Mathematical Learning Disability

Cognitive Conditions, Development and Predictions

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Abstract

The purpose of the present thesis was to test and contrast hypotheses regarding the cognitive conditions that support the development of mathematical learning disability (MLD). The following hypotheses were tested in this thesis: a) the domain-general deficit hypothesis—the deficit is primarily localized to domain-general systems such as working memory; b) the number sense deficit hypothesis—the deficit is localized to the innate approximate number system (ANS), c) the numerosity coding deficit hypothesis—the deficit is localized to an exact number representation system, d) the access deficit hypothesis—the deficit occurs in the mapping between symbols and the innate number representation system (e.g., ANS) and e) the multiple deficit hypothesis—MLD is related to more than one deficit.

Three studies examined the connection between cognitive abilities and arithmetic. Study one and three compared different groups of children with or without MLD (or the risk of MLD). Study two investigated the connection between early number knowledge, verbal working memory and the development of arithmetic ability.

The results favor the multiple deficit hypothesis; more specifically, the results indicate that number sense deficit together with working memory functions constitute risk-factors for the development of MLD in children. A simple developmental model based on von Asters and Shalev´s (2007) model and the present results that seeks to aid understanding of the development of MLD in children is suggested.

CONTENTS

List of papers

This thesis is based on the following papers:

I. Andersson, U., & Östergren, R. (2012). Number magnitude processing and basic cognitive functions in children with mathematical learning disabilities. *Learning and Individual Differences*. doi:10.1016/j.lindif.2012.05.004

II. Östergren, R., & Träff, U. (2013). Early number knowledge and cognitive ability affect early arithmetic ability. *Journal of Experimental Child Psychology*, 115, 405–421. doi: 10.1016/j.jecp.2013.03.007

III. Östergren, R., Skagerlund, K., & Träff, U. (Submitted). Cognitive conditions of children at risk of developing mathematical learning disabilities

Introduction

Arithmetic ability is important in a modern society, and mathematical skill prior to school entry is one of the best predictors of educational success (Duncan et al. 2007). Although they are difficult to estimate, the costs of low numeracy have been estimated at £763 million each year in the United Kingdom alone (Every Child a Chance Trust, 2009). Low numeracy can be related to special educational needs, educational failure, antisocial behavior etc. However, low numeracy is not believed to be the single cause of youth crime, rather it is believed to be a piece of a causal puzzle that involves educational failure for the individual, which is a risk factor. Thus, problems with learning basic math, such as arithmetic, could possibly result in high personal costs to the individual and also to society. Mathematical learning disability (MLD), developmental dyscalculia (DD), mathematical disability (MD), and mathematical difficulties are all names for a phenomena that entails problems with learning basic math or arithmetic. Hereafter, MLD will be used in the present thesis to refer to these difficulties; however, several different views exist on the conditions that facilitate the development of MLD. Many different conditions can result in low numeracy, including poor schooling, poor home environment and poor cognitive disposition. MLD can be viewed as a description of the cognitive aspects that lead to low numeracy. What type of cognitive ability serves as the foundation of numeracy? Is there a single deficit that underlies MLD, or is MLD the result of several deficits? The accumulation of knowledge concerning MLD could aid interventions that would save our community both money and human resources. To investigate these types of questions, we first need to examine the theories surrounding the cognitive abilities that serve as the foundation of arithmetic ability.

Typical development of number and basic mathematical cognition

What is the mechanism that underlies children´s numerical development? How do children represent and process numbers? What cognitive systems support the development of arithmetic ability? The following section describes several relevant theories that address these types of questions.

Approximate number system (ANS).

Humans and other species have the ability to represent numbers (e.g., sets of items) in an analog magnitude manner (Carey, 2009; Dehaene, 2011). This nonverbal ability to represent numerical magnitude in an approximate fashion is called the approximate number system (ANS) (Butterworth, 2010; Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). The representation of numbers is dependent on the ratios between items; for example, a 6-item set can be distinguished from an 18-item set immediately after birth in humans (Izard, Sann, Spelke, & Streri, 2009). During the first year of development, the ANS rapidly matures, as evidenced by the ability to discriminate the ratios of 1:3 and 2:3. The ANS continues to develop until 20 years of age, at which time the ratio of 7:8 is discriminable (Halberda & Feigenson, 2008; Piazza, 2010). The Weber fraction is a measurement of the acuity of the ANS and is derived from Weber's law; i.e., the smallest detectable difference between two stimuli (Piazza, 2010). A simple formula for the internal Weber fraction is (A-B)/B, where A is the larger number, and B is the smaller number; the Weber fraction could be applied to, for example, the difference between 8 and 7 dots (Halberda & Feigenson, 2008). A metaphor that is commonly used to describe the function of the ANS is a mental number line (Dehaene, 2011); numbers are spatially oriented on a mental number line with zero to the left and ascending numbers to the right. It has been suggested that the mental number line is the foundation of Arabic numerals and counting words (Dehaene 2011; Piazza, 2010). Evidence for this notion primarily comes from a number of numerical effects that have been found in experimental settings. The distance effect (Moyer & Landauer, 1967) refers to the observation that it takes longer to compare which digit is numerically larger when the numerical distance between the two digits being compared is small compared to when that distance is large (e.g., 5 – 6 vs. 5 – 9). This effect supports the notion of an underlying mental number line with an analogue magnitude representation of numbers; if the digits being compared are close to each other, it is harder to discriminate between them (the discrimination is more time consuming), but if they are far apart on the mental number line, discrimination is easier due to the reduced degree of overlap (Deheane, 1992; Gallistel & Gelman, 1992; Moyer & Landauer, 1967) or scalar variability (Gallistel & Gelman, 2000; Whalen, Gallistel, & Gelman, 1999).

The problem size-effect is another numerical effect that provides support for the notion of an analog mental number line (Dehaene, 1992; Hinrichs, Yurko, Hu, 1981). The problem size-effect refers to the fact that it takes longer to compare larger digits (e.g., 9 vs. 8) than it does to compare smaller digits (e.g., 1 vs. 2). Larger digits have a greater overlap due to the logarithmic nature of the analogue mental number line; i.e., the number line becomes more compressed for larger numbers.

The link between the ANS and mathematics is often viewed as a mapping between a biological primary system and an exact symbolic system of numbers (Feigenson & Halberda, 2013). This mapping process has been described by theoretical developmental models of number cognition as presented below (e.g., von Aster & Shalev, 2007; Geary, 2013).

Parallel individuation

The parallel individuation system is a working memory system that is used as a primary system to represent numbers in the lower range; i.e., it is typically used to represent numbers less than 3 and never used to represent numbers over 4. The parallel individuation system represents numbers in an exact way, regardless of sensory modality if those numbers are an object (called an object file). The *object tracking system* is another term for the *object file* system (Carey, 2009). The parallel individuation system model can explain some data from habituation studies performed on human infants because infants can discriminate between 2 and 3 but not between 1 and 4 items, although the ratio between the numbers suggests that the ANS should be able to discriminate these numbers. However, when such aspects as size and shape are controlled for, infants typically do not discriminate based on numerical features (Carey, 2009). The parallel individuation model is not a system that only represents numbers, rather other aspects, such as the sizes and shapes of stimuli, can be differentiated by this system.

The parallel individuation system and the ANS seem to be present early in development in humans; which system is activated is most likely governed by context; e.g., the number of stimuli etc. (Carey, 2009). Parallel individuation has been suggested to be part of a domain-general system, in contrast to the ANS, which is considered to be a domain-specific system (Piazza, Fumarola, Chinello, & Melcher, 2011). The interplay between the two systems can perhaps be explained by contextual factors. Attentional load, working memory load or perceptual cues (close together) and the number of items could trigger the engagement of the ANS for lower numbers rather than the parallel individuation system (Hyde, 2011).

Enriched parallel individuation and bootstrapping

When parallel individuation is combined with the set-based quantification system, the result is enriched parallel individuation (Le Corre & Carey, 2007). The set-based quantification system represents the commonalities in a set and within individual items; for example, the ability to distinguish singular from plural (i.e., ordinality). Enriched parallel individuation makes it possible to distinguish 3-4 items (or sets) and maintain some order between the sets (e.g., 3 are more than 1). Along with the count list, enriched parallel individuation formulates the rule that for every step in the counting sequence, 1 is added to the former set, which results in a set of cardinality that is +1 of the previous set. This notion, that a child first constructs a concept of exact numbers that corresponds to the integers, has been labeled as a form of Quinian bootstrapping (Carey, 2009; 2011). It is this bootstrapping process that allows for the representation of number over the limited capacity of the parallel individuation system that is 3 in young children and 4 in older children/adults. The mapping of counting words and digits does not directly involve the ANS; rather, it has been hypothesized that mapping onto the ANS does not occur until an exact representation of number is in place (Noël & Rousselle, 2011).

Numerosity coding

One major argument against enriched parallel individuation and the bootstrapping account is the involvement of language. According to Carey (2009), language ability does play a role in the concept of exact numerosity; however, studies of children with specific language disorder have revealed no evidence of problems with basic-level number processing (Butterworth, 2010). An alternative theory that represents numbers in an exact way is the system of numerosity coding (NC) (Butterworth, 2010). NC is a neural network model that has primarily been tested in simulations (Zorzi, & Butterworth, 1999; Zorzi, Stoianow, & Umiltà, 2005). This model states that numbers are represented as a discrete set of neuron-like elements; i.e., one is represented by activating one element, and two is represented by activating two elements etc.. NC does not have the limited capacity of parallel individuation (or enriched parallel individuation), and it also accounts for such numerical effectsas the distance effect and problem size effect. NC has been proposed to be a third system, in addition to the ANS and parallel individuation (or OTS), upon which arithmetic ability is based (Butterworth, 2010). Thus, the central condition of developing arithmetic ability is not an approximate analog number representation but instead an exact discrete representation of numbers.

Number knowledge: when symbols connect with innate number representations
Number knowledge (or early number knowledge) is the ability to process digits and counting words when they have been mapped onto the innate number representations system (i.e., the ANS) and is also referred to as number sense (Jordan, Kaplan, Oláh, & Locuniak, 2006; National Mathematics Advisory Panel, 2008). Other vocabulary used to refer to the same ability includes, number competence and symbolic, secondary, and verbal number competence, and the explicit number system (Geary, 2013; Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan, Ramineni, & Locuniak, 2009). The present thesis will use number knowledge to refer to the abilities that include the use of symbols and reserve the term number sense for the ANS. Number knowledge can be divided into two parts, although it forms one ability. One part is primarily biological and requires no deliberate practice for development (Geary, 1995; Jordan & Levine, 2009). This part is the innate system of the ANS. The other part is secondarily biological, requires deliberate practice for development and is composed of the symbolic number system.

A number of theoretical models describe the integration of these parts of number knowledge. The developmental model of number acquisition proposed by von Aster and Shalev (2007) (see *Figure* 1) includes four steps: the first step begins with the innate ability to represent numbers (e.g., the ANS), the second step is the formation of connections with the verbal number system (number words), the third step is the Arabic numeral system (digits), and the fourth and final step is the mental number line (or the symbolic mental number line); i.e., when the number words and digits of step two and three have been fully mapped onto the ANS). According to this model, this four-step-developmental process relies upon the support of domain-general abilities such as working memory.

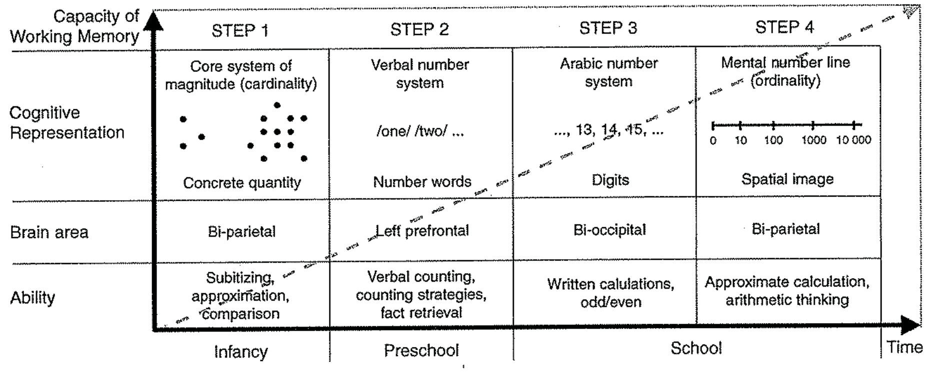

Figure 1: *Four-step-developmental model of numerical cognition. Shaded area below broken line: 'increasing working memory.'*

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Figure 1 is from von Aster and Shalev´s (2007) article in *Developmental Medicine & Child Neurology*, *49*, 868–873.

Another model that espouses similar developmental ideas was presented by Geary (2013). This three-step model also starts with the ANS and the mapping of number words and digits as supported by attentional control. This model also supports the construction of early explicit number system knowledge. Intelligence is also a supportive function in the final phase of this model.

Furthermore, the pathway model of LeFevre et al. (2010) distinguishes between an early number representation system that mostly relates to numerical magnitude processing (symbol-independent) and a symbolic number system that is mostly affected by linguistic abilities and spatial attention.

Early number knowledge seems to be an important predictor of mathematical skills early in a child's school career. This relation is also evident after domain-general abilities are controlled for (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak, & Jordan, 2008; Träff, in press). However, it is often the case that some type of arithmetic calculation test is used to measure number knowledge (Aino, Ee, Lim, Hautamäki, & Van Luit, 2004; Lago & DiPerna, 2010; National Mathematics Advisory Panel, 2008; Purpura, Hume, Sims, & Lonigan, 2011). For example, Jordan and coworkers measure of number knowledge includes as many as three calculation tasks: nonverbal arithmetic, story problems and number combinations. Other tests include more implicit aspects of calculation; for example Geary et al.'s (2009) number sets test for nonverbal calculations, as in LeFevre et al. (2010). The inclusion of calculation can be problematic for predicting young children`s arithmetic skills due to the risk of using the same skill as both the predictor and the criteria (Schneider, Grabner, & Paetsch, 2009). However, previous studies that have not included calculation in the number knowledge concept have also shown that the number knowledge concept predicts mathematical achievement (Chard et al., 2005; Clarke & Shinn, 2004; Krawejski & Schneider, 2009b; Träff, in press).

Domain-general abilities

What role do domain-general abilities have in relation to the development of mathematical skill? Intelligence, language abilities, working memory, executive function/attention, attentional control, phonological awareness/ability, spatial ability, and processing speed are all abilities that have been related to mathematical achievement (Alloway, & Passolonghi, 2011; Swanson, 2004). These abilities also overlap, to greater or lesser extents, with one another.

Intelligence is chiefly defined in the field of mathematical development in the same theoretical tradition as Spearman's (1904) introduction of the g-factor. A typical study in the field of numerical cognition will often include some part of a test battery, for example Wechsler's scales, that is used to estimate both verbal (crystalized) and/or nonverbal or perceptual (fluid) abilities as a proxy for intelligence (e.g., Geary, 2011). The theoretical overlap with other constructs occurs primarily for working memory and executive functions. It has been debated whether working memory capacity is the same as intelligence (see Ackerman, Beir & Boyle, 2005; Beir & Ackerman, 2005; Kane, Hambrick & Conway, 2005; Oberauer, Schultze, Wilhelm & Süb, 2005). In a large study, intelligence accounted for slightly more than half of the variance in mathematical performance (Deary, Strand, Smith, & Fernandes, 2007). Many studies have found some relation between both crystallized and fluid intelligence and mathematics (see Geary, 2011 for a review).

Working memory is the ability to simultaneously store and process/manipulate information in a goal-directed way. Several theories exist as to how the ability is structured (see Baddeley, 2012 for a review of the multicomponent model, and Conway, Jarrold, Kane, Miayke & Towse, 2008 for more process orientated theories**).** One structure divides working memory into aspects that are based on the tasks used to measure the construct, if the tasks are verbal or visual in nature (see Raghubar, Barnes, & Hecht, 2010 for review). Such distinctions often incorporate aspects of executive functions, executive attention and attentional control into the concepts of verbal working memory and visuospatial working memory. Executive attention has been proposed to be the source of the variation in working memory capacity (Kane, Conway, Hambrick, & Engle, 2008). Executive attention processes are hard to distinguish from attentional control**,** which is an important construct in Geary´s (2013) recent theoretical proposal of the development of early explicit number system knowledge. Attentional control and executive attention in the area of number/mathematic cognition should be interpreted as referring to the same process.

The relative contribution of number knowledge and domain-general ability

The relation between domain-general ability (e.g., verbal working memory) and number knowledge and arithmetic ability has been the subject of numerous studies (Cirino, 2011; Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010; Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, et al., 2010; Geary, 2011; Krajewski & Schneider, 2009a; LeFevre et al., 2010; Passolunghi & Lanfranchi, 2012; Träff, in press). Theoretically, it makes more sense to view domain-general abilities as a supportive framework for number knowledge in accordance with the model of von Aster and Shalev (2007).

Some studies have, however, found that different constellations of domain-general and domain-specific number abilities underlie different aspects of basic mathematics. Number combination skills are supported by number knowledge but not by domain-general abilities (e.g., working memory), whereas word problem solving skills are supported by both domain-general skills and number knowledge skills (Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010; Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, et al., 2010; Träff, in press).

The aforementioned two studies by Fuchs and coworkers used the number set test (Geary et al., 2009) and a number line estimation task (Siegler & Opfer, 2003; Siegler & Booth, 2004) as measures of number knowledge. Regarding the growth of procedural calculation skills, only number knowledge was a predictor, but the growth of word problem solving was predicted by both domain-general abilities and number knowledge (Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, et al., 2010). Using the same measures of number knowledge (the number set test and number line estimation) Geary (2011) found that number knowledge, along with working memory, predicted mathematical skill. Phonological ability has sometimes been found to be a domain-general ability that supports the development of number knowledge (Krajewski & Schneider 2009a) that indirectly supports arithmetic ability via number knowledge. Krajewski and Schneider (2009a) also found that phonological ability supports a lower-level aspect of number knowledge (number words that do not refer to quantity), in contrast to visuospatial working memory, which supports a higher-level aspect of number knowledge (number words that refer to quantity). Passolunghi and Lanfranchi (2012) found no evidence that phonological ability was important for either number knowledge or mathematics; rather, these authors found that working memory, intelligence and processing speed were important. Working memory has an indirect effect, via number knowledge and verbal IQ, on mathematical achievement, and processing speed has a direct effect on mathematic achievement. Additionally, LeFevre et al. (2010) found that, depending on the outcome measure, different abilities are important; linguistic ability is related to number naming but not to nonlinguistic arithmetic, and spatial attention is related to all mathematical outcomes. The indirect role of domain-general abilities via number knowledge in arithmetic ability has also been reported by Cirino (2011). These empirical investigations seem to suggest that working memory (both verbal and nonverbal), and sometimes phonological ability, support number knowledge and that, depending on the task, these factors also partially support mathematical skills. This notion is in line with developmental models of number knowledge (Geary, 2013; von Aster & Shalev, 2007)

Purpose and Aim

How are these theories of number cognition related to MLD? Most researchers in the area of MLD and mathematical cognition would most likely acknowledge these theories, but they would not agree on how the different cognitive abilities are related to MLD. The aim of the present thesis was to investigate the underlying conditions of developing MLD. As a part of that overall aim, the impacts of domain-general abilities, such as verbal working memory, and the domain-specific number knowledge ability on to arithmetic ability were also investigated. Thus, this work seeks to further the understanding of which abilities (domain-general abilities or specific number abilities) support arithmetic ability in general. This question is directly related to MLD, as the different hypotheses about the origin of MLD can, in principle, be divided into domain-general cognitive deficits and domain-specific number deficits. In the next section, I will describe the specific hypotheses that originated from the above-mentioned theories. The approach of this thesis to the specific hypotheses was to test them simultaneously and contrast as many as possible against each other. Studies II and III are part of the same large longitudinal project, and study I was performed on a different sample. Before the specific hypotheses are described, a brief history of MLD and some of the theories regarding different subtypes of the disability will be presented.

Mathematical Learning Disability

Mathematical learning disability (MLD) is understudied compared to reading disorder or dyslexia. A search of the Psych INFO database for articles published between 2006 and 2012 using the search terms "reading disorder", "reading disability", and dyslexia (i.e., the same search terms used by Gersten, Clarke, and Mazzocco (2007)) resulted in 1873 articles in peer-reviewed journals. When acalculia, dyscalculia, "mathematics disorder", "mathematics disability", "mathematics difficulties", "arithmetic disorders" and "mathematics disorders" were used as search terms, 182 articles were found. For every article that is published in the field of MLD, roughly 10 articles about reading disability are published.

The study of mathematical disability began neurological case studies of people with brain injuries in the early 20th century; however, as a research field, the study of mathematical disability began later with Kosc´s classification from 1970 (Kosc, 1974; Gersten et al., 2007). The classification of Kosc from 1970 was the first on DD, and describes it as a disorder that affects brain regions that are responsible for mathematic ability, and without any other disorder affecting mathematic ability; it has also a genetic or congenital origin. This classification used the discrepancy between IQ and skill level, meaning that a child with MLD should have a mathematical ability that is lower than expected for his/her age and learning history and that that child does not suffer from a deficit in general mental ability.

Subtypes.

Different subtypes of MLD have been suggested by a number of researchers. For example, Geary (2004) proposed three subtypes: the *procedural* subtype, the *semantic memory* subtype, and the *visuo-spatial* subtype. Persons with the procedural subtype of MLD often make use of immature strategies (e.g., counting fingers for simple calculations), often make procedural errors, display poor understanding of the concepts that underlie procedures, and display problems with the sequencing more complex procedures. The most obvious feature of persons with the semantic memory subtype of MLD is difficulties with arithmetic facts; i.e., retrieving answers to simple arithmetic problems (e.g., 3 + 4) from long-term memory. Persons with the visuo-spatial subtype of MLD have problems with spatial-numerical information (i.e., the spatial representation of numerical information) and with numerical information that is represented with Arabic digits. For example, multicolumn arithmetic problems and the interpretation of place values in multi-digit numbers are difficult for people with this subtype of MLD.

The idea of subtypes is not new, Kosc (1974) suggested the following six different subtypes of developmental dyscalculia: verbal dyscalculia, practognostic dyscalculia, lexical dyscalculia, graphical dyscalculia, ideognostical dyscalculia, and operational dyscalculia. This author also defined pseudo-dyscalculia, which is not a form of dyscalculia (mathematical anxiety would fall under this label). All types of DD should be the result of neurological developmental dysfunctions of the brain area that supports mathematical skills. Kosc (1974) also noted that the subtypes could be manifested alone or in combination in people suffering from DD.

Wilson and Dehaene (2007) listed three potential subtypes of MLD with specific neurological origins. The first subtype was referred to as a deficit in verbal symbolic representation; people with this subtype have problems learning and retrieving arithmetic facts and may have problems with counting sequence. The brain areas that are relevant for this subtype are the left frontal and/or temporal language areas and the left basal ganglia and angular gyrus. The second subtype involves a deficit in executive functions that originates from frontal lobe dysfunction and results in problems with procedural usage, strategy and arithmetic fact retrieval. The third subtype involves a deficit in spatial attention and is related to posterior superior parietal dysfunction and the object file or object tracking system; thus, subitizing should be affected in people with this subtype. The third subtype is perhaps most difficult to separate from number sense deficit (which has been proposed to the core deficit of MLD) because of the connection between numerical and spatial representations.

Recently, Price and Ansari (2013) suggested a division between secondary and primary DD. Children with primary DD are those with the most severe form of mathematical difficulties that are caused by a developmental impairment of the neurological foundations of numerical magnitude processing. Children who do not suffer from primary DD but still have difficulties due to other factors, such as working memory deficits, poor schooling, behavioral attention problems, etc., are labeled as having secondary DD.

A similar division has also been suggested by Henik, Rubinsten and Ashkenazi (2012). These authors defined DD as a disorder that is caused by a deficit in the processing of numerical quantities (on a cognitive level) and a deficit in the IPS (on a biological level). Henik et al. (2012) labeled those that exhibit a deficit in the IPS that causes both a deficit in numerical processing and attention as MLD. Finally, these researchers suggested that deficits in arithmetic and attention could be due to a deficit in executive functions that originates from a deficit in frontal lobe areas. The same line of division was proposed by Kaufmann and von Aster (2012); in their work, DD is the label for a specific disability that originates from a deficit in the IPS and results in a core deficit in number and quantity processing. MLD, on the other hand, can be the result of multiple deficits involving functions such as working memory, attention, number processing, etc.

Another description of the potential subgroups of MLD involve the application of a variable approach and the specification of different representational components of numerical processing. Moeller, Fischer, Cress, and Nuerk (2012) suggested six forms of numerical representations. *Visual number form,* is the ability to read digits, and deficit in this ability should be manifested through difficulty processing symbolic numerical information while non-symbolic processing remains intact; s*emantic representation of numerical magnitude* is the ability to automatically access the semantic meaning of numbers via both symbolic and non-symbolic stimuli. Deficits in this representational form are similar to those of the number sense. *Verbal numerical representation* is the basis of the formation of arithmetic facts in long-term memory. Deficits in this area result in severe impairments related to arithmetic facts. *Spatial representation of numerical magnitude* is often described as the mental number line, and deficits in this representation, which combines numerical and spatial information, are seen in placements on the number line. *Representation of the place-value structure of the Arabic numeral system* represents multi-digit numbers via the correct representation of the Arabic numeral system's base 10 structure. Incorrect representations of units, such as tens and hundreds, results in deficits in all types of number processing that involve multi-digit numbers. A potential symptom of this type of deficit is the inability to utilize the carry effect during the calculation of addition problems. *Strategic, conceptual and procedural components* represent the application of the representations mentioned earlier. A child that does not display difficulties in individual representation forms but does display such difficulties when those forms need to be combined would fall into this category.

Consensus has not been reached regarding subgroups/subtypes of MLD. In empirical investigations, the prevalent approach does not distinguish between subgroups; rather, researchers usually employ some type of cut-off criteria (e.g., the 10 or 15 percentile, or 1 or 2 standard deviations) in one or more mathematical tests to divide children into different groups (e.g., MLD, low achievers and typical achievers). The division of children into subgroups is more of a theoretical perspective that is also often connected to some specific cause for each subgroup (e.g., IPS deficits). In the next section, the specific hypotheses about the underlying conditions of MLD that have been empirically evaluated will be described.

Hypothesis about the “underlying” condition of MLD

Domain-general cognitive deficit

As previously mentioned, domain-general abilities, such as intelligence, working memory and the executive function of working memory, predict mathematical achievement. If a child has trouble learning mathematics, it is reasonable to assume this difficulty is a good predictor of poor achievement. David C. Geary, an authority in the area of MLD, has suggested that at least one cause of MLD is a deficit in working memory (Geary, 2004; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). The number of studies that have found some support for domain-general deficits (e.g., working memory deficits) is rather large, which precludes an exhaustive review in this section. However, a number of studies have used an impressive design that is worth mentioning. Geary (2012) used a longitudinal approach over five years and followed children from six to eleven years of age. Children who scored below the 25^{th} percentile on a broad measure of mathematical achievement were classified as having MLD. In this study, support for verbal working memory deficits and some domain-specific deficits (differences in the number set test, which contains both symbolic and non-symbolic items) was obtained. The children with MLD also differed in class attention. Murphy, Mazzocco, Hanich, and Early (2007) also used a longitudinal approach to examine children from five to eight years of age. To be classified as having MLD, children needed to score below the 10^{th} percentile at least twice over the course of four year. These authors found deficits in working memory and also found that children with MLD exhibited different growth trajectories in mathematics than did children who were classified as low or typical achievers. Geary, Baily, Littlefield, Wood, Hoard and Nugent (2009) investigated children between kindergarten and grade three using latent trajectory analysis to classify the children as MLD, LA, TA and HA. These authors found that the MLD children seemed to have deficits in phonological, visuo-spatial and central executive working memory functions and some type of domain-specific deficit in number knowledge. Some reviews have concluded that children with MLD seem to have deficits in all three core systems of working memory (i.e., central executive, the phonological loop, and the visuo-spatial sketchpad) (Geary, 2011). However, Geary (2010) explicitly recognize a deficit in number sense in both MLD and low achieving children (including MD). The review of Raghubar, Barnes and Hecht (2010) on working memory and mathematics suggested that it is too early to say that working memory causes mathematical difficulties (these authors included studies that defined MDs at more inclusive cut-offs up to the 35^{th} percentile). To summarize, it is impossible to have a simple causal relation that only exists between working memory and MLD, but it is quit realistic to assume that working memory deficits can contribute to MLD.

Number sense deficit

This hypothesis states that the deficit originates in the horizontal intra-parietal sulcus (HIPS) (Rubinsten & Henik; 2009, Wilson & Dehaene, 2007). According to this view, the number sense is also called the approximate number system (ANS) (Dehaene, 2011; Piazza, 2010). As previously mention, according to this hypothesis, the ANS is the foundational system of number knowledge and arithmetic ability. A deficit in the ANS will result in impairments in both the symbolic and non-symbolic processing of numbers. Thus, children with deficits in the ANS will have trouble with approximate non-symbolic number discrimination and the use of symbolic number tasks, such as deciding which digit is the larger of two digits, making estimates on a visual number line, calculating, etc. (Wilson & Dehaene, 2007). The number sense deficit is the only hypothesis that explicitly states that children should perform poorly on non-symbolic approximate number comparison tasks. A number of studies that have directly compared different hypotheses about the underlying condition of MLD have provided support for the number sense hypothesis (Desoete, Ceulman, De Weerdt, & Pieters, 2012; Landerl, Fussenger, Moll & Willburger, 2009; Mazzocco, Feigenson & Halberda, 2011; Mejias, Mussolin, Rousselle, Grégoire & Noël, 2012; Mussolin, Mejias & Noël, 2010; Piazza et al, 2010; Price, Holloway, Räsänen, Vesterinen & Ansari, 2007). Desoete et al. (2012) found support for a deficit in non-symbolic approximate number comparison in five to six year old children. These authors used two time points and a 10th percentile cut-off to classify the children as having MLD. The deficit in the non-symbolic number comparison task was not present when the children were in grade 2 (seven to eight years old). Landerl, Fussenger et al. (2009) used 1 SD below age norms on arithmetic test on one occasion as a cut-off to classify ten year-old children as having MLD and found that the MLD group differed from the control group on a non-symbolic approximation task and on the number line estimation task. Mazzocco et al. (2011) used a longitudinal approach to classifying children as having MLD; at least two measurement points, the children had to score below the 10th percentile on math achievement tests to be classified as MLD. These authors also included low achievers (LAs, those between the 11th and 25th percentiles), typical achievers (TAs; 25th-95th percentile) and high achievers (HAs above 95th percentile). These authors found that fourteen to fifteen year olds with MLD performed worse on non-symbolic approximate number comparisons compared to the other groups. Mejias et al. (2012) found that a group of MLD children performed worse in both non-symbolic and symbolic number comparisons. These authors classified ten year-olds as MLD if they performed below the 15th percentile of a large sample (n=390) on one test occasion. Interestingly, the MLD children performed worse on the symbolic number task than the non-symbolic number task, suggesting an aggravation of the number sense deficit when the symbolic system is included. Mussolin et al. (2010) investigated children that were ten to eleven years old and used the 15$^{\mathbf{th}}$ percentile on a multiplication fluency test on one test occasion as the cut-off to classify the children as having MLD. These authors found more pronounced distance effects in both symbolic and non-symbolic number comparisons in the MLD group. Piazza et al. (2010) used 2 SD below the mean of an age-matched norm group on one occasion to classify 10-year-old children having MLD; the MLD group performed worse than the control group on a non-symbolic approximate number comparison task. Price et al. (2007) investigated twelve year-olds and classified them as having MLD if they performed below 1.5 SD of the control group mean on an arithmetic test on one occasion. These authors found a group difference in non-symbolic approximate number comparison, and the MLD group also made more errors and displayed a more pronounced distance effect. The overall empirical support for the number sense deficit comes from studies that have used children that are around ten years old and a cut-off around the 10th percentile on one test occasion, and the effect sizes span from small to large. Pure response time has rarely generated any support for a number sense deficit.

Numerosity coding deficit

A slightly different hypothesis regarding the origin of MLD is the numerosity coding hypothesis. Numerosity coding deficits are also called number module deficits (Butterworth, 2005; 2010). This hypothesis, similar to the number sense hypothesis, states that the deficits of children with MLD are due to a deficit in the innate number representation and processing system. The localization in the nervous system is the same area as the number sense; namely the IPS (Butterworth, 2010). However, in contrast to the ANS, numerosity coding represents numbers exactly and not approximately (Butterworth, 2010). Deficits in the numerosity coding system should therefore be observed during the exact enumeration of dots when the number of dots is over the subitizing range (i.e., greater than 3-4). Due to the idea that numerosity coding coexists with the ANS, deficits should not affect the approximate judgment of dots in the same number range (i.e., greater than 4) (Butterworth, 2010). The empirical support for numerosity coding comes from studies that have tested exact enumeration either within the subitizing range and or within the counting range. Landerl et al. (2004) investigated children at the age of nine using a rather strong cut-off criteria of 3 standard deviations from the control group mean either on response times or on error rates on one occasion to define the MLD group. In the study of Landerl et al. (2004) a small and marginally significant (p = .06) difference in the slopes of the response patterns of children counted dots (counting range = 4 to 10 dots) was found. Another study that lends support to the numerosity coding deficit hypothesis is the study of Iuculano et al. (2008). These authors investigated eight to nine year-old children with MLD. To be classified as having MLD, the child had to perform below age norms on either a symbolic number comparison task or a dot enumeration task from the Dyscalculia Screener test battery. One of the two children identified as having MLD showed impairment in exact non-symbolic comparison. The third study that lends support to the numerosity coding deficit (or number module deficit) hypothesis is a study of Schleifer and Landerl (2011) that investigated children between eight and ten years of age with MLD. These authors used a cut of criteria of 1.5 standard deviations below the national norm on arithmetic tests (both fluency and mental arithmetic test) on one occasion to be labeled as suffering from MLD. The children with MLD had greater slopes in the subitizing range on a dot enumeration task. Fischer, Gebbhardt and Hartnegg (2008) investigated children between the age of seven and seventeen and used a cut-off criteria of scoring below 16^{th} percentile on a arithmetic test given on one occasion to classify children as suffering from MLD. These authors found that children in the MLD group were less efficient, as measured by response times and error rates in both the subitizing and counting range. Desoete and Grégoire (2006) found that 33% of a sample of eight year olds classified as having MLD based on clinical assessment and scores 2 SD below age norms on one occasion had subitizing deficits.

Together, the empirical support for the numerosity coding deficit hypothesis is mostly grounded in studies that have been performed on children around eight to ten years old who performed between 1.5-3 standard deviations below control groups or the national norm on one occasion. The findings are all based on response times, are seldom based on correctness, and have generally involved rather weak effect sizes.

Access deficit

A third domain-specific hypothesis, the access deficit hypothesis, states that the problem that underlies MLD is in the connection between the innate systems of magnitude representation (i.e., the ANS or numerosity coding) and numerals (symbols) (Rousselle & Noël, 2007). The connection deficit causes the slower response times that often children with MLD of display during numerical comparisons. According to the access deficit hypothesis, children with MLD have difficulty in accessing numerical information from symbols but do not have difficulty with non-symbolic number processing (De Smedt &Gilmore, 2011; Rousselle & Noël, 2007; Wilson & Dehaene, 2007). This hypothesis has primarily been contrasted with the number module hypothesis. The empirical support for the access deficit hypothesis is comparable to that for the ANS and numerosity coding hypotheses.

Rousselle and Noël (2007), who originally suggested the access deficit hypothesis, found that a sample of seven year-old children with MLD only differed from controls on symbolic tasks and not on non-symbolic tasks in terms of response times. Rousselle and Noël (2007) used the 15th percentile on a mathematical test that required both numerical knowledge and calculation and was administered on one occasion as the cut-off criteria to be labeled as having MLD. Iuculano et al. (2008) (described above) found that the low numeracy group differed from the control group only in symbolic comparison tasks and not in non-symbolic tasks as did one of two individuals with MLD. Landerl and Kölle (2009) investigated children with MLD between eight and ten years old who were selected based on low performance (below 1.5 SD) compared to age norms on an arithmetic test that administered on one occasion. These authors also used response times, and the MLD group differed only in symbolic tasks and not in non-symbolic tasks. De Smedt and Gilmore (2011) corroborated previously reported findings (cf. Landerl & Kölle, 2009; Rousselle & Noël, 2007) in a sample of six year-olds classified as having MLD by their performance below the 16th percentile on a broad mathematical test that was administered on one occasion. However, the group difference in the symbolic task was rather small.

Recently, Desoete et al. (2012) provided some support for the access deficit hypothesis by demonstrating a group effect (in response times) of medium size between second graders with and without MLD in symbolic number comparison.

The overall empirical picture that supports the access deficit hypothesis is generated from studies that have investigated rather young children, six to ten years of age, and used response time as the dependent measure. The classification of children as having MLD is usually based on mathematical tests administered on one test occasion and a cut-off criteria at or below the 10th or 15th percentile.

A developmental account that criticizes the defective ANS hypothesis, but is consistent with the access deficit hypothesis, has been suggested by Noël and Rousselle (2011). The main argument of this account is that non-symbolic deficits occur subsequent to the presence of symbolic deficits in development. It is only after the child has constructed the representation of exact numbers that the child starts to map that representation to the ANS. It is the parallel individuation system that is the base of this development; the child constructs sets with the help of these systems and then develops an exact representation of numbers. This process is also called enriched parallel individuation. Noël and Rousselle (2011) developmental a model of MLD that stipulates that children with MLD will first suffer from a deficit at the symbolic level and after that, as a result of deficient calibration of the ANS, a deficit in the ANS will emerge. However, the empirical support for this perspective is weak. Mejias et al. (2012) argued that their result, obtained with ten year olds, supports the notion that the ANS deficit is a result of problems with exact number representation. According to this developmental perspective, one should expect to find an ANS deficit in ten year-old children with MLD due to the lack of refinement of the system, which is refined in their peers; one should not expect to find this deficit in younger children (see Noël & Rousselle, 2011).

Single or multiple deficits

None of the main hypotheses regarding domain-specific deficits (the access deficit, number sense deficit or numerosity coding deficit/number module deficit) have obtained conclusive evidence. Indeed, some of previously mentioned studies have produced results that lend support to more than one hypothesis (e.g., Desoete, Ceuleman, De Weerdt, & Pieters, 2012; Desoete & Grégoire, 2006; Iuculano et al. 2008). Thus, an alternative account to the single deficit perspective is that MLD is caused by multiple deficits (Dowker, 2005;Henik, Rubinsten & Ashkenazi, 2012; Rubinsten & Henik, 2009; von Aster & Shalev, 2007). The same neurological deficit (such as an IPS deficit) could lead to different deficits at both the cognitive and behavioral levels. Children with MLD can simultaneously suffer from more than one deficit, while some children with MLD may have different single deficits that result in in the symptoms of MLD.

Over the course of development, different deficits can give rise to different profiles that, in turn, affect the development of new deficits. To simply assume that the deficit observed in adults is the same deficit in young children is a rather strong and most likely faulty assumption in regards to MLD (Ansari, 2010). The developmental process needs to be taken into account when theorizing about the origin of MLD in children.

A theoretical model that may capture the multifaceted picture of MLD is von Asters and Shalevs (2007) (see also Kaufmann & von Aster, 2012) four-step developmental model of numerical cognition. Step 1 is the innate number representation systems of the ANS and OTS, step 2 is the verbal number system, step 3 is the learning of numerical symbols (e.g., the Arabic numerals), and finally step 4 is the symbolic mental number line (number knowledge). A distortion in any of the four steps could possibly lead to the development of MLD. Another model that also describes the effects of the interaction between systems and contextual factors on development is the developmental calculation model of Kaufman, Wood, Rubinsten and Henik (2011). Their tentative model is based on neuroimaging findings and states that, over the course of development, number representation becomes more integrated and more dependent on the IPS area. These authors also suggest that domain-general factors have a greater contribution in children than in adults. Additionally, the Geary (2013) models describes development as progressing from an innate number representation system that connects with verbal and symbolic systems via a mapping process that is supported by domain-general attentional control. This process results in a number knowledge that is also supported by the domain-general abilities of both attentional control and intelligence. The present research is based on the idea that humans are born with some type of nonverbal number representation system that provides the foundation for further development of number abilities such as number knowledge. The importance of more general abilities, such as working memory, must also be recognized because of their supportive function during development. The purpose of the present thesis was to contrast these hypotheses against and thereby investigate the types of cognitive conditions that are foundational in the development of MLD in children. With the above hypothesis in mind, the empirical investigations sought to test and contrast the different hypotheses about the foundation of MLD. Table 1 shows the relations between the different hypotheses and the three empirical studies.

Studies	**Domain-general/cognitive deficit**	**Number sense deficit**	**Numerosity coding deficit**	**Access deficit**	**Multiple deficits**
Study I	Should find a deficit in one or multiple cognitive functions. For example, verbal WM, visuo-spatial WM, semantic fluency, color naming, word recall.	Should find a deficit in the ANS (number sense). Without any deficit in domain general abilities.	Should find a deficit in numerosity coding. No deficits in the ANS or domain general abilities are expected.	Should find an intact non-symbolic processing and domain-general abilities and a deficits in symbolic processing.	Should display more than one deficit.
Study II	Tests of verbal WM and number knowledge and their relation to arithmetic ability and the development of arithmetic ability. Study the domain-general vs. domain-specific aspects of the hypotheses. If domain-general aspects seem to be more important, these aspects are perhaps be more likely to be the origin of MLD; however, if number knowledge is more important, one of the domain-specific hypotheses could be more likely to be correct. Number knowledge incorporates all three hypotheses; the number sense, numerosity coding, and access deficit hypotheses, and verbal WM stands for the domain-general hypothesis.				
Study III	Should find a deficit in verbal WM and possibly in non-verbal intelligence, spatial ability, phonological ability, processing speed.	Should find a deficit in the ANS (number sense) and no deficit in domain-general abilities.	Should find a deficit in numerosity coding, and no deficit in the ANS or domain-general abilities are expected.	Should find intact non-symbolic processing and domain-general abilities and a deficit in symbolic processing.	Should display more than one deficit.

Empirical studies

Study I

Background and aim

As previously mentioned, MLD is a learning disability that has been studied less extensively than reading disability. There are a number of theories and hypotheses regarding the origin(s) of MLD in children. The most investigated and theoretically anchored hypotheses are the domain-general hypothesis, the number sense deficit, the object tracking hypothesis, the access deficit and the numerosity-coding deficit. The domain-general deficit hypothesis states that MLD (primarily) originates from a deficit in working memory, executive function or the memory systems (Geary, 2004). The number sense deficit states that the foundational deficit of MLD is in the innate system of representing numbers, called the approximate number system (ANS; Wilson & Dehaene, 2007). A second innate system that can represent small numbers is called object tracking system (OTS) (Piazza, 2010), and this system is a potential candidate for a core deficit in MLD (Wilson & Dehaene, 2007). In addition to the two previous systems, the ANS and OTS, Butterworth (2010) has suggested a third system, called the numerosity coding system. This system is believed to represent exact numbers but is not confined to small numbers ≤ four. Another hypothesis is the access deficit hypothesis, which states that MLD is the result of a deficit in the connection between digits, counting words and the innate number representation systems (Rousselle & Noël, 2007). The last hypothesis states that the origin of MLD encompasses multiple deficits (Wilson & Dehaene, 2007). The aim of the first study was to test the above-mentioned hypotheses of the origin of MLD in a sample of 11-13-year-old children.

Method

A total of 63 children between 11-13 years old were tested for mathematical competence, cognitive function and number processing. The MLD group consisted of 20 children who scored 1.5 SD below the mean of the control group on the mathematical screening measure and required special instruction in their ordinary schooling.

Results and discussion

Regarding the domain general hypothesis, the MLD group differed from the children in the control group in the visual-matrix span task and the color-naming task. Regarding the OTS deficit and numerosity coding hypotheses, only support for a deficit in the OTSs of the MLD group was obtained. This support came from the slower performance of the MLD group in the subitizing task. However, no difference was detected in the enumeration task.

A number of results were supportive of the number sense (or ANS) deficit hypothesis; the MLD group exhibited a larger distance effect than the control group in the one-digit comparison task. Moreover, the results from the number line estimation task were in line with predictions made by the hypothesis; the estimations of the MLD group were less linear than those of the control group. However, no group effect was detected on the dot magnitude discrimination task, and the MLD group was slower to name one and two digits, which supports the access deficit hypothesis. Further support comes from the finding that the MLD group performed slower in the number magnitude comparison tasks. However, the difference in speed remained even after controlling for the symbolic system (naming digits). Moreover, the subitizing effect (see above) is difficult to interpret in the context of the access deficit hypothesis. Furthermore, the results from the number line estimation task are, to some extent, problematic because the estimations of the MLD group were less linear than those of the control group even after controlling for the number comparison task (i.e., controlling for the connection between digits and magnitude representations).

In summary, the results of study I are in line with the OTS deficit, the number sense deficit, and the domain-general deficit. Overall, these results are also in accordance with the hypothesis of multiple deficits.

Study II

Background and aim

Both domain-general and domain-specific abilities support the learning of mathematics and arithmetic. The domain-specific ability of early number knowledge is important for the early acquisition of mathematical skill (National Mathematics Advisory Panel, 2008). Number knowledge (as previously mentioned) can be divided into two abilities, primary and secondary biological ability (Geary, 1995). Examples of primary abilities are the ANS and the OTS, as they develop without deliberate practice. Secondary biological abilities do not develop without deliberate practice and include abilities such as the comprehension of counting words, symbolic numerical comparison and arithmetic operations. Number knowledge is an ability that emerges through the interplay between primary and secondary abilities as described in von Aster and Shalev's (2007) model. The development of number knowledge is also supported by domain-general abilities, including working memory, which can be important when children learn the counting words etc. Both abilities are important predictors of mathematical skills (Geary, 2011). Study II was conducted to investigate the effects of preschool number knowledge and verbal working memory on arithmetic ability both in preschool and in first grade**.** The following hypotheses were tested:

1) Preschool number knowledge and domain-general cognitive ability (i.e., verbal working memory) should independently affect preschool arithmetic calculation ability.

2) Number knowledge and verbal working memory during preschool should independently affect arithmetic ability in the first grade and the growth in arithmetic ability.

Method

The study was part of a large longitudinal project. An unselected sample of 315 children attending preschool was tested over two consecutive years. A test battery of 30 tasks was administered individually. However, the present study only reports on 12 of these tasks. Naming Arabic numerals, counting forward, counting backward and number line estimation served as indicators of number knowledge. Complex word repetition, segment subtraction and word fluency served as indicators of verbal working memory. The matrix reasoning task was used as a control for non-verbal intelligence. Simple verbal addition and subtraction tasks were used as indicators of arithmetic calculation ability. Structural equation modeling was used to test different models.

Results and discussion

Three models were tested (the last model is displayed in Figure 2.). All models showed good model fit (χ^2 value, $p > .05$). The first model tested the effects of verbal working memory and number knowledge on arithmetic ability in preschool while controlling for nonverbal intelligence. This model showed that both abilities uniquely affected arithmetic ability. The second model tested the effects the two abilities on arithmetic ability one year later in the first grade. Both number knowledge and verbal working memory affected arithmetic ability in grade 1. The third, and final, model also incorporated arithmetic ability in preschool, which made it possible to test the effect of growth in arithmetic ability from preschool to first grade. Only number knowledge had an effect on growth in arithmetic ability, verbal working memory only indirectly affected growth through number knowledge.

The results are in accordance with theoretical models, such as von Aster and Shalev's (2007) model. Study II highlights the importance of number knowledge ability in the development of early arithmetic ability. Although verbal working memory is an important ability, this ability was found to affect development only indirectly.

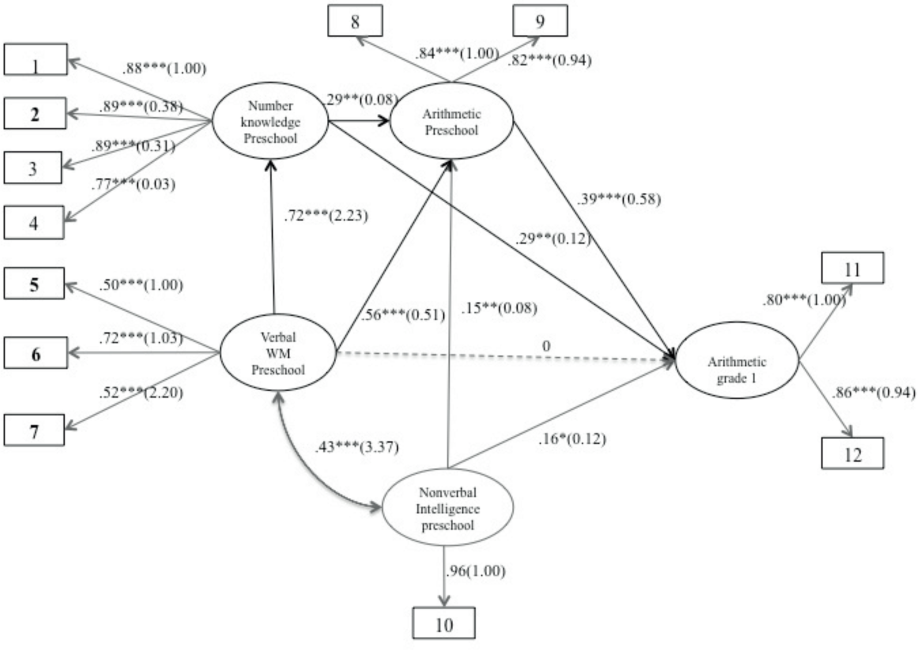

Figure 2. Final model C. 1. Naming Arabic numerals. 2. Counting forward. 3. Counting backward. 4. Number line estimation. 5. Complex word repetition. 6. Segment subtraction. 7. Word fluency. 8. Story problem addition. 9. Story problem subtraction. 10. Matrix reasoning. 11. Story problem addition grade 1. 12. Story problem subtraction grade 1. A test of the relationships between number knowledge, verbal working memory, nonverbal intelligence, arithmetic ability in preschool and arithmetic ability in grade 1. Completely standardized maximum likelihood parameter estimates and unstandardized values are in parentheses. Error terms were left out of the figure for visual clarity. *p < .05, **p < .01, ***p < .001.

Study III

Background and aim

Competing theories and hypotheses about the origin of MLD in children exist. Both domain-general and domain-specific abilities have been found to be important aspects in the development of arithmetic ability. Domain-general abilities, such as working memory, may be the source of MLD (Geary, 2004); however, it is also possible that some of the innate number representation systems, such as the ANS, are involved in MLD (Dehaene, 2011). MLD could also be due to the connection between symbols and the representational system (Rousselle & Noël, 2007). Few studies have used longitudinal approach's to categorize children as having MLD, and even fewer studies have also simultaneously investigated high achievers when studying the origin(s) of MLD. Similar to study I, the aim of study III was to test the following hypotheses regarding the origin of MLD: the domain-general deficit, number sense deficit, numerosity-coding deficit, access deficit and multiple deficits hypotheses. Additionally, study III incorporated a group of high achievers (HA). Another important aim of study III was the investigation of the development of MLD in the same sample of children. Are the children at risk of MLD and the children in the typical achiever (TA) group different regarding their development of both domain-specific and general abilities? A similar question was also investigated in relation to the children in the HA group.

Method

A sub-sample of the larger project used in study II was used in study III. Ninety-five children were divided into three groups: MLD (N = 13), TA (N = 57), and HA (N = 25). The group classification was based on scores on arithmetic tasks that were administered to children once in the first grade and again in the second grade. The criterion for inclusion in the MLD group was scores below the 15th percentile. The TA group was composed of children who scored between the 26-74th percentile, and the HA group was composed of children who scored above the 85th percentile at both time points.

Results and discussion

In the matrix reasoning task, segment subtraction task, and number line estimation task, the MLD group performed worse than the TA group, and the

HA group performed better than the TA group. The HA group performed better than the other groups on the digit matching and color naming tasks and showed a tendency towards better performance on the phonological fluency task. The MLD group performed worse than the TA and HA groups on the quantitative discrimination, digit comparison, and mental rotation task. The MLD group also displayed a more pronounced problem size effect.

To test which abilities in preschool predict future group membership (i.e., membership in the MLD, TA or HA groups), a multinomial logistic regression was calculated. The TA group was used as the reference category, and predictors representing domain-general abilities were the complex word repetition, digit matching and matrix reasoning tasks. Domain-specific abilities were represented by the number line estimation, digit comparison and calculation ability tasks. The only task that predicted MLD group compared to TA group membership was the digit comparison task. However, the calculation, number line estimation and complex word repetition tasks predicted HA group compared to TA group membership.

The developmental trajectories of the different groups were also examined. All three groups exhibited similar trajectories, but these trajectories occurred at different achievement levels (there was a main effect of group). The MLD group displayed the poorest performance followed by the TA group. The HA group showed the best performance on the matrix reasoning and segment subtraction task. For the digit comparison tasks (one and two digit) and number line estimation task, an interaction between time and group was detected. Thus, the developmental trajectories were somehow different across groups. Further analysis revealed that the HA group did not improves as much as the other two groups over time. Thus, the HA group seemed to have developed an efficient number knowledge faster and earlier than either the TA and MLD groups.

The results from study III indicate that children at risk of developing MLD in second grade have a deficit in the number sense (ANS) and are also impaired in spatial processing. The MLD group in second grade also had vulnerabilities related to other domain-general abilities, such as nonverbal intelligence and phonological ability, that may have enhanced their number sense deficits. Children with a chance of developing into high achievers were superior in terms of domain-general abilities that most likely support the development of more sophisticated mental number line (and thus number knowledge).

Discussion

Is it a single deficit that serves as the main condition for developing MLD or multiple deficits? Is it the same deficit over the course of development? The present results support the notion of multiple deficits as conditions for developing MLD. In both studies I and III, when studying children either with MLD or at risk of developing MLD, the results indicated multiple deficits. Some of the deficits were domain general, such as in visuo-spatial working memory and retrieval from long term memory. Others were domain specific, such as deficits in the OTS and the ANS. The notion of multiple deficits is also highlighted by the results from study II suggesting that both domain-general abilities and number knowledge support arithmetic ability. Verbal working memory seems to have a more indirect role via number knowledge. In study III, we found differences among all three groups on measures of domain-general abilities, phonological ability and nonverbal intelligence. A unique deficit for the MLD group in special ability was also found. Domain-specific deficits were present in the form of a deficit on measures of the ANS and perhaps in numerosity coding as well as in the more complex number knowledge ability. In sum, it is likely that children with MLD have multiple conditions that serve as a vulnerability for developing MLD. When taking a developmental perspective on MLD, it is obvious that many different conditions may or can result in MLD. *Figure* 3 shows a model anchored in the models of Geary (2013), Kaufman et al. (2011) and von Aster and Shalev (2007), incorporating as well the results from the present thesis.

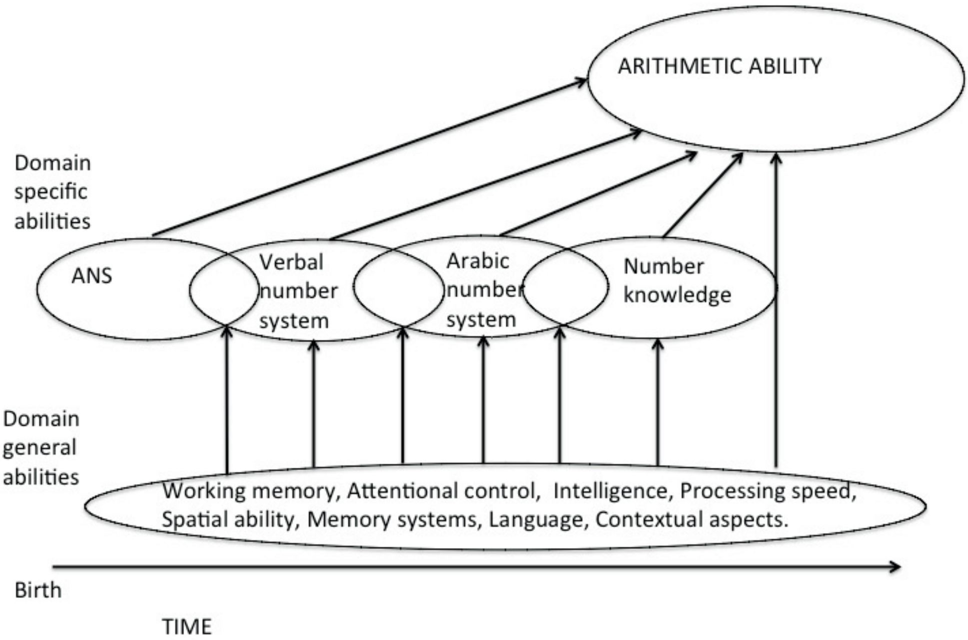

Figure 3. A theoretical model of the development of *number knowledge* and its relation to arithmetic ability.

The development of number knowledge starts with an innate system of number representation such as the ANS. Next, verbal numbers (counting words) are connected or mapped onto the ANS, and shortly thereafter, the digits are mapped onto the ANS. The formation of number knowledge takes place after the verbal and Arabic number systems have been mapped onto the ANS. With development, arithmetic ability becomes dependent on increasingly more complex number abilities, along with increasingly more complex domain-general abilities. *Figure 3* shows that number knowledge is interconnected with the three systems: Arabic numbers, verbal numbers and the ANS. Once the chain has been connected, it becomes increasingly more difficult to differentiate the subsystems from the whole (cf. Dehaene, 1992). Domain-general abilities do, however, support both number knowledge (as a whole) and arithmetic ability, with different types of functions depending on the task. Thus, for example, verbal calculation would be supported by verbal working memory, both directly and via number knowledge, due in part to the verbal nature of such a calculation task. Accordingly, the domain-general abilities are likely to be important when the different connections (e.g., ANS to verbal system) are made. Many different deficits could result in MLD. A deficit in number knowledge could develop from an intact ANS as a result of weak verbal working memory that does not provide support when the verbal and symbolic systems connect with the ANS. Each connection, indicated by arrows in *Figure* 3, could potentially dysfunction, resulting in low arithmetic ability, and, if sufficiently low, what would be defined as MLD. It is also possible that other connections could compensate for a dysfunctional connection, resulting in an arithmetic ability that does not cross the threshold for MLD. A child could possibly have a deficit in the ANS but compensate with a strong verbal working memory, resulting in a "good enough" arithmetic ability. It is also important to recognize that there are numerous other contextual factors, such as family environment, schooling, relations with others and so forth, that most likely support the development of number knowledge. In Geary´s terms (1995), number knowledge is an ability that should be classified as being a biological secondary ability, that is, it requires deliberate practice in order to develop. In sum, the present thesis could not find exclusive support for any of the main hypotheses, except for the multiple-deficits hypothesis. Based on the model in *Figure 3*, the notion of subgroups is plausible. For instance, according to Price and Ansari (2012), only children with a deficit in the ANS would be considered to have primary developmental dyscalculia. All other possible deficits in number knowledge, such as deficits in domain-general abilities, that result in low arithmetic ability would thus be classified as secondary developmental dyscalculia. All of the subgroups mentioned could potentially be identified, due to the many roads that could lead to MLD. In the next section, I will discuss the present results in accordance with each hypothesis.

Domain-general deficit

The present results from all three studies fit well with the conclusion that vulnerabilities in the domain-general abilities (e.g., working memory, spatial ability, phonological ability) serve as a partial condition for developing MLD, but they interact with domain-specific abilities (see study II; Geary, 2011; Raughubar et al., 2010). Thus, domain-general abilities are important but are unlikely to be the only source for developing MLD. (It is likely that some parts of the domain-specific abilities would need to interact with the vulnerability in the domain-general ability.) This conclusion is also in line with the results from study II and other empirical investigations that show the importance of both domain-general and domain-specific abilities (Geary, 2012). Previous research indicates that domain-general abilities have an indirect effect on arithmetic ability, thus serving as supportive structures in the development of domain-specific abilities, such as number knowledge (Cirino, 2011; Krajewski & Schneider 2009ab; Passolungi & Lanfranchi, 2012). It should be noted, however, that different outcome measures could indicate different impacts from domain-general abilities (Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010; Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, et al., 2010). The theoretical model displayed in *Figure 3* shows the supportive, sometimes direct, relation of domain-general abilities to arithmetic ability. The role that domain-general abilities (e.g., working memory) serve over the course of development in relation to MLD is likely to vary over time, depending on time and learning period (whether at an early or a later stage). The impact of a deficit in the domain-general abilities depends on several factors: how many of the connections are affected, which ability is deficient and to what extent. Again, consider all the connections from the domain-general level in the model (Figure 3) and imagine one or more of those connections being broken. The impact of such a deficit could be extensive. For example, an executive function deficit in the mapping between the ANS and the symbolic systems would result in difficulties in understanding the meaning of digits and uncertainty about the relation between the numbers. On a lesser scale, a deficit in processing speed in the direct connection to number knowledge results in slowness when using number knowledge skills but with intact comprehension.

Number sense deficit

Both studies I and III support the notion of a deficit in the ANS, as seems evident among children with MLD. In the MLD group, the ANS seemed fuzzy or imprecise, compared to their peers, resulting in more overlap between the number representations. This result is in line with previous research (Desoete, Ceulman, De Weerdt, & Pieters, 2012; Landerl, Fussenger, Moll & Willburger, 2009; Mazzocco, Feigenson & Halberda, 2011; Mejias, Mussolin, Rousselle, Grégoire & Noël, 2012; Mussolin, Mejias & Noël, 2010; Piazza et al, 2010; Price, Holloway, Räsänen, Vesterinen & Ansari, 2007). However, the combined results from all three studies suggest that a number sense deficit is probably not the only source for MLD. As previously mentioned, both studies I and III found support for domain-general deficits. The results of study II indicated that verbal working memory is important both for number knowledge and arithmetic. Looking at the model again (*Figure 3*), it is clear that a deficit in the ANS (an imprecise ANS) would be a foundational flaw, affecting all aspects of number knowledge development and directly affecting some aspects of arithmetic ability. Most of the effect, however, would be indirect through effects on different aspects of number knowledge ability (Wilson & Dehaene, 2007). This implies several possible ways to compensate for a deficit in the ANS during development. However, without some sort of compensation (e.g., an excellent verbal working memory), the deficit would likely have a severe impact on arithmetic ability. With regard to the notion that a deficit in the ANS will result in a very severe arithmetic disability (Price & Ansari, 2012; Wilson & Dehaene, 2007), such a strong and straight line between ANS deficit and arithmetic deficit is unlikely to be correct if the possibility of compensation is taken into account.

Numerosity coding deficit
The overall results are partially in favor of a deficit in numerosity coding. Both studies I and III found some evidence that exact number representations in both the subitizing range and above did not function as well in the MLD group compared to the TA group, consistent with the results of other studies (Desoete & Grégoire, 2006; Fischer et al., 2008; Schleifer & Landerl, 2011). As with the number sense deficit, the finding that the MLD groups in both studies I and III displayed domain-general deficits is problematic for the hypothesis. The numerosity coding hypothesis also predicts that MLD children should not display a deficit in non-symbolic approximate number comparison (Butterworth, 2010). However, direct evidence against this prediction is presented in study III.

How can the results be explained in terms of the model (see *Figure 3*) proposed in this thesis? A subitizing deficit could potentially reflect domain-general problems such as a deficit in working memory, as parallel individuation (or the OTS) has been suggested as a domain-general ability (Piazza, et al. 2011). Perhaps a domain-general deficit in the working memory system inhibits the formation of working memory models, such as parallel individuation. Instead, the ANS is engaged in the subitizing task, with results look like a deficit in the exact representation of numbers. In the counting range, all three groups (MLD, TA, HA) differed from each other in study III, possibly reflecting a poorer domain-general ability and poorer number knowledge ability in general (e.g., the verbal system), as the responses on the test were in the verbal format.

In sum, it is unlikely that numerosity coding is the single core deficit that underlies MLD, based on the results from the present thesis. The strongest argument against the hypothesis is the finding of domain-general deficits and ANS deficits.

The access deficit

The access deficit hypothesis has primarily been contrasted against the numerosity coding deficit (number module deficit) hypothesis (e.g., Rousselle & Noël, 2007; De Smedt & Gilmore, 2011). The strongest prediction from the access deficit hypothesis is that children with MLD should not display any deficits in non-symbolic approximate number comparison ability (no deficit in the ANS) together with the prediction that children with MLD should display a variety of problems in the number domain, such as a number sense deficit or numerosity coding deficit (Rousselle & Noël, 2007; Wilson & Dehaene, 2007). The support provided by the present results for the access deficit hypothesis is rather weak, due to direct evidence against intact non-symbolic number comparison in the MLD group (study III) and the more indirect evidence of ANS deficit (study I). Moreover, the domain-general deficits found in studies I and III are problematic for the access deficit hypothesis. Whereas the developmental model (see *Figure. 3*) allows for the possibility of a deficit in the connection between the ANS and the verbal/Arabic symbolic level, with resulting poorer number knowledge in children with MLD, the results of the current three studies suggest that a deficit could easily originate either from a deficit in the ANS (negatively affecting the connection) or from a deficit in domain-general abilities that support the connection (e.g., verbal working memory). As with the numerosity coding deficit hypothesis, the access deficit hypothesis cannot explain the data, and in fact, some of the data directly dispute the hypothesis.

Multiple deficits

A deficit in the innate number sense (ANS) along with deficits in domain-general abilities are, by definition, multiple deficits. The clearest result from the present thesis is that the MLD group suffers from multiple deficits, as both studies I and III showed both domain-general and domain-specific deficits. A possible explanation is a common neurological deficit, either in the area of the IPS, in the fronto-parietal networks or in the frontal area (Henik, Rubinsten & Ashkenazi, 2012; Rubinsten & Henik, 2009). Due to the developmental process (as displayed in *Figure* 3), it is possible that one deficit results in the development of another (new) deficit. A deficit in the working memory processes could thus result in a deficit in number knowledge due to a lack of support when the ANS, verbal and symbolic systems integrate in the development of number knowledge. Noël and Rousselle provide another possible explanation (2011), suggesting that children first form an exact number representation through enriched parallel individuation (Carey, 2009; 2011), which increases the precision of the ANS. If the exact representation does not develop typically, it is possible that the precision of the ANS would be affected. Either way, it is likely that deficits can affect each other and can leave lesser or larger developmental footprints depending on the presence or absence of other compensatory conditions, such as abilities or contextual factors. In sum, the multiple deficits hypothesis receives support from all three studies in the present thesis. More specifically, the multiple deficits consist of a domain-general deficit in working memory and/or spatial ability together with a domain-specific deficit in the ANS.

Limitations, practical issues and implications for education

The present thesis is foremost a research project that investigates theoretical questions regarding the etiology of MLD. However, some limitations and potential practical implications need to be addressed. One obvious limitation with the present thesis is the use of different cut-off criteria in studies I and III, as this presents a risk of studying slightly different populations. However, the use of different classifications (e.g., belonging to an MLD group or not) is the rule and not the exception in the research field of MLD. Overall, the recommendation is to use a longitudinal approach when classifying children as having MLD, as study III highlights the variability that exists in arithmetic achievement over time. This is especially so during the early years of schooling when children's arithmetic ability can change dramatically over a short period of time. The risk of including false positives when using a single measurement is something that needs to be recognized in future studies. The same variability is also an argument for using the term "children at risk of developing MLD" (as in study III), instead of classifying them as having the disability. We should use our terminology with caution, as we risk labeling a child too early in development. I suggest using "child at risk of developing MLD" rather than "child with MLD" during the child's first two years of school. The question of using diagnostic labels for children in the educational setting has been debated, with some proponents stating it is a common way of speaking in our culture (Gillum, 2012). However, labels can have their drawbacks, sometimes doing more harm than good (Lauchlan & Boyle, 2007). I suggest acting with care in relation to using such labels in educational settings, as it first needs to be proven that they are beneficial. To connect special resources to a label like MLD would probably be a mistake in younger children. The present thesis has shown that MLD is likely to have a multifaceted origin. The consequence must be the realization of the relatively small gain in classifying children as having or not having MLD.

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