Haptic Interaction with Deformable Objects

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Abstract

The integration of haptics into virtual environments has triggered a new era by allowing interaction with virtual objects through force feedback in a number of fields. Medicine has been the field with highest potential benefit through improved realism and immersion. Not only have virtual environments become superior to traditional medical training methods due to cost-efficiency, repeatability, and objective assessment but the idea of surgery rehearsal by using patient specific data has been raised as well.

Achieving sufficient realism in haptics has been a significant challenge due to performance requirements. In order to provide a stable and smooth feedback to the user, the update rates of force feedback need to be in the range of 1 kHz, which restricts the solution time for real-time interactive applications. Realism, on the other hand, demands advanced algorithms capable of simulating physical properties. These advanced algorithms have a high computational burden, taking significant amounts of time and their real-time use, therefore, mostly requires simplification of the virtual scene affecting realism.

During palpation, information is transferred to the hand from the local neighbourhood of contact. In deformation simulations, it is therefore common to use a multiresolution scheme, where the local region is modelled with a higher resolution than more distant regions, and at higher update rates. This approach saves computational power, however the less elaborate modelling in the more remote regions affects accuracy. This thesis presents a pipeline to analyse the error introduced by multiresolution techniques. The idea is to estimate how simulation parameters lead to different error magnitudes, as a preprocessing step. This information can subsequently be used for monitoring the error in real-time, or for adjusting simulation parameters to keep the error under a desired limit.

There is a trade-off between accuracy/error and computation time required. In an ideal situation, this error should be kept under perceivable levels. Levels of perception is a topic that has been surveyed in psychophysics among other aspects of touch. It has been shown that differences smaller than a ratio of a reference signal, such as force or stiffness, cannot be perceived. Evaluating the exact value of this ratio, however, is non-trivial since there are many secondary factors having a significant impact, such as the multimodal input. This thesis presents the analysis of some factors affecting the sense of touch that were shown to have such impact. Effects of exploratory procedures on stiffness perception were examined through user studies, followed by another study indicating the significant effects of stiffness gradient.

Medical data, such as MR and CT, has much higher resolution than is practically used
for deformable meshes. It has been common practice to model deformation behaviour by a mesh with lower resolution than is used for visual representation. Lastly, this thesis presents an approach to introduce high-resolution information. The proposed algorithm allows for the detection of inhomogeneous structures beneath a surface. This can be applied in situations similar to the diagnosis of tumours by palpation. The approach is independent of mesh structure and resolution, and can be integrated into any proxy-based haptic rendering algorithm. This makes the algorithm a complementary choice for deformation simulation.
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Part A

Context of the Work
Chapter 1

Introduction

The rapid growth rate of technology in the information era we are witnessing has transformed the way we think of computers, which occupy bigger roles in our daily lives. The first seeds of our addiction have been spread through mobile phones which were used for communication purposes only. The developments in the technology, motivated by the size of the potential of the market—each individual around the world—has led to the evolution of mobile phones to mobile computers. This shift in the concept of computers away from the desktop version composed of a screen, a keyboard and a mouse has not only introduced great benefits but brought new demands for human computer interaction as well.

The ability to carry one’s computer around has increased the functionality of the computer but, on the other hand, made the traditional interaction methods obsolete. Despite the functionality of numerous traditional devices having been gathered into a single device by touch screen technology, more intuitive ways of interaction have always been needed. The growth in business has also encouraged researchers to develop affordable devices for the end user. Among these devices the Nintendo Wii [113], Kinect [52], MYO [66], Leap Motion [56] can be named as examples which create change through their intuitive interaction methods. These devices have numerous application areas including computer games, art and design, device control, basic computer interaction amongst many others.

Striving for intuitive human computer interaction, one cannot ignore the sense of touch, *haptics*. Attempts to provide haptic feedback have been quite few when compared with the advances in visual and audio channels, due to the relatively young lifetime of haptics research. The main contribution of haptics to perception is through providing information about the material and geometrical properties of objects, while spatial and temporal information are mainly transferred by the visual and audio channels. There are two main afferent subsystems: cutaneous and kinaesthetic, which are involved in haptics perception [57]. Cutaneous feedback is perceived through the skin by the mechano- and thermo-receptors while the mechano-receptors in the muscles, tendons and joints provide the kinaesthetic feedback. These two types of input are combined to create a haptic visualization of the environment in the brain. The existing haptic hardware technology can be considered from a similar point of view of haptic perception. There are two main types of haptic hardware solutions: First; a surface structure providing cutaneous cues, so called tactile feedback, to the skin to simulate features such as texture or temperature, and, secondly, a set of robotic arms providing kinaesthetic cues through force feedback in
order to simulate features like stiffness, viscosity, mass etc. Haptic texture, for example, is simulated by integration of tactile feedback devices into touch screen technologies [93]. Due to their higher price the kinaesthetic devices, on the other hand, are mostly used in research labs or virtual environments established at institutions such as hospitals. Sensable’s Phantom [92], Force Dimension’s Omega [31], Novint’s Falcon [71] can be named among the most popular kinaesthetic haptic devices.

A new era has been triggered by the integration of haptics into virtual environments. Not only has the immersion effect been enriched but also the ability to feel virtual objects has brought various functionalities. The additional force feedback has been employed in different contexts for various aims. Guidance has been one main use where the movement of the user is governed by the force in scenarios like aiming for a target or following a path. Haptic user interfaces, computer aided design [74], stroke rehabilitation [75], guided multi-user haptic applications are among examples of providing guidance through haptics. An alternative use of force feedback has been in the enhancement of visualization, especially in extremely small or large scales. For example simulating the attraction and repulsion forces between atoms [44] or gravitational forces in the solar system are potential educational areas for haptics. One of the most prominent uses of haptics, however, has been in the field of medicine through improving realism. Due to increased realism, virtual environments have become a better alternative for medical training instead of traditional methods.

Practical training in medicine has traditionally been accomplished through the use of plastic models or cadavers. These methods, however, have several drawbacks related to cost efficiency, objective performance assessment and repeatability of scenarios. In many training scenarios, plastic models and cadavers can only be used once and it can be quite expensive and difficult to find these for a specifically desired scenario. Training in a virtual environment, however, allows both a limitless number of repetitions and the ability to design a specific scenario while the significant amount of its cost is initial. In addition to being more cost efficient, the repeatability also allows standardization in training quality by being able to use identical scenarios to test the trainees. Moreover, the performance assessment can be based on digital metrics in virtual environments which not only brings objectivity but also provides instant feedback to the trainee. The distribution of forces on

Figure 1.1: Examples of commercial kinaesthetic haptic devices. From left to right, Sensable’s Premium, Force Dimension’s Omega and Novint’s Xio.
different tissue regions, the hand movements near crucial areas, the drilled volume of tooth bone are some examples which can visually be presented immediately after the training procedure creating a more efficient and interactive communication between the trainee and the instructor. Besides medical training, the use of patient specific data offers the opportunity of rehearsal immediately before an operation. Assuming a high degree of realism in these rehearsals, this can provide the ability to foresee possible problems and reassurance to the surgeons, meaning virtual environments have great potential to significantly affect real lives. Virtual environments in medicine, therefore, can have superiority over traditional training methods in many aspects, which has also been demonstrated [95] in the literature.

One of the most popular uses of virtual environments is surgery simulations, particularly for minimally invasive surgeries (MIS) where operations are carried out through tiny incisions in the skin, a cavity or an anatomical opening without larger cuts. A long thin tube or a similar device is used to perform the desired tasks and observe the operation region indirectly with the help of a camera attached to it. The image from the camera is magnified and viewed on displays during the surgery. The instruments are fixed at some point which limits the movements and the force feedback and monitors deployed for visual feedback are features naturally fitting virtual simulation environments. Simulation of MIS is, therefore, an active commercial market including hepatic [25], laparoscopic [16, 105], endoscopic gynaecology [102], endoscopic sinus [79], intestinal [80] and knee [35, 43] surgeries. Compared with open surgeries, however, the importance of force feedback is much smaller in MIS due to the limited movement and force magnitude ranges.

In open surgeries, unlike MIS, the surgeon has a direct vision and contact with the patient and often larger incisions and cuts through the body are required. The freedom in motion and the direct visual and haptic feedback make the simulation of open surgeries much more challenging than MIS. A cataract surgery, in which the original lens is replaced by an artificial lens implant, is simulated in [21] while a virtual eye model [85] as well as abdominal trauma surgery [15] were proposed for medical simulation. In [72], interactions with complex human joints, as in the case of the human shoulder, were simulated for virtual arthroscopy and physiotherapy palpation. Among the widely performed operations in medicine are suturing and needle based operations. Although these are relatively simpler procedures, an appropriate training is still crucial and these procedures have also found place in virtual environments [61, 70, 111].

In spite of the great potential of virtual environments in medicine, the spread of their use as a training method at medical institutions is quite slow. The main reason for this slow progress is insufficient realism which is so important if virtual environments are to replace traditional methods in medical training. It is essential to keep standards high in the field since inappropriate surgeon training might have serious consequences. Immersion is a crucial factor for delivering the trainee as realistic an experience as possible, while the realism of the force feedback is important for gaining the right hands-on habits. To create a stronger immersion effect, it is common to deploy haptic devices within a workbench environment [38, 45, 94, 101] which collocates the haptic and visual workspaces, as illustrated in figure 1.2 to imitate interaction in a natural way. Achieving sufficient realism in
Introduction

Figure 1.2: An immersive setup displaying a 3D virtual image collocated with the workspace of the haptic device via a semi-transparent mirror and stereo glasses.

force feedback, on the other hand, has been a bottleneck for scenarios including materials with complex physical characteristics.

1.1 Research Challenges

Following the global principle saying, “Nothing comes free in this world...”, the force feedback has brought its challenges as well. Our somatosensory system can perceive stimuli up to 1 kHz [30]. This requires an update rate for the force feedback to be in the same range to be able to provide a stable and smooth feeling of touch. Providing force feedback for interactive applications, therefore, requires the solution of haptic algorithms in a limited time. The choice of haptic algorithms, on the other hand, determines the realism which depends on the complexity of the scene. In the field of soft tissue deformation, for example, advanced algorithms need to be deployed in order to simulate complex material properties of soft tissues such as non-linearity, viscoelasticity, and anisotropy [86, 116]. The deployment of these advanced algorithms for sufficiently large resolution data comes with a computation burden, therefore meeting the limited time requirement to provide a stable force feedback becomes non-trivial.

The most significant challenge has been achieving a compromise between realism and performance. A stable interaction in real-time is a must for haptic applications, therefore the common approach has been to maintain the desired refresh rates while sacrificing realism. This sacrifice happens by choosing a fast algorithm which cannot model the desired physical properties and/or using a low-resolution simulation space which is not sufficient to represent the details accurately. Finding novel haptic algorithms and optimization of mathematical models to enable use of sufficiently large resolution simulations has been a popular track in the haptics community.

Continuing the discussion on the realism in medical simulations, one has to mention that neither determining the degree of realism required, nor evaluation of existing systems has been easy. One issue is the large number of descriptive reports, with few outcome-
1.2 Overview of the thesis

There are studies [34, 42, 76, 87, 114] which have tried to address this issue by evaluation or performance comparison of the existing simulations. The source of the problem, however, is the difficulty in determining and quantifying the requirements for simulations, which was conceptually discussed in [5, 53, 112]. In addition to the need for more validation and assessment of the systems, as emphasized in [53], the lack of human factor studies and the necessity for the development to be ‘needs driven’ instead of technology driven [5] must be addressed as well.

A common approach used to evaluate surgery simulations is to have the surgeons in the field test the simulation and ask about their qualitative opinion. This approach is obviously not the most reliable method due to the subjectivity factor during the process. Objective evaluation of haptic systems, generally, is a topic which requires widely accepted standards in the haptic community. Although some work is going on [97] to create such standards, a widely accepted procedure for evaluating proposed algorithms is still not in place.

In order to develop effective haptic applications, in addition to validation and evaluation, we also need a better understanding of our perception mechanism. All of our experiences can be considered as an interpretation of the world around us by our brain. To accomplish such a complex interpretation, our brain continuously synthesizes input from different modalities such as visual, audio, haptic, smell, taste etc. A comprehensive understanding of this process in the brain has a great potential to shape our civilization, by changing the magnificent products of ours like art and technology. Due to the complexity of this process, however, we are still far from a complete understanding of the organ that leads our lives.

Along the way to an understanding of our brain, the branch of science known as psychophysics has been taking tiny steps, partly focusing on the perception of touch. There have been numerous questions including, but not limited to, perception limitations, factors affecting perception, and the contribution of multimodalities to perception. In parallel with the advances in the haptic hardware technology, these questions have mostly focused on properties such as mass, stiffness, viscosity, shape, size, length etc. In addition to understanding our nature, the research in psychophysics also aims to improve the design and development of haptic hardware and software solutions, however, many mysteries about our perception mechanism still exist.

1.2 Overview of the thesis

The thesis is divided into two parts where the aim of the first part is to introduce the context, provide necessary background and present the literature survey to the reader. In the second part the contributions are summarized in three chapters. In chapter 3, studies on accuracy analysis of soft tissue deformation are discussed. The control and assessment of error in real-time is related to perception limits in order to optimize adaptive simulation parameters. Chapter 4 summarizes the findings of perception studies surveying the stiffness perception affected by different ways of touch and the effect of stiffness gradient on
perception. An algorithm providing high-resolution complementary information about the inhomogeneities beneath a surface during surface exploration is summarized in chapter 5. Finally, a summary of contributions is followed by a discussion and future of deformation simulation in chapter 6.

1.3 Overview of the papers

**Paper I:** The *asynchronous regions* which are parts of a deformable mesh solved at different update rates and by different methods are introduced. The aim of *asynchronous regions* is to update the local neighbourhood of the contact with higher refresh rates while performing the deformation calculations for more remote regions less frequently.

**Paper II:** An abstract pipeline to analyse the force error due to use of multiresolution solution in both time and the spatial domain is proposed. The uses of the pipeline are summarized as: to create an explicit correlation between the error and simulation parameters, to monitor the force error in real-time without significant computation burden and to adjust the multiresolution simulation parameters to keep the force error below desired limits.

**Paper III:** This paper has surveyed the effects of different ways of touching on stiffness perception through evaluation studies. The continuity of touch and the direction of motion over the surface have been examined as two basic aspects of various exploratory procedures.

**Paper IV:** In this paper the significance of stiffness gradient during continuous contact on stiffness perception has been examined through user studies.

**Paper V:** In this paper, the widely used haptic rendering algorithm, virtual coupling, for surface exploration is modified to provide high-resolution complementary information about inhomogeneities beneath the surface. The energy variations in the spring-damper system due to varying stiffness are compensated for to maintain passivity. The compensating terms, providing stiffness information, are shown to improve shape recognition underneath a surface.

**Paper VI:** In this paper the algorithm proposed in the previous paper is extended to deformable meshes and tested in this new context, through user studies.
Chapter 2

Background

The work presented in this thesis is composed of contributions to different branches related to deformation simulation including mathematical modelling of physically-based deformation, perception of touch, and haptic rendering. This chapter starts with a discussion of the theoretical background on continuum mechanics for soft tissue modelling which aims to provide the reader with a better understanding of the underlying fundamentals about deformation. This is followed by an overview of techniques used in real-time deformation simulation, primarily focusing on the physically-based applications. Finally, a literature survey of the perception studies related to touch is presented.

2.1 Continuum Mechanics for Soft Tissue Modelling

Continuum mechanics studies the response of materials to different loading conditions of deformation. Materials are assumed to be continuously distributed within the entire space of their body rather than as being composed of molecular or crystal structures. Basic concepts defined in continuum mechanics are commonly used in the modelling of anatomical structures. The aim of this section is to present a general outline for deriving a mathematical deformation model. The reader is assumed to have basic knowledge of tensor algebra and derivation of some basic deformation properties were not included in the discussion. For more elaborate analysis of continuum mechanics and tensor algebra, the reader is referred to the book [13] on non-linear continuum mechanics for finite element analysis.

Deformation refers to motion of a body, a set of particles. This motion can be analysed in two different ways. The first alternative is to describe relevant quantities in the current frame of deformation which varies continuously with deformation. This method is called an Eulerian description which focuses on the behaviour at a spatial position. For instance, if a time-varying density is considered, a change in the time will result in a different density being observed at the same spatial position, possibly occupied by a different particle. Eulerian description is the most common technique employed in fluid mechanics simulations.

The second alternative is to describe relevant properties in the initial configuration, before the actual deformation occurs. This method is called a material or Lagrangian description which has its focus on the behaviour of a material particle. In the case of the time-varying density, a different density of the same particle is observed in this description. The use
of Lagrangian description is common in solid mechanics which will be the focus throughout
the rest of this section.

### 2.1.1 Deformation Gradient

Deformation can be considered as the morphing of a body from an initial configuration to
another configuration. Quantities such as distance between two points, the surface area
or volume will change during this morphing. The relationships between such quantities
between initial and current configuration, during deformation, are defined by a second order
tensor called the deformation gradient. It is a key quantity for various strain measures by
defining the relative spatial positioning of two neighbouring particles during deformation
in terms of their relative spatial positioning before deformation. This can be explained
by the help of figure 2.1 which shows the motion of three particles during deformation.
The deformation is defined by a mapping function, \( \phi \), in the figure such that the current
position of a particle, \( x \), is a function of the initial position, \( X \), and time:

\[
x = \phi(X, t)
\]

(2.1)

Considering two points, \( p \) and \( q_1 \), the difference vector from one point to the other after
the deformation, \( l_1 \), can be written as:

\[
l_1 = x_{q_1} - x_p = \phi(X_p + L_1, t) - \phi(X_p, t)
\]

(2.2)
where \( \mathbf{L}_1 \) refers to the difference vector before the deformation. The deformation gradient can then be defined as:

\[
F = \frac{\partial \phi}{\partial \mathbf{X}} \tag{2.3}
\]

The difference vector between the two points in the current configuration can be written in terms of the difference vector between the points in the initial configuration with the help of the deformation gradient as:

\[
\mathbf{l}_1 = F \mathbf{L}_1 \tag{2.4}
\]

By transforming vectors from one configuration to another, the deformation gradient is said to be a two-point tensor. The relationship it defines between two difference vectors is used in various strain measures.

### 2.1.2 Strain Measures

Deformation can be considered as a special case of motion of particles in a free space. The case where the distance between each particle is preserved is called a rigid body motion. Deformation, on the other hand, can be defined as strain referring to how much a motion differs from a rigid body motion. There are different measures used for strain, two most common ones will be defined in this section.

**Cauchy-Green Tensor:**

Considering the scalar product of two difference vectors, \( \mathbf{L}_1 \) and \( \mathbf{L}_2 \), in figure 2.1, one can define a deformation measure involving both the stretch, the change in length, and the change in the enclosed angle between the difference vectors. Writing the scalar product in the current configuration in terms of the difference vectors in the initial configuration by using equation 2.4:

\[
\mathbf{l}_1 \cdot \mathbf{l}_2 = (FL_1)^T FL_2 = L_1^T F^T F L_2 = L_1 \cdot C L_2 \tag{2.5}
\]

the right *Cauchy-Green tensor* can be defined in terms of the deformation gradient as:

\[
C = F^T F \tag{2.6}
\]

**Lagrangian-Green Tensor:**

If one writes the change in the scalar product in terms of the difference vectors in the initial configuration:

\[
\mathbf{l}_1 \cdot \mathbf{l}_2 - \mathbf{l}_1 \cdot \mathbf{L}_2 = L_1 \cdot C L_2 - L_1 \cdot \mathbf{L}_2 = L_1 \cdot (C - I) L_2 = 2L_1 \cdot E L_2 \tag{2.7}
\]

the *Lagrangian-Green Tensor*, given in terms of the *Cauchy-Green tensor*, can be defined as:

\[
E = \frac{1}{2}(C - I) \tag{2.8}
\]
Both tensors presented above depend on the deformation gradient which is dependent on the coordinate system in which it is defined. The strain, however, is a physical measure of deformation which should be independent of the coordinate system. Certain features of a second order tensor, called invariants, are independent of the reference coordinate system used. The three invariants of the strain tensor $\varepsilon$ are:
\[
\begin{align*}
I_1\varepsilon &= \text{tr}(\varepsilon) \\
I_2\varepsilon &= \frac{1}{2}[\text{tr}(\varepsilon^2) - (\text{tr}(\varepsilon))^2] \\
I_3\varepsilon &= \det(\varepsilon)
\end{align*}
\]

While defining strain, these invariants are used to keep the strain independent of the reference frame where the deformation gradient is created.

### 2.1.3 Stress Measures

Deformation is propagated through a body via internal forces between particles produced as a reaction to the external forces acting upon the body. Stress is a measure of these internal forces which are analysed within infinitely small areas of imaginary internal surfaces. The use of infinitely small areas, in the limit, defines the stress at each point within the body which results in a second order tensor at that point. In this section the widely used \textit{Cauchy stress tensor} is introduced followed by the \textit{Piola-Kirchhoff stress tensors}.  

![Figure 2.2: Traction forces and stress during deformation.](image)
2.1 Continuum Mechanics for Soft Tissue Modelling

Cauchy Stress Tensor:

If an internal imaginary surface with an area $da$ and a normal $n$ around a point $p$ is considered in the current deformation configuration, as illustrated in figure 2.2, the traction vector at point $p$ corresponding to the normal direction $n$ is defined as:

$$ t(n) = \lim_{da \to 0} \frac{dr}{da} $$

(2.10)

where $dr$ is the resultant force acting on the surface. One can observe that the normal of the defined surface does not have to show the same direction as the resultant force at point $p$. This leads to the definition of the Cauchy stress tensor, $\sigma$, at that point as:

$$ t(n) = \sigma n $$

(2.11)

The Cauchy stress tensor, therefore transforms the normal of any cross-sectional surface to the traction vector caused by the resultant force acting on that point. The physical meaning of it can better be understood by combining the equations 2.10 and 2.11 into:

$$ dr = t da = \sigma n da $$

(2.12)

which shows that the Cauchy stress tensor is a measure of force per unit area in the current deformation configuration.

First Piola-Kirchhoff Stress Tensor:

The Cauchy stress tensor defines the relation between the normal of an internal surface and the traction vector in the current deformed configuration. A similar relation is defined by the First Piola-Kirchhoff stress tensor, between $N$ and $T(N)$ as:

$$ T(N) = PN $$

(2.13)

where $T(N)$ is the traction vector acting over an area, $dA$, in the initial configuration. The First Piola-Kirchhoff stress tensor also relates the area in the initial configuration to resultant force in the current deformation configuration [13] formulated as:

$$ dr = PNdA = TdA $$

(2.14)

In order to derive the explicit expression for the First Piola-Kirchhoff stress tensor, one needs to know the relation between the areas in the initial and current deformation configurations [13] which is:

$$ dan = JdAF^{-T}N $$

(2.15)

where $J$ is the determinant of the deformation gradient. Deriving equation 2.13 from equation 2.11 by the help of equations 2.12, 2.14 and 2.15 leads to:

$$ T = \frac{da}{dA} \sigma n = \frac{da}{dA} \sigma J \frac{dA}{da} F^{-T}N = J\sigma F^{-T}N $$

(2.16)

and the First Piola-Kirchhoff stress tensor can be written as:

$$ P = J\sigma F^{-T} $$

(2.17)
Second Piola-Kirchhoff Stress Tensor:

By defining a relationship between the area in the initial configuration and the resultant force in the current deformation configuration, the First Piola-Kirchhoff stress tensor does not completely belong to any of the configurations. The resultant force in the initial configuration can be obtained from the resultant force in the current deformation configuration by using the deformation gradient as;

$$dR = F^{-1}dr = F^{-1}PN\,dA$$

(2.18)

where the $dR$ refers the resultant force in the initial configuration. The Second Piola-Kirchhoff stress tensor can therefore be defined as;

$$S = F^{-1}P = JF^{-1}\sigma F^{-T}$$

(2.19)

which defines the force per unit area in the initial configuration.

2.1.4 Principle of Virtual Work

The formulations of a well known technique, Finite Element Modelling (FEM), are established on the principle of virtual work in solid mechanics. This requires the consideration of the virtual work, $\delta w$, done by the residual force, $r$, per unit volume and time. These residual forces are a result of external forces acting on the body. Figure 2.3 illustrates the external body forces $f$ per unit volume and traction forces $t$ acting on a deforming volume $v$ and its surface boundary $\partial v$. The sum of the residual forces over the whole volume can be written as a sum of the body and traction forces as;

$$\int_v r \, dv = \int_{\partial v} t \, da + \int_v f \, dv$$

(2.20)

By using the Gauss theorem and equation 2.11;

$$\int_v r \, dv = \int_{\partial v} \sigma n \, da + \int_v f \, dv = \int_v (\text{div}\sigma + f) \, dv$$

(2.21)

where div refers to the divergence operator. The virtual work can be expressed in terms of residual forces and virtual velocity $\delta v$ and then be integrated over the whole body. In case of equilibrium, the total amount of virtual work should diminish leading to;

$$\delta W = \int_v \delta w \, dv = \int_v r \cdot \delta v \, dv = \int_v (\text{div}\sigma + f) \cdot \delta v \, dv = 0$$

(2.22)

By using the property of divergence;

$$\text{div}(\sigma \delta v) = (\text{div}\sigma) \cdot \delta v + \sigma : \nabla \delta v$$

(2.23)
and Gaussian theorem, the virtual work can be rewritten as:

$$\delta W = \int_{\partial V} \mathbf{n} \cdot \sigma \delta \mathbf{v} \, da - \int_{V} \sigma : \nabla \delta \mathbf{v} \, dv + \int_{V} f \cdot \delta \mathbf{v} \, dv = 0 \quad (2.24)$$

Finally, by writing the first term of the right hand side in terms of traction forces, thanks to symmetry of $\sigma$ and expressing the gradient of the virtual velocity as the symmetric virtual rate of deformation $\delta \mathbf{d}$, the spatial virtual work equation can be obtained as:

$$\delta W = \int_{V} \sigma : \delta \mathbf{d} \, dv - \int_{V} f \cdot \delta \mathbf{v} \, dv - \int_{\partial V} t \cdot \delta \mathbf{v} \, da = 0 \quad (2.25)$$

This equation, showing the equilibrium during deformation, defines a basis for FEM equations. The first term referring to the internal virtual work depends on work conjugacy, which implies the work can be expressed as a multiplication of a pair of work conjugate quantities. In equation 2.25, it is written in the current deformation configuration as a product of the Cauchy stress tensor and the rate of deformation. This principle is used in defining the strain-stress relation, explained below, such that the stress and strain measures in this relation have to be work conjugate.

### 2.1.5 Strain - Stress Relation

The virtual work principle introduced above formulates the equilibrium conditions in terms of stress. The stress throughout the body results from the deformation of the body, there-
fore it is related to a deformation measure which is strain. There are several mathematical models defined for this relation between strain and stress, which are referred to as constitutive equations. These models are directly related to the type of the deformation behaviour to be simulated. The behaviour where the time-independent stress is defined as a function of the current state of deformation and is referred to as elastic behaviour. The case where the deformation results in permanent change in the body configuration is called plastic behaviour while the time-dependency of the stress-strain relation is known as viscoelastic behaviour. Materials in nature deform in various ways and the choice of the mathematical model is crucial to simulate the deformation behaviour correctly. A special type of elastic behaviour, hyperelasticity, where the virtual work done by the stresses depends only on the initial state at time $t_0$ and the final state at time $t$ is presented in this section.

In order to evaluate the elastic potential $\psi$ per unit volume at any time $t$, the virtual work done by the stress can be integrated by using a work conjugate pair of stress and strain measures. The most common work conjugate stress-strain pairs are: the First Piola-Kirchhoff stress tensor and the rate of deformation gradient, the Second Piola-Kirchhoff stress tensor and the Lagrangian-Green Tensor, and the Cauchy stress tensor and the rate of deformation. The use of the first two pairs is discussed here in the context of elastic potential;

$$\psi = \int_{t_0}^{t} P : \dot{F} \, dt = \int_{t_0}^{t} S : \dot{E} \, dt = \int_{t_0}^{t} \frac{1}{2} S : \dot{C} \, dt$$  \hspace{1cm} (2.26)

At this point it might be useful to recall that the stress measures in equation 2.26, $P$ and $S$, can be written as a function of their work conjugates, therefore making the elastic potential also a function of the corresponding strain measures. The rate of change of the elastic potential can therefore be written as;

$$\partial \psi(F) = \frac{\partial \psi}{\partial F} : \dot{F}$$
$$\partial \psi(E) = \frac{\partial \psi}{\partial E} : \dot{E}$$  \hspace{1cm} (2.27)
$$\partial \psi(C) = \frac{\partial \psi}{\partial C} : \dot{C}$$

By combining the last two equations, one can write;

$$\left( \frac{1}{2} S - \frac{\partial \psi}{\partial C} \right) : \dot{C} = 0$$  \hspace{1cm} (2.28)

In summary, defining a relation between the stress and strain first requires defining an elastic potential $\psi$ as a function of a chosen strain measure. The function $\psi$ is constructed in the light of physical experiments in order to model observed deformation behaviour. The rate of change of $\psi$ with respect to the chosen strain measure is then related to the corresponding stress tensor, which is a work conjugate of the chosen strain. A well-known hyperelastic material, for instance, is the St. Venant Kirchhoff model which has an elastic
potential as as function of the Lagrangian-Green Tensor;

$$\psi(E) = \frac{1}{2} \lambda (\text{tr}(E))^2 + \mu E : E$$

(2.29)

where \( \lambda \) and \( \mu \) are material coefficients. The corresponding work conjugate pair, Second Piola Kirchhoff stress tensor can be evaluated by the help of 2.28 as;

$$S = \lambda \text{tr}(E) I + 2\mu E$$

(2.30)

This mathematical model shows linear behaviour, therefore, it is realistic assuming the strain is kept smaller.

Compressible Materials:

There is no restriction on the strain during deformation of compressible materials, which implies that equation 2.28 must hold for any arbitrary rate of change of the strain tensor. Therefore the term in the parentheses must diminish for any strain tensor, which allows to rewriting of the Second Piola-Kirchhoff stress tensor as;

$$S = 2 \frac{\partial \psi}{\partial C}$$

(2.31)

In our daily lives, many materials show an isotropic behaviour which means identical behaviour in any material direction, which requires that the stress-strain relation be independent of any reference frame. This relation depends on the defined elastic potential which is a function of a second order strain tensor. Recalling the discussion of the invariants of second order tensors in section 2.1.2, one needs to define the elastic potential as a function of invariants in order to model isotropic behaviour. Choosing the Cauchy-Green tensor as a strain measure, the Second Piola-Kirchhoff stress tensor can be written as;

$$S = 2 \frac{\partial \psi}{\partial C} = 2 \frac{\partial I_1}{\partial C} \frac{\partial I_1}{\partial C} + 2 \frac{\partial I_2}{\partial C} \frac{\partial I_2}{\partial C} + 2 \frac{\partial I_3}{\partial C} \frac{\partial I_3}{\partial C}$$

(2.32)

where \( I_1, I_2 \) and \( I_3 \) refer to the invariants of the Cauchy-Green tensor which are given together with their derivatives as;

$$I_1 = \text{tr}(C) = C : I \quad ; \quad \frac{\partial I_1}{\partial C} = I$$

$$I_2 = \text{tr}(CC) = C : C \quad ; \quad \frac{\partial I_2}{\partial C} = 2C$$

$$I_3 = \text{det}(C) = J^2 \quad ; \quad \frac{\partial I_3}{\partial C} = J^2 C^{-1}$$

(2.33)

Compressible materials are a special case of hyperelasticity. An example is compressible Neo-Hookean material with an elastic potential function as;

$$\psi = \frac{\mu}{2} (I_1 - 3) - \mu \ln J + \frac{\mu}{2} (\ln J)^2$$

(2.34)

and the Second Piola-Kirchhoff stress tensor can be rewritten, with the help of equation 2.32, as;

$$S = \mu (I - C^{-1}) + \mu (\ln J) C^{-1}$$

(2.35)
Incompressible Materials:

In order to talk about incompressibility, one needs an elaborate analysis of strain and stress measures in relation to volume change during deformation. Considering a volume element represented by three orthogonal vectors one can evaluate the volume by triple product of these vectors. A comparison of this volume element between initial and current deformation configuration can be done by deformation gradient. The volume change during deformation can be formulated by:

\[ dv = JdV \]  

(2.36)

where \( J \) is the determinant of the deformation gradient [13]. The deformation gradient is considered as composed of two components: volumetric and distortional during incompressibility analysis. The distortional component implies no change in volume, therefore, has a determinant of unit value. To meet this requirement, the distortional component, \( \hat{F} \), is related to the deformation gradient as:

\[ \hat{F} = J^{-1/3} F \]  

(2.37)

Since the deformation gradient is the basic building block for the strain tensors, all of these tensors can be decomposed into volumetric and distortional components. Symbols with an additional cap will be used to refer to the distortional components of the quantities. Similarly to strain, stress can also be decomposed into volumetric and distortional components as:

\[ \sigma = \hat{\sigma} + pI; \quad \text{tr}(\hat{\sigma}) = 0; \quad p = \frac{1}{3}\text{tr}(\sigma) \]  

(2.38)

where \( p \) is called the hydrostatic pressure. Like the Cauchy stress tensor, the Piola-Kirchhoff stress tensors can also be written as two components by the help of equations 2.17 and 2.19;

\[ P = J \hat{\sigma} F^{-T} + pJF^{-T} = \hat{P} + pJF^{-T} \]
\[ S = JF^{-1}\hat{\sigma} F^{-T} + pJC^{-1} = \hat{S} + pJC^{-1} \]  

(2.39)

In order to evaluate the hydrostatic pressure, one can take the double contraction of the above equations by \( F \) and \( C \), respectively, to obtain:

\[ \hat{P} : F = 0 \]
\[ \hat{S} : C = 0 \]  

(2.40)

which is then used with equation 2.39 to give hydrostatic pressure as:

\[ p = \frac{1}{3} J^{-1} \hat{P} : F \]
\[ p = \frac{1}{3} J^{-1} \hat{S} : C \]  

(2.41)
2.1 Continuum Mechanics for Soft Tissue Modelling

Going back to the discussion of strain-stress relation in the case of incompressible materials, the term in the brackets in equation 2.28 is not always zero since the rate of the strain tensor is no longer arbitrary. The fact that the deformation gradient must have a unit determinant, therefore zero rate of change of determinant, for incompressibility, requires the following restriction on the rate of strain tensor:

\[ \frac{1}{2} J C^{-1} : \dot{C} = 0 \quad (2.42) \]

Comparing this relation with equation 2.28 and interpreting the double contraction operator for the second order tensors as a dot product of vectors, one can write;

\[ \frac{1}{2} S - \frac{\partial \psi}{\partial C} = \gamma \frac{J}{2} C^{-1} \quad (2.43) \]

where \( \gamma \) is an unknown scalar coinciding with the hydrostatic pressure under certain circumstances. By re-arranging the equation Second Piola-Kirchhoff stress tensor can be obtained as;

\[ S = 2 \frac{\partial \psi(C)}{\partial C} + \gamma J C^{-1} \quad (2.44) \]

The resemblance of the equations 2.44 and 2.39 can be further analysed in order to relate the unknown scalar \( \gamma \) with the hydrostatic pressure. Rewriting the Second Piola-Kirchhoff stress tensor in equation 2.41 as;

\[ p = \frac{1}{3} J^{-1} S : C = \frac{1}{3} J^{-1}[2 \frac{\partial \psi}{\partial C} + \gamma J C^{-1}] : C = \gamma + \frac{2}{3} J^{-1} \frac{\partial \psi}{\partial C} : C \quad (2.45) \]

which shows \( \gamma \) and hydrostatic pressures coincide only if the double contraction in the second term diminishes. This can be met if the elastic potential function is homogeneous of order 0, that is, \( \psi(\alpha C) = \psi(C) \) for any arbitrary constant \( \alpha \) [13]. With some algebra, one can show that the use of distortional component of the Cauchy-Green stress tensor, \( \tilde{C} \), satisfies this condition. Therefore the strain-stress relation for incompressible materials can be finally expressed as:

\[ S = 2 \frac{\partial \psi(\tilde{C})}{\partial \tilde{C}} + p J C^{-1} \quad (2.46) \]

The mathematical model for incompressible Neo-Hookean material can be obtained by setting \( J \) to one in equation 2.34 as:

\[ \psi(C) = \frac{1}{2} \mu (\text{tr}(C) - 3) \quad (2.47) \]

and the corresponding Second Piola Kirchhoff stress tensor then becomes:

\[ S = 2 \frac{\partial \psi(\tilde{C})}{\partial \tilde{C}} + p J C^{-1} = \mu J_{hc}^{-1/3} (I - \frac{1}{3} I_{hc} C^{-1}) + p J C^{-1} \quad (2.48) \]
2.1.6 Soft Tissue Characteristics

The discussion up to this point in this section has been on the mathematical models aiming to define physical properties of materials observed in nature. One needs to keep in mind that these are theoretical models which have not yet been matured enough to be able to define the deformation behaviour completely. Complex physical properties of materials contribute to deformation behaviour in real life. It is common to observe non-linear behaviour in soft tissues, including both geometric and material non-linearity. The first of these refers to the non-linearity between the strain and the displacements of the deformed body, while the second is defined between strain and stress. Simulation of non-linearity introduces additional computation burden since it requires update of the strain and stress measures used to define the internal virtual work during deformation.

Viscoelastic behaviour has been observed in soft tissues like the brain [27, 64] and liver [86]. In the case of viscoelasticity, the stress-strain relation not only depends on the current deformation configuration but is also affected by the history of the deformation. The different paths during loading and unloading phases in stress characteristics result in dissipation of stored energy, which is absorbed by the material. This phenomenon is referred to as hysteresis in the literature. In addition, the speed of the loading also determines the path followed in the stress-strain relationship. Increasing strain over time in the case of constant stress, creep, and decreasing stress over time in the case of constant strain, relaxation, are additional characteristics of viscoelasticity. Simulation of these concepts requires keeping track of the deformation history which not only affects the real-time performance considerations but demands a significant amount of memory resources as well. Another characteristic which is sometimes observed in soft tissues is anisotropy meaning that the deformation behaviour of the material becomes direction dependent.

2.2 Real-Time Deformation Simulations

Simulating a deformable object in a computer environment has been a challenging field. In addition to mathematical models to define physical behaviour, digital limitations have to be taken into account. Efficient application of the mathematical models on the data structures representing the object, limited memory, performance requirements and visualization can be named among the significant concerns. To date, it has not been possible to achieve realistic behaviour as observed in real life but several methods have been developed, leading to an evolution which may yield a solution.

Work on deformation of objects started in the second half of the 1980’s, before which only rigid objects could be visualized. Free form deformation was presented in [90]. This was followed by the study [106] using material properties in the deformation for the first time. Since then animations, object modelling, time-varying medical visualization, fluid mechanics and surgery simulations have become some of the main applications in the deformation simulation.

It is out of scope of this document to attempt to present a complete analysis of de-
formation algorithms, however, an overview of the techniques is given in addition to a brief explanation of the mesh-based methods which are most commonly used in object deformation. This is followed by a survey of studies on real-time deformation simulation using FEM, which is the method related to the work presented in this thesis. In case the reader is interested in a more detailed explanation of various methods, the state of the art reports [36, 63, 67] provide an elaborate discussion of deformation simulation.

2.2.1 Overview of the Methods

The methods in the literature can be categorized into two main categories: Physical and non-physical models. Non-physical models use only the geometrical properties of the objects. The talent of the designer, therefore, has a more important role than the physical principles. These are mostly employed when the artistic appeal of the simulation dominates realism concerns, as in the case of animations in movies and computer games. The most common techniques are parametric curves, and free form deformation. The parametric methods use a series of control points to deform the curves, surfaces or volumes. Bezier curves and NURBS (Non-Uniform Rational B-Splines) are among the most well-known parametric techniques. In the case of free form deformation, on the other hand, the space in which the object lies is deformed.

Physical methods, obviously, provide more realistic behaviour since they depend on the principles observed in nature to some extent. *Eulerian* methods, in general, have found more application areas in various fluid simulations while simulation of solid mechanics has favoured the *Lagrangian* methods. Among the *Lagrangian* methods, mesh-based methods have been a popular choice where the object is represented by a mesh composed of a set of points and element volumes, or areas, defined between these points.

**Mass - Spring Method:**

In the mass-spring method, the object is modelled by a number of discrete mass points which are connected to each other with springs (or spring-damper systems) and Newton’s laws of motion are employed on these springs:

\[
m_i \dddot{u}_i + b_i \dddot{u}_i + f^\text{int}_i = f^\text{ext}_i
\]  

(2.49)

where \(m_i, b_i, f^\text{int}_i, f^\text{ext}_i, \) and \(u_i\) refer to mass, damping, internal and external forces, and displacement of the mass point \(i\), respectively. The internal force acting on a particle is calculated by summing the forces of the springs connected to the mass point by:

\[
f^\text{int}_i = \sum_j k_j^i (|x_{ij}| - l_{ij}) \frac{x_{ij}}{|x_{ij}|}
\]  

(2.50)

where \(k_j^i, x_{ij}, l_{ij}\) correspond to stiffness, displacement and initial distance between two points, respectively. The set of resulting differential equations, 2.49, is solved by integrating the time-step employing explicit or implicit solvers. Corresponding displacements for
the mass points are calculated by this procedure in response to given force inputs. The stiffness of the springs is used to simulate hardness/softness of the material while additional components like damping have also been used in attempts to simulate viscoelasticity.

The mass-spring model is a fast and simple technique compared to continuum-based techniques, however, it has some major drawbacks affecting the realism of the deformation. The major drawback is that the spring parameters are not easy to tune to accurately represent measured material properties. In most of the earlier cases the parameters were set experimentally, while the use of learning algorithms [10] and comparison with FEM [11, 77] have also become common more recently. Another drawback is that modelling incompressible volumes and thin surfaces persistent to bending is difficult to realize. The mass-spring method, therefore, is mostly considered to be a simple and fast algorithm for interactivity but not for attaining a realistic deformation behaviour.

Chain - Mail Method:

The chain-mail method was introduced in [37] which depends on the idea of representing the model as a set of chains hold together. In short, each chain can move freely within a specified boundary and, when this boundary is violated, the chain causes its neighbour to move as well. The algorithm is composed of two main steps. First, the displacements of a chain unit and its neighbours are calculated and, if this chain unit causes any of its neighbours to move, the same procedure is repeated for the neighbour. The whole chain is traversed in this way. Second, a relaxation step is carried out, where the chain is relaxed locally until it reaches a valid state of minimum energy.

The major advantage of the chain-mail method is its simplicity and performance as in the mass-spring model. The original algorithm, however, suffered from two major drawbacks: The restriction to rectilinear grids and the inability to simulate inhomogeneity. The former was addressed in [59], and the latter in [88]. Modelling inhomogeneity, by basically changing the boundaries of the chains, introduces substantial extra computational load. The original algorithm has proved that it is enough to pass each chain unit element only once, which is not valid for the inhomogeneous case. This problem was solved by using a sorted list during neighbourhood movement. As a result, as in the mass-spring method, the main advantage is the performance while the realism is typically poor because of the difficulties in the adjustment of the parameters.

Continuum - Based Methods:

The continuum-based methods consider the deformable object as a solid body with mass and energy continuously spread throughout the object. For the solution of deformation in a computational environment, a discrete computational representation is needed. After creation of the continuum-based models as discussed in section 2.1, a discretization of these models into a finite state vector is performed. Principles for FEM, a widely used continuum-based method, will be used to explain this discretization.
2.2 Real-Time Deformation Simulations

The main aim of the discretization is to define a relation between the mesh used to represent the object in the digital environment and the fundamental principles of continuum mechanics. Recalling that the deformation gradient is a key building block depending on position, the discretization of position would be sufficient to represent stress and strain measures used in the spatial virtual work equation 2.25 which defines a basis of FEM. The discretization of the external forces in this equation will finally lead to the FEM partial differential equations.

Considering a single element, the position of a point within the element can be interpolated from the positions of the nodes in the element as:

$$x = \sum_{a=1}^{n} N_a(x_a(t))$$  (2.51)

where $x_a$ refers to displacement of the node, $a$, and the summation is performed for all the nodes within the element. $N_a$ is called the standard shape function referring to the weight distribution of each node within the volume of the element, therefore $N_a$ is a function of three variables, $\xi_1, \xi_2, \xi_3$, referring to 3 dimensions. Deformation gradient has been defined as the derivative of the deformation mapping with respect to the initial configuration at a point, equation 2.3, and the deformed mapping at such a point actually refers to position of that point, $x$. The deformation gradient can be rewritten as:

$$F = \frac{\partial x}{\partial X} = \frac{\partial}{\partial X} \sum_{a=1}^{n} N_a(x_a(t)) = \sum_{a=1}^{n} x_a \otimes \frac{\partial N_a}{\partial X}$$  (2.52)

where the partial derivative of the shape function with respect to the initial configuration position can be calculated as:

$$\frac{\partial N_a}{\partial X} = ((\frac{\partial X}{\partial \xi}))^{-T} \frac{\partial N_a}{\partial \xi}$$  (2.53)

By this summation, all the nodes within an element contribute to the resulting deformation gradient which is a second order tensor. The definition of strain and stress measures which are obtained from the deformation gradient does not demand additional consideration than has been presented in section 2.1, until we reach the stage of discussing the spatial virtual work equation. Rewriting equation 2.25, by introducing the interpolation for the virtual rate of deformation in terms of virtual velocity [13], for only one node within a single element $e$:

$$\delta W^{(e)} = \int_{v^{(e)}} \sigma : (\delta \mathbf{v}_a \otimes \nabla N_a) \, dv - \int_{v^{(e)}} f \cdot (N_a \delta \mathbf{v}_a) \, dv - \int_{\partial v^{(e)}} t \cdot (N_a \delta \mathbf{v}_a) \, da$$  (2.54)

where $\nabla$ refers to partial derivative with respect to $X$. By rearranging the terms:

$$\delta W^{(e)} = \delta \mathbf{v}_a \cdot (\int_{v^{(e)}} \sigma \nabla N_a \, dv - \int_{v^{(e)}} N_a f \, dv - \int_{\partial v^{(e)}} N_a t \, da) = \delta \mathbf{v}_a \cdot (T^{(e)} - F^{(e)})$$  (2.55)
where $T_a^{(e)}$ and $F_a^{(e)}$ correspond to the contribution of a single element $e$ on internal and external forces, respectively, of a single node $a$. A summation is carried out for all nodes within a single element, followed by a second summation of contributions from other elements which ultimately gives total forces, $T_a$ and $F_a$, on a single node $a$. For the equilibrium to be met the total amount of virtual work for the whole mesh must diminish, therefore we can rewrite the virtual work as:

$$\delta W = \sum_{a=1}^{N} \delta v_a \cdot (T_a - F_a) = 0 \quad (2.56)$$

Recalling that the above equation has to hold for any arbitrary virtual velocity, one can conclude that the internal and external forces need to be equal. By changing the notation to fit in the general FEM terminology and adding the dynamic components, the ultimate FEM equations can be obtained as:

$$M \ddot{u} + C \dot{u} + K(u)u = F \quad (2.57)$$

where $M$ and $C$ refer to mass and damping matrices while $u$ and $F$ refer to displacement and external force vectors respectively. The last term on the left hand side represents the internal forces, $T_a$ in equation 2.56. The most common way to solve this equation system is discretization of simulation time into time-steps and deploying iterative solvers to evaluate the displacements of the nodes in the mesh to a set of applied forces on the nodes.

### 2.2.2 Deformation Simulation with FEM

FEM is considered to be the method providing the most realistic deformation behaviour by allowing adjustment of material properties in a deterministic way. The use of FEM in real-time simulations, however, is not trivial. Applications providing force feedback via haptic devices have high performance demands such that the refresh rate for the force calculation needs to be in the range of 1 kHz in order to deliver a continuous and stable force feedback to the user. This requirement creates a compromise between realism and performance due to the computational burden of FEM. Researchers, therefore, have been developing various optimization techniques to be able to deploy FEM in real-time interactive applications. This section presents an overview of the studies deploying FEM in deformation simulation, especially in the field of medicine.

The studies [32, 47, 61, 65, 69, 91, 118] use linear FEM to simulate deformation. Static condensation is applied to group the solution system into boundary and non-boundary condition nodes in [32]. The known boundary node displacements and non-boundary node forces are used to evaluate the unknowns. The banded structure of the stiffness matrix is exploited in [8] to simulate suturing and the work is improved in [9, 61]. In [91], FEM calculations are done offline during the preprocessing step to evaluate constraints to simulate tissue cutting, then the constraints for the nodes along the cutting path are moved and the model is updated by using discontinuous discrete free form deformations during the simulation. In [118] modification of the tissue cutting was permitted but to
2.2 Real-Time Deformation Simulations

achieve this, the possible operation area is restricted to a local region in the preprocessing step. Another common and efficient optimization technique for only static simulations is condensation [14] which simplifies the solution matrix by removing the interior nodes by keeping their effect.

Soft tissue has proven to show complex behaviour like non-linearity, viscoelasticity and anisotropy as discussed in section 2.1.6. Taking into account these physical properties in FEM simulations introduces additional computational burden. In the case of non-linearity the stiffness matrix, $K(u)$ in equation 2.57 becomes a function of displacement varying during deformation, in the case of viscoelasticity the deformation response becomes time dependent requiring keeping the history of the deformation and anisotropy requires simulating different responses of tissue to different directions.

A pre-processing or an optimization method is needed for the solution of non-linear systems. The idea of recording the response of the system for all nodes to a unit input in pre-processing is implemented in [22]. The linear displacement equations are solved for the whole mesh in the pre-processing and the superposition principle is applied to the recorded data to calculate a combination of the linear response in real-time. Non-linear and anisotropic tissues are modelled in [78]. Due to the computational burden the non-linearity brings, non-linear components are added only for a local neighbourhood of the contact area. A similar idea was exploited in [12] where a number of samples were acquired for deformation, applied force magnitude and force direction by using a force sensor. Strain-stress relations for each measurement were interpolated in strain space to synthesize deformation in real-time.

The principle of recording the impulse response has been used in several other studies. In [89] this idea was exploited to model linear viscoelasticity. The principle was used, together with local dynamic regions, in a hybrid model to allow cutting of the soft tissue in [26], which restricts the cutting region depending on specific surgery types in the pre-calculation step. The hybrid model with a restricted operation region and adaptive meshes were implemented in [43] and [119].

When making modifications to the topology, maintaining the refresh rates becomes a challenge. In addition to modifying the physical solution system as in non-linear cases, the visual update of the model takes significant amount of time. In the case of FEM, re-meshing with sufficient resolution is not reasonable in real-time, however one suitable solution is Extended FEM (XFEM). XFEM allows the update of the interpolation functions used during the creation of the initial mesh. In the case of cutting, instead of recalculating the elements, discontinuities are allowed to be introduced to the interpolation functions. [110] is an example exploiting XFEM for simulation of tissue cutting.

Another common technique to increase speed performance is deploying an adaptive multiresolution mesh [17, 24, 40, 47, 51, 54, 68, 119] with a higher resolution near the local neighbourhood of the contact. There are two main different approaches to implementation of multiresolution of FEM. The first one is to evaluate and store the matrices of the different levels of detail independently. The second one is to use hierarchical Finite Elements. In the case of hierarchical Finite Elements, during the calculation of element stiffness matrix for a level, the shape functions of all coarser levels are also blended into the current level with
varying weights. The idea is to retain the behaviour of the coarser levels in the behaviour of high-resolution regions. The latter technique is used in [40, 54, 68] by updating the stiffness functions real-time to change the resolution adaptively. In addition to level of detail in the spatial domain, solving different regions with different frequencies is also an efficient technique. The level of detail is combined in both the time and spatial domains in [3, 24]. Multigrid methods and hierarchical collision detection have also been exploited in the determination of boundary conditions of deformable objects in [73].

2.3 Perception of Touch

In addition to research in haptic hardware and software solutions, studies in psychophysics are contributing to the knowledge about our mechanism for perception of touch. The widespread use of haptics raises new research questions about this mechanism. Our understanding in this area plays an important role in facilitating the design of more effective solutions in haptics, therefore it is crucial that we have a fundamental knowledge about perception of touch to guide the developments in the related field.

The perception of touch, as discussed earlier, is a rather complex mechanism, involving various types of cues contributing to our perception during exploration of an object. Material properties such as stiffness, mass density, damping, and temperature, and geometrical properties like shape, volume, texture, and size are among the significant cues we can perceive by touch. In addition, there are numerous affecting factors related to the way we explore objects including but not limited to the part of the body we use, the positioning of muscles, and additional modalities (visual, audio). Research has been going on to understand the effects of these features, however it is not an easy task due to the complexity of the nature of touch our in real life.

Perception of texture has been studied in [28, 82]. The force cues were shown to dominate the identification of such shape features as bumps or holes regardless of surface geometry. This was further researched in [28] by decoupling force and position cues. User studies indicated a higher contribution from position cues for more convex high arches. Effects of force shading on perception of bumps and holes was also surveyed in [28]. The effect of shading was found to be dependent on curvature and the number of polygons.

During exploration with touch, we are not able to feel differences less than a ratio of a reference signal. This ratio is referred to as just noticeable difference, JND, in psychophysics. JND is either described through a psychometric curve that expresses the probability of correctly identifying a stimulus as a function of the size of the stimuli, or as the size required to be able to correctly identify the stimulus at a predetermined probability. JND of viscosity has been surveyed in [6, 50]. Subjects were asked to match a viscosity value they were feeling in one arm and with a reference value presented to the other arm in [50]. The procedure was repeated for 10 different reference viscosity values and the mean JND for their 11 subjects was 34%. While in [6], for a reference viscosity of 120 Ns/m, the mean JND was 13.6% for their 3 subjects. In the same study, the mean JND for mass discrimination was 21% for their 3 subjects with a reference mass of 12 kg.
2.3 Perception of Touch

The effects of different muscle groups on perception of force magnitude were examined in [48]. Results indicated that the perceived force magnitude varies between different muscle groups. Forces produced by the index finger flexors were consistently overestimated in magnitude when matched by elbow flexion forces, and elbow forces were underestimated when matched by flexing index finger. Jones also concluded that the index finger flexors are the most precise matching muscle group while the elbow flexors are the least.

Another study, [104], surveyed JND of length for different reference lengths. For a reference length of 10 mm, mean JND was found to be 1 mm, while it increased to 2.4 mm when the reference length was increased to 80 mm. Number of subjects in these experiments, however, was not clearly stated. It was mentioned that three or more subjects have performed the experiments. JND for compliance and force were also compared between two scenarios, one with a fixed displacement and the other with roving displacement setup. The results showed that roving displacement substantially increased the JND for both force and compliance discrimination.

Perception of force direction was analysed in [4] by varying visual inputs presented together with a haptic impulse. Experiments were repeated for three conditions; haptic impulse only, haptic and congruent visual impulse, and haptic and incongruent visual impulse. Comparison of JNDs showed that the congruent visual impulse improves perception while the incongruent has a worsening effect.

Stiffness (compliance), by providing cues about the hardness/softness of a material, is one of the most significant cues contributing to the perception by touch. Thus, there exist various studies exploring different aspects of stiffness perception in the psychophysics.

In [49], the JND of stiffness was examined for 8 different intensities. 10 subjects were asked to adjust the stiffness of a motor attached to one arm to match the stiffness of a computer-controlled motor attached to the other arm. The relation between the reference stiffness and the matching was linear. This linearity resulting in a constant ratio of JND for varying reference magnitudes is known as the Weber fraction in the literature. The Weber fraction observed for stiffness was 23% for the 10 subjects. In [103], work and force cues were dissociated from compliance cues by the help of their specially designed equipment during compliance studies. By this, effects of different types of cues on compliance discrimination were analysed separately. Their results suggested that compliance discrimination is significantly based on work and force cues. In another study [108], the stiffness JND was shown to worsen significantly without surface deformation.

Studies [99, 117] have explored the effect of visual information on stiffness perception. A dominance of visual feedback over kinaesthetic sense of hand position was demonstrated in [99]. Compliant objects that are further away were perceived to be softer in the case of haptic feedback alone [117], while the addition of the visual information reduced the bias. In another study [41], stiffness discrimination was compared between three different conditions: visual feedback, proprioceptive feedback and their combination. The results of their experiments, carried out by 10 subjects, showed Weber fractions of 0.056 for visual, 0.036 for proprioception and 0.039 for their combination. The results indicated the importance of proprioception for stiffness perception.

Further studies [33, 55, 100] have examined the effects of exploratory procedures on
stiffness perception. In [100], the contribution of tactile and kinaesthetic cues were kept separate for deformable and non-deformable objects. The results showed that the tactile information alone is sufficient for discrimination of deformable objects while additional kinaesthetic feedback is necessary for compliant objects with non-deformable surfaces. The effect of using a tool was explored in a similar study [55] where additional kinaesthetic cues were found to be necessary for all types of objects when a tool was used for exploration. Squeezing a deformable object between thumb and index finger was explored in [33, 83]. In [83], tactile information was found to be negligible when squeezing objects between thumb and index finger.

2.4 Summary

Deformation simulation is a field which faces challenges from different research areas. Understanding the basic principles of mathematical models used to define deformation behaviour is crucial. Explaining every concept in detail has not been the aim of this chapter, instead, a brief discussion was presented to make reader familiar with the basic ideas. The reader, however, is referred to other sources such as [13, 120] for more elaborate discussion. The fundamentals on continuum-mechanics was followed by a literature overview of practical applications aiming to achieve interactivity in real-time. Finally, an overview of psychophysics studies which focus on important concepts related to perception of touch was given.
Part B

Contributions
Deformation simulation is a challenging multi-disciplinary field demanding consideration of different issues from various branches. One needs a comprehensive understanding of mathematical models used to simulate deformation in addition to knowledge on software technologies like parallel processing and GPU programming in order to achieve real-time performance of these mathematical models. Another challenge is determining the parameters of these complex mathematical models to match the physical material properties in real life. This includes measurements on the soft tissues and internal organs in the medical field where great efforts have been spent in the latest decades. Psychophysics, in addition to the efforts for haptic hardware and software solutions, has also been trying to understand the human perception mechanism for the sake of some enlightenment within the relatively young research field, haptics.

The focus in the beginning of this research was to achieve a model to provide physically-based realistic deformation behaviour. Among several methods, FEM was considered to be the ideal choice since it depends on a physical approach of continuum mechanics. The physics-based approach allows the use of scientific data such as measured physical properties of soft tissues in determination of the parameters of the model. The computational burden of FEM, however, requires optimization in order to meet the real-time requirements of an interactive simulation with a sufficiently high-resolution mesh. The use of multiresolution in the spatial and time domains is a common technique to model the local neighbourhood of the contact more elaborate while sacrificing the quality of the more remote regions in order to save computational power. This saved computational power, however, does not come for free since it introduces error, decreasing the accuracy of the deformation behaviour. The analysis of the accuracy with respect to multiresolution techniques was not shown enough attention in real-time deformation simulation field where stability has been a bigger concern. The contributions from the beginning phase of my research are, therefore, about the error analysis of adaptive deformation.

Being able to measure the accuracy of the deformation in real-time has potential benefits for improvement. This real-time information can be exploited with the findings from psychophysics in order to keep the error below perception limits, therefore to achieve simulations which cannot be distinguished from the ones without error while still saving computational power. This shifted my interest towards perception of touch, especially the perception of stiffness which significantly contributes to our perception by providing hardness/softness cues about a material. There is a large amount of research about stiffness
perception some of which examines the JND of stiffness while others [33, 55, 83, 100] research the factors affecting the perception. One can observe a large variance of the stiffness JNDs in the literature which can be explained by the differences between the scenarios, methods and environments used in the studies. This has raised the question: “How is the stiffness perception affected by the mode of exploration?” Considering the numerous modes of interaction of touch that we use in our daily lives, one can realize the complexity of the concept. An analysis of the exploratory procedures and perception was presented in Lederman’s study [57] which has become an inspiration to focus on this topic more. After the error analysis, therefore, the research focused on an examination of the effects of different ways of touching on stiffness perception through scenarios related to medical situations like palpation of a soft tissue, tissue cutting, insertion of a needle into a vein etc.

Towards the last phases of the research, a way to gather my previous findings and apply them to FEM deformation was sought. One of the problems with FEM deformation simulation was, I thought, the use of low-resolution meshes to simulate deformation. The popular imaging formats in medicine like MR and CT have much larger resolutions and deploying lower resolution meshes for deformation therefore causes great loss of detail. In order to address this issue, I modified the widely used haptic rendering principle, virtual coupling, to provide high-resolution complementary information about the underlying structures during exploration of a surface. During this modification, I exploited the findings from my perception studies on stiffness perception. The significance effect of stiffness gradient and worse discrimination in the case of lateral movement over a surface were considered to improve the force feedback during exploration of the surface.

In this part of the thesis I present the contributions of my research in three different categories: Error Analysis for Adaptive Deformation, Stiffness Perception and Dynamic Virtual Coupling. A brief discussion is given in the beginning of each chapter followed by the summaries of the related papers which can be found at the end of the thesis. A summary and the future work also follow in the conclusions chapter.
Chapter 3

Error Analysis for Adaptive Deformation

FEM is, as mentioned earlier, considered to provide most realistic deformation behaviour but requires a great deal of computational power. The use of FEM in real-time simulations, therefore, requires a compromise between accuracy and stability. Achieving higher accuracy needs higher resolution meshes composed of larger number of nodes. This means longer solution times conflicting with the stability issues which require smaller time-steps during real-time simulation. Several optimization techniques have been developed in order to meet real-time performance while solving the FEM equations for sufficiently high-resolution meshes. The use of multiresolution in the time and spatial domains, solving different parts of the mesh with different resolutions and different update rates, is one of these techniques. The purpose is, by modelling the local neighbourhood of the contact with higher resolution and solving with higher update rates, to dedicate the computational power to regions around the contact point which are more significant for deformation behaviour. The use of lower resolutions and update rates in the more remote regions, however, introduces some error affecting the accuracy of the deformation. This error has not been shown enough interest in real-time deformation simulations where maintaining the stability has been seen as a more important concern. The main theme of the contributions summarized in this chapter is, in addition to presentation of the multiresolution deformation, the analysis of accuracy and its potential benefits.

3.1 Paper I

Aims

The aim of this paper was to introduce asynchronous regions which are different parts of FEM that can be solved with different update rates and different mathematical solvers and examine their effects on the accuracy of the force feedback during deformation. The motivation is to solve the deformation in the local neighbourhood around the contact, the primary region, with higher update rates while solving the more remote regions, the secondary regions, with lower update rates in order to save some computational power. The ability to use different solvers for different regions allows deployment of explicit solvers for the primary region and implicit ones for the secondary regions. In the case of explicit solvers the differential equations can be written independently from each other allowing
A linear FEM model requiring solution of a system of partial differential equations was used to simulate deformation:

$$\mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{f}$$

(3.1)

where \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) refer to mass, damping and stiffness matrices, and \( \mathbf{f} \), \( \mathbf{u} \), \( \dot{\mathbf{u}} \), \( \ddot{\mathbf{u}} \) refer to force, displacement, velocity and acceleration vectors, respectively. To achieve better performance, mass and damping matrices were diagonalized and iterative mathematical solvers which discretize simulation time into time-steps were preferred to solve equation 3.1. The idea of asynchronous regions depends on solving deformation for the separate parts of the model with different refresh rates which requires dividing the solution of the displacements in equation 3.1 to different parts and solving them with different time-steps. In order to facilitate the visualization of this concept, the placement of asynchronous regions on a model and stiffness matrix is illustrated in figure 3.1. The black dots on the stiffness matrix refer to the non-zero values showing the sparsity of the stiffness matrix and different colours refer to the entries of the matrix for the nodes in different regions. During
the solution of displacements of a region, $\mathbf{u}$, the displacements of the nodes at the border of the neighbouring regions are needed for the deformation to propagate between the regions. The stiffness matrix coefficients for these nodes in the neighbouring regions refer to the non-zero entries in the white regions of the stiffness matrix. In the case of explicit solvers, since it is possible to write the differential equations explicitly, the displacements of the neighbouring nodes can be used directly in matrix-vector multiplication. The implicit solvers, on the other hand, consist of dependent equations requiring the solution of a matrix system. The partial solution for a region therefore requires subtracting the contribution of the neighbouring nodes from the force vector, $\mathbf{f}$, on the right hand side of the equation 3.1 after being multiplied by the corresponding stiffness matrix entries. Only after this process can the local matrix system for the region be solved in a traditional way.

Lower update rates in the secondary regions act as a low pass filter to the applied input. In the case of extremely low update rates in the secondary regions compared to the applied input, the propagation of deformation from the primary region to the secondary regions diminishes until the displacements of secondary regions are evaluated. The changes in the displacements of the nodes of secondary regions, therefore become insignificant causing an artificial increase in the output force. Another issue related to the use of lower secondary regions update rates is the ‘moving wall’ effect. The nodes in the secondary regions stay constant while the nodes in the primary region are being calculated creating a ‘wall’ effect and the next calculation of the displacements of the secondary regions’ nodes would end up in a sudden jump creating the effect that this wall is moving. To address these issues interpolation was applied for the displacements of the secondary regions nodes between two consecutive updates.

The allocation of the asynchronous regions on the model is updated in real-time when the contact node is changed. For the most efficient use of the asynchronous regions, the size of the primary region is also adapted depending on the amount of strain in real-time. A linear mapping function was used to increase the size of the primary region with increasing amount of strain while the use of any function is possible. The size of the primary regions, however, is obviously bounded from above by the computational power and the number of nodes.

### Results

Experiments were carried out to study the effect of the asynchronous regions on the deformation behaviour, more specifically on the force feedback. The aim of the experiments was to identify the relationship between the primary region size, secondary region frequency and the force error for various types of input applied. The reference force was evaluated by solving the deformation of the whole model at the highest resolution, when subject to a given input at 1 kHz and recording the force response. In order to decouple the calculations from the computational power limitation, the calculations were performed offline and pre-recorded strain inputs were applied through a simulated haptic device. The same input was applied to the model when varying the primary region size and secondary regions update rates and the force response was compared with the reference force to extract the
Error Analysis for Adaptive Deformation

Table 3.1: The performance and force error comparison of the asynchronous regions with the whole analytical solution. N is the number of nodes solved at 1 kHz, f is the frequency of the secondary regions.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>f</th>
<th>Average Solution Time(ms)</th>
<th>Speed Gain(%)</th>
<th>Peak Force(N)</th>
<th>Force Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Sol.</td>
<td>40</td>
<td>-</td>
<td>0.175</td>
<td>-</td>
<td>1.65</td>
<td>-</td>
</tr>
<tr>
<td>Asynch. Regions</td>
<td>30</td>
<td>500</td>
<td>0.123</td>
<td>29.7</td>
<td>1.66</td>
<td>0.5</td>
</tr>
<tr>
<td>Regions</td>
<td>30</td>
<td>50</td>
<td>0.094</td>
<td>46.3</td>
<td>1.76</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>500</td>
<td>0.059</td>
<td>66.3</td>
<td>1.70</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>30</td>
<td>0.017</td>
<td>90.3</td>
<td>2.86</td>
<td>7.3</td>
</tr>
</tbody>
</table>

force error. The speed gain and the error are shown in table 3.1 for different values of the primary region sizes and secondary region update rates. A second set of experiments was performed to examine the behaviour of the force error for adaptable and non-adaptable asynchronous regions. A continuous sinusoid strain input with constant frequency but increasing amplitude was applied to the model. For adaptable asynchronous regions, the size of the primary region was increased by increasing strain, while it was kept constant for the non-adaptable case. The results showed a significantly smaller force error for the adaptable case, as expected.

Contributions

Although the idea of using different update rates was not completely new, a comprehensive analysis of the deformation behaviour from the accuracy point of view had been missing. In addition to proposing asynchronous regions, experiments were performed to survey their effects on accuracy for various parameters. In addition, the ability to deploy different solvers for different regions allows the use of unconditionally stable implicit solvers for secondary regions with smaller update rates and the use of easily modifiable explicit solvers for the primary region where tissue cutting is more likely occur. The independent asynchronous solution of different parts also opens up the possibility of modelling computationally expensive complex physical behaviour, such as non-linearity or viscoelasticity, in the primary region alone instead of throughout the whole model. In summary, this paper showed the potential of the asynchronous regions for haptic applications by examining the speed performance gain and force error.

3.2 Paper II

Aims

Analysis of the accuracy has been paid scant attention in the deformation simulation field in spite of the popularity of multiresolution techniques. Although the error analysis is widely studied in the context of FEM [1, 7, 39, 81], the common practice in real-time deformation simulation field has been presenting the maximum number of nodes which can be solved in real-time while maintaining the stability. The analysis of element sizes,
time-steps and their effects on deformation behaviour have been underestimated. The aim of this study is, therefore, to propose an abstract pipeline for error analysis which can be applied to different types of deformation techniques and consider various types of deformation parameters. As a case study, the pipeline was applied to the existing FEM simulation to create an error mapping for the model.

Work

The essence of the pipeline is to calculate an offline error mapping in order to minimize additional computation for error estimation in real-time. The offline error mapping consists of applying a Monte Carlo simulation by varying a set of input parameters to a given deformation model and recording the response of the model for each sample. The response for each sample is then compared to a reference response in order to create the error mapping. Since the error in this context refers to the variations in the force feedback caused by the deployment of multiresolution techniques, the reference response was calculated by recording the force response of the whole mesh with highest resolution and with a sufficiently low enough time-step to maintain stability. The error for each sample was, therefore, caused by changing the element size and the time-step used in deformation calculation.

The idea is summarized in figure 3.2 which illustrates three common types of input in deformation simulations: user input, material properties, adaptive simulation parameters. The amount of strain, input frequency or the contact node can be named as examples of user input while material properties include elasticity, mass, damping and poisson ratio. The reference force was evaluated for a range of these parameters by recording the response of the whole mesh with the reference resolution and time-step. The same procedure was then repeated by changing the adaptive simulation parameters, distributions of element sizes and time-steps throughout the mesh, to create error mapping. There are two main uses of the error mapping in real-time: To assess the quality of the simulation by monitoring the error and to keep the error below some desired limits by adjusting the adaptive simulation parameters. One way to determine the error in real-time from the error mapping is to transcribe it into a lookup table. The varying values for the user input, material properties
Figure 3.3: The online use of error mapping: For a given set of user input, material properties and adaptive simulation parameters the error is determined in real-time for quality assessment or adjustment of adaptive simulation parameters for maintaining the error under desired limits.

and adaptive simulation parameters in real-time can then be used to determine the error via the lookup table. The estimated error can then either be used for monitoring purposes only, warning the user when the simulation error has exceeded the limit, or be fed back to the deformation model to adjust the distribution of element sizes and time-steps. This adjustment, however, also needs to consider the stability of the simulation since the time-step and number of nodes in a real-time simulation are also limited by the computation power.

Results

The pipeline was applied to linear FEM by using a cubic model composed of cubic elements as illustrated in figure 3.4(a). Multiresolution in the spatial domain was combined with

Figure 3.4: (a) The cubic model used in the experiments composed of $13 \times 13 \times 13$ nodes (b) The use of parallel coordinates for an overview of correlation between the force error and parameters.
the asynchronous regions which can use different time-steps and different solvers for different parts of the model. Two regions, one primary region and one secondary region were employed throughout the whole procedure. The parameters varied were the time-step and the element size of the secondary region, the size of the primary region as adaptive simulation parameters, the amount of strain and input frequency as user input, and elasticity, mass, damping and poisson ratio as material properties. An error mapping composed of over 50000 samples with 9 variables was created as a result of the Monte Carlo simulation. Visualizing the relationships between the error and the several parameters with such large amount of data is not easy but parallel coordinates is a popular visualization technique to provide an overview in addition to providing the possibility to identify a partial correlation between the error and an individual parameter interactively. Figure 3.4(b) is an example where the correlation between poisson ratio and the error was displayed by fixing the values of the other material properties. While parallel coordinates are useful for an overview of the results, in order to quantify the correlations statistical analysis were performed. The results of the Spearman partial correlation test show the strength of correlation between the force error and each individual parameter in table 3.2.

<table>
<thead>
<tr>
<th>Spearman Error Correlation</th>
<th>Coefficient</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson Ratio</td>
<td>0.5488</td>
<td>0</td>
</tr>
<tr>
<td>Mass</td>
<td>0.0053</td>
<td>0.2152</td>
</tr>
<tr>
<td>Damping</td>
<td>-0.1691</td>
<td>0</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.2712</td>
<td>0</td>
</tr>
<tr>
<td>Input Strain</td>
<td>0.158</td>
<td>0</td>
</tr>
<tr>
<td>Input Frequency</td>
<td>-0.1244</td>
<td>0</td>
</tr>
<tr>
<td>Level of Detail</td>
<td>0.7033</td>
<td>0</td>
</tr>
<tr>
<td>Pr. Reg. Size</td>
<td>-0.6243</td>
<td>0</td>
</tr>
<tr>
<td>Sec. Reg. Freq.</td>
<td>0.0003</td>
<td>0.9403</td>
</tr>
<tr>
<td>Force</td>
<td>-0.1827</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.2: The partial correlation between force error and the parameters: The range of Spearman correlation coefficient is between -1 and 1 and the higher the absolute value of the coefficient the stronger the correlation. The significance of the results were checked by a t-test whose p-values are also shown. The correlation coefficients are interpreted as small between 0.0 and 0.1, medium between 0.1 and 0.3, and large between 0.5 and 1.0 [29].
Contributions

A pipeline is proposed to survey the varying accuracy of the force feedback due to multiresolution with respect to a number of parameters. This pipeline was applied to linear FEM model as a case study and the explicit correlations between the individual parameters and the force error was obtained. The explicit knowledge about the accuracy of force in relation to the various parameters individually, has the potential to facilitate choosing the right deformation model depending on the type of application. Different error characteristics of a deformation model for different ranges of material properties, for instance, would let the user consider another deformation model with lower error for that range of material properties. In addition, monitoring the error in real-time to assess the quality of the simulation and to adjust the multiresolution parameters to keep the error below the desired limits are the other alternative uses of the error mapping. It is a well-known fact that human beings have perception limitations, such that we are not able to detect differences less than a certain percent of a reference value, leading to the concept of JND. By keeping the error below the JND, applications with force feedback indistinguishable from the ‘reference’ force can be developed while saving some computational power at the same time.
Chapter 4

Stiffness Perception

Exploring objects by touching provides various types of information such as texture, hardness, temperature, weight, size, shape etc., and the choice of the type of exploration has a significant role in our perception. Different types of exploration, referred to as exploratory procedures in [57], have different efficiencies for acquiring a specific type of information. For instance, contour following is an efficient way of exploration to recognize the shape of an object. Lateral motion and texture, applying pressure and hardness, static contact and temperature, unsupported holding and weight are among the other examples of exploratory procedure-information pairs where the procedure is an effective way of obtaining the desired information [57]. Therefore, the type of the information we are looking for determines the exploratory procedure we use during touch.

In real life, the exploration does not usually happen in the form of a single exploratory procedure neither does our brain intentionally choose one. We are eager to optimize the efficiency of interaction to get the desired information in the most effective way by combining several exploratory procedures naturally. Understanding the mechanism behind the perception of touch is, therefore, not easy while continuous attempts have been going on in psychophysics. With the growing use of haptics in different areas like virtual environments and mobile applications, the importance of understanding the perception mechanism has become more obvious. The importance of comprehending the mechanism behind the perception of touch for the development of more efficient and effective haptic hardware and software solutions is hard to over-state.

Perception of stiffness being a material property significantly contributing to our perception of hardness/softness an object was focused in the studies presented in this chapter. First paper surveyed how the stiffness perception is affected by the way we touch by considering some basic aspects of exploratory procedures and the second paper performed some experiments to examine the effect of stiffness gradient on perception.

4.1 Paper III

Aims

The aim of this work was to study the effects of exploratory procedures on stiffness perception by considering two basic aspects of touch: direction of motion and the transition
42 Stiffness Perception

of stiffness. Exploration of a surface has two principle directions of motion: Normal and lateral direction. The former refers to pushing into the surface by applying pressure and the latter refers to moving over the surface topography as in the case of contour following. The second aspect, the transition of stiffness, also has two types: comparing the stiffness of two objects by separately touching them, which was referred to as discontinuous contact, and detecting the change in stiffness during exploration without losing the contact with the object, which was referred to as continuous contact. The main motivation of surveying the effects of these aspects on stiffness perception is that they are often observed in real life scenarios where detecting stiffness changes might be important. Diagnosing a tumour beneath the soft tissue by touching, tissue cutting during surgery, detecting a vein during needle insertion are some examples in the medical field where detecting the stiffness changes during continuous contact is crucial to accomplish the task.

Work

The JND of stiffness was examined under three different stiffness comparison scenarios designed by considering the two aspects of touch as illustrated in figure 4.1. In **Discontinuous Pressure Comparison — DP** the stiffness of different surfaces were compared by separately applying pressure along the surface normal direction to each of them. This is the most common scenario surveyed in stiffness perception studies [55, 99, 100]. The **Continuous Pressure Comparison — CP** scenario refers to detecting the stiffness change during palpation of a surface along its normal axis. This is a very common situation in real life since many materials we palpate have either non-linear elastic characteristics or inhomogeneities under their surface. The non-linearity of soft tissue, feeling the bone under the fat layer, feeling the veins during needle insertion are among several cases from the medical field where the stiffness changes during palpation. In the last scenario, **Continuous Lateral Comparison — CL**, the stiffness change was supposed to be discriminated during

![Figure 4.1: Two aspects of contact during stiffness change: Direction of Motion and Transition of Stiffness.](image)

The discontinuity during touch refers to touching discrete objects separately in order to discriminate the stiffness difference between them. While continuity corresponds to stiffness change during contact with the same object. The continuous stiffness change has been surveyed as two types: Lateral stiffness change and stiffness change along the surface normal axis during applied pressure, which is a special case of non-linearity.
a lateral movement over the surface. The stiffness change in this scenario is similar to a surgeon exploiting the stiffness variations while cutting the soft tissue during surgery in order to determine the path of the cut. This scenario, however, should not be confused with the fingers touching a textured surface to obtain tactile cues since it is more relevant to contour following providing more kinaesthetic cues than tactile cues. There is another possible combination of different types of the two aspects, discontinuous lateral, however this combination was not surveyed since it does not have as much importance to real life situations as the others.

Results

An evaluation study with the three scenarios explained above was performed in a virtual environment composed of a framework used to collocate a 3D visual representation of the scene with the workspace of a haptic device, figure 1.2. For each scenario the subjects were presented a number of trials where they were asked to choose the harder box among three boxes. Two of the boxes always had the reference stiffness while the third one was different in a way depending on the scenario, see figure 4.2. The different box had a harder stiffness value than the reference stiffness in scenario DP. In scenario CP, it was composed of two stiffness regions modelling non-linearity as two piecewise-linear functions as shown in figure 4.2(b). The region closer to the surface had the reference stiffness while the region at the bottom always had a harder stiffness. Figure 4.2(c) illustrates scenario CL where the different box had the harder region on its left hand side. In scenario CL the subjects were asked to make a lateral sweeping movement over the surface while they were asked to push down on the surface in the other two scenarios.

In order to find the JND of each subject for each scenario a one-up-two-down adaptive staircase method [58] was used to vary the harder stiffness. The method starts with an initial stiffness difference and changes this difference, depending on the responses of the subject, such that it gradually converges to the perception limit of the individual with a certainty of 70%. The evaluation was performed as a within subjects design with one independent variable, stiffness, and three different scenarios for 12 subjects. Three scenarios were carried out as separate sessions the order of which were balanced by a

![Figure 4.2](image-url)

Figure 4.2: Three virtual boxes were presented to the subjects during each individual trial. The subjects were asked to find the single harder box among the three and the location of the harder box was randomized during the experiment. The way the harder box differed from the others depended on the type of the experiment. Interpolation is applied around the transition region in scenario CP and CL to prevent an obvious discontinuity in stiffness.
The JND values of each subject were analysed for all three scenarios by a Friedman ANOVA test showing a significant difference between the scenarios, Chi-Square(12, 2) = 12.667, $p = 0.001$. The Wilcoxon signed-rank test showed a significant difference between scenarios $DP$ and $CP$, $z=-3.061$ $p<0.001$ and between scenarios $CP$ and $CL$, $z=-4.71$ $p=0.003$ but no significant difference between scenarios $DP$ and $CL$. The mean value of the JND for the $CP$ is $5.23\pm3.01\%$, while the $DP$ and $CL$ scenarios have a mean JND of $12.91\pm6.93\%$ and $15.77\pm9.59\%$ respectively. The results show that the stiffness perception is significantly better in scenario $CP$ than the other two scenarios.

**Contributions**

The contribution of this paper is the study indicating significant effects of different aspects of palpation on stiffness discrimination. Effects of two different aspects of touch, continuity and axis of motion have been surveyed. Considering the two aspects, detection of stiffness variations were compared for three different touch scenarios: separate stiffness comparison, continuously applied pressure and continuous lateral movement. Significant results were obtained showing better performance of stiffness perception with continuously applied pressure. The results have indicated better perception in the case of continuous contact and in the case of applying pressure along the surface normal axis than that for lateral motion over the surface. These outcomes were discussed together with the related concepts of haptic memory [96] and force constancy [18].
4.2 Paper IV

Aims

The effect of stiffness gradient on perception was studied in this paper. One of the motivations was the outcome of the previous paper indicating superior stiffness perception in the case of continuous contact. Another motivation was the frequent occurrence of continuous contact as a natural way of exploring objects in real life. There have also been studies in the medical field [46, 109, 115] showing the relation between the tissue characteristics and the stiffness gradient. This has also shown potential benefits of examining the stiffness gradient and how it affects perception.

Work

To examine the effect of stiffness gradient, three different position dependent stiffness scenarios were designed with different gradient characteristics. In all three scenarios, a surface with two different stiffness regions was considered such that along a lateral movement on the surface, the stiffness was changed from the softer to the harder with a transition region in between. The three scenarios varied in the interpolation function used in the transition region while the difference between the two stiffness levels and the width of the transition region were considered to be the same. The interpolation functions were designed to have different stiffness gradient magnitudes as illustrated in figure 4.4. The three different interpolation functions, linear, cosine and scaled tanh, have different gradient magnitudes as shown in the figure. The aim was to compare the JNDs of these scenarios for a significant difference caused by the different gradient magnitudes in the interpolation region, which would indicate an effect of stiffness gradient in addition to the difference in stiffness levels.

A varying stiffness as a function of position can also be considered as a function of time during a continuous contact. A real interaction would naturally be expressed by stiffness variation in spatial domain, however different hand velocities during touch result in different

![Figure 4.4: The interpolation functions with their respective spatial derivatives.](image-url)
stiffness-time characteristics, therefore stiffness gradient, for the same object. Therefore one needs to consider both the spatial and time domains in the case of varying stiffness for perception analysis. The pilot studies, performed by controlled hand velocities, included both position and time dependent stiffness variations and the contribution of stiffness gradient from both domains on the stiffness JND was observed. The hand velocities were, therefore, controlled during the experiments while the stiffness was varied as a function of position since it is the natural way of touching we experience in our daily lives.

Results

A user study was performed in two stages: In the first stage the JND of stiffness was compared between the three scenarios with different stiffness gradient characteristics as described above, and in the second stage the width of the transition region between two stiffness regions, with a constant stiffness difference in between, was varied to produce different stiffness gradients during JND comparison. The experiments were performed in a virtual environment using the same equipment as in the previous paper which was illustrated in figure 1.2. In both stages, the subjects were presented three virtual boxes two of which had uniform reference stiffness while the third one had a non-uniform stiffness with two different stiffness levels and a transition region between them, as shown in figure 4.5(a). The subjects were asked to make a sweeping movement sideways across the surface and choose the box with the non-uniform stiffness among the three. Since the speed of the sweeping movement is a concern for stiffness gradient analysis, as discussed above, the subjects were asked to match the speed of an animated series of balls moving along sideways with the reference velocity, figure 4.5(b). The aim was not to exactly match the speed of the animation but to provide a reference speed in order not to have significantly larger differences of hand speed between the subjects to prevent different stiffness gradients in time domain. The experiments were performed by 12 subjects.

In the first stage, evaluation was performed within-subjects with one independent vari-
able, stiffness, and three scenarios with different interpolation functions: Linear, cosine and tanh as described above. The three scenarios were carried out as separate sessions the order of which was balanced by a Latin-square procedure for each subject. The JND of stiffness was found in each scenario for each subject by varying the harder stiffness of the non-uniform box depending on the responses from the subject. A one-up-two-down adaptive staircase [58] starting with an initial stiffness difference and converging to the perception limit of the individual was deployed to find the JND.

In the second stage the stiffness difference between the two stiffness levels were kept constant at the JND value for each subject, as found using the linear scenario in the first stage. The width of the transition region was varied for a number of trials presented to the subject and the probability of the correct responses of finding the non-uniform box was measured for each transition width to create the psychometric function with respect to stiffness gradient.

The analysis of the JND of stiffness for each subject from the first stage by a repeated measures ANOVA showed a significant difference between the three scenarios, F(2,22) = 9.059, p = 0.001. In order to determine the conditions significantly differing from each other, Bonferroni corrected pairwise comparisons were performed as a post-hoc test. A significant difference was observed between the tanh scenario and the other two scenarios.

The mean value of the JND for the cosine scenario is 14.7 ± 8.9%, while the linear and tanh scenarios have a mean JND of 17.95 ± 9.62% and 10.86 ± 6.24% respectively. The results of the first stage showed a significantly better discrimination for the scenario with the highest stiffness gradient. Among the other two scenarios, linear and cosine, a better discrimination was observed for the one with the higher stiffness gradient while the statistical significance was lacking. The absence of a significant difference between these two scenarios can be explained by their relatively closer gradient magnitudes compared with the gradient of the tanh scenario which has a much larger stiffness gradient. The psychometric function obtained from the second stage also showed a direct relation between the probability of detecting the stiffness differences and the stiffness gradient being consistent with the results of the first stage.

**Contributions**

The results showed that the stiffness perception is affected by the stiffness gradient and not only the difference between stiffness levels. This result can be seen as complementary to the context-dependency of stiffness perception, where the Weber fraction is observed in stiffness JND [19]. In other words, the JND of the stiffness is not only affected by the reference stiffness value by being a fraction of it but also dependent on how the variation occurs. This outcome can potentially be exploited in simulation scenarios where detecting stiffness variations are crucial. During exploration of a model, stiffness gradient can be considered in force calculations. This can be accomplished in various forms such as emphasizing subtle variations for guidance purposes or improving boundary detection. Some scenarios which might benefit from this concept can be named as needle insertion, bone drilling, and exploration of tissues for malign growth.
Chapter 5

Dynamic Virtual Coupling

Achieving real-time interactive performance in deformation simulation is non-trivial because of the computational burden required for the solution of the mathematical models. This burden often limits the resolution of the deforming mesh, preventing the use of the whole information in the data. It is common practice to use a lower resolution mesh to simulate deformation and a higher resolution mesh for visual rendering to meet the real-time requirement, for example [2]. The common formats for the medical data, however, such as CT and MR, usually have much higher volumetric resolution than the resolution of deformation models. This results in a large amount of information loss in the case of medical simulations. In scenarios like detecting a tumour beneath the soft tissue, a decayed region in a tooth, or tissue cutting, high-resolution information might be crucial for the accomplishment of the task.

In order to address these issues, an algorithm is introduced providing high-resolution information beneath a surface complementary to the surface information during exploration of the surface. This new algorithm, which is called Anisotropic Virtual Coupling — AVC, can be combined with any surface rendering algorithm including texture and friction rendering since it is a modification of a widely-used haptic rendering principle, virtual coupling [20].

The principle of virtual coupling depends on evaluating force feedback by a spring-damper system defined between the haptic probe and its virtual representation, the proxy. Use of the proxy instead of the haptic probe in rendering algorithms provides more control over the force feedback in addition to simplifying the problem of ensuring stability of the haptic display [60]. The traditional use of virtual coupling in surface exploration is by means of restricting the proxy movements on the surface to prevent it from surface penetration [84, 121]. Updating the proxy position on the surface such that it would follow the haptic probe and the spring-damper force pulling the probe towards the proxy provide the sensation of touching the surface.

The AVC introduces anisotropy, the property of being direction dependent, by modifying the stiffness of the system depending on the direction between proxy and probe. A ray cast through the data along the proxy-probe direction is used to gather information from beneath the surface which is then used to modulate the surface stiffness. In addition to being decoupled from the mesh resolution, this modulation allows detection of inhomogeneities under the surface.
This dynamic modulation of the stiffness by direction affects the stored potential energy of the spring-damper system. Energy conservation, therefore, needs to be considered both to follow a physics-based approach and to maintain passivity of the haptic display. To address this issue, AVC is extended to Anisotropic Virtual Coupling with Energy Conservation — AVCEC by additional force components compensating for the energy changes. The first paper described in this chapter explains these two algorithms, AVC and AVCEC, and presents the results of the evaluation studies assessing their performance and their effects on perception while the second paper focuses on the extension of these principles to deformable models.

5.1 Paper V

Aims

The aim of this paper was to introduce AVC and AVCEC providing high-resolution haptic information about the data underneath a surface complementary to the surface information. The functionality of AVC was tested in a scenario where AVC was deployed during exploration of a surface covering two rigid bodies. One of the rigid bodies was obscured from above by the other and user studies were performed to evaluate whether the obscured rigid body can be located during exploration of the surface. A second user study surveyed the effect of AVCEC on the shape recognition by providing gradient information in a scenario where rigid bodies with different shapes were located underneath the surface being explored.

Work

The simulation of surface exploration by virtual coupling, employing a spring-damper system between proxy and probe allows simulation of different compliances. The stiffness of the spring-damper system refers to the perceived surface stiffness and varying this stiffness depending on the proxy position on the surface is traditionally used to provide different hardnesses in different parts of a surface model. In the algorithm proposed, AVC, the direction between the proxy and the probe is used in addition to the proxy position in determination of the stiffness. A ray cast through the data along the proxy-probe direction is used in a scenario where rigid bodies are placed under the surface as illustrated in figure 5.1(a). The ray ends where it either hits a rigid body or the ground. The stiffness of the spring is then modulated depending on the length of the ray. The spring-damper system is modelled such that the stiffness is inversely proportional to the length of the ray. The rigid bodies along the ray resulting in shorter ray lengths would cause higher stiffness, therefore allowing the user feel the location and the depth of the rigid bodies along the direction of movement. Since modification is applied to surface stiffness determined by the proxy position, the ability to detect the rigid bodies beneath becomes complementary to the feeling of the surface.
The scenario in figure 5.1(a) was designed to emphasize the use of AVC for detecting even partially obscured rigid bodies. The fact that the smaller rigid box in the figure is obscured by another rigid body from the top view does not prevent the detection of the smaller box by the user in the AVC case, proving the benefits of directional information. The use of friction in the proxy update algorithm [84] causes the proxy to lag behind the probe along the direction of hand movement over the surface. Varying the direction of movement, therefore, results in varying proxy-probe direction allowing the user to find a direction where the cast ray hits the smaller rigid body. This holds in the case of detecting partially obscured rigid bodies where at least at one point on the surface, there should not be any obstacle between the proxy and the rigid body along the proxy-probe direction. In the case where a rigid body is completely obscured, for instance completely covered by another rigid body, the AVC can be modified, if desired, to perform information gathering techniques along the ray as in volume rendering to allow detection of completely obscured rigid bodies. The calculation of the spring force between the proxy and the probe depends on the well-known Hooke’s Law which compensates for potential energy changes stored in the spring. The modification of the spring stiffness affecting the stored potential energy, therefore, requires consideration of the energy changes if a physics-based approach is to be followed. Energy conservation was implemented in order to keep the system passive by evaluating additional force components to compensate for the energy changes in AVCEC. Potential energy stored in a spring refers to the area under the force curve in a force-
displacement graph and in our case can be formulated as:

\[
E(\vec{x}_{py}, \vec{x}_{pb}) = \int_{0}^{D} k(\vec{x}_{py}, \vec{x}_{pb}) r \, dr = k(\vec{x}_{py}, \vec{x}_{pb}) \frac{D^2(\vec{x}_{py}, \vec{x}_{pb})}{2}
\]  

(5.1)

where \( E \) is the energy function depending on the proxy position on the surface, \( \vec{x}_{py} \) in \( \mathbb{R}^2 \), and the probe position \( \vec{x}_{pb} \) in \( \mathbb{R}^3 \), \( D(\vec{x}_{py}, \vec{x}_{pb}) \) is the distance between the proxy and the probe and \( k(\vec{x}_{py}, \vec{x}_{pb}) \) is the stiffness function. One can calculate the term compensating for the energy change by:

\[
-\nabla E(\vec{x}_{py}, \vec{x}_{pb}) = -k(\vec{x}_{py}, \vec{x}_{pb}) D(\vec{x}_{py}, \vec{x}_{pb}) \vec{d} - \nabla k(\vec{x}_{py}, \vec{x}_{pb}) \frac{D^2(\vec{x}_{py}, \vec{x}_{pb})}{2}
\]

(5.2)

where \( \vec{d} \) is the unit vector along the proxy-probe direction. The first term refers to the well known spring force depending on the Hooke’s Law and the second term refers to the force for energy compensation due to stiffness variation. \( \nabla E \) is a vector in \( \mathbb{R}^5 \) affecting both the proxy and the probe movement. Since the proxy movements are restricted to a 2D surface, the proxy and probe components of \( \nabla E \) were evaluated separately.

The proxy component, \( \vec{F}_{py} \), is calculated as the partial derivative of the potential energy with respect to proxy movement resulting in a two dimensional lateral force on the surface of exploration. Since this force was caused by the movements of the proxy, it is fed back to the algorithm [84] used to update the proxy movements. \( \vec{F}_{py} \) is calculated by using the formula:

\[
\vec{F}_{py} = -\nabla \vec{x}_{py} k(\vec{x}_{py}, \vec{x}_{pb}) \frac{D^2(\vec{x}_{py}, \vec{x}_{pb})}{2}
\]

(5.3)

The derivative of the stiffness was evaluated by using a 2D Sobel kernel around the proxy position on the surface. Figure 5.1(b) illustrates the rays cast to calculate the stiffness values corresponding to the boundaries of the Sobel kernel by green lines. These stiffness values were weighted by the kernel coefficients to evaluate the stiffness gradient in equation 5.3.

The probe component for the energy compensation, \( \vec{F}_{pb} \), is a three dimensional vector calculated as the partial derivative of the energy compensation with respect to the probe position as:

\[
\vec{F}_{pb} = -\nabla \vec{x}_{pb} k(\vec{x}_{py}, \vec{x}_{pb}) \frac{D^2(\vec{x}_{py}, \vec{x}_{pb})}{2}
\]

(5.4)

The stiffness gradient in equation 5.4 was calculated by mapping a 2D Sobel kernel spherically around the proxy-probe direction as illustrated in figure 5.1(b). The blue lines refer to the rays cast to calculate the stiffness values for the boundaries of the Sobel kernel. The resulting torque after the application of the Sobel was converted to a force and fed back to the haptic device.

**Results**

Two different experiments were designed; one to assess the performance of AVC on detecting rigid bodies underneath a surface and another to survey the effect of AVCEC on shape
Figure 5.2: (a) The rigid box hidden under the flat rectangle in region 3, which was not visible to the test subjects. (b) A screen-shot from the AVC experiment showing what the subjects were seeing.

recognition. Both experiments were carried out in a virtual environment using the same setup deployed in the previous studies, figure 1.2. For each trial in both experiments, the subjects were presented with an opaque virtual box which was covering different structures within. The content of the virtual box and the task varied depending on the experiment, as explained below, and the procedure was carried out for 10 subjects.

In the first experiment, the *AVC Experiment*, the task was to locate a rigid box which was partially obscured by a rigid flat plate where both the rigid bodies were covered by the virtual box as shown in figure 5.2(a). The rigid box was always located under the rigid flat plate but the position of it was randomly varied to stay in one of the four quadrants of the plate when viewed from the top. The figure illustrates the case where the bottom-left quadrant was chosen for the rigid body. The top surface of the virtual box was completely opaque making the rigid structures underneath invisible and the boundaries of the four quadrants were visually displayed as a texture on the surface, figure 5.2(b). The scenario was designed such that it was impossible to detect the rigid box obscured by the rigid flat plate by traditional uses of virtual coupling. With AVC, however, varying the proxy-probe direction made it theoretically possible to intersect the rigid box, allowing the subjects to change the direction of their lateral movement to search for the rigid box. The subjects were presented 40 trials and asked to make a lateral movement over the surface to find the rigid box in one of the four quadrants. The percentage of the correct responses for all subjects resulted in a mean value of 98% showing the obvious functionality of AVC to detect partially obscured rigid bodies. In the second experiment, *AVCEC Experiment*, the effect of energy conservation on the shape recognition was surveyed. During the trials
of AVCEC, it was observed that detection of the boundaries was easier with the energy conserving term providing stiffness gradient information. A scenario where a rigid flat plate was placed in the virtual box, similar to the one in the first experiment, was used. The task was to guess the shape of the rigid flat plate varying among circle, triangle, rectangle, hexagon and pentagon during exploration of the top surface of the box covering it. The experiment was carried out as a within-subject design with two conditions: with and without energy conservation, referred to as energy condition and non-energy condition respectively. The conditions were carried out separately and each of them included 25 trials where the five different shape types were distributed balanced but randomly among them.

The success rate and time spent for exploration were compared both for each shape type individually and in total. Higher success rates and lower exploration times in the mean values were observed for the energy condition. In order to check for significance and quantization non-parametric tests were performed on the data. A Wilcoxon signed-rank test showed that the energy condition has significantly higher success rate than the non-energy condition for the shapes circle ($Mdn_{energy} = 100\%$, $Mdn_{non-energy} = 30\%$, $z=-2.84$, $p<0.01$, $r=-0.40$), triangle ($Mdn_{energy} = 100\%$, $Mdn_{non-energy} = 40\%$, $z=-2.75$, $p<0.01$, $r=-0.39$) and rectangle ($Mdn_{energy} = 90\%$, $Mdn_{non-energy} = 20\%$, $z=-2.69$, $p<0.01$, $r=-0.38$). The analysis for the whole data set resulted in significantly higher performance for the energy condition ($Mdn_{energy} = 66\%$, $Mdn_{non-energy} = 28\%$, $z=-2.81$, $p<0.01$, $r=-0.18$) also. The significance analysis of the exploration time was also similar to the success rate such that significance was observed only for the shapes; circle, triangle and rectangle. The absence of significance for the shapes, hexagon and pentagon, can be explained by the increasing complexity with increasing number of edges. An analysis of the distribution of the wrong answers for these two shapes showed consistency between the responses and number of edges. This can be interpreted as the ability of silhouette approximation for complex shapes where the exact shape can not be detected.

Contributions

Two new algorithms, AVC and AVCEC, were proposed and evaluated to increase the quality of haptic exploration of virtual surfaces. The motivation was to provide complementary high-resolution information about the data beneath the surface in order to facilitate detection of inhomogeneities. The results of the experiments showed that the AVC algorithm works for detecting rigid bodies even though they are partially obscured by other structures. The results showed that shape recognition of the objects underneath the surface was significantly better in the case where energy conserving terms were added. In addition to enhancing the surface exploration experience, these algorithms are easily adaptable to any haptic surface rendering algorithm since they depend on the main principle of virtual coupling behind many surface rendering algorithms.
5.2 Paper VI

Aims

In deformation simulations, it has been common practice to evaluate deformation with lower resolution meshes although higher resolution medical data such as MR and CT exist. This results in a significant amount of information loss degrading the detection of variations within the deformable shape beneath the surface. The previous study has shown the ability to detect partially obscured rigid bodies beneath a soft tissue by AVC and improved shape perception by AVCEC. The aim of the work in this paper is to integrate these algorithms with deformable meshes to enhance the force feedback during palpation of a deformable surface. By this integration, information loss due to the use of lower resolution deformable meshes is aimed to be partially captured. An evaluation study has also aimed to show potential benefit of AVC and AVCEC in a scenario similar to detecting the position of a dislocated bone under the skin.

Work

The boundaries of deformable meshes are often represented in the form of a surface. Applying force or displacement input to a subset of the surface mesh nodes has been the most common way to simulate interaction. To employ virtual coupling on deformable surfaces, one needs to evaluate the position of the proxy over the surface, which has been widely covered in the literature. The technique described in [84] is used in this paper. In addition to evaluation of the proxy position, a relation between the virtual coupling force and the surface mesh nodes needs to be defined. Distribution of force input among the surface nodes is calculated by applying low pass filtering on the virtual coupling force over the
Interaction with a deformable shape occurs between virtual coupling and the surface. The deformation algorithm and the inner mesh nodes are, therefore, decoupled from the virtual coupling. Being based on virtual coupling, AVCEC also exploits this feature. In order to modulate the stiffness of the virtual coupling and evaluate the energy conserving terms, AVCEC casts rays through the data along the proxy-probe direction. This requires no additional consideration in the case of AVCEC, since both the proxy and the probe positions are known by the virtual coupling algorithm [84]. Evaluation of energy conserving terms requires movement of the proxy position over the surface. This movement is calculated on the plane defined by the surface element with which the proxy is in contact.

Inhomogeneities within a deformable shape are discussed in two categories. First, dynamic inhomogeneities which deform together with the deformable shapes by which they are covered are considered. A tumour within soft tissue, cartilage structures in the ear or nose can be named as dynamic inhomogeneities. Deformation of these inhomogeneities is simulated consistently with the deformable shape by using a mapping relating them to the deformable shape. Information gathered along the rays cast through the data is evaluated by considering the up to date deformed version. Different structures can be used to represent inhomogeneities like implicit surfaces, voxel arrays, or surface geometries. In this paper, surface geometries are implemented to represent inhomogeneities. For each ray cast through the data, collision checking against the surfaces of the inhomogeneity...
geometries is used to modulate the stiffness. Second, static inhomogeneities which are not deformable themselves but affecting the deformation behaviour by being within the shape are considered. A rigid bone under fat layer, for instance, affects how soft tissue is deformed although it is static itself. These structures are simulated in the deformable mesh by fixing the mesh nodes of the deformable shape. Their static structure facilitates collision checks for the rays cast by AVCEC. One can observe both static and dynamic inhomogeneities within a deformable shape in figure 5.4.

Results

An experiment was performed to asses the additional benefit AVCEC brings to a FEM-based deformation simulation. The scenario was designed to simulate a dislocated bone beneath the skin, as illustrated in figure 5.3. The aim of the experiment was to compare the success rate and time spent for the task of finding the location of the dislocated bone under the skin during a lateral movement over the surface between two conditions: A FEM-based deformation, the FEM condition, and FEM-based deformation enhanced with AVCEC algorithm, the AVCEC condition.

The experiment was carried out in a virtual environment using the same setup deployed in the previous studies, figure 1.2. Subjects were presented with a deformable mesh, figure 5.3, for each trial and told to make a lateral contour following movement over the surface. To represent bone structure beneath soft tissue, a rigid body was located within the deformable mesh. Mesh nodes spatially covered by the rigid body were fixed and direct force feedback from the rigid body was disabled. Feeling the rigid body, bone, was therefore achieved through only fixed nodes of the FEM-based deformation behaviour. Geometry of the rigid body was chosen to simulate a displaced bone structure. Right hand side of the
rigid body was placed closer to the tissue surface as illustrated in figure 5.3 and the depth
difference between the two sides was kept constant. For each trial, the horizontal location
of the bone dislocation was randomly varied among three different positions: left, right,
and middle. The task was to find the location of the bone dislocation during a lateral
movement over the surface of the deformable mesh. The independent variable was the
horizontal position of the bone dislocation and the experiment was carried out as within
subjects design for two conditions, FEM and AVCEC, which were performed separately
following a balanced order. Each subject was presented 40 trials for each condition, and
the procedure was repeated for 10 subjects.

Data was analysed for each subject to compare success rates and average trial times
of two conditions: The FEM and the AVCEC. One can observe the significantly higher
success rate of AVCEC, which is 100% for each subject in figure 5.5. For the FEM con-
dition, on the other hand, a mean value of 33.5±7.84% was observed. Besides its higher
success rate, the AVCEC condition resulted in lower average trial times with a mean value
of 8.64±4.8 seconds, while the FEM condition had 17.84±7.36 as average trial time. Sig-
nificance of the results can be observed in the figure showing the performance of each
individual subject. Checking significance of the data by statistical analysis was therefore
not carried out.

Contributions
The algorithms introduced in paper V, providing information about inhomogeneities be-
neath a surface, were integrated with a deformable mesh simulation. The main motivation
was the use of low-resolution deformable meshes resulting in a significant amount of in-
formation loss beneath the surface. A user study was, therefore, carried out to asses the
benefit of AVCEC in detecting subtle variations in the bone structure under the skin. Suc-
cess rates and time spent for finding a dislocated bone under the soft tissue were compared
between two conditions: FEM-based deformation and AVCEC integrated FEM-based de-
formation. The results have shown significant improvement of detecting inhomogeneities
which can not be detected by a FEM-based deformation alone.
Chapter 6

Conclusions

In the beginning of this work, the focus was primarily on the effects of adaptive simulation on accuracy of force feedback. As the work progressed, the analysis of the force error has led to questioning the importance of the error, therefore shifting the focus towards the psychophysics side. As related to deformation, different aspects of stiffness perception have been studied. The effects of different modes of exploration on stiffness perception have been examined, followed by a study indicating the significant effect of the stiffness gradient on perception. Finally, a novel algorithm to detect inhomogeneities beneath the soft tissue was introduced. The main motivation was the common use of low-resolution meshes in deformation simulations in spite of the availability of medical data such as CT and MR with much higher resolutions. The proposed algorithm, during virtual surface exploration, provides complementary high-resolution information from beneath the surface in order to detect inhomogeneous structures.

In the rest of this chapter, the contributions of the thesis will be summarized which will be followed by a discussion on the topic. Finally, the most important issues in the future of deformation simulation will be presented.

6.1 Summary of Contributions

The contributions of the thesis which have been elaborated in the previous chapters can be listed as;

- **Asynchronous regions**: asynchronous solutions of different parts of a deformable object by different refresh rates modelled by different resolutions. This is used in the solution of the local neighbourhood of the contact with higher resolutions and update rates, while lower resolution meshes were solved less frequently in the more remote regions. The decoupled structure of the regions also allows deploying different types of solvers, implicit or explicit, in different regions.

- An abstract pipeline to analyse the accuracy of deformation with respect to numerous types of inputs.

- A case study performed by using the proposed pipeline: The force error caused by adaptivity in spatial and time domain was analysed for a number of parameters.
Three types of parameter were considered during the analysis: User Input, Material Properties, and Adaptivity Parameters. User input was composed of input frequency and strain, material properties were composed of elasticity, mass, damping and poisson ratio while the adaptivity parameters included the update rate and resolution of the secondary region and the size of the primary region, where the primary region refers to the local neighbourhood of the contact and the secondary region refers to the more remote regions.

- As a result of the case study: Explicit correlations between force error and material and simulation properties were obtained. Online use of calculated error mapping was discussed for monitoring the quality of the simulation in real-time or adjusting the simulation parameters in order to keep the error below some desired limits.
- The effect of exploratory procedures on stiffness perception was examined by considering two aspects: continuity of contact and the axis of motion on a surface.
- The stiffness perception was found to be better during continuous contact, detecting a stiffness transition, than discontinuous contact, with a separate comparison of stiffness values.
- Stiffness perception was found to be better along the normal axis of a surface compared to lateral movement over the surface.
- The gradient of the stiffness in addition to the magnitude of stiffness change was shown to have an effect on perception.
- An algorithm, AVC, providing high-resolution information to allow probing of inhomogeneous structures underneath a surface was proposed.
- An energy conserving version of the previous algorithm, AVCEC, which compensates for the energy variations of the spring-damper system of virtual coupling principle through additional force components was introduced to maintain passivity.
- It was shown that the energy compensating force components, by providing gradient information, improve the shape recognition for the objects beneath a surface.

### 6.2 Conclusions

This thesis has considered different aspects of deformation simulation. Accuracy analysis for force feedback, stiffness perception and detection of high-resolution inhomogeneities underneath a surface can be named as issues considered in this work.

The most significant challenge in the deformation simulation is achieving sufficient realism together with the desired refresh rates. Several optimizations techniques have been proposed, one of which is adaptivity which models important parts of an object more elaborately during deformation. This, obviously, has an impact on the accuracy of deformation.
6.2 Conclusions

A pipeline to perform an error analysis was introduced and a case study was carried out to create a force error mapping with respect to various simulation parameters. One outcome from this case study is the explicit correlations between the force error and individual parameters. The knowledge about the relation between error and a single parameter has a potential benefit in determining the simulation techniques depending on the type of the application. Another use of the pipeline is quality assessment by monitoring the error in real-time. The real-time error can be fed back to the simulation to adjust adaptivity parameters to keep the error below some desired limits. This adjustment, however, is limited by the stability concerns.

The real-time error monitoring and parameter adjustment can benefit the perception limits such that the error can be kept below perceivable differences. The nature of touch, however, does not allow a simplified analysis since it is composed of several factors affecting the perception simultaneously. Exploration of an object by touch mostly happens in the form of combination of several movements like contour following, applying pressure, weighting etc. There are studies surveying our perception limits [49, 104], however these are mostly carried out using simplified scenarios considering one aspect of touch at a time. In paper III, our perception limits were shown to be affected by the way we touch. Two basic aspects of touch, continuity and axis of motion, creating three different scenarios were surveyed: Discontinuous Pressure, through separate comparison of stiffness values, Continuous Pressure, detecting stiffness change along the direction of the applied pressure, and Continuous Lateral detecting lateral stiffness change over a surface. One of the two main outcomes of this study was a better stiffness perception in the case of applying pressure along the normal axis than a lateral sweeping movement over the surface. The other outcome was the better perception in the case of continuous contact than comparing different stiffness values separately. These outcomes show the significance of the touch scenario for perception analysis, therefore one can emphasize the need for a comprehensive understanding of perception before we aim for quantization of our perception limits. The perception during continuous contact was further studied in paper IV which showed the significant effect of stiffness gradient on perception. This result could have beneficial applications considering related medical studies [46, 109, 115] discussing the direct relation between the stiffness gradient and tissue characteristics. The stiffness gradient can, for instance, contribute to the force feedback in different ways to emphasize the underlying information within the data.

The last contributions of the thesis aim to enhance virtual volumetric exploration through surface probing by providing high-resolution complementary information. During exploration of a surface in real life, we mostly feel stiffness variations due to inhomogeneity of the object. In the virtual world, the use of low-resolution meshes in deformation simulation, however, decreases the accuracy of feeling these inhomogeneities. In addition, medical data such as CT and MR have high volumetric resolution which causes a great deal of information loss by employment of low-resolution deformable meshes. To solve this problem, I modified the virtual coupling principle such that information from along a ray cast in the proxy-probe direction is used to modulate the surface stiffness. User studies showed that this allows location of inhomogeneous structures under a virtual surface even
if they are partially obscured by another structure. Due to varying stiffness of the virtual coupling, energy variations affect the passivity of the haptic system. In order to maintain passivity, energy compensation is applied by providing gradient information. This gradient information was shown to improve shape recognition of structure beneath a surface. This was expected considering the outcome from paper IV about the significant effect of gradient on perception.

One major feature of the proposed algorithm is that it is independent of the resolution of the surface, therefore it has great potential to be used together with low-resolution deformable meshes. Underlying inhomogeneities in high-resolution volume data, tissue tumour for instance, can be felt during deformation as well. The fact that it depends on a widely accepted principle, virtual coupling, makes the use of the algorithm possible for many surface rendering techniques from texture rendering to friction rendering. The modulation of the surface stiffness rather than overwriting it also keeps the surface information in addition to the volume information underneath.

6.3 Future Work

The ultimate aim for deformation simulations is to be able to achieve a realistic deformation for high-resolution in an immersive environment. The current focus is mostly on simulation of complex physical properties such as non-linearity and viscoelasticity. In addition to optimizing existing models, more efficient novel techniques will probably be developed to tackle the problem. The use of specialized hardware like the GPU has already received some attention, for example [21, 23, 62], in addition to the use of multi-core processor architectures [107]. When these advances allow using higher resolutions for deformation, interaction with inhomogeneous materials will likely receive further consideration. Measurement of material properties to create a volume mapping as in elastography (see, for example [98]) should be further integrated into the simulations. I hope also that more focus will be shown to fill the gap between psychophysics and technology, such that the findings from psychophysics will be considered more in the design and development of devices and algorithms.
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