Robust finite-frequency H2 analysis

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Abstract
Finite-frequency $\mathcal{H}_2$ analysis is relevant to a number of problems in which a priori information is available on the frequency domain of interest. This paper addresses the problem of analyzing robust finite-frequency $\mathcal{H}_2$ performance of systems with structured uncertainties. An upper bound on this measure is provided by exploiting convex optimization tools for robustness analysis and the notion of finite-frequency Gramians. An application to a comfort analysis problem for an aircraft aeroelastic model is presented.

Keywords: Optimization, Control theory
Robust finite-frequency $H_2$ analysis

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Abstract—Finite-frequency $\mathcal{H}_2$ analysis is relevant to a number of problems in which a priori information is available on the frequency domain of interest. This paper addresses the problem of analyzing robust finite-frequency $\mathcal{H}_2$ performance of systems with structured uncertainties. An upper bound on this measure is provided by exploiting convex optimization tools for robustness analysis and the notion of finite-frequency Gramians. An application to a comfort analysis problem for an aircraft aeroelastic model is presented.

I. INTRODUCTION

Research on robust $\mathcal{H}_2$ analysis has been carried out for roughly forty years. A survey of the field is given in [1]. Algorithms based on Riccati equations or linear matrix inequalities are presented in for instance [2], [3], [4], [5], [6], [7], [8] and [9]. However, in certain applications it may not be so informative to compute an $\mathcal{H}_2$ measure over the entire frequency range. If the disturbances and all other input signals have a limited frequency range, it is in many cases more relevant to consider a measure computed over this limited range only.

In [10] new measures for stability and performance analysis of aircrafts were suggested. One of the measures was for turbulence response and is $\mathcal{H}_2$-like and formulated over a limited frequency range. Measures like this are used in the aerospace industry today. Other areas where robust finite-frequency $\mathcal{H}_2$ analysis can be applied are structural dynamics, acoustics and colored noise disturbance rejection.

How to compute the $\mathcal{H}_2$ norm over a finite frequency interval, for a system without uncertainty, is described in [11]. A short review of the procedure is given in Section II-A. The key step is to compute the finite-frequency observability Gramian. This is accomplished by first computing the regular observability Gramian and then scale it by a system dependent matrix. Hence, when we consider robust $\mathcal{H}_2$ analysis over a finite frequency interval we do not only have to compute an estimate of the worst case observability Gramian but we also have to find a well defined system corresponding to this Gramian. Most of the methods referenced above, for computing the robust $\mathcal{H}_2$ norm, do not deliver such a system.

The contribution of this paper is to provide an upper bound on the robust finite-frequency $\mathcal{H}_2$ performance of systems with structured uncertainties, by combining the notion of finite-frequency Gramians with convex optimization tools commonly used in robust control.

The rest of the paper is organized as follows. In Section II the problem formulation and some background information is given. In Section III it is shown how to compute an upper bound on the finite-frequency $\mathcal{H}_2$ norm. Dynamic scaling can be introduced to tighten the upper bound. The procedure for doing so is presented in Section IV. In Section V, the proposed technique is validated by means of a numerical example and an application concerning comfort analysis for an aeroelastic aircraft model. Finally, conclusions are drawn and presented in Section VI.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the state-space system:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} c(t)$$

and denote its transfer function $G(s)$. The $\mathcal{H}_2$ norm of (1) is defined as

$$\|G(j\omega)\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Tr} \{G(j\nu)^*G(j\nu)\} d\nu = \int_0^\infty \text{Tr} \left\{ B^T e^{A^T t} C^T C e^{A t} B \right\} dt$$

$$= \text{Tr} \left\{ B^T W_o B \right\},$$

where $W_o$ is the observability Gramian of the system. To compute the observability Gramian we can solve the Lyapunov equation

$$A^T W_o + W_o A + C^T C = 0.$$

Using Parseval’s identity we can also express the observability Gramian as:

$$W_o = \frac{1}{2\pi} \int_{-\infty}^{\infty} H^*(\nu) C^T C H(\nu) d\nu,$$

where $H(\nu) = (j\nu I - A)^{-1}$. In this paper, we are interested in the finite-frequency $\mathcal{H}_2$ norm, defined as

$$\|G(j\omega)\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Tr} \{G(j\nu)^*G(j\nu)\} d\nu.$$

A. Finite-frequency observability Gramian

To study the robust finite-frequency $\mathcal{H}_2$ problem, it is necessary to introduce the notion of finite-frequency Gramians [11]. The finite-frequency observability Gramian in the
The next lemma provides a way to compute $W_o(\omega)$ in (6), in terms of the regular observability Gramian $W_o$.

**Lemma 1:** The finite-frequency observability Gramian can be computed as

$$W_o(\omega) = L(\omega)^*W_o + W_oL(\omega),$$

where $W_o$ is the regular observability Gramian computed as in (3) and

$$L(\omega) = \frac{1}{2\pi} \int_{-\omega}^{\omega} H(\nu)C^TCH(\nu)d\nu.$$ (7)

The aim of this paper is to compute the robust finite-frequency $\mathcal{H}_2$ norm of system (9), i.e.

$$\sup_{\Delta \in \Delta} \| S(M;\Delta) \|^2_{2,\omega} = \frac{1}{2\pi} \sup_{\Delta \in \Delta} \text{Tr} \int_{-\omega}^{\omega} S(M;\Delta)^*S(M;\Delta)d\nu.$$ (13)

By exploiting the results in [7]-[8], it is possible to compute an upper bound on the regular robust $\mathcal{H}_2$ norm, i.e.

$$\sup_{\Delta \in \Delta} \| S(M;\Delta) \|^2_{2,\omega} = \frac{1}{2\pi} \sup_{\Delta \in \Delta} \text{Tr} \int_{-\omega}^{\omega} S(M;\Delta)^*S(M;\Delta)d\nu.$$ (13)

Let us by $\mathcal{X}$ denote the set of all Hermitian block-diagonal operators which commute with $\Delta \in \Delta$. These operators are often called scaling matrices.

**Lemma 2:** The system $S(M, \Delta)$ has robust $\mathcal{H}_2$ norm less than $\gamma^2$ if there exist a matrix $X \in \mathcal{X}$, Hermitian matrices $P_-, P_+ \in \mathbb{R}^{n \times n}$ and a Hermitian matrix $W_o \in \mathbb{R}^{n \times n}$, satisfying

$$\begin{bmatrix}
P_- & 0 & X > 0, \\
A P_- + A^T P_+ & B_q X B_q^T & P_- C^T + B_q X D^T \\
C P_- + D X B_q^T & D X D^T - \begin{bmatrix} X & 0 \\
0 & I \end{bmatrix} & < 0,
\end{bmatrix}$$

$$W_o \begin{bmatrix} I & P_+ \end{bmatrix} > 0,$$

$$\text{Tr} \left\{ B_o^T W_o B_w \right\} < \gamma^2.$$ (14)

**Proof:** See [7]-[8].

Problem (14) is a SemiDefinite Program (SDP) and can be solved by applying standard convex optimization tools [5].

The condition provided by Lemma 2 is in general conservative, since it guarantees robust $\mathcal{H}_2$ performance for complex nonlinear time-varying uncertainties [8], while we only consider real linear time-invariant ones. If the conditions of Lemma 2 are satisfied, then it is also possible to find a state-space representation of a system given by a spectral factorization related to the solution of (14). The procedure is described in the following Lemma.

**Lemma 3:** Let $P_+, P_-, X, W_o$, be the solution of (14). Define $\hat{M} = \begin{bmatrix} \hat{M}_1 & \hat{M}_2 \end{bmatrix}$ such that

$$\begin{bmatrix}
\hat{M}_1 \\
\hat{M}_2
\end{bmatrix} = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix} = \begin{bmatrix}
A & B_q \\
C_p & D_p & 0
\end{bmatrix}$$

$$\begin{bmatrix}
B_q \\
D_p & 0
\end{bmatrix},$$

$$\begin{bmatrix}
0 \\
D_q & 0
\end{bmatrix}.$$ (11)

For a simplier notation we drop the dependence on $\omega$. Then, the transfer function matrix from $w$ to $z$ is given by the upper LFT [12]

$$S(M;\Delta) = M_{22} + M_{21}(I - M_{11}\Delta)^{-1}M_{12}.$$ (12)

$$\hat{M}_1 = X^{-\frac{1}{2}} M_1 X^{-\frac{1}{2}}, \quad \hat{M}_2 = X^{\frac{1}{2}} M_2 X^{-\frac{1}{2}}.$$ (15)

and there exists a spectral factor $N \in \mathcal{R} \mathcal{H}_\infty, N^{-1} \in \mathcal{R} \mathcal{H}_\infty$, such that $I - \hat{M}_1 \hat{M}_1^* = NN^*$, and $\|N^{-1}M_2\|^2_2 < \gamma^2$. 

$$\| \hat{M}_1 \|_{\infty} = ||X^{-\frac{1}{2}} M_1 X^{-\frac{1}{2}}||_{\infty} < 1.$$ (16)
A state-space realization of $N^{-1}\hat{M}_2$ is given by:

$$N^{-1}\hat{M}_2 = \begin{bmatrix} A + (P_-C^T + B_qXD^T)R^{-1}C & B_w' \\ R^{-1}C & 0 \end{bmatrix}^{-1},$$

where

$$R = \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} - DXD^T.$$

Moreover, $W_o$ is the observability Gramian of $N^{-1}\hat{M}_2$. 

**Proof:** See [7]-[8].

### III. AN UPPER BOUND ON THE ROBUST FINITE-FREQUENCY $H_\infty$ NORM

The results from Lemma 2 and Lemma 3 allow us to derive an upper bound on the frequency gains for all frequencies $\omega$ and for all uncertainties $\Delta \in \Delta$. This upper bound can be used to obtain an estimate of the robust finite-frequency $H_\infty$ norm. In the next theorem we present how to compute this estimate, based on the results above.

**Theorem 3.1:** Let $P_\pm, P_- X, W_o$ be the solution of (14). Then an upper bound on the robust finite-frequency $H_\infty$ norm is given by:

$$\sup_{\Delta \in \Delta} \|S(M;\Delta)\|_{\infty,\omega} \leq \text{Tr} \left\{ B_o^T(W_o \mathcal{L}(\omega) + \mathcal{L}(\omega)W_o)B_w \right\},$$

where

$$\mathcal{L}(\omega) = \frac{j}{2\pi} \ln \left[ (A + j\omega I)(A - j\omega I)^{-1} \right],$$

and $R$ as in (18).

**Proof:**

Let $P_+, P_- X, W_o$ be the solution of (14) and set

$$Y = (N^{-1}\hat{M}_2)^*(N^{-1}\hat{M}_2),$$

where $N$ and $\hat{M}_2$ are defined according to Lemma 3. By means of simple algebraic manipulations we get

$$Y = (N^{-1}\hat{M}_2)^*(N^{-1}\hat{M}_2) = \hat{M}_2^* \left( I - \hat{M}_1^*\hat{M}_1 \right)^{-1} \hat{M}_2.$$  

From (23) and (16), one has that:

$$\begin{bmatrix} I - \hat{M}_1^*\hat{M}_1 > 0 \\ \hat{M}_2^*\hat{M}_2 - Y + \hat{M}_2^*\hat{M}_1 \left( I - \hat{M}_1^*\hat{M}_1 \right)^{-1} \hat{M}_1^*\hat{M}_2 = 0. \end{bmatrix}$$

Let us define

$$T = \begin{bmatrix} I - \left( I - \hat{M}_1^*\hat{M}_1 \right)^{-1} \hat{M}_1^*\hat{M}_2 \\ 0 \\ I \end{bmatrix}.$$

By exploiting (24), one gets

$$T^* \begin{bmatrix} \hat{M}_1^*\hat{M}_1 - I \\ \hat{M}_2^*\hat{M}_1 \\ \hat{M}_2^*\hat{M}_2 - Y \end{bmatrix} T = \begin{bmatrix} \hat{M}_1^*\hat{M}_1 - I \\ \hat{M}_2^*\hat{M}_1 \\ 0 \end{bmatrix} \leq 0.$$
where the upper left block of (33) can be expressed as
\[
\begin{bmatrix}
X(\omega) & 0 \\
0 & I
\end{bmatrix} - M_1(j\omega)X(j\omega)M_1(j\omega)^* \\
= \left\{ \begin{bmatrix}
\Psi & 0 \\
0 & I
\end{bmatrix} \right\}
\begin{bmatrix}
X(\omega) & 0 \\
0 & I
\end{bmatrix}
\left\{ \begin{bmatrix}
M_{11} & 0 \\
0 & M_{21}
\end{bmatrix} \right\} X
\left\{ \begin{bmatrix}
M_{11} & 0 \\
0 & M_{21}
\end{bmatrix} \right\}^*.
\]
By introducing the transfer function matrix
\[
\hat{C}(sI - \hat{A})^{-1}\hat{B}_q + \hat{D} = \left( \begin{bmatrix}
M_{11} & 0 \\
0 & M_{21}
\end{bmatrix} \right)
\] (35)
and setting \( \mathcal{Y} = [0 \ I]^T \), (34) can be reformulated as
\[
\left\{ \begin{bmatrix}
\hat{C}(sI - \hat{A})^{-1} I \\
\hat{B}_q \\
\hat{D}
\end{bmatrix} \right\}
\begin{bmatrix}
-X \ 0 \ 0 \\
0 \ X \ 0 \\
0 \ 0 \ I
\end{bmatrix}
\left\{ \begin{bmatrix}
\hat{B}_q \\
I \\
0
\end{bmatrix} \right\}^*.
\] (36)
Notice that, from (11) and (32), the matrices of the realization (35) can be expressed as
\[
\hat{A} = \begin{bmatrix}
A & B_qC_\nu & 0 \\
0 & A_\nu & 0 \\
0 & 0 & A_\nu
\end{bmatrix},
\hat{B}_q = \begin{bmatrix}
0 & B_q & 0 \\
I & 0 & 0 \\
0 & I & 0
\end{bmatrix},
\hat{C} = \begin{bmatrix}
C & DC_\nu & C_0 \\
0 & D & 0
\end{bmatrix},
\hat{D} = \begin{bmatrix}
0 & D & 0 \\
0 & I & 0
\end{bmatrix},
\] (37)
where \( \hat{A} \in \mathbb{R}^{\tilde{n}_s \times \tilde{n}_s}, \hat{B}_q \in \mathbb{R}^{\tilde{n}_s \times \tilde{d}}, \hat{C} \in \mathbb{R}^{(p+d) \times \tilde{n}_s}, \hat{D} \in \mathbb{R}^{(p+d) \times \tilde{d}}, \tilde{n}_s = 2n_{\nu} + n, \) and \( \tilde{d} = 2n_{\nu} + 2d. \)
By introducing the affine function of \( X \)
\[
\Pi(X) = \begin{bmatrix}
\hat{B}_q \\
\hat{D} \\
\mathcal{Y}
\end{bmatrix}
\begin{bmatrix}
-X & 0 & 0 \\
0 & X & 0 \\
0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
\hat{B}_q \\
I \\
0
\end{bmatrix}
\begin{bmatrix}
\hat{B}_q \\
I \\
0
\end{bmatrix}^T
\]
\[
= \begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{21} & \Pi_{22}
\end{bmatrix},
\] (38)
with \( \Pi_{11} \in \mathbb{R}^{\tilde{n}_s \times \tilde{n}_s}, \Pi_{12} \in \mathbb{R}^{\tilde{n}_s \times (d+p)}, \) and \( \Pi_{22} \in \mathbb{R}^{(d+p) \times (d+p)}, \) the following lemma can be stated.

**Lemma 4:** The system \( S(M, \Delta) \) has robust \( H_2 \) norm less than \( \gamma^2 \) if there exist a matrix \( X \in \mathbb{R}^{(n_{\nu} + d) \times (n_{\nu} + d)} \) such that \( X(s) \in \mathcal{X}, \) Hermitian matrices \( P_-, P_+ \in \mathbb{R}^{\tilde{n}_s \times \tilde{n}_s}, Q \in \mathbb{R}^{n_{\nu_2} \times n_{\nu_2}}, \) and a Hermitian matrix \( \tilde{W}_\alpha \in \mathbb{R}^{\tilde{n}_s \times \tilde{n}_s}, \) satisfying
\[
\begin{cases}
P_- > 0, & Q > 0, \\
A_\nu Q + QA_\nu^T & QC_\nu^T \\
C_\nu Q & 0
\end{cases} < X < 0,
\]
\[
\begin{cases}
\hat{A}P_- + P_- \hat{A}^T & P_- \hat{C}^T \\
\hat{C}P_- & -\Pi(X) < 0,
\end{cases}
\]
\[
\begin{cases}
\hat{A}P_+ + P_+ \hat{A}^T & P_+ \hat{C}^T \\
\hat{C}P_+ & -\Pi(X) < 0,
\end{cases}
\]
\[
\begin{cases}
\tilde{W}_\alpha & I \\
P_+ - P_- & > 0,
\end{cases}
\]
\[
\text{Tr}\left\{ \begin{bmatrix}
B_w & 0 \\
0 & \tilde{W}_\alpha
\end{bmatrix} \begin{bmatrix}
B_w \\
0
\end{bmatrix} \right\} < \gamma^2.
\] (39)

**Proof:** See [7].

By following the same approach adopted in Section III for constant scalings, it is possible to introduce a suitable system which bounds the \( H_2 \) performance of the considered LFR (9)-(10) from above. Given the solution of the SDP (39), the bounding system has a state-space realization
\[
\tilde{N}^{-1} \tilde{M}_2 = \begin{bmatrix}
\tilde{A} - (\Pi_{112} - P_- \hat{C}^T)\Pi_{22}^{-1} \hat{C} \\
\Pi_{22}^{-1/2} C
\end{bmatrix}
\begin{bmatrix}
B_w \\
0
\end{bmatrix},
\] (40)
and its observability Gramian is given by \( \tilde{W}_\alpha \). This allows one to formulate the extension of Theorem 3.1 to the case of dynamic scalings.

**Theorem 4.1:** Let \( P_+, P_- \), \( X, \tilde{W}_\alpha \), be the solution of (39). Then an upper bound on the robust finite-frequency \( H_2 \) norm is given by:
\[
\sup_{\Delta \in \mathcal{D}} \|S(M; \Delta)\|_{L_2, \omega}^2 \leq \text{Tr} \left\{ \begin{bmatrix}
B_w \\
0
\end{bmatrix}^T \left( \tilde{W}_\alpha \hat{\omega} \hat{\omega}^* + \hat{\omega} \hat{\omega}^* \tilde{W}_\alpha \right) \begin{bmatrix}
B_w \\
0
\end{bmatrix} \right\},
\] (41)
where
\[
\hat{\omega} = \frac{j}{2\pi} \ln \left( (\hat{A} + j\hat{\omega}I)(\hat{A} - j\hat{\omega}I)^{-1} \right),
\] (42)
and
\[
\tilde{A} = \hat{A} - (\Pi_{112} - P_- \hat{C}^T)\Pi_{22}^{-1} \hat{C}.
\] (43)

**V. NUMERICAL EXAMPLES**

In this section we discuss the application of the presented techniques to two case studies: an academic example, for which the actual robust finite-frequency \( H_2 \) norm is known, and an example derived from the clearance of flight control laws of civil aircraft. All SDPs have been solved by SDPT3 [14] under YALMIP [15].

**Example 5.1:** Consider system (9), with
\[
A = \begin{bmatrix}
-2.5 & 0.5 & 0 & -50 & 0 \\
0 & -1 & 0.5 & 0 & 0 \\
0 & -0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & -100 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
B_0 = \begin{bmatrix}
0.25 & -0.5 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
D = \begin{bmatrix}
0 \\
1 & 0 \\
0 \\
0 \\
0
\end{bmatrix},
\] (44)
\( \Delta(\delta) = \delta I_2 \) and \(-1 \leq \delta \leq 1 \). This system is known to have robust \( H_2 \) norm equal to \( \gamma^2 = 1.5311 \), attained for \( \delta = 0.25 \).

Figure 1 reports the gain plots of the system for different values of the uncertain parameter \( \delta \), while Figure 2 shows the finite-frequency \( H_2 \) norm obtained by numerical integration for different values of \( \omega \) and \( \delta \). It can be observed that the contribution to the \( H_2 \) norm from the high-frequency peak at \( 100 \text{ rad/s} \) is not negligible. Let us assume that we want to compute the robust finite-frequency \( H_2 \) norm of the system.
with $\bar{\omega} = 50 \text{ rad/s}$, in order to skip the contribution of the high-frequency peak. It can be checked that the worst-case finite-frequency $H_2$ norm is still attained for $\delta = 0.25$ and takes the value

$$\sup_{\Delta \in \Delta} \| S(M, \Delta) \|_{2,50}^2 = 0.8919$$

In the analysis the dynamic scaling $\Psi(s)$ in (32) has been chosen as

$$\Psi(s) = \left[ \frac{(s-p)^{n_p-1}}{(s+p)} I_2 \quad \frac{(s-p)^{n_p-2}}{(s+p)} I_2 \quad \cdots \quad \frac{1}{(s+p)} I_2 \right]$$

with $p = 150$.

Figure 3 reports the true value of the robust finite-frequency $H_2$ norm (thick line) and the upper bounds returned by Theorem 4.1 for different degrees $n_\psi$ of the dynamic scaling (45).

An alternative approach for approximating the finite-frequency $H_2$ norm is to low-pass filter the system output.

In this analysis, filters have been considered of the form

$$F(s) = \frac{\hat{p}^{n_p}}{(s + \hat{p})^{n_p}}$$

where $\hat{p}$ is devised in such a way that the gain at $\omega \leq \bar{\omega} = 50 \text{ rad/s}$ is decreased by less than 1%. In Table I the results obtained by the application of Lemma 4 on the filtered system are reported, while Table II present the results obtained by the application of Theorem 4.1 on the non filtered system.

<table>
<thead>
<tr>
<th>$n_\psi$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$|G|_{2,50}^2$</td>
<td>1.2634</td>
<td>1.2233</td>
<td>1.1953</td>
<td>1.1934</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE II Example 5.1: finite-frequency robust $H_2$ norm returned by Theorem 4.1.
the level of fullness of the fuel tanks and it is normalized in the range \([-1, 1]\). The resulting overall uncertain system is modeled as an LFR (9)-(10) with \(n = 21\) states and \(\Delta\) block of size \(d = 14\).

The considered model has been derived in the frequency range between 0 and 15 rad/s and has no physical meaning outside this range. Hence, the comfort criterion boils down to robust finite-frequency \(H_2\) computation for the considered LFR with \(\bar{\omega} = 15\) rad/s. Figure 4 reports the gain plot of \(\|S(M, \Delta)\|\) for different values of the parameter \(\delta\).

Figure 5 shows the values of the robust finite-frequency \(H_2\) norm obtained by applying the technique of Theorem 3.1 with constant scalings in 100 “portions” of the uncertainty interval \([-1, 1]\) (partitioning the uncertainty interval allows one to reduce the conservatism). The asterisks in Figure 5 represent the values of the \(H_2\) norm obtained by numerical integration, for fixed values of the uncertain parameter \(\delta\).

VI. CONCLUSIONS

The paper has proposed a technique for the computation of an upper bound to the robust finite-frequency \(H_2\) performance of systems affected by structured uncertainties. The main result is formulated in terms of a convex optimization problem and the use of dynamic scaling matrices is exploited in order to reduce the conservatism of the bound. The present work has been motivated by the evaluation of a comfort criterion for an aeroelastic model of an aircraft, but the same technique can be exploited in a wide variety of applications in which a priori information is available on the frequency spectrum of the input signals. Future research will concern the investigation of suitable criteria for choosing the basis functions for the dynamic scaling matrices and the study of efficient strategies for partitioning the parametric uncertainty domain, in order to achieve a good trade-off between conservatism and computational burden.

REFERENCES