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Amplifier-Aware Multiple-Input Multiple-Output Power Allocation

Daniel Persson, Thomas Eriksson, and Erik G. Larsson

Abstract—We propose multiple-input multiple-output (MIMO) transmitter power allocation which takes dissipation in the power amplifiers into account. We derive the equations of the general problem with full channel state information (CSI), discuss its challenges, and supply solutions in two special cases, namely for a multiple-input single-output channel with a linear beamformer constraint, and for a parallel MIMO channel. The proposed algorithms show substantial gains in terms of rate and total consumed power compared to previous state of the art over a wide range of parameter settings, and have low computational complexities.

Index Terms—Multiple-input multiple-output (MIMO), power amplifiers, power dissipation, power allocation

I. INTRODUCTION

Today's cellular multiple-input multiple-output (MIMO) communication systems increase both robustness and capacity [1]. If the transmitter has full knowledge of the channel, it can apply power allocation in order to optimize the transmission according to different criteria. For single-user (SU) MIMO with circularly symmetric complex Gaussian noise with equal variance for the i.i.d. real and imaginary components (Gaussian) noise and full channel state information (CSI) at the transmitter and receiver, Telatar shows in [2] that singular value decomposition-based diagonalization of the channel together with waterfilling transmit power allocation achieves capacity. A special case of the MIMO channel is the multiple-input single-output (MISO) channel, where the above strategy reduces to maximum-ratio transmission (MRT) [1, p. 180].

A sum-power constraint is not realistic in that it does not take the maximum output constraint of the individual amplifiers into account. Per-antenna constraints are considered for SU MIMO in [3]. The papers [4], [5], [6], [7], and [8] investigate the multi-user downlink channel with per-antenna constraints. For the SU MISO channel, the capacity under per-antenna constraints but without a total power constraint was established on closed form in [9].

In the above works, the mutual information was maximized under an assumption of a limited output power. However, in many applications it is desirable to instead limit the total consumed power, consisting of both output power and losses in the transmitter chain. As we discuss in Sec. II, the scientific literature agrees to a rather high extent that the power amplifier is the largest source of losses in the transmitter, and there is also agreement on what analytic form the losses have.

A. Our contribution

In this work, we utilize the analytic expression of amplifier losses in Sec. II to design MIMO beamforming schemes. Our contributions are the following.

- For MIMO transmitter optimization, we propose to minimize the power amplifier total consumed power as defined in Sec. II, instead of the sum of the output power.
- We formulate the general SU MIMO consumed power optimization problem, and discuss its challenges. We take per-antenna output power constraints into consideration as well.
- We supply solutions in two special cases, namely for a MISO channel with a linear beamformer constraint, and for a parallel MIMO channel. Signal processing computational complexity is also associated with a power cost. We therefore quantify the computational complexities of the precoding schemes and show that these are low. We observe that our MISO solution, differently from the traditional MRT beamforming, is such that some antennas in general are turned off. Turning off whole radio frequency chains saves additional energy in filters, mixers, and cooling systems. We hope the proposed solutions will inspire future solutions to the general MIMO case.

Scalars are denoted by lower-case letters, vectors are denoted by bold-face lower-case letters, and matrices are denoted by bold-face upper-case letters. A vector $\mathbf{x}$ of length N has components x_i , where $i = 1, \dots, N$, and a matrix $\mathbf{X}$ of size $M \times N$ has elements $x_{m,n}$, with $m = 1, \dots, M$ and $n = 1, \dots, N$.

II. POWER AMPLIFIER MODELING

It is well known that in a communication link, the majority of the power losses stems from the last parts of the transmitter chain, particularly the power amplifier [10]–[12]. In the literature, there are some efforts to derive a general analytical expression for the losses and efficiency of power amplifiers. As an example, it has been shown that an ideal class B amplifier has a peak efficiency of

$$\frac{P}{P_{\text{cons}}} = \frac{\pi}{4}, \quad (1)$$

which is obtained when the generated output power P is at its maximum P_{max} , see [13, Equation (6.94)]. In (1), P_{cons} is the overall consumed power. All powers are calculated as means over the constellation. For lower output powers, it is shown by many authors, e.g., [13, Equation (6.93)], that the efficiency reduces with the square root of the output power, according to

$$\frac{P}{P_{\text{cons}}} = \eta_{\text{max}} \sqrt{\frac{P}{P_{\text{max}}}}, \quad (2)$$

where $\eta_{\text{max}} \in [0, 1]$ is the maximum power efficiency obtained only when $P = P_{\text{max}}$. If $\eta_{\text{max}} = 0$, all power is dissipated. In

[14], Mikami et al. hold that the efficiency can be written as

$$\frac{P}{P_{\text{cons}}} = \eta_{\text{max}} \left(\frac{P}{P_{\text{max}}} \right)^\epsilon, \quad (3)$$

where ϵ is a parameter that can vary between $\frac{1}{2}$ and 0.64. Björnemo [15, Equation (2.14)] agrees to this model, and shows its similarity with other models in the literature [16]. Tsuori and Wulich [17, Equation (6)] and Wulich [18, Equation (6) and Table I] give a similar expression, which can be reworked into the expression above, with $\epsilon = 0.52, \dots, 0.54$ depending on the amplifier class. In our own laboratory at Chalmers University, efficiency measurements of a class D amplifier [19] has indicated that the equation is indeed quite useful and accurate, using $\epsilon = 0.5$. Thus, in this paper we will use the efficiency expression as above, with $\epsilon = 0.5$. Solving for the consumed power, which is the power we seek to minimize, we have

$$P_{\text{cons}} = \frac{1}{\eta_{\text{max}}} \sqrt{PP_{\text{max}}}, \quad (4)$$

so that the consumed power is proportional to the square root of the output power.

III. MIMO TRANSMISSION WITH CONSUMED POWER CONSTRAINT

For the transmission, we have a SU baseband channel model given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{u}, \quad (5)$$

where $\mathbf{x} \in \mathbb{C}^{N_T \times 1}$ is the transmitted vector, $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix, $\mathbf{u} \in \mathbb{C}^{N_R \times 1}$ is the channel noise vector, and $\mathbf{y} \in \mathbb{C}^{N_R \times 1}$ is the received vector. The transmitted symbol vector fulfills $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^\dagger]$, and $E[\mathbf{u}\mathbf{u}^\dagger] = N_0\mathbf{I}$, where $\mathbf{I} \in \mathbb{C}^{N_R \times N_R}$ is the identity matrix. We consider full CSI at the transmitter and receiver. Given (4), we want to optimize the mutual information [2] under total and per-antenna power constraints

$$\begin{aligned} & \underset{\mathbf{Q}}{\text{maximize}} && \log_2 \left(\left| \mathbf{I} + \frac{1}{N_0} \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger \right| \right) \\ & \text{subject to} && \frac{\sqrt{P_{\text{max}}}}{\eta_{\text{max}}} \sum_{i=1}^{N_T} \sqrt{Q_{i,i}} \leq P_{\text{tot}} \\ & && Q_{i,i} \leq P_{\text{max}}, \quad i = 1, \dots, N_T \\ & && \mathbf{Q} \geq 0, \end{aligned} \quad (6)$$

where P_{tot} is the total consumed power. Even if considering the aforementioned strategies [4], [6], [7], [9], in Sec. I, it is not known how to solve (6) (especially with the sum power constraint involving the sums of square roots of the individual antenna output powers) with low computational complexity. In this paper, we treat two special cases of practical interest and give low computational complexity solutions. The first considered case is a MISO channel, i.e., $N_R = 1$, and we set $\mathbf{H} = \mathbf{h}^T$. Even with $\mathbf{H} = \mathbf{h}^T$, a low-complexity solution to (6) is an open problem. We also impose a rank-1 constraint on $\mathbf{Q}$, i.e., we choose to work with linear beamformers, see Sec. IV.

In the second special case of interest, we assume parallel

Gaussian channels, i.e., $\mathbf{H} = \text{diag}([h_{1,1}, \dots, h_{N_T, N_T}]^T)$, see Sec. V. Our hope is that these solutions will inspire future solutions to the general problem (6).

IV. LINEAR BEAMFORMING FOR THE MISO CHANNEL

With $N_R = 1$, $\mathbf{H} = \mathbf{h}^T$, and the rank-1 constraint, we factorize $\mathbf{Q} = \mathbf{a}\mathbf{a}^\dagger$ and rewrite (6) as

$$\begin{aligned} & \underset{\mathbf{a}}{\text{maximize}} && \log_2 \left(1 + \frac{1}{N_0} |\mathbf{h}^T \mathbf{a}|^2 \right) \\ & \text{subject to} && \frac{\sqrt{P_{\text{max}}}}{\eta_{\text{max}}} \sum_{i=1}^{N_T} |a_i| \leq P_{\text{tot}} \\ & && |a_i|^2 \leq P_{\text{max}}, \quad i = 1, \dots, N_T. \end{aligned} \quad (7)$$

We observe that maximizing the maximand in (7) corresponds to maximizing the expression $|\mathbf{h}^T \mathbf{a}|^2$. Since the phase of $\mathbf{a}$ is no part of the constraint and thus can be chosen freely, we observe that $|\mathbf{h}^T \mathbf{a}|$ is maximized if the components are all aligned, i.e., $\arg(h_1 a_1) = \arg(h_2 a_2) = \dots = \arg(h_{N_T} a_{N_T})$, i.e., we strive for an alignment of the contributions from the different transmit antennas at the receiver. We can thus write

$$\mathbf{a} = \begin{bmatrix} \frac{h_1^*}{|h_1|} b_1 \\ \vdots \\ \frac{h_{N_T}^*}{|h_{N_T}|} b_{N_T} \end{bmatrix}, \quad (8)$$

where $\mathbf{b} \in \mathbb{R}^{N_T \times 1}$ with elements greater than or equal to 0. Equation (8) is up to an arbitrary phase factor that we have chosen to be 1. Given (8), we simplify (7) to

$$\begin{aligned} & \underset{\mathbf{b}}{\text{maximize}} && \sum_{i=1}^{N_T} |h_i| b_i \\ & \text{subject to} && \sum_{i=1}^{N_T} b_i \leq \tilde{P}_{\text{tot}} \\ & && b_i \leq \tilde{P}_{\text{max}}, \quad i = 1, \dots, N_T \\ & && b_i \geq 0, \quad i = 1, \dots, N_T, \end{aligned} \quad (9)$$

where $\tilde{P}_{\text{tot}} = \frac{\eta_{\text{max}} P_{\text{tot}}}{\sqrt{P_{\text{max}}}}$ and $\tilde{P}_{\text{max}} = \sqrt{P_{\text{max}}}$.

Solution: We note that (9) is a linear, thus convex optimization problem. In addition, it exhibits extra structure in terms of the $|h_i|$ being all positive. Our strategy for finding the optimal solution to (9) is the following. We make an ansatz, and subsequently show that the ansatz fulfills the Karush-Kuhn-Tucker (KKT) conditions [20, p. 244]. Since the KKT conditions are fulfilled, the solution must be optimal, since (9) is a convex optimization problem.

Our ansatz is the following. We set

$$N_{\text{full}} = \left\lfloor \frac{\tilde{P}_{\text{tot}}}{\tilde{P}_{\text{max}}} \right\rfloor. \quad (10)$$

The parameter N_{full} is a number of antennas that will be fed with maximum power. We extract the N_{full} largest antenna gains $|h_i|$, and set the power for the corresponding antennas to the maximum. In the rest of the development, and without loss of generality, we assume that the $|h_i|$ are sorted in order

of decreasing magnitude. We then have

$$b_i = \tilde{P}_{\max}, \quad i = 1, \dots, N_{\text{full}}. \quad (11)$$

For the $N_{\text{full}} + 1$ -th largest antenna gain, we set the power to

$$b_{N_{\text{full}}+1} = \tilde{P}_{\text{tot}} - N_{\text{full}}\tilde{P}_{\max}, \quad (12)$$

and we set the power on the remaining antennas as

$$b_i = 0, \quad i = N_{\text{full}} + 2, \dots, N_T. \quad (13)$$

Verifying that our ansatz consisting of (11), (12), and (13) is actually fulfilling the KKT conditions is straight-forward and left out because of lack of space. That this ansatz is optimal is in fact also given directly by inspection: Start with no allocated power. Since the total consumed power cost of increasing output power on an antenna always is the same in (9), and since the maximand in (9) is linear, the maximum is given by always increasing output power on the antenna corresponding to the maximum $|h_i|$ among those antennas which not yet has full output power. We can preprocess the calculation of (10) offline.

Differently from the standard MISO maximum-ratio combining power allocation solution [1, p. 180], the procedure above allows us to turn off antennas while operating optimally, which is beneficial in cases where dissipated power per antenna is significant. *This also gives us the possibility to turn off whole radio frequency chains with filters and mixers, which saves additional power.* The proposed power allocation algorithm has computational complexity equal to that of sorting N_T numbers, which is of the order $N_T \log(N_T)$.

V. PARALLEL GAUSSIAN CHANNELS

In the second case, we assume parallel Gaussian channels, i.e., $\mathbf{H} = \text{diag}([h_{1,1}, \dots, h_{N_T, N_T}]^T)$. In this case, the Hadamard inequality [2] gives that *for every power allocation $\mathbf{Q}$ which is admissible by the power constraints*, the maximum value of $\log_2 \left(\left(\mathbf{I} + \frac{1}{N_0} \mathbf{H} \mathbf{Q} \mathbf{H}^H \right) \right)$ is given by a diagonal $\mathbf{Q}$. Therefore, we set $\mathbf{Q} = \text{diag}([b_1^2, \dots, b_{N_T}^2]^T)$ with the b_i real and positive, and we can rewrite (6) as

$$\begin{aligned} \underset{\mathbf{b}}{\text{minimize}} \quad & - \sum_{i=1}^{N_T} \log_2 \left(1 + \frac{1}{N_0} b_i^2 |h_{i,i}|^2 \right) \\ \text{subject to} \quad & \frac{\sqrt{P_{\max}}}{\eta_{\max}} \sum_{i=1}^{N_T} b_i \leq P_{\text{tot}} \\ & b_i^2 \leq P_{\max}, \quad i = 1, \dots, N_T \\ & b_i \geq 0, \quad i = 1, \dots, N_T. \end{aligned} \quad (14)$$

Solution: The present problem is not convex. More specifically, the minimand in (14) is not convex in the b_i . The minimand is convex in $\beta_i = (b_i)^2$, but the sum power constraint is a concave function in terms of β_i . The problem (14) can however be solved by uniform quantization of b_i -space with a step length Δ , and using the standard dynamic programming approach, which is well understood in the mathematical literature [21]. It is equivalent to formulate the dynamic programming solution as forward or backward recursion. We choose to use forward recursion.

We follow the approach in [21, p. 577] closely. Using dynamic programming language, $i = 1, \dots, N_T$ are referred to as steps. The b_i are now labeled steering variables. States s_i describe the resource being used up to and including step i . We define the transfer function $s_i = s_{i-1} + b_i$ to describe the relation between states at step $i - 1$ and i , and the resource used at step i . We introduce $r_i(b_i) = -\log_2 \left(1 + \frac{1}{N_0} b_i^2 |h_{i,i}|^2 \right)$ and the optimal value function $f_i(s_i) = \min_{b_1, \dots, b_{N_T}} \left(\sum_{k=1}^i r_k(b_k) \right)$. The recursion relation is $f_i(s_i) = \min_{b_i} (r_i(b_i) + f_{i-1}(s_{i-1}))$. The sought value is $f_{N_T}(s_{N_T})$. We have the boundary conditions $s_0 = 0$ and $s_{N_T} = \lfloor \tilde{P}_{\text{tot}} / \Delta \rfloor$. The limitation $0 \leq b_i \leq \lfloor \tilde{P}_{\max} / \Delta \rfloor$ means that $s_{i-1} \leq s_i \leq s_{i-1} + \lfloor \tilde{P}_{\max} / \Delta \rfloor$. Calculations are performed from step 1 to N_T . The b_i resulting in the optimal value function $f_i(s_i)$ are stored for each state s_i for each step i , and the globally optimal b_i are finally given by backtracking from s_{N_T} by means of the transfer function. Given the quantization by means of Δ , the dynamic programming converges to the global optimum. In the experiments, the solution converges if $\Delta \rightarrow 0$. The dynamic programming has computational complexity roughly on the order of $N_T \tilde{P}_{\text{tot}} \tilde{P}_{\max} / \Delta^2$, which is a lot less than the brute force complexity $(\tilde{P}_{\text{tot}} / \Delta)^{N_T}$ for relevant values of the parameters in question.

VI. EXPERIMENTS

We choose the experimental parameters as follows. The number of transmit antennas $N_T = 4$. The noise variance $N_0 = 1$, and $\eta_{\max} = 0.55$ which is a realistic value, cf., parameter choices with [17, Equation (6)]. In each figure, $P_{\text{tot}} = \frac{P_{\max} N_T}{\eta_{\max}} [0.1, 0.2, \dots, 1]^T$, i.e., we investigate the whole range of power allowed by the individual antenna energy constraints. The channels $\mathbf{h}$ and $\mathbf{H}$ are Rayleigh fading, i.e., $\mathbf{h}$ in Sec. IV and the diagonal of $\mathbf{H}$ in Sec. V have i.i.d. elements $\sim \mathcal{CN}(0, 1)$. We estimate mean rate by means of a Monte Carlo simulation with 100 channel realizations per P_{tot} point.

In Fig. 1, we illustrate the MISO beamforming from Sec. IV. We set $P_{\max} = 18$ dB. The proposed method is compared to MRT-like allocation. We set $\mathbf{Q} = \mathbf{z}^* \mathbf{z}^T$, with $\mathbf{z} = g \left(\frac{\tilde{P}_{\text{tot}}}{\sum_{i=1}^{N_T} |h_i|} \mathbf{h} \right)$, where $g(a)$ produces a vector with element i equal to $\max(|a_i|, \sqrt{P_{\max}}) \exp(j \arg(a_i))$. We also compare to uniform i.i.d. allocation with $\mathbf{Q} = \text{diag} \left(\frac{\tilde{P}_{\text{tot}}}{N_T^2} [1, \dots, 1]^T \right)$. The proposed method increases performance compared to the methods of comparison over the whole range of investigated transmit P_{tot} values.

In Fig. 2, we illustrate the MIMO beamforming from Sec. V. We set $N_R = 4$, $P_{\max} = 25$ dB, and $\Delta = \tilde{P}_{\text{tot}} / 500$. The proposed method is compared to two methods. The first is standard waterfilling-like allocation with two fixes. The standard sum output power constraint is switched for the sum consumed power constraint in (14). Moreover, for each water-increase, the power on each antenna is limited by the per-antenna constraints in (14). We also compare to uniform i.i.d. allocation with $\mathbf{Q} = \text{diag} \left(\frac{\tilde{P}_{\text{tot}}}{N_T^2} [1, \dots, 1]^T \right)$. The proposed method increases performance compared to the methods of

Fig. 1. Comparison of allocation strategies in terms of average rate on the Rayleigh-fading MISO channel. The whole range of P_{tot} permitted by the single-antenna constraints is visualized. The number of antennas $N_T = 4$ and $\eta_{\text{max}} = 0.55$. Additional parameters and the methods of comparison are described in detail in Sec. VI.

Fig. 2. Comparison of allocation strategies in terms of average rate on the parallel Rayleigh-fading MIMO channel. The whole range of P_{tot} permitted by the single-antenna constraints is visualized. The number of antennas $N_T = N_R = 4$ and $\eta_{\text{max}} = 0.55$. Additional parameters and the methods of comparison are described in detail in Sec. VI.

comparison over the whole range of investigated transmit P_{tot} values. The MISO and MIMO results remained the same in essence when changing the number of antennas, P_{max} , and η_{max} .

VII. CONCLUSION

We propose to take dissipated energy into account for MIMO beamforming using adequate power amplifier modeling. The general MIMO problem is formulated. We discuss the challenges related to solving this problem, and give the solution for two special cases, the linear beamformer on the MISO channel, and secondly, power allocation for a parallel MIMO channel. The MISO problem turns out to be linear, while the MIMO problem is of non-convex type. For the MISO case, a new beamforming algorithm, with computational complexity on the order of $N_T \log(N_T)$, is proposed. This solution is in

general such that the antennas with the weakest channel gains will be turned off. This allows us to turn off whole radio frequency chains with filters and mixers, which saves additional power. For the MIMO case, a dynamic programming approach is used. This strategy has computational complexity on the order of $N_T \tilde{P}_{\text{tot}} \tilde{P}_{\text{max}} / \Delta^2$. Given the assumptions, i.e., linear MISO beamforming, and a parallel MIMO channel, the solutions are optimal. Relatively to previous state-of-the-art, the proposed algorithms give significant performance benefits in comparison with other methods, in terms of increased rate for a fixed total consumed power, or in terms of reduced total consumed power for a fixed rate, over wide ranges of total consumed power.

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