2}^M \\ a_{s,x,c3}^M + \dot{\theta}_{c3}^2 L_1 \\ a_{s,z,c3}^M \end{pmatrix}}_{b}, \quad (34)$$

where index ci, $i = 1, 2, 3$ indicates from which robot configuration the measurements come from. Equation (34) has more rows than unknowns, hence the solution to (34) is given by the solution to the optimisation problem

$$\arg\min_{l} ||b - Al||_2^2, \quad (35)$$

which has the analytical solution

$$\hat{l} = \left(A^T A\right)^{-1} A^T b. \quad (36)$$

There exist better numerical solutions to (34) than (36), e.g. `l=A\b` in MATLAB. The procedure to estimate the position of the accelerometer is summarised in Algorithm 2.

Algorithm 2 *Estimation of the mounting position*

1) *Measure the acceleration of the tool* $[a_s^M]_s$ *and the angular velocity* $\dot{\theta}$ *for the three different configurations in Figures 3, 4 and 5 when* θ *varies from* θ_{min} *to* θ_{max} *with constant angular velocity.*
2) *Construct* A *and* b *in* (34).
3) *Solve* (34) *with respect to* l*, for example according to* (36).

V. Experimental Results

In this section the proposed orientation and position estimation method described in the two algorithms in Sections III and IV is evaluated using experimental data. For Algorithm 1, the data, i.e., the acceleration values, are collected during 4 s for each one of the six configurations in Figure 2 using a sample rate of 2 kHz. For Algorithm 2, the arm angular velocity $\dot{\theta}$ for joint 1 and the acceleration measurements are collected when the robot is in the three different configurations according to Figures 3, 4 and 5. The arm angular velocity for joint 1 is computed from the motor angular velocity $\dot{\theta}_m$ using,

$$\dot{\theta}_m = \tau\dot{\theta}, \quad (37)$$

where τ is the gear ratio. In the position estimation experiments data are collected during 4 s in each one of the three configurations, but it is only the constant angular velocity part of the data that is used. The same sample rate as before is used, i.e., 2 kHz. The accelerometer used in the experiments is a triaxial accelerometer from Crossbow Technology, with a range of ± 2 g, and a sensitivity of approximately 1 V/g [7]. The accelerometer is connected to the measurement system of the robot controller, and hence the acceleration and motor angular velocity can be synchronised and measured with the same sampling rate.

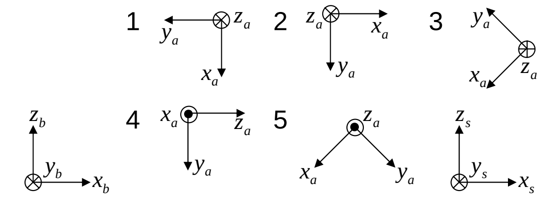


Fig. 6. Orientation for the five mounting positions that were used to evaluate the two algorithms. The orientation of the base coordinate system and the desired coordinate system are also shown.

Five different mounting positions and different orientations of the accelerometer have been used for evaluation of Algorithms 1 and 2. The actual physical orientation of the sensor was measured using a protractor, see Figure 6, where the orientation of the desired sensor coordinate system also is shown.

Algorithm 1 was applied to the five test cases presented above and the result $\hat{R}$, $\hat{\kappa}$ and $\hat{\rho}_0$ can be seen in Table I. From Figure 6 we have that the rotation matrix R in (1) should resemble

$$R^1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, R^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$R^3 = \begin{pmatrix} -a^3 & -b^3 & 0 \\ 0 & 0 & 1 \\ -c^3 & d^3 & 0 \end{pmatrix}, R^4 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

$$R^5 = \begin{pmatrix} -a^5 & b^5 & 0 \\ 0 & 0 & -1 \\ -c^5 & -d^5 & 0 \end{pmatrix},$$

where a, b, c and d are positive numbers that should be close to $\cos(45°) \approx 0.7071$. The superscript indicates the test number. A rotational difference between the measured rotation matrix R^i and the estimated matrix $\hat{R}^i$ can be computed using the corresponding unit quaternions $\mathfrak{q}^i$ and $\hat{\mathfrak{q}}^i$. The rotation angle ϑ^i of $\mathfrak{q}_\Delta^i$ from $\mathfrak{q}_\Delta^i = \left(\mathfrak{q}^i\right)^{-1} * \hat{\mathfrak{q}}^i$, which should be small, is a good measure of the difference between R^i and $\hat{R}^i$. See e.g. [5] for a short introduction to quaternions. The resulting rotation angle ϑ^i for the five test cases can be seen i Table II. The difference is small in all cases but for test 3 and 5 a larger deviation can be seen. One explanation for this is that it is more difficult to mount the accelerometer in a configuration not aligned with the robot tool, as seen in Figure 1.

It is more difficult to obtain true values for the parameters κ and ρ_0. To verify them, the measured acceleration for all five test cases in configuration 1, in Figure 2, is transformed from $Ox_a y_a z_a$ to $Ox_s y_s z_s$, which results in three constant signals $a_{s,x}^M$, $a_{s,y}^M$ and $a_{s,z}^M$ for the three axes of the accelerometer. Figure 2 shows that the measured acceleration in frame $Ox_s y_s z_s$ should resemble $a_{s,x} = 0$, $a_{s,y} = 0$ and $a_{s,z} = g$. Subtracting $a_{s,j}$ from the mean of $a_{s,j}^M$, $j = x, y, z$, gives an error for the transformed acceleration. A diagram of the errors for each coordinate axis in $Ox_s y_s z_s$ is shown in Figure 7. The diagram shows the median as the central mark, the edges of the box are the 25th and 75th percentiles and

TABLE I

ESTIMATED PARAMETERS IN (1) USING ALGORITHM 1 FOR FIVE DIFFERENT TEST CASES.

Test	$\hat{\kappa}$	$\hat{\rho}_0$	$\hat{R}$
1	9.91	$\begin{pmatrix} 25.05 \\ -23.75 \\ 24.26 \end{pmatrix}$	$\begin{pmatrix} -0.0138 & -0.9998 & -0.0170 \\ -0.0094 & -0.0169 & 0.9998 \\ -0.9999 & 0.0140 & -0.0092 \end{pmatrix}$
2	9.91	$\begin{pmatrix} -23.89 \\ -24.03 \\ 25.11 \end{pmatrix}$	$\begin{pmatrix} 0.9999 & -0.0070 & -0.0131 \\ 0.0129 & -0.0276 & 0.9995 \\ -0.0073 & -0.9996 & -0.0275 \end{pmatrix}$
3	9.91	$\begin{pmatrix} 34.80 \\ -23.73 \\ 3.07 \end{pmatrix}$	$\begin{pmatrix} -0.6348 & -0.7724 & -0.0208 \\ -0.0027 & -0.0247 & 0.9997 \\ -0.7727 & 0.6347 & 0.0135 \end{pmatrix}$
4	9.91	$\begin{pmatrix} -24.46 \\ 24.86 \\ 23.74 \end{pmatrix}$	$\begin{pmatrix} 0.0169 & -0.0139 & 0.9998 \\ -0.9992 & -0.0355 & 0.0164 \\ 0.0353 & -0.9993 & -0.0145 \end{pmatrix}$
5	9.92	$\begin{pmatrix} -3.91 \\ 24.95 \\ 33.81 \end{pmatrix}$	$\begin{pmatrix} -0.6314 & 0.7751 & 0.0209 \\ -0.0269 & 0.0050 & -0.9996 \\ -0.7750 & -0.6318 & 0.0177 \end{pmatrix}$

TABLE II

THE ROTATION ANGLE ϑ INDICATES HOW CLOSE THE ESTIMATED AND MEASURED ROTATION MATRICES ARE TO EACH OTHER. THE MATRICES ARE IDENTICAL IF $\vartheta = 0°$

Test	1	2	3	4	5
ϑ	1.4°	1.8°	5.8°	2.4°	6.0°

the dashed lines extend to the most extreme error. The errors are small and, as expected, the errors are larger in x and y due to the higher sensitivity to orientation errors in these axis when measuring gravity along the z-axis. The bias in x can be explained by a systematic error in orientation due to the robot elasticity and gravitational force acting on the robot in the evaluation position, see Figure 1.

Algorithm 2 was also applied for the five test cases. Figure 8 shows how the measured data, i.e., the acceleration in $Ox_sy_sz_s$ and the arm angular velocity, can look like when the robot is in the configuration according to Figure 3. Note that it is only the sequence where the angular velocity is constant, in this case around 3 rad/s, that is used. From Figure 3 we see that the acceleration in the z-direction only originate from the gravity which is verified by Figure 8(a). We also see that the acceleration due to the circular motion should be in the negative x-direction and in the positive

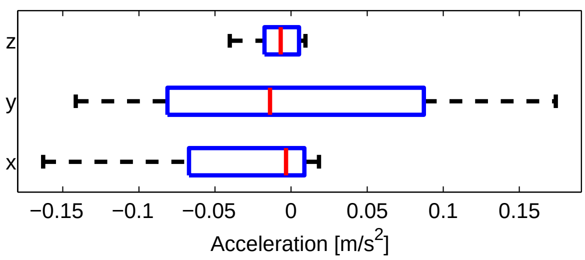
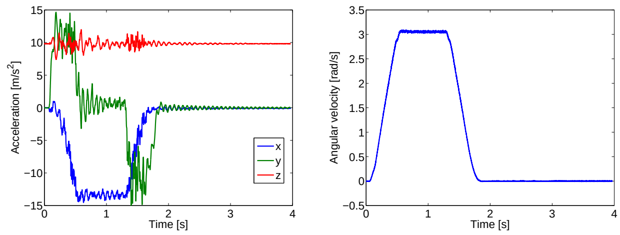


Fig. 7. Diagram of the transformation errors in the x-, y- and z-direction for (1) in configuration 1 (Figure 2) for all five test cases. The central mark is the median, the edges of the box are the 25th and 75th percentiles and the dashed lines extend to the most extreme error.

TABLE III

ESTIMATED POSITIONS $\hat{l}$ OF THE ACCELEROMETER IN THE COORDINATE SYSTEM $Ox_wy_wz_w$ FOR FIVE DIFFERENT MOUNTING POSITIONS. Δ IS THE ERROR RELATIVE THE MEASURED POSITION l^M.

Test	Estimated position (l) [cm]	$\Delta = \hat{l} - l^M$ [cm]
1	$(35.20 \quad 6.27 \quad 15.50)^T$	$(0.2 \quad 2.3 \quad -1.0)^T$
2	$(14.20 \quad 5.82 \quad 16.85)^T$	$(-0.3 \quad -1.2 \quad 1.8)^T$
3	$(36.33 \quad 6.29 \quad 21.38)^T$	$(-1.7 \quad 2.3 \quad -1.6)^T$
4	$(29.19 \quad 1.60 \quad 5.86)^T$	$(2.2 \quad 1.6 \quad 0.4)^T$
5	$(34.75 \quad -3.91 \quad 16.50)^T$	$(-0.7 \quad 0.1 \quad 1.0)^T$



(a) Measured acceleration in $Ox_sy_sz_s$. (b) Measured arm angular velocity.

Fig. 8. Measured data, to be used to estimate the position l, for test 1 when the robot is in the configuration according to Figure 3.

y-direction which is the case in Figure 8(a). Hence, the transformation from $Ox_ay_az_a$ to $Ox_sy_sz_s$, given by the identified parameters in (1), is correct.

The estimated position $\hat{l}$ for the five test cases can be seen in Table III. Note that $\hat{l}_2$ for test five is negative which comes from the fact that the sensor is placed on the other side of the weight than was used in the derivation in Section IV. The table also shows the error Δ between $\hat{l}$ and the measured position l^M. The position was always measured using a tape measure to the centre of the accelerometer, since the position of the origin of the accelerometer's coordinate system inside the sensor is unspecified. Considering the accuracy of the measurements and the uncertainty of the origin of the accelerometer coordinate system the result in Table III is considered as acceptable. The actual requirement of the result, in terms of position and orientation accuracy, will depend on the application where the accelerometer is used. A more detailed investigation of the requirement for the accuracy in the dynamic position and orientation estimation of the tool position, such as described in [8], is left as future work.

VI. CONCLUSIONS

A method to find the position and orientation of a triaxial accelerometer mounted on a six DOF robot is presented. The method is divided into two main steps, where in the first step, the orientation is estimated by finding the transformation from the actual coordinate system of the accelerometer, with unknown orientation, to a new coordinate system with known orientation. It is also possible to find the sensitivity and the bias parameters. The estimation of the orientation is based on static measurements of the gravity vector when the accelerometer is placed in different orientations using the six DOF robot arm. In the second step of the method, the mounting position of the accelerometer in a robot fixed coordinate system is computed using several experiments where the robot is moving with constant speed. Finally, the method is evaluated on experimental data. The resulting position and orientation accuracy are evaluated using measurements on the physical system. The orientation error is in the range 1 to 6 degrees and the position error up to 2 cm. The accuracy is sufficient in experiments with dynamic position and orientation estimation of the tool position using sensor fusion methods, such as extended Kalman filter and particle filter.

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REFERENCES

[1] E. L. Renk, W. Collins, M. Rizzo, F. Lee, and D. S. Bernstein, "Calibrating a triaxial accelerometer-magnetometer—using robotic actuation for sensor reorientation during data collection," *Control Systems Magazine*, vol. 25, no. 6, pp. 86–95, December 2005.

[2] S.-h. P. Won and F. Golnaraghi, "A triaxial accelerometer calibration method using a mathematical model," *IEEE Transactions on Instrumentation and Measurement*, vol. 59, no. 8, pp. 2144–2153, Aug 2010.

[3] B. K. P. Horn, H. M. Hilden, and S. Negahdaripour, "Closed-form solution of absolute orientation using orthonormal matrices," *Journal of the Optical Society of America*, vol. 5, no. 7, pp. 1127–1135, July 1988.

[4] B. K. P. Horn, "Closed-form solution of absolute orientation using unit quaternions," *Journal of the Optical Society of America*, vol. 4, no. 4, pp. 629–642, April 1987.

[5] L. Sciavicco and B. Siciliano, *Modelling and Control of Robot Manipulators*, 2nd ed. London, UK: Springer, 2000.

[6] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. John Wiley & Sons, 2005.

[7] Crossbow Technology, "Accelerometers, High Sensitivity, LF Series, CXL02LF3," Jan. 2004, http://www.xbow.com.

[8] R. Henriksson, M. Norrlöf, S. Moberg, E. Wernholt, and T. B. Schön, "Experimental comparison of observers for tool position estimation of industrial robots," in *Proceedings of 48th IEEE Conference on Decision and Control*, Shanghai, China, December 2009, pp. 8065–8070.

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Method to Estimate the Position and Orientation of a Triaxial Accelerometer Mounted to an Industrial Manipulator

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Sammanfattning
Abstract

A novel method to find the orientation and position of a triaxial accelerometer mounted on a six degrees-of-freedom industrial robot is proposed and evaluated on experimental data. The method consists of two consecutive steps, where the first is to estimate the orientation of the sensor data from static experiments. In the second step the sensor position relative to the robot base is identified using sensor readings when the sensor moves in a circular path and where the sensor orientation is kept constant in a path fixed coordinate system. Once the accelerometer position and orientation are identified it is possible to use the sensor in robot model parameter identification and in advanced control solutions. Compared to previous methods, the sensor position estimation is completely new, whereas the orientation is found using an analytical solution to the optimisation problem. Previous methods use a parameterisation where the optimisation uses an iterative solver.

Nyckelord
Keywords

Robotics, Accelerometer, Estimation