Frequency-Domain Identification of Continuous-Time Output Error Models Part I - Uniformly Sampled Data and Frequency Function Estimation

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Abstract

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Keywords: System Identification

Frequency-Domain Identification of Continuous-Time Output Error Models

Part I - Uniformly Sampled Data

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Abstract

This paper treats identification of continuous-time output error (OE) models based on sampled data. The exact method for doing this is well known both for data given in the time and frequency domains. This approach becomes somewhat complex, especially for non-uniformly sampled data. We study various ways to approximate the exact method for reasonably fast sampling. While an objective is to gain insights into the non-uniform sampling case, this paper only gives explicit results for uniform sampling. *Copyright ©2005 IFAC*

Key words: Continuous-time systems; Parameter estimation; Continuous-time OE

1 Introduction

In this contribution we shall discuss identification of possibly grey-box structured linear continuous time models from discrete-time measurements of inputs and outputs. This as such is a well known problem and discussed, e.g. in [2]. Several techniques for identification of continuous time models are also discussed in, among many references, [7], [11], [4].

The “optimal” solution is well known as a Maximum-likelihood (ML) formulation. It consist of computing the Kalman filter predictions of the output at the sampling instants by sampling the continuous time model over the sampling instants. These predictions are functions of the parameters in the continuous time model and by minimizing the sum of squared prediction errors with respect to the parameters, the Maximum likelihood estimate is obtained in case of Gaussian disturbances. For equidistantly sampled data, this method is also implemented in the SYSTEM IDENTIFICATION TOOLBOX, [3].

No method can be better, in theory, asymptotically as the number of data tends to infinity, than this maximum likelihood method. However, it may encounter numerical problems at fast sampling, and it may be computationally demanding for irregularly sampled data.

We shall therefore here investigate some approximations based on frequency domain data that may be useful alternatives to the basic ML method. For relevant references on frequency domain identification see, e.g. [6]. While an important objective for us is to gain insights into the case of irregular sampling we shall here concentrate on equidistant sampling to bring out the essential issues.

A straightforward method for frequency domain identification of a continuous-time input output model is the following. First, equidistantly sampled input and output data $\{u(kT_s), y(kT_s)\}_{k=1}^{N_t}$ are acquired. Then, the discrete-time Fourier transforms of the sampled data is computed as follows

$$\begin{cases} U(e^{i\omega_n T_s}) = T_s \sum_{k=1}^{N_t} u(kT_s) e^{i\omega_n T_s}, \omega_n = \frac{2\pi}{T_s} n, \\ Y(e^{i\omega_n T_s}) = T_s \sum_{k=1}^{N_t} y(kT_s) e^{i\omega_n T_s}, n = 0, \dots, \lfloor \frac{N_t-1}{2} \rfloor. \end{cases}$$

After that, the parameters would be identified by minimizing the sum of the square of the difference of the

measured and expected frequency response as follows [5]

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{N_{\omega}} \sum_{k=1}^{N_{\omega}} \left| Y(e^{i\omega_k T_s}) - \hat{Y}(e^{i\omega_k T_s}, \theta) \right|^2 \quad (1)$$

$$\hat{Y}(e^{i\omega_k T_s}, \theta) = G_d(e^{i\omega_k T_s}, \theta) U(e^{i\omega_k T_s}), \quad k = 1..N_{\omega} \quad (2)$$

where N_{ω} represented the number of frequency components. The pivotal element of this construction is the discrete-time transfer function $G_d(z)$ which governs the frequency response $G_d(e^{i\omega T_s})$ of the system (therefore relating the frequency content of the sampled input to that of the sampled output). In some cases, this so called *pulse transfer function* might be problematic. As the rate of sampling increase, the relationship between the continuous- and discrete-time system can become more or less ill-conditioned. Mainly, this is due to the gathering of the poles of the discrete-time system around the value 1 in the complex plane. This will produce numerical difficulties while mapping back to the continuous-time parameters. We would therefore like to investigate robust alternatives to the exact $G_d(e^{i\omega T_s})$, which can circumvent such a problem by using the continuous-time frequency response $G_c(i\omega)$ directly. Another, maybe more important reason for studying such approximations is that they will provide insight into how one can deal with non-uniformly sampled data in frequency domain identification.

2 Outline

Most of the material in this paper is based on the properties of the discrete-time transfer function $G_d(z)$. This entity, which will be introduced in Section 3, is also called the *pulse transfer function*, since it is the discrete-time transfer function for the magnitudes of individual pulses in the input pulse train.

The relationship between the pulse and continuous-time transfer function can be represented in terms of an infinite summation. In Section 4, the effect of truncating this sum during system identification from uniformly sampled data is investigated. The problems associated with the most basic form of truncation, a straight forward replacement of the pulse transfer function by the continuous-time transfer function are also illustrated.

While the method of approximation in Section 4 is the truncation of a sum, the means in Section 5, 6 and 7 is that of approximating the transfer function $G_c(s)$. Here, two different methods are derived, where the last one can be interpreted as a form of estimate of the continuous-time Fourier transform of the output $y(t)$.

3 Pulse Transfer Function

Consider a discrete-time system G_d as in Figure 1, consisting of a zero-order hold circuit H at the input, a

continuous-time system G_c and a sampling circuit at the output. Then, the discrete-time *pulse transfer function*

Fig. 1. Input and sampling setup for a continuous-time system.

G_d can also be written as

$$G_d(e^{i\omega T_s}, \theta) = \left(\frac{1 - e^{-i\omega T_s}}{T_s} \right) \sum_{k=-\infty}^{\infty} \frac{G_c(i\omega + i\frac{2\pi}{T_s}k, \theta)}{i\omega + i\frac{2\pi}{T_s}k}. \quad (3)$$

This result can be found in the classical book by [10] on computer controlled systems. However, since the result is central for this paper, a brief summary of the thoughts leading to it will follow below.

First, define the *Dirac “comb” function* as

$$m(t) = \sum_{k=-\infty}^{\infty} \delta(t - T_s k)$$

where $\delta(t)$ is the *Dirac delta function*. Define the operator $\cdot^*$ such that for an arbitrary function v

$$v^*(t) = v(t)m(t) \quad (4)$$

and for the Laplace transform

$$V^*(s) = \int_0^{\infty} v^*(t) e^{-st} dt. \quad (5)$$

Then, one can denote

$$\begin{aligned} u^*(t) &= u(t)m(t) \\ y^*(t) &= y(t)m(t) \end{aligned}$$

as the sampled versions of the input and output and their Laplace transforms. If the input is assumed to be zero-order hold its sampled version will pass through a hold circuit with transfer function

$$H(s) = \frac{1 - e^{-sT_s}}{s}$$

before entering the system G . Let the combination of the hold circuit and the system be defined as

$$F(s) = G(s)H(s).$$

Then, the relationship between the sampled versions of the output is

$$y^*(t) = m(t)F(s)u^*(t)$$

where p is the differentiation operator. If f is the impulse response of the system F , the sampled versions of the input and output will be related as

$$y^*(t) = (f(t) * u^*(t))^* = m(t) \int_{-\infty}^{\infty} f(t - \tau)m(\tau)u(\tau)d\tau.$$

At the same time

$$\begin{aligned} (f^*(t) * u^*(t)) &= \int_{-\infty}^{\infty} m(t - \tau)f(t - \tau)m(\tau)u(\tau)d\tau \\ &= \int_{-\infty}^{\infty} m(t)f(t - \tau)m(\tau)u(\tau)d\tau \end{aligned}$$

since $m(t - kT_s) = m(t)$ and $m(\tau) \neq 0$ only for $\tau = kT_s$ [10]. Hence

$$y^*(t) = (f(t) * u^*(t))^* = (f^*(t) * u^*(t))$$

and an analogous expression holds for the Laplace transform

$$Y^*(s) = [F(s) * U^*(s)]^* = F^*(s)U^*(s).$$

Since the transforms of the discrete input and output are trivial

$$\begin{aligned} U^*(s) &= \int_0^{\infty} u(t)m(t)e^{-st}dt = \sum_{k=0}^{\infty} u(kT_s)e^{-skT_s} \\ Y^*(s) &= \int_0^{\infty} y(t)m(t)e^{-st}dt = \sum_{k=0}^{\infty} y(kT_s)e^{-skT_s} \end{aligned}$$

the only term of real interest is F^* for which

$$F^*(s) = \int_0^{\infty} f(t)m(t)e^{-st}dt \quad (6)$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} F(s + \frac{2\pi}{T_s}k) \quad (7)$$

because the product in the time domain can be expressed as a convolution in the frequency domain

$$\begin{aligned} F^*(s) &= \frac{1}{i2\pi} \int_{\gamma-i\infty}^{\gamma+i\infty} F(v)M(s-v)dv \\ &= \frac{1}{i2\pi} \int_{\gamma-i\infty}^{\gamma+i\infty} F(v) \frac{1}{1 - e^{-(s-v)T_s}} dv. \end{aligned}$$

Placing the path of integration between the poles of F and M and completing it by a large semi-circle enclosing the poles of M , residue calculus can be used under some mild conditions. The poles of M will be located at the zeros of $e^{T_s(s-v)} = 1$ which are

$$v = s + i\frac{2\pi}{T_s}k, \quad k = \dots, -1, 0, 1, \dots$$

The residues can be proved to be

$$\frac{1}{T_s} F(s + \frac{2\pi}{T_s}).$$

Since $F(s) = G_c(s)H(s)$ and

$$\begin{aligned} H(s + i\frac{2\pi}{T_s}k) &= \frac{1 - e^{-(s+i\frac{2\pi}{T_s}k)T_s}}{s + i\frac{2\pi}{T_s}k} \\ &= \frac{1 - e^{-sT_s}e^{i2\pi k}}{s + i\frac{2\pi}{T_s}k} \\ &= \frac{1 - e^{-sT_s}}{s + i\frac{2\pi}{T_s}k}. \end{aligned}$$

Hence the expression

$$G_d(e^{i\omega T_s}, \theta) = \left(\frac{1 - e^{-i\omega T_s}}{T_s} \right) \sum_{k=-\infty}^{\infty} \frac{G_c(i\omega + i\frac{2\pi}{T_s}k, \theta)}{i\omega + i\frac{2\pi}{T_s}k}$$

follows from (6). For a more detailed but similar discussion we refer the reader to Theorems 4.1 and 4.2 in the book by [10].

4 Truncating the Pulse Transfer Function

A drawback connected with using the formula in (3) for estimation is of course the infinite sum. Good approximations can however be achieved with a limited number of terms, when the continuous-time system is strictly proper. Then, for each individual frequency ω , the contribution from higher order terms of the sum will approach zero as $N_f \rightarrow \infty$. In Figure 2 the second-order continuous-time OE-model

$$G_c(s) = \frac{b_0s + b_1}{s^2 + a_1s + a_2} \quad (8)$$

with true parameters $a_1 = 2$, $a_2 = 3$, $b_0 = 1$ and $b_1 = 0.5$ has been estimated using the method described in (1). The duration of the data set was $T = 1000$ s, the sampling time was $T_s = 1$ s and a random excitation signal was used. During estimation, the discrete-time counterpart $G_d(e^{i\omega T_s})$ to the continuous-time frequency response function $G_c(i\omega)$ has been approximated as fol-

Fig. 2. Bode diagram comparing a noise free estimate (dashed) of the true system $G(s) = \frac{s+0.5}{s^2+2s+3}$ (solid) using the method in (1) when $N_f = 0$ in (9). The figure illustrates the frequency-domain bias which could occur if the higher order terms in (9) are not taken into account.

lows

$$G_d(e^{i\omega T_s}, \theta) = \left(\frac{1 - e^{-i\omega T_s}}{T_s} \right) \sum_{k=-N_f}^{N_f} \frac{G_c(i\omega + i\frac{2\pi k}{T_s}, \theta)}{i\omega + i\frac{2\pi k}{T_s}}. \quad (9)$$

In Figure 2, we have used $N_f = 0$ in (9), which means that we have assumed that the discrete- and continuous-time frequency response functions are almost equal. The figure illustrates the frequency-domain bias which could occur if the higher order terms in (9) are not taken into account. In Table 1 the parameter values for the same estimate as in Figure 2 are illustrated.

Table 1

Identified parameters for the transfer function $G_c(s) = \frac{s+0.5}{s^2+2s+3}$. Here $N_f = 0$ in (9) which means that one assumes that $G_d(e^{i\omega T_s}) = G_c(i\omega) \frac{H(i\omega)}{T_s}$. The sampling interval is $T_s = 1$ s and the parameters are biased.

Method	a_1	a_2	b_0	b_1
True System	2	3	1	0.5
Estimated ($N_f = 0$)	1.3605	4.7870	0.5266	1.4300

4.1 Numerical Illustration

In Figure 3 the parameters of the model in (8) have been estimated using $N_f = 0, \dots, 10$ in the expression for the frequency response of the pulse transfer function in (9). As can be seen, the parameter bias decreases as N_f increases. This is also illustrated numerically in Table 2. In Table 3 the effect of both the number N_f in (9) and

the sampling times T_s are shown. As can be seen the bias also decrease with the sampling rate.

Fig. 3. Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus the number N_f of higher order terms in (9), $T_s = 1$ s. The figure illustrates how the bias decreases as N_f increases.

Table 2

Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus the number N_f of higher order terms in (9), $T_s = 1$ s. The figure illustrates how the bias decreases as N_f increases.

N_f	$a_1 = 2$	$a_2 = 3$	$b_0 = 1$	$b_1 = 0.5$
0	1.3605	4.7870	0.5266	1.4300
1	1.7908	3.1375	0.9334	0.6408
2	1.9061	3.0930	0.9740	0.5906
3	1.9673	3.0706	1.0006	0.5640
4	1.9844	3.0028	1.0075	0.5333
5	1.9838	3.0063	1.0016	0.5289
6	1.9932	3.0207	1.0065	0.5280
7	1.9941	3.0324	1.0048	0.5307
8	1.9985	3.0263	1.0062	0.5252
9	2.0054	3.0263	1.0061	0.5259
10	2.0080	3.0189	1.0096	0.5161

It should be noticed that the system in (8) is among the most difficult cases when using (9). Quite many high order terms may be needed at high frequencies since the gain there,

$$|G_c(i\omega)| \approx \frac{b_0}{|\omega|}, \quad (10)$$

is decreasing slowly with increasing ω .

Table 3

Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus the number N_f of higher order terms in (9) and the sampling time T_s . The table illustrates how the bias decreases as N_f increases and T_s decreases.

T_s	0.02	0.1	0.5	1	1.5
$N_f = 1$					
1	1.0047	1.0003	1.0093	0.9263	0.3645
0.5	0.4957	0.5057	0.5416	0.6516	0.5723
2	2.0288	2.0019	1.9882	1.7926	1.0915
3	2.9968	3.0109	3.0391	3.1560	2.7964
$N_f = 2$					
1	1.0047	1.0000	1.0072	0.9791	0.5087
0.5	0.4957	0.5050	0.5245	0.5822	0.5119
2	2.0288	2.0018	1.9962	1.9114	1.3011
3	2.9968	3.0103	3.0215	3.0696	2.6539
$N_f = 5$					
1	1.0047	0.9997	1.0038	1.0036	0.6869
0.5	0.4957	0.5045	0.5109	0.5341	0.4856
2	2.0288	2.0017	1.9998	1.9831	1.5824
3	2.9966	3.0098	3.0087	3.0260	2.6876
$N_f = 10$					
1	1.0047	0.9995	1.0021	1.0049	0.8129
0.5	0.4957	0.5042	0.5056	0.5170	0.4878
2	2.0288	2.0016	2.0004	1.9969	1.7655
3	2.9967	3.0096	3.0042	3.0127	2.7935

5 Approximating the Continuous-Time System

In Section 4 the mode of approximation was the systematic truncation of the sum in (3). Another way to alter this relationship in a controlled manner would be to slightly change the properties of the continuous-time system G_c . Assume that G_c has the following transfer function

$$G_c(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}. \quad (11)$$

Then, for ω above the bandwidth of the system, we have

$$G_c(i\omega) \approx \frac{b_0}{(i\omega)^\ell} \quad (12)$$

where $\ell = n - m$ is the so called *pole excess* or *relative degree* of the system. This means that, at high frequencies or over small time intervals the system approximately behaves as a set of ℓ integrators in series with a gain b_0 .

The high frequency part of the transfer function is usually called the *roll-off* and the relative degree basically

tells us that the output $y(t)$ of the system at high sampling rates almost behaves as if it would be ℓ times continuously differentiable.

5.1 Sampling Zeros and Euler-Frobenius Polynomials

Assume that the system in (11) can be represented as

$$G_c(s) = K \frac{(s - z_1)(s - z_2) \dots (s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_n)} \quad (13)$$

where K is the static gain, $\{p_k\}_{k=1}^n$ are the continuous-time poles and $\{z_k\}_{k=1}^m$ are the continuous-time zeros. Then, we know that the relative degree of the discrete-time pulse transfer function will always be 1 and the continuous-time poles will move to the discrete-time poles $\{e^{p_k T_s}\}_{k=1}^n$ and the continuous-time zeros will move to the discrete-time zeros $\{e^{z_k T_s}\}_{k=1}^m$.

The remaining $\ell - 1 = n - m - 1$ zeros, which are called *sampling zeros* will approach the roots of the well known *Euler-Frobenius polynomials* $\Pi_\ell(z)$ [9,13]. In fact, if the system is composed of only ℓ integrators in series and a static gain b_0 such that

$$G_c(s) = \frac{b_0}{s^\ell} \quad (14)$$

then, the corresponding pulse transfer function would be

$$G_d(z) = b_0 \frac{\Pi_\ell(z)}{\ell! \left(\frac{z-1}{T_s}\right)^\ell}. \quad (15)$$

The Euler-Frobenius polynomials and their properties have a long pedigree dating back to the days of Euler in the 18th century, containing celebrated names such as Sobolev [8] and Froebenius [1]. In fact Frobenius [1] showed the interesting recursive relationship for these polynomials

$$\Pi_{\ell+1} = (1 + \ell z) \Pi_\ell(z) + z(1 - z) \frac{d}{dz} \Pi_\ell(z) \quad (16)$$

$$\Pi_0(z) = 1. \quad (17)$$

For practical purposes, the expansion

$$\Pi_\ell(z) = b_1^\ell z^{\ell-1} + b_2^\ell z^{\ell-2} + \dots + b_\ell^\ell, \quad \ell \geq 1 \quad (18)$$

where

$$b_k^\ell = \sum_{l=1}^k (-1)^{k-l} l^\ell \binom{\ell+1}{k-l}, \quad k = 1, \dots, \ell \quad (19)$$

is more suitable [9] and for $\ell = 1, \dots, 4$ the polynomials have the following appearance

$$\Pi_1(z) = 1 \quad (20)$$

$$\Pi_2(z) = z + 1 \quad (21)$$

$$\Pi_3(z) = z^2 + 4z + 1 \quad (22)$$

$$\Pi_4(z) = z^3 + 11z^2 + 11z + 1 \quad (23)$$

$$\Pi_5(z) = z^4 + 26z^3 + 66z^2 + 26z + 1. \quad (24)$$

The Euler-Frobenius polynomials appears during spline interpolation and will therefore be discussed in more depth in the companion paper mentioned before. From now to the end of the chapter, the relationships in (12) and (15) will be exploited together with the expression for the pulse transfer function G_d in (3) in order to produce interesting approximations.

6 Approximating the Roll-Off Behavior

In the previous section we used the summation formula (9) in order to estimate parameters. In this section, we will try to find a good approximation of the sum which is easier to calculate. For instance, define

$$F_{dc}^{(\ell)}(i\omega) = \frac{\Pi_\ell(e^{i\omega T_s})}{\ell! \left(\frac{e^{i\omega T_s} - 1}{T_s} \right)^\ell} - \frac{1}{(i\omega)^\ell} \frac{H(i\omega)}{T_s} \quad (25)$$

where $\Pi_\ell(z)$ are the Euler-Frobenius polynomials described in 5.1 and

$$H(i\omega) = \frac{1 - e^{-i\omega T_s}}{i\omega} \quad (26)$$

is the continuous-time transfer function of a zero order hold circuit. Assume, as before that the system G_c , defined in (8) has relative degree $\ell = n - m$. Then, we know that the high frequency gain will be

$$b_0 = \lim_{\omega \rightarrow \infty} G_c(i\omega)(i\omega)^\ell. \quad (27)$$

In the theorem below we use (27) and (25) in order to derive a new approximation of the discrete-time pulse transfer function $G_d(e^{i\omega T_s}, \theta)$

Theorem 1 Assume that $\ell \geq 1$, then as $T_s \rightarrow 0$

$$G_d(e^{i\omega T_s}, \theta) \rightarrow G_c(i\omega, \theta) \frac{H(i\omega)}{T_s} + b_0 F_{dc}^{(\ell)}(i\omega)$$

for each ω where $F_{dc}^{(\ell)}(i\omega)$ is defined as in (25) and b_0 as in (27). Also, H and $F_{dc}^{(\ell)}$ are independent of the parameters in θ .

PROOF. First of all we have

$$\frac{b_0}{\left(\frac{i\omega + i\frac{2\pi}{T_s}k}{T_s} \right)^\ell} \rightarrow 1$$

as $T_s \rightarrow 0$ for $k \neq 0$. The explanation is that only the high frequency part of the transfer function will be visible in this frequency range. This, in turn, means that the discrete-time pulse transfer function in (3) can be approximated with

$$G_d(e^{i\omega T_s}, \theta) \rightarrow G_c(i\omega, \theta) H(i\omega) + b_0 \sum_{k \neq 0} \frac{1 - e^{-i\omega T_s}}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}}$$

as $T_s \rightarrow 0$. Here H is defined as in (26). Then, from Lemma 3.2 in [12]

$$\frac{e^{i\omega T_s} \Pi_\ell(e^{i\omega T_s})}{\ell! \left(\frac{e^{i\omega T_s} - 1}{T_s} \right)^{\ell+1}} = \sum_{k=-\infty}^{\infty} \frac{1}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}}$$

and by multiplying by $(1 - e^{-i\omega T_s})/T_s$ and then subtracting the central term where $k = 0$ we get

$$F_{dc}^{(\ell)}(i\omega) = \frac{\Pi_\ell(e^{i\omega T_s})}{\ell! \left(\frac{e^{i\omega T_s} - 1}{T_s} \right)^\ell} - \frac{1}{(i\omega)^\ell} \frac{H(i\omega)}{T_s}.$$

This completes the proof.

The terms on the right-hand side of expression (25) allow for a few interesting reflections. Let

$$F_{dc}^{(\ell)}(i\omega) = F_1(e^{i\omega T_s}) - F_2(i\omega).$$

Then, the first term

$$F_1(e^{i\omega T_s}) = \frac{\Pi_\ell(e^{i\omega T_s})}{\ell! \left(\frac{e^{i\omega T_s} - 1}{T_s} \right)^\ell},$$

can be interpreted as the frequency response of a system made up of a chain of ℓ integrators where the input is subject to zero-order hold and the input and output are sampled. For more details, the reader is referred to Lemma 1 in the seminal paper on sampling zeros by Åström et.al[9]. The second term

$$F_2(i\omega) = \frac{H(i\omega)}{(i\omega)^\ell}$$

represents the continuous-time transfer function of a hold circuit followed by the same chain of integrators.

6.1 Numerical Illustration

In Figure 4 the parameters of the continuous-time system in (8)

$$G(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} \quad (28)$$

where $b_1 = 1$, $b_2 = 0.5$, $a_1 = 2$ and $a_2 = 3$ have been estimated using the method

$$\begin{aligned} \hat{\theta} = \arg \min_{b_0, \theta} \frac{1}{N_\omega} \sum_{k=1}^{N_\omega} & \left| Y_d(e^{i\omega_k T_s}) - \hat{Y}_d(e^{i\omega_k T_s}, \theta) \right|^2 \\ \hat{Y}_d(e^{i\omega_k T_s}, \theta, b_0) &= \hat{G}_d(e^{i\omega T_s}, \theta) U_d(e^{i\omega_k T_s}) \\ \hat{G}_d(e^{i\omega T_s}, \theta) &= G_c(i\omega_k, \theta) H(i\omega) + b_0 F_{dc}^{(\ell)}(i\omega_k) \end{aligned} \quad (29)$$

with different sampling intervals T_s . The process has in all cases been observed during $T = N_t T_s = 1000$ s and the excitation signal is random and binary. Frequency domain data up to the Nyquist frequency have been used such that

$$\omega_k = \frac{2\pi}{T_s} k, \quad k = 1 \dots N_\omega \quad (30)$$

where $N_\omega = \frac{N_t - 1}{2}$. The estimated parameter values for

Fig. 4. Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus T_s . The method used is that of (29).

the system are also found in Table 4. As can be seen from both Figure 4 and Table 4 the approximation will be good at moderate sampling rates.

7 Estimating the Continuous-Time Fourier Transform

Assume that there is no output noise in the continuous-time model in Figure 1. The frequency domain relation-

Table 4

Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus T_s . The method used is that of (29).

T_s	$a_1 = 2$	$a_2 = 3$	$b_0 = 1$	$b_1 = 0.5$
0.1	1.9996	3.0055	0.9992	0.5020
0.2	1.9966	3.0075	0.9980	0.5023
0.3	1.9895	3.0116	0.9953	0.5045
0.4	1.9768	3.0161	0.9894	0.5068
0.5	1.9609	3.0309	0.9802	0.5117
0.6	1.9272	3.0418	0.9645	0.5192
0.7	1.8862	3.0356	0.9421	0.5225
0.8	1.8245	3.0183	0.9118	0.5234
0.9	1.7522	3.0135	0.8723	0.5331
1.0	1.6652	2.9433	0.8194	0.5225

ship between the sampled input and output will then be

$$Y_d(e^{i\omega T_s}) = G_d(e^{i\omega T_s}, \theta) U_d(e^{i\omega T_s}) + \text{transients}$$

where G_d is defined by (3) if the input is assumed to be piecewise constant. In continuous-time, the corresponding relationship would be characterized by

$$Y_c(i\omega) = G_c(i\omega, \theta) U_c(i\omega) = G_c(i\omega, \theta) H(i\omega) U_d(e^{i\omega T_s}),$$

since the connection between the continuous- and discrete-time Fourier transforms of the input is

$$U_c(i\omega) = H(i\omega) U_d(e^{i\omega T_s})$$

due to the hold circuit H in (26). This way of thinking will open for the estimation of the continuous-time Fourier transform of the output of the systems. Assume that we know the exact parameter values θ_0 . Then, we could compute the exact continuous-time Fourier transform of the output as

$$Y_c(i\omega) = \frac{G_c(i\omega, \theta_0) H(i\omega)}{G_d(e^{i\omega T_s}, \theta_0)} Y_d(e^{i\omega T_s}). \quad (31)$$

However, since we wish to identify the parameters, the knowledge of θ_0 is quite unrealistic. Therefore, assume as before, that the system in (11) is strictly proper, stable and of relative degree $\ell = n - m$. Further assume that the sampling time T_s is such that the rate of sampling is above the system bandwidth. Define

$$F_{\ell+1, T_s}^c(i\omega) = \frac{T_s \left(\frac{e^{i\omega T_s} - 1}{i\omega T_s} \right)^{\ell+1}}{e^{i\omega T_s} \Pi_\ell(e^{i\omega T_s})} \quad (32)$$

where $\Pi_\ell(z)$ are the Euler-Frobenius polynomials in Section 5.1.

Fig. 5. Comparison of $G_c(i\omega)H(i\omega)/G_d(e^{i\omega T_s})$ (solid) in (31) and $F_{\ell+1,T_s}^c$ (dotted) in (32) with $\ell = 2$ for the continuous-time system $G_c(s) = \frac{1}{s^3 + 2s^2 + 3s + 4}$. The sampling rate is $T_s = 0.5$. For moderately high frequencies, the difference is quite small.

In Figure 5 we can see that the frequency domain difference between

$$\frac{G_c(i\omega)H(i\omega)}{G_d(e^{i\omega T_s})} \quad (33)$$

in (31) and $F_{\ell+1,T_s}^c$ (32) for the transfer function

$$G(s) = \frac{1}{s^3 + 2s^2 + 3s + 4} \quad (34)$$

is quite small. The generality of this observation is also verified by the following theoretical result.

Theorem 2 Assume $F_{\ell+1,T_s}^c$ is defined as in (32) with $\ell \geq 1$. Then, for each ω

$$\frac{G_c(i\omega, \theta_0)H(i\omega)}{G_d(e^{i\omega T_s}, \theta_0)} \rightarrow F_{\ell+1,T_s}^c(i\omega)$$

with the rate $\mathcal{O}(T_s^{\ell+1})$ as $T_s \rightarrow 0$.

PROOF. First of all we have

$$\frac{G_c\left(i\omega + i\frac{2\pi}{T_s}k\right)}{\frac{b_0}{(i\omega + i\frac{2\pi}{T_s}k)^\ell}} \rightarrow 1 \quad (35)$$

as $T_s \rightarrow 0$ if $k \neq 0$ and b_0 is defined as in (27). This has

the consequence that

$$\frac{G_c(i\omega)H(i\omega)}{G_d(e^{i\omega T_s})} \rightarrow \frac{\frac{G_c(i\omega)}{i\omega}}{\frac{G_c(i\omega)}{i\omega} + \sum_{k \neq 0} \frac{b_0}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}}}$$

as $T_s \rightarrow 0$ if we insert (35) in (3). From Lemma 3.2 in [12] it is also clear that

$$F_c^\ell(i\omega) = \frac{\frac{1}{(i\omega)^{\ell+1}}}{\frac{1}{(i\omega)^{\ell+1}} + \sum_{k \neq 0} \frac{1}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}}}.$$

By putting the two previous expressions on a common denominator, we get the following relation

$$\frac{G_c(i\omega)H(i\omega)}{G_d(e^{i\omega T_s})} - F_{\ell+1,T_s}^c(i\omega) \rightarrow F_{\ell+1,T_s}^c(i\omega)R(i\omega)S(i\omega)$$

where

$$R(i\omega) = \frac{1 - G_c(i\omega)\frac{(i\omega)^\ell}{b_0}}{\frac{G_c(i\omega)}{i\omega} + \sum_{k \neq 0} \frac{b_0}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}}}$$

and

$$S(i\omega) = \sum_{k \neq 0} \frac{b_0}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}}.$$

Since F and R are bounded in ω and the terms of S are bounded as

$$\left| \frac{1}{(i\omega + i\frac{2\pi}{T_s}k)^{\ell+1}} \right| \leq \left| \frac{T_s^{\ell+1}}{(i\omega T_s + i2\pi k)^{\ell+1}} \right| \leq C(\omega) \left(\frac{T_s}{k} \right)^{\ell+1}$$

if $k \neq 0$, the result

$$\left| \frac{G_c(i\omega)H(i\omega)}{G_d(e^{i\omega T_s})} - F_{\ell+1,T_s}^c(i\omega) \right| \leq |F_{\ell+1,T_s}^c(i\omega)| |R(i\omega)| |S(i\omega)| \leq C(\omega) T_s^{\ell+1}$$

follows.

This result opens for the estimation of the continuous-time Fourier transform of the output as

$$\hat{Y}_c(i\omega) = F_{\ell+1,T_s}^c(i\omega)Y_d(e^{i\omega T_s}) \quad (36)$$

which can be interpreted as assuming that $y(t)$ behaves as an ℓ times continuously differentiable function between the sampling instants. The parameters can then

be estimated from $\hat{Y}_c$ by the continuous-time method

$$\hat{\theta} = \arg \min_{\theta} \sum_{k=1}^{N_{\omega}} \left| \hat{Y}_c(i\omega_k) - G_c(i\omega_k, \theta) U_c(i\omega_k) \right|^2. \quad (37)$$

This two-stage process of first estimating the continuous-time spectrum and then estimating the system parameters, has the advantage of not directly involving the exact discrete-time frequency response $G_d(e^{i\omega T_s})$.

7.1 Numerical Illustration

In Figure 6 the parameters of the continuous-time system in (8)

$$G(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} \quad (38)$$

where $b_1 = 1$, $b_2 = 0.5$, $a_1 = 2$ and $a_2 = 3$ have been estimated using the method described in (36) and (37) using different sampling intervals T_s . The process has always been observed during $T = N_t T_s = 1000s$ and the excitation signal is random and binary. Frequency domain data up to the Nyquist frequency have been used, and therefore we have

$$\omega_k = \frac{2\pi}{T_s} k, \quad k = 1 \dots N_{\omega} \quad (39)$$

where $N_{\omega} = \frac{N_t - 1}{2}$. The estimated parameter values are

Fig. 6. Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus T_s . The method used is that of (37).

also found in Table 5. As can be seen from both Figure 4 and Table 4 the approximation will be quite good at moderate sampling rates.

Table 5

Identified parameters for the system $G_c(s) = \frac{s+0.5}{s^2+2s+3}$ versus T_s . The method used is that of (37).

T_s	$a_1 = 2$	$a_2 = 3$	$b_0 = 1$	$b_1 = 0.5$
0.1	1.9910	3.0021	0.9949	0.5033
0.2	1.9684	3.0170	0.9800	0.5152
0.3	1.9270	3.0284	0.9539	0.5274
0.4	1.8824	3.0542	0.9202	0.5531
0.5	1.8180	3.0436	0.8767	0.5579
0.6	1.7498	3.0501	0.8232	0.5737
0.7	1.6830	3.0506	0.7657	0.5904
0.8	1.6036	3.0727	0.6994	0.6126
0.9	1.5101	3.0596	0.6272	0.6164
1.0	1.4255	2.9974	0.5526	0.6040

8 Comparisons

In Sections 4, 6 and 7 new, approximation based, methods for identification of continuous-time input-output models have been introduced. In this section we will illustrate how the different approaches perform for some systems with different sampling intervals. In all cases we will simulate the models

$$G_1(s) = \frac{s + 0.5}{s^2 + 2s + 3} \quad (40)$$

$$G_2(s) = \frac{1}{s^3 + 2s^2 + 3s + 4} \quad (41)$$

$$G_3(s) = \frac{4}{s^3 + 4s^2 + 4s + 4}, \quad (42)$$

$$(43)$$

with a piecewise constant input where the amplitudes of the constant segments have a Gaussian distribution. No noise was added to output in the simulations. The results of this comparison is provided in Table 6, Table 7 and Table 8.

8.0.1 Approach 1

The first, and simplest, approach is to push one's luck and assume that the data is sampled so fast that it can be considered band-limited. Thereby one would use the method

$$\hat{\theta} = \arg \min_{\theta} \sum_{k=1}^{N_{\omega}} \left| Y_d(e^{i\omega_k T_s}) - G_c(i\omega_k, \theta) U_d(e^{i\omega_k T_s}) \right|^2. \quad (44)$$

If the true system is of low pass character, this assumption may be more plausible for the output than for the input. We will label this *Approach 1*.

8.0.2 Approach 2

Another reasonable approximation which was presented earlier, is to use just the central term (i.e. $N_f = 0$ in (9)) in the approximation of the discrete-time frequency response. This would then mean that we can apply the method

$$\hat{\theta} = \arg \min_{\theta} \sum_{k=1}^{N_{\omega}} \left| Y_d(e^{i\omega_k T_s}) - G_c(i\omega_k, \theta) \frac{H(i\omega_k)}{T_s} U_d(e^{i\omega_k T_s}) \right|^2. \quad (45)$$

Compared to Approach 1, we have a continuous-time method where the piecewise constant input is correctly translated to continuous time and a band limited assumption on the output is used. This is in line with the assumption that the system is low pass in relation to the sampling interval. We label this *Approach 2*.

8.0.3 Approach 3

An obvious variant of the above approach is to involve more terms in (9). We call this *Approach 3*. Clearly, as $N_f \rightarrow \infty$ we approach the “correct” method.

$$\hat{\theta} = \arg \min_{\theta} \sum_{k=1}^{N_{\omega}} \left| Y_d(e^{i\omega_k T_s}) - G_d(i\omega_k, \theta) U_d(e^{i\omega_k T_s}) \right|^2 \quad (46)$$

where

$$G_d(e^{i\omega T_s}, \theta) = \left(\frac{1 - e^{-i\omega T_s}}{T_s} \right) \sum_{k=-N_f}^{N_f} \frac{G_c(i\omega + i\frac{2\pi}{T_s}k, \theta)}{i\omega + i\frac{2\pi}{T_s}k}. \quad (47)$$

8.0.4 Approach 4

In (36), a way to estimate the continuous-time Fourier transform of the output $Y_c(i\omega)$ using the pre-filter in (32) was devised.

$$\hat{Y}_c(i\omega) = F_{\ell+1, T_s}^c(i\omega) Y_d(e^{i\omega T_s}) \quad (48)$$

After the transform was found, the continuous-time parameters could be acquired using the relationship in (37),

$$\hat{\theta} = \arg \min_{\theta} \sum_{k=1}^{N_{\omega}} \left| \hat{Y}_c(i\omega_k) - G_c(i\omega_k, \theta) U_c(i\omega_k) \right|^2 \quad (49)$$

where

$$U_c(i\omega_k) = H(i\omega_k) U_d(e^{i\omega_k T_s}) \quad (50)$$

We call this *Approach 4*.

8.0.5 Approach 5

Finally, the method in (29) from Section 6

$$\begin{aligned} \hat{\theta} &= \arg \min_{b_0, \theta} \frac{1}{N_{\omega}} \sum_{k=1}^{N_{\omega}} \left| Y_d(e^{i\omega_k T_s}) - \hat{Y}_d(e^{i\omega_k T_s}, \theta) \right|^2 \\ \hat{Y}_d(e^{i\omega_k T_s}, \theta, b_0) &= \hat{G}_d(e^{i\omega_k T_s}, \theta) U_d(e^{i\omega_k T_s}) \\ \hat{G}_d(e^{i\omega_k T_s}, \theta) &= G_c(i\omega_k, \theta) H(i\omega) + b_0 F_{dc}^{(\ell)}(i\omega_k) \end{aligned}$$

where the higher order terms in (3) ($k \neq 0$) are approximated by ℓ order integrators

$$F_{dc}^{(\ell)}(i\omega) = \frac{\Pi_{\ell}(e^{i\omega T_s})}{\ell! \left(\frac{e^{i\omega T_s} - 1}{T_s} \right)^{\ell}} - \frac{1}{(i\omega)^{\ell}} \frac{H(i\omega)}{T_s} \quad (51)$$

we call *Approach 5*.

8.0.6 Final Remark

It should be noted that in practice it may be essential to limit the fit of all the estimation methods to frequencies that do not extend all the way to the Nyquist frequency, since the observations may be less reliable at higher frequencies. Another reason is that $F_{\ell+1, T_s}^c$ in (32) will tend to infinity at the Nyquist frequency for ℓ being even (the sampled multi-integrator will then have a zero at the Nyquist frequency).

9 Summary

In this paper, different approaches to direct frequency domain estimation of continuous-time transfer functions based on sampled data have been presented. If the input inter-sample behavior is known this can be done without approximation. In particular for piecewise constant excitations, there are well known, but somewhat complicated formulas for this. We have investigated various frequency domain approximation of the exact transformation that are simpler to use and give good approximations, at least for sufficiently fast sampling rates. Essentially, these approximations are based on replacing the true parameter dependent system with a number of integrators that equal the pole excess of the system. This line of thought is continued in the following paper.

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Table 6

Results for the system $\frac{s+0.5}{s^2+2s+3}$. This system has a pole excess of 1 and a bandwidth of 8.60 rad/s.

T_s	0.02	0.1	0.5	1	1.5
Appr 1					
1	0.9753	0.8794	0.2614	-0.5413	-0.3764
0.5	0.5137	0.5529	1.4552	1.4623	0.5553
2	1.9443	1.7432	1.3928	1.5599	1.0350
3	2.9972	2.9670	4.8357	5.6280	3.1000
Appr 2					
1	1.0002	1.0020	0.9879	0.5292	-0.1094
0.5	0.5002	0.5046	0.6266	1.4389	1.0510
2	2.0006	2.0003	1.9033	1.3682	0.7592
3	3.0005	3.0046	3.1556	4.8057	4.2858
Appr 3 $N_f = 10$					
1	1.0007	1.0029	1.0088	1.0070	0.8288
0.5	0.4998	0.5058	0.5112	0.5180	0.4944
2	2.0018	2.0084	2.0133	2.0073	1.7852
3	2.9988	3.0085	3.0154	3.0123	2.8269
Appr 4					
1	0.9999	0.9949	0.8791	0.5541	0.1864
0.5	0.5000	0.5013	0.5272	0.6046	0.5351
2	2.0003	1.9929	1.8353	1.4233	1.0898
3	3.0000	2.9986	2.9659	3.0044	2.8335
Appr 5					
1	1.0047	0.9994	0.9866	0.8223	0.4488
0.5	0.4956	0.5040	0.5020	0.5244	0.4341
2	2.0288	2.0015	1.9760	1.6736	1.1949
3	2.9966	3.0093	3.0037	2.9537	2.3787

Table 7

Results for the system $\frac{1}{s^3+2s^2+3s+4}$. This system has a pole excess of 3 and a bandwidth of 2.1 rad/s.

T_s	0.02	0.1	0.5	1	1.5
Appr 1					
1	0.9805	0.9096	0.6692	0.4873	0.3584
2	1.9550	1.7887	1.2413	0.8993	0.7554
3	2.9805	2.9084	2.6481	2.4271	2.2400
4	3.8937	3.5078	2.2317	1.4376	1.0490
Appr 2					
1	1.0017	1.0000	0.9996	0.9857	0.9050
2	2.0031	2.0000	1.9993	1.9729	1.8921
3	3.0018	3.0000	2.9997	2.9900	2.9346
4	4.0061	4.0000	3.9987	3.9458	3.7830
Appr 3 $N_f = 2$					
1	0.9983	1.0039	1.0092	0.9957	0.9979
2	2.0001	2.0100	2.0161	1.9907	2.0124
3	3.0003	3.0032	3.0080	3.0033	3.0019
4	3.9991	4.0172	4.0299	3.9778	4.0190
Appr 4					
1	1.0018	1.0000	1.0015	1.0268	0.9641
2	2.0032	1.9998	2.0002	1.9821	1.5172
3	3.0019	3.0000	3.0002	3.0044	2.8956
4	4.0063	3.9996	4.0013	3.9758	3.1243
Appr 5					
1	1.0346	0.9993	1.0028	0.9949	0.9900
2	2.1001	2.0012	2.0092	1.9930	2.0436
3	3.0554	2.9998	3.0040	2.9966	2.9994
4	4.1943	4.0038	4.0172	3.9863	4.0891

Table 8

Estimated values of a for the model $\frac{a}{s^3+a^2s^2+as+a}$ with true value $a = 4$. This system has a pole excess of 3 and a bandwidth of 0.75 rad/s.

T_s	0.02	0.1	0.5	1	1.5
Appr 1	4.0081	4.0388	4.2330	4.4837	4.7438
Appr 2	3.9988	3.9935	3.9988	3.9995	4.0020
Appr 3 ($N_f = 5$)	4.0123	4.0041	4.0092	4.0017	4.0501
Appr 4	3.9987	3.9935	3.9988	3.9999	4.0038
Appr 5	3.9444	3.9855	3.9989	3.9951	3.9689

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Titel Frequency-Domain Identification of Continuous-Time Output Error Models Part I - Uniformly Title Sampled Data and Frequency Function Estimation		
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Sammanfattning Abstract <p>This paper treats identification of continuous-time output error (OE) models based on sampled data. The exact method for doing this is well known both for data given in the time and frequency domains. This approach becomes somewhat complex, especially for non-uniformly sampled data. We study various ways to approximate the exact method for reasonably fast sampling. While an objective is to gain insights into the non-uniform sampling case, this paper only gives explicit results for uniform sampling.</p>		
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