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Achievable Outage Rate Regions for the MISO Interference Channel

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Abstract—We consider the slow-fading two-user multiple-input single-output (MISO) interference channel. We want to understand which rate points can be achieved, allowing a non-zero outage probability. We do so by defining four different outage rate regions. The definitions differ on whether the rates are declared in outage jointly or individually and whether the transmitters have instantaneous or statistical channel state information (CSI). The focus is on the instantaneous CSI case with individual outage, where we propose a stochastic mapping from the rate point and the channel realization to the beamforming vectors. A major contribution is that we prove that the stochastic component of this mapping is independent of the actual channel realization.

Index Terms—Achievable rate region, beamforming, interference channel, MISO, outage probability.

I. INTRODUCTION

WE study the two-user multiple-input single-output (MISO) interference channel (IC), consisting of two transmitter (TX) - receiver (RX) pairs (or links). The transmissions are concurrent and cochannel; hence, they interfere with each other. The TXs employ multiple antennas and the RXs a single antenna. We assume that the channels are flat and slow fading and we say that a link is in *outage* if the IC experiences fading states that cannot support a desired data rate. The fundamental question raised is how to define the *outage rate region*. That is, which rate points can be achieved with a certain probability? For multi-user systems, such as the IC, broadcast channel (BC), and multiple-access channel (MAC), one can consider *common* or *individual* outage. We declare a common outage if the rate of at least one link cannot be supported (see, e.g., [2] for the BC). We declare an individual outage if a specific link is unable to communicate at the desired rate.

So far, studies of outage rate regions have been restricted to the single-antenna BC and MAC for which the outage capacity regions for instantaneous channel side information (CSI) were given in [2] and [3], respectively. For statistical CSI, the MAC and BC were studied in [4] and [5], respectively. The instantaneous rate region for the MISO IC is well-understood (see, e.g., [6] and [7]). In [1], we defined the regions for individual and common outage for statistical CSI and common

outage for instantaneous CSI. For the Gaussian IC in the high signal-to-noise ratio (SNR) regime, recent research activities have explored the diversity-multiplexing trade-off (DMT) (see e.g., [8] for characterization of the two-user IC). In [9], our results in [1] were used to approximately perform weighted sum-rate maximization under outage constraints for the MISO IC with statistical CSI. Also for statistical CSI, outage probabilities in the multiple-input multiple-output (MIMO) IC were given in closed form and an outage-based robust beamforming algorithm was proposed in [10].

In this paper, we propose and analyze achievable outage rate regions for the MISO IC. The results generalize those of [2]–[5] both in the sense that the BC and MAC are special cases of the IC, and in that we treat the multiple antenna case. Since we allow a non-zero outage probability, our results extend those of [6] and [7] where outage was not allowed. In contrast to the DMT analysis, e.g., [8], our results are valid for any SNR regime. For completeness, we consider common and individual outage for both instantaneous and statistical CSI, but we focus on the individual outage rate region for instantaneous CSI, which we did not treat in [1]. A challenge is how to handle the scenario where either of the rates can be achieved, but not simultaneously. We solve this by proposing a stochastic mapping of the beamforming vectors that depends on the rates and the channels. We prove that the randomness of the mapping is independent of the channel realization. Compared to [1], the statistical CSI definitions extend the single-stream transmission scheme to multi-stream. The definitions are valid for arbitrary assumptions on the channel distribution.

II. SYSTEM MODEL

We assume that the RXs treat the interference as additive Gaussian noise. Also, RX_i experiences additive Gaussian thermal noise with variance σ_i^2 . TX_i employs n antennas and uses a Gaussian vector codebook with covariance Ψ_i . By $\mathbf{h}_{ij} \in \mathbb{C}^n$, we denote the slow-fading conjugated channel vector between TX_i and RX_j and we assume that the channels $\{\mathbf{h}_{ij}\}_{i,j=1}^2$ are statistically independent. We let $\mathbf{h}$ denote a specific realization of the channels, i.e., $\mathbf{h} = [\mathbf{h}_{11}^T, \mathbf{h}_{12}^T, \mathbf{h}_{21}^T, \mathbf{h}_{22}^T]^T$. By $\mathbf{H}$ we denote a random channel with pdf $f_{\mathbf{H}}(\mathbf{h})$. The achievable rate, in bits per channel use, of link i is

$$R_i(\mathbf{h}, \Psi_i, \Psi_j) = \log_2 \left(1 + \frac{\mathbf{h}_{ii}^H \Psi_i \mathbf{h}_{ii}}{\mathbf{h}_{ji}^H \Psi_j \mathbf{h}_{ji} + \sigma_i^2} \right).$$

We bound the transmit power to $\text{trace}\{\Psi_i\} \leq 1$. For statistical CSI, multi-stream transmission, i.e., $\text{rank}\{\Psi_i\} \leq n$ is optimal in general. However, for instantaneous CSI, single-stream transmission, i.e., $\text{rank}\{\Psi_i\} = 1$, is optimal [11].

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For the latter case, we set $\Psi_i = \mathbf{w}_i \mathbf{w}_i^H$, where $\mathbf{w}_i$ is the beamforming vector with $\|\mathbf{w}_i\|^2 \leq 1$. For instantaneous CSI, the corresponding rate is denoted $R_i(\mathbf{h}, \mathbf{w}_1, \mathbf{w}_2)$.

III. OUTAGE RATE REGION FOR INSTANTANEOUS CSI

We assume that the TXs have instantaneous CSI and therefore can adapt the beamforming vectors to the current fading state. The definition of the common outage rate region given in Sec. III-A, was first proposed in [1]. Here, we give it for completeness. The definition of the individual outage rate region is novel and is given in Sec. III-B.

For a given channel realization $\mathbf{h}$, the achievable instantaneous rate region is the set of rate points that can be achieved by using any pair of feasible beamforming vectors, i.e.,

$$\mathcal{R}(\mathbf{h}) \triangleq \bigcup_{\|\mathbf{w}_i\|^2 \leq 1, i=1,2} (R_1(\mathbf{h}, \mathbf{w}_1, \mathbf{w}_2), R_2(\mathbf{h}, \mathbf{w}_1, \mathbf{w}_2)).$$

By $R_i^{\text{SU}}(\mathbf{h}) \triangleq \log_2(1 + \|\mathbf{h}_{ii}\|^2/\sigma_i^2)$, we denote the single-user (SU) rate for link i , i.e., the maximum rate achieved in absence of interference when TX _{i} uses its matched-filter (MF) beamforming vector $\mathbf{w}_i^{\text{MF}} \triangleq \mathbf{h}_{ii}/\|\mathbf{h}_{ii}\|$.

A. Common Outage Rate Region for Instantaneous CSI

We denote by $\mathcal{R}_{\text{inst}}^{\text{com}}(\epsilon)$ the sought common outage rate region for instantaneous CSI. If a rate point $(r_1, r_2) \notin \mathcal{R}(\mathbf{h})$, i.e., it is not achievable, we say that the IC is in outage.

Definition 1. Let $\epsilon > 0$ denote the common outage probability specification. Then, $(r_1, r_2) \in \mathcal{R}_{\text{inst}}^{\text{com}}(\epsilon)$ if there exists a deterministic mapping $(\mathbf{w}_1(\mathbf{h}, r_1, r_2), \mathbf{w}_2(\mathbf{h}, r_1, r_2))$ with $\|\mathbf{w}_i\|^2 \leq 1, i = 1, 2$, such that $\Pr\{r_1 < R_1(\mathbf{H}, \mathbf{w}_1, \mathbf{w}_2), r_2 < R_2(\mathbf{H}, \mathbf{w}_1, \mathbf{w}_2)\} \geq 1 - \epsilon$.

To determine if (r_1, r_2) is achievable for a channel realization $\mathbf{h}$, we can solve a scalar, quasi-concave rate maximization problem that takes r_1 as input and returns r_2^* [12]. If $r_2^* \geq r_2$, then (r_1, r_2) is achievable and the solution of the rate maximization problem gives us the enabling beamforming vectors.¹ Note that the beamforming vectors, which depend on both the channel realization and the rate point, are not unique. Equivalently to Def. 1, we say that $(r_1, r_2) \in \mathcal{R}_{\text{inst}}^{\text{com}}(\epsilon)$ if $\Pr\{(r_1, r_2) \in \mathcal{R}(\mathbf{H})\} \geq 1 - \epsilon$. Since $\Pr\{(r_1, r_2) \in \mathcal{R}(\mathbf{H})\}$ decreases with respect to one rate when the other is fixed, a point on the outer boundary of $\mathcal{R}_{\text{inst}}^{\text{com}}(\epsilon)$ has an outage probability equal to ϵ .

B. Individual Outage Rate Region for Instantaneous CSI

We denote by $\mathcal{R}_{\text{inst}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ the sought individual outage rate region for instantaneous CSI. In contrast to the common outage rate region, we assume that when the rate of one link cannot be achieved, the corresponding TX is switched off. In such occasion, the other link does not experience interference, hence it has increased chances of achieving its desired rate.

¹By the symmetry of the problem, we can equivalently choose r_2 as input and r_1^* as output of the optimization. Then (r_1, r_2) is feasible if $r_1 \leq r_1^*$.

Definition 2. Let $\epsilon_1, \epsilon_2 > 0$ denote the individual outage probability specifications. Then, $(r_1, r_2) \in \mathcal{R}_{\text{inst}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ if there exists a stochastic mapping $(\mathbf{w}_1(\mathbf{h}, r_1, r_2), \mathbf{w}_2(\mathbf{h}, r_1, r_2))$ with $\|\mathbf{w}_i\|^2 \leq 1, i = 1, 2$, such that $\Pr\{r_1 < R_1(\mathbf{H}, \mathbf{w}_1, \mathbf{w}_2)\} \geq 1 - \epsilon_1$ and $\Pr\{r_2 < R_2(\mathbf{H}, \mathbf{w}_1, \mathbf{w}_2)\} \geq 1 - \epsilon_2$.

In the following, we motivate Def. 2 by proposing a stochastic mapping $(\mathbf{w}_1(\mathbf{h}, r_1, r_2), \mathbf{w}_2(\mathbf{h}, r_1, r_2))$ and conditions for having $(r_1, r_2) \in \mathcal{R}_{\text{inst}}^{\text{ind}}(\epsilon_1, \epsilon_2)$. First, we focus on a given rate point (r_1, r_2) and a realization of the channels and determine whether the rates r_1 and r_2 are achievable or not and outline the stochastic mapping. Either none of the rates is achievable, or both of them are achievable, or only one of them is achievable. We formalize this by serially performing the following checks:

1) Is $r_1 > R_1^{\text{SU}}(\mathbf{h})$ and $r_2 > R_2^{\text{SU}}(\mathbf{h})$? If yes, we have case $\mathbb{A}$: none of r_1 and r_2 is achievable, we set $\mathbf{w}_1 = \mathbf{w}_2 = \mathbf{0}$, and both links are in outage.

2) Is $(r_1, r_2) \in \mathcal{R}(\mathbf{h})$? If yes, we have case $\mathbb{B}$. We find $(\mathbf{w}_1, \mathbf{w}_2)$ by solving the rate maximization problem in [12]. Note that this is the only case where the MISO IC is not in outage under the common outage specification.

3) Is $r_1 > R_1^{\text{SU}}(\mathbf{h})$ or $r_2 > R_2^{\text{SU}}(\mathbf{h})$? If $r_2 > R_2^{\text{SU}}(\mathbf{h})$, we have case $\mathbb{C}_1$: r_1 is achievable with $\mathbf{w}_1 = \mathbf{w}_1^{\text{MF}}$ while r_2 is in outage and we set $\mathbf{w}_2 = \mathbf{0}$. If $r_1 > R_1^{\text{SU}}(\mathbf{h})$, we have case $\mathbb{C}_2$: r_2 is achievable with $\mathbf{w}_2 = \mathbf{w}_2^{\text{MF}}$ while r_1 is in outage and we set $\mathbf{w}_1 = \mathbf{0}$. If neither $r_1 > R_1^{\text{SU}}(\mathbf{h})$ nor $r_2 > R_2^{\text{SU}}(\mathbf{h})$, we have case $\mathbb{D}$: there is an ambiguity; both rates can be achieved, but not simultaneously. Here, we decide that one of the links is active while the other is in outage. For example, we could always decide in favor of link 1. But, as we illustrate in Sec. V and Fig. 1, this is a suboptimal strategy. In order to have as large region as possible, we make this binary decision at random, e.g., by flipping a biased coin. We let $I \in \{1, 2\}$ be the outcome of the coin flip and assume that the coin has bias $\Pr\{I = 1 | \mathbf{H} = \mathbf{h}; r_1, r_2\} = 1 - \Pr\{I = 2 | \mathbf{H} = \mathbf{h}; r_1, r_2\}$. That is, the coin bias depends on both the channel realization and the specific rate point.

4) Is $I = 1$? We have case $\mathbb{D}_1$: we achieve r_1 while r_2 is in outage by using the same beamforming vectors as in $\mathbb{C}_1$. Is $I = 2$? We have case $\mathbb{D}_2$: we achieve r_2 while r_1 is in outage by using the same beamforming vectors as in $\mathbb{C}_2$.

Second, we consider the entire set of channel realizations and a specific rate point (r_1, r_2) . We define $\mathcal{S}_{\mathbb{X}}(r_1, r_2)$ to be the set of channel realizations for which (r_1, r_2) falls under case $\mathbb{X} \in \{\mathbb{A}, \mathbb{B}, \mathbb{C}_1, \mathbb{C}_2, \mathbb{D}\}$. Therefore, we have

$$\mathcal{S}_{\mathbb{A}}(r_1, r_2) \triangleq \{\mathbf{h} : r_1 > R_1^{\text{SU}}(\mathbf{h}), r_2 > R_2^{\text{SU}}(\mathbf{h})\}, \quad (1)$$

$$\mathcal{S}_{\mathbb{B}}(r_1, r_2) \triangleq \{\mathbf{h} : (r_1, r_2) \in \mathcal{R}(\mathbf{h})\}, \quad (2)$$

$$\mathcal{S}_{\mathbb{C}_1}(r_1, r_2) \triangleq \{\mathbf{h} : r_1 \leq R_1^{\text{SU}}(\mathbf{h}), r_2 > R_2^{\text{SU}}(\mathbf{h})\}, \quad (3)$$

$$\mathcal{S}_{\mathbb{C}_2}(r_1, r_2) \triangleq \{\mathbf{h} : r_1 > R_1^{\text{SU}}(\mathbf{h}), r_2 \leq R_2^{\text{SU}}(\mathbf{h})\}, \quad (4)$$

$$\mathcal{S}_{\mathbb{D}}(r_1, r_2) \triangleq \{\mathbf{h} : r_1 \leq R_1^{\text{SU}}(\mathbf{h}), r_2 \leq R_2^{\text{SU}}(\mathbf{h}), (r_1, r_2) \notin \mathcal{R}(\mathbf{h})\}. \quad (5)$$

Since the events $\mathbb{A}, \mathbb{B}, \mathbb{C}_1, \mathbb{C}_2$, and $\mathbb{D}$ are mutually exclusive, we can conclude that the sets defined in (1)–(5) are disjoint and span the set of all channel realizations.

The probability that a channel realization belongs to each

of the sets defined in (1)–(5) is $P_{\mathbb{X}}(r_1, r_2) \triangleq \Pr\{\mathbf{H} \in \mathcal{S}_{\mathbb{X}}(r_1, r_2)\}$ for $\mathbb{X} \in \{\mathbb{A}, \mathbb{B}, \mathbb{C}_1, \mathbb{C}_2, \mathbb{D}\}$. Especially, we have

$$P_{\mathbb{A}}(r_1, r_2) = \Pr\{r_1 > R_1^{\text{SU}}(\mathbf{H})\} \Pr\{r_2 > R_2^{\text{SU}}(\mathbf{H})\}, \quad (6)$$

$$P_{\mathbb{B}}(r_1, r_2) = \Pr\{(r_1, r_2) \in \mathcal{R}(\mathbf{H})\}, \quad (7)$$

$$P_{\mathbb{C}_1}(r_1, r_2) = \Pr\{r_1 \leq R_1^{\text{SU}}(\mathbf{H})\} \Pr\{r_2 > R_2^{\text{SU}}(\mathbf{H})\}, \quad (8)$$

$$P_{\mathbb{C}_2}(r_1, r_2) = \Pr\{r_1 > R_1^{\text{SU}}(\mathbf{H})\} \Pr\{r_2 \leq R_2^{\text{SU}}(\mathbf{H})\}, \quad (9)$$

$$P_{\mathbb{D}}(r_1, r_2) = \Pr\{r_1 \leq R_1^{\text{SU}}(\mathbf{H})\} \Pr\{r_2 \leq R_2^{\text{SU}}(\mathbf{H})\} \\ - \Pr\{(r_1, r_2) \in \mathcal{R}(\mathbf{H})\}, \quad (10)$$

where (6), (8), and (9) follow by the independence of the random variables $R_1^{\text{SU}}(\mathbf{H})$ and $R_2^{\text{SU}}(\mathbf{H})$. For (10) we use that

$$\begin{aligned} & \Pr\{r_1 \leq R_1^{\text{SU}}(\mathbf{H}), r_2 \leq R_2^{\text{SU}}(\mathbf{H})\} \\ &= \Pr\{r_1 \leq R_1^{\text{SU}}(\mathbf{H}), r_2 \leq R_2^{\text{SU}}(\mathbf{H}) | (r_1, r_2) \in \mathcal{R}(\mathbf{H})\} \\ & \quad \times \Pr\{(r_1, r_2) \in \mathcal{R}(\mathbf{H})\} \\ & \quad + \Pr\{r_1 \leq R_1^{\text{SU}}(\mathbf{H}), r_2 \leq R_2^{\text{SU}}(\mathbf{H}), (r_1, r_2) \notin \mathcal{R}(\mathbf{H})\} \\ &= \Pr\{(r_1, r_2) \in \mathcal{R}(\mathbf{H})\} + P_{\mathbb{D}}(r_1, r_2), \end{aligned}$$

since $\Pr\{r_1 \leq R_1^{\text{SU}}(\mathbf{H}), r_2 \leq R_2^{\text{SU}}(\mathbf{H}) | (r_1, r_2) \in \mathcal{R}(\mathbf{H})\} = 1$. It is straightforward to verify that the probabilities in (6)–(10) sum up to one. For case $\mathbb{D}_i$, we introduce the joint mixed distribution $f_{\mathbf{H}, I}(\mathbf{h}, i; r_1, r_2) = \Pr\{I = i | \mathbf{H} = \mathbf{h}; r_1, r_2\} f_{\mathbf{H}}(\mathbf{h})$. The interpretation is that the coin bias depends on both the realization of the channels and the rate point. We have

$$\begin{aligned} P_{\mathbb{D}_i}(r_1, r_2) &\triangleq \Pr\{\mathbf{h} \in \mathcal{S}_{\mathbb{D}}(r_1, r_2), I = i; r_1, r_2\} \\ &= \int_{\mathcal{S}_{\mathbb{D}}(r_1, r_2)} \Pr\{I = i | \mathbf{H} = \mathbf{h}; r_1, r_2\} f_{\mathbf{H}}(\mathbf{h}) d\mathbf{h}. \end{aligned} \quad (11)$$

Based on the discussion above, we write the outage constraint for link i in Def. 2 as

$$\begin{aligned} & \Pr\{R_i(\mathbf{H}, \mathbf{w}_1, \mathbf{w}_2) > r_i\} \\ &= P_{\mathbb{B}}(r_1, r_2) + P_{\mathbb{C}_i}(r_1, r_2) + P_{\mathbb{D}_i}(r_1, r_2) \geq 1 - \epsilon_i. \end{aligned} \quad (12)$$

Third, we focus on the coin bias $\Pr\{I = i | \mathbf{H} = \mathbf{h}; r_1, r_2\}$.

Proposition 1. *The coin bias can be chosen independently of the realization of the channels, i.e., $\Pr\{I = 1 | \mathbf{H} = \mathbf{h}; r_1, r_2\} = p(r_1, r_2)$ and $\Pr\{I = 2 | \mathbf{H} = \mathbf{h}; r_1, r_2\} = 1 - p(r_1, r_2)$.*

Proof: According to Def. 2, $(r_1, r_2) \in \mathcal{R}_{\text{inst}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ if (12) is satisfied for $i = 1, 2$. By inserting (11) into (12), we get the equivalent conditions

$$\begin{aligned} & 1 - \epsilon_1 - P_{\mathbb{B}}(r_1, r_2) - P_{\mathbb{C}_1}(r_1, r_2) \leq P_{\mathbb{D}_1}(r_1, r_2) \\ &= \int_{\mathcal{S}_{\mathbb{D}}(r_1, r_2)} \Pr\{I = 1 | \mathbf{H} = \mathbf{h}; r_1, r_2\} f_{\mathbf{H}}(\mathbf{h}) d\mathbf{h} \\ &\leq P_{\mathbb{B}}(r_1, r_2) + P_{\mathbb{C}_2}(r_1, r_2) + P_{\mathbb{D}}(r_1, r_2) - 1 + \epsilon_2, \end{aligned} \quad (13)$$

where the last inequality follows since $P_{\mathbb{D}_1}(r_1, r_2) = P_{\mathbb{D}}(r_1, r_2) - P_{\mathbb{D}_2}(r_1, r_2)$. By selecting $\Pr\{I = 1 | \mathbf{H} = \mathbf{h}; r_1, r_2\}$ appropriately, we can force the integral in (13) to assume any value in $[0, P_{\mathbb{D}}(r_1, r_2)]$. If we restrict $\Pr\{I = 1 | \mathbf{H} = \mathbf{h}; r_1, r_2\} = p(r_1, r_2)$ for some function of (r_1, r_2) that does not depend on $\mathbf{H}$, then likewise, we can force the integral to assume any value in $[0, P_{\mathbb{D}}(r_1, r_2)]$ as well. ■

By using the result of Prop. 1, we can write (13) as

$$\begin{aligned} & 1 - \epsilon_1 - P_{\mathbb{B}}(r_1, r_2) - P_{\mathbb{C}_1}(r_1, r_2) \leq P_{\mathbb{D}}(r_1, r_2)p(r_1, r_2) \\ & \leq P_{\mathbb{B}}(r_1, r_2) + P_{\mathbb{C}_2}(r_1, r_2) + P_{\mathbb{D}}(r_1, r_2) - 1 + \epsilon_2 \end{aligned} \quad (14)$$

If we can find a bias $p(r_1, r_2) \in [0, 1]$ which satisfies (14), then we have $(r_1, r_2) \in \mathcal{R}_{\text{inst}}^{\text{ind}}(\epsilon_1, \epsilon_2)$. In order to have $0 \leq p(r_1, r_2) \leq 1$, the following conditions must be satisfied: a) The lower bound in (14) is less than $P_{\mathbb{D}}(r_1, r_2)$. b) The upper bound is non-negative. c) The lower bound is smaller than the upper bound. Hence, using the fact that the probabilities (6)–(10) sum up to one, we have the conditions

$$\epsilon_1 \geq P_{\mathbb{A}}(r_1, r_2) + P_{\mathbb{C}_2}(r_1, r_2), \quad (15)$$

$$\epsilon_2 \geq P_{\mathbb{A}}(r_1, r_2) + P_{\mathbb{C}_1}(r_1, r_2), \quad (16)$$

$$\epsilon_1 + \epsilon_2 \geq 1 + P_{\mathbb{A}}(r_1, r_2) - P_{\mathbb{B}}(r_1, r_2). \quad (17)$$

If all conditions in (15)–(17) are satisfied, we choose $p(r_1, r_2)$ according to (14) and the rate point (r_1, r_2) lies in the individual outage rate region. Otherwise, (r_1, r_2) does not belong to the outage rate region. To give some interpretation, we insert (6)–(9) into (15)–(17), and get

$$\epsilon_1 \geq \Pr\{r_1 > R_1^{\text{SU}}(\mathbf{H})\}, \quad (18)$$

$$\epsilon_2 \geq \Pr\{r_2 > R_2^{\text{SU}}(\mathbf{H})\}, \quad (19)$$

$$\begin{aligned} & \epsilon_1 + \epsilon_2 \geq \Pr\{r_1 > R_1^{\text{SU}}(\mathbf{H})\} \Pr\{r_2 > R_2^{\text{SU}}(\mathbf{H})\} \\ & \quad + \Pr\{(r_1, r_2) \notin \mathcal{R}(\mathbf{H})\}. \end{aligned} \quad (20)$$

It is apparent that $\Pr\{r_1 > R_1^{\text{SU}}(\mathbf{H})\}$ and $\Pr\{r_2 > R_2^{\text{SU}}(\mathbf{H})\}$ are decreasing with r_1 and r_2 , respectively. Also, $\Pr\{(r_1, r_2) \notin \mathcal{R}(\mathbf{H})\}$ increases when one of the rates increases but the other is fixed. Therefore, we conclude that points on the outer boundary of the outage rate region must satisfy at least one of the inequalities (18)–(20) with equality. Another observation is that (18) and (19) are the trivial outage constraints for the SU points, i.e., SU MISO channel, whereas (20) gives the shrinkage of the outage rate region due to interference. Note that, equivalently to Def. 2, we can define $\mathcal{R}_{\text{inst}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ as the set of rate points which satisfy (18)–(20).

IV. OUTAGE RATE REGIONS FOR STATISTICAL CSI

We assume that the TXs only have knowledge of the channels' statistical distribution. That is, the TXs have statistical CSI and can only adapt their transmit covariance matrices to the channel statistics. Therefore, the TXs design the transmit covariance matrices once and use them for all fading states. The definitions are given for completeness; for details we refer to [1]. We give definitions for the common and individual outage rate regions in Secs. IV-A and IV-B, respectively.

A. Common Outage Rate Region for Statistical CSI

We denote by $\mathcal{R}_{\text{stat}}^{\text{com}}(\epsilon)$ the sought common outage rate region for statistical CSI and define it as follows.

Definition 3. Let $\epsilon > 0$ denote the *common* outage probability specification. Then, $(r_1, r_2) \in \mathcal{R}_{\text{stat}}^{\text{com}}(\epsilon)$ if there exists a deterministic mapping $(\Psi_1(f_{\mathbf{H}}(\mathbf{h}), r_1, r_2), \Psi_2(f_{\mathbf{H}}(\mathbf{h}), r_1, r_2))$ with $\text{trace}\{\Psi_i\} \leq 1$ for $i = 1, 2$ such that $\Pr\{R_1(\mathbf{H}, \Psi_1, \Psi_2) > r_1, R_2(\mathbf{H}, \Psi_1, \Psi_2) > r_2\} \geq 1 - \epsilon$.

Fig. 1. Outer boundaries of the outage rate regions for the MISO IC. Also, we illustrate the effect of choosing optimal bias for the individual outage rate region for instantaneous CSI.

Note that the transmit covariance matrices Ψ_1 and Ψ_2 depend on both the channel statistics and the actual rate point.

B. Individual Outage Rate Region for Statistical CSI

We denote by $\mathcal{R}_{\text{stat}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ the sought individual outage rate region for statistical CSI. We allow one link to be in outage while the other is not. Since the TXs do not know whether the transmission is in outage or not, a TX continues transmitting even when the link is in outage.

Definition 4. Let $\epsilon_1, \epsilon_2 > 0$ denote the individual outage probability specifications. Then, $(r_1, r_2) \in \mathcal{R}_{\text{stat}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ if there exists a deterministic mapping $(\Psi_1(f_{\mathbf{H}}(\mathbf{h}), r_1, r_2), \Psi_2(f_{\mathbf{H}}(\mathbf{h}), r_1, r_2))$ with $\text{trace}\{\Psi_i\} \leq 1$ for $i = 1, 2$ such that $\Pr\{R_i(\mathbf{H}, \Psi_1, \Psi_2) \geq r_i\} \geq 1 - \epsilon_i$.

V. NUMERICAL EXAMPLE

We illustrate the outage rate regions given in Defs. 1–4. The TXs employ $n = 2$ antennas each and we model $\mathbf{h}_{ij} \in \mathbb{C}^n$ as a zero-mean complex-symmetric Gaussian vector with covariance $\mathbf{Q}_{ij}$. We assume that $\sigma_1^2 = \sigma_2^2 = 0.5$ and $\epsilon = \epsilon_1 = \epsilon_2 = 0.1$. For a given set of channel covariance matrices $\{\mathbf{Q}_{ij}\}$, we depict the regions in Fig. 1.

We use exhaustive-search methods to generate the regions. For instantaneous CSI, we make a grid of rate points. Then, for each rate point, we estimate the probabilities (18)–(20) by running Monte-Carlo simulations. For determining if $(r_1, r_2) \in \mathcal{R}(\mathbf{h})$, we use the fast method given in [12]. For statistical CSI we draw beamforming vectors randomly. Using results from [13], we compute the probabilities in Defs. 3 and 4 in closed form. For each pair of beamforming vectors, we determine the rate points that meet the outage specifications. We find the outer boundary via a brute-force comparison among all computed rate points.

We observe that the individual outage regions are larger than the corresponding common outage regions and the instantaneous CSI regions are larger than the corresponding statistical CSI regions. These results are expected since common outage

is more restrictive than individual outage and instantaneous CSI is always better than statistical CSI. These results are true in general, but we omit the proof due to space limitations.

We also illustrate the effect of choosing the bias according to (14). Area 1 is the gain, compared to the common outage case, from including the obvious cases $\mathbb{C}_1$ and $\mathbb{C}_2$. Area 2 (or 3) is the gain from solving the conflict by always choosing in favor of link 1 (or 2), i.e., by switching off deterministically. Area 4 is the gain from randomly switching off the transmissions using the bias according to (14).

VI. CONCLUSIONS

We defined four outage rate regions for the MISO IC. The definitions correspond to different scenarios of channel knowledge and outage specification. We observe that neither the definitions depend on the channels' distributions nor they are restricted for Gaussian coding. On the other hand, for Gaussian coding and channels, we have efficient methods for illustrating the regions. Whereas the definitions for statistical CSI assume that interference is treated as noise, the definitions for instantaneous CSI are valid for any achievable rate region and could potentially be extended to the MIMO IC.

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