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Enabling Uncertainty Quantification of Large Aircraft System Simulation Models

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Abstract

A common viewpoint in both academia and industry is that that Verification, Validation and Uncertainty Quantification (VV&UQ) of simulation models are vital activities for a successful deployment of model-based system engineering. In the literature, there is no lack of advice regarding methods for VV&UQ. However, for industrial applications available methods for Uncertainty Quantification (UQ) often seem too detailed or tedious to even try. The consequence is that no UQ is performed, resulting in simulation models not being used to their full potential.

In this paper, the effort required for UQ of a detailed aircraft vehicle system model is estimated. A number of methodological steps that aim to achieve a more feasible UQ are proposed. The paper is focused on 1-D dynamic simulation models of physical systems with or without control software, typically described by Ordinary Differential Equations (ODEs) or Differential Algebraic Equations (DAEs). An application example of an aircraft vehicle system model is used for method evaluation.

1 Introduction to Uncertainty Quantification

Uncertainty Quantification (UQ) or Uncertainty Analysis (UA) refers to the process of identifying, quantifying, and assessing the impact of uncertainty sources embedded along the development and usage of simulation models. According to Roy and Oberkampf (2011), all uncertainties originate from three key sources:

1. *Model inputs*: e.g. input signals, parameters, and boundary conditions.
2. *Numerical approximations*: e.g. due to the numerical method used by the solver.
3. *Model form*: e.g. model simplifications or fidelity level of underlying equations.

This is in line with the definitions provided by Coleman and Steele (2009). Commonly, a distinction is made between aleatory uncertainty (due to statistical variations, also referred to as variability, inherent uncertainty, irreducible uncertainty, or stochastic uncertainty) and epistemic uncertainty (due to lack of information, also referred to as reducible uncertainty or subjective uncertainty). See

Padulo (2009) for an extensive literature review of uncertainty taxonomies.

It may be questioned whether the term *uncertainty* or *error* should be used, and in the literature these terms are sometimes used interchangeably. To avoid misinterpretation, uncertainty here refers to the nature of the source, i.e. if it is aleatory, epistemic or a mixture, and is often characterized as a Probability Density Function (PDF) or an interval. Error on the other hand does not concern the nature of the source, and is often seen as a single realization of an uncertain entity.

For the common case of using measurement data for validation purposes, the uncertainties of the data used for validation are as important as the uncertainties of the model itself. Sometimes the uncertainty of the validation data is deemed too hard to assess and is ignored without justification, or simply understood as the measurement error of a specific sensor. The following basic equations provide the fundamental relationships between the simulation result *S*, the validation data D, the validation comparison error E, and the true (but unknown) value T. The error in the simulation result δ_S and the error in the validation data δ_D are also defined. The equation variables may be either time-series or single values, such as steady-state values. The equations originate from Coleman and Steele (2009), and corresponding equations are found in Oberkampf and Roy (2012).

$$E = S - D \tag{1.1}$$

$$\delta_S = S - T \tag{1.2}$$

$$\delta_D = D - T \tag{1.3}$$

Hence, the validation comparison error is the combination of all errors in the model and in the validation data.

$$\begin{aligned} E &= (\delta_S + T) - (\delta_D + T) \\ &= \delta_S - \delta_D \end{aligned} \tag{1.4}$$

With the three model uncertainty sources described at the beginning of this section, the error in the simulation result can be defined as follows.

$$\delta_S = \delta_{S\,input} + \delta_{S\,num} + \delta_{S\,model} \tag{1.5}$$

In addition to sensor measurement error (which may also include A/D conversion implying finite resolution), the total error in the validation data may depend on various characteristics of the physical test setup, e.g. uncertain boundary conditions, experimental simplifications, or placement of sensors. An example might be when comparing airstream temperatures obtained from a 1-D simulation model with experimental results. In such a case, the model typically does not take local effects and inhomogeneous flow patterns into account. Therefore, to obtain useful validation data, the placement of the temperature sensor should be carefully chosen, e.g. in terms of downstream distance from a mixing point or radial positioning in a pipe. To emphasize this, the equations provided by Coleman and Steele (2009) can be expanded by defining the total error in the validation data as a combination of sensor measurement error $\delta_{D\,sensor}$ and errors due to the experimental setup $\delta_{D\,setup}$ itself.

$$\delta_D = \delta_{D\,sensor} + \delta_{D\,setup} \tag{1.6}$$

Combining equations (1.4) to (1.6) and solving for the model form error $\delta_{S\,model}$ yields:

$$\begin{aligned} \delta_{S\,model} = E &- (\delta_{S\,input} + \delta_{S\,num}) \\ &+ (\delta_{D\,sensor} + \delta_{D\,setup}) \end{aligned} \tag{1.7}$$

There are methods to estimate $\delta_{S\,input}$ and $\delta_{S\,num}$, but according to Coleman and Steele (2009) no way to independently observe or calculate the effects of $\delta_{S\,model}$. In most cases, the knowledge of $\delta_{D\,sensor}$ is available, but the knowledge of $\delta_{D\,setup}$ is often limited. In some sense, the error due to the experimental setup $\delta_{D\,setup}$ is the experimental counterpart to the model form error of the simulation $\delta_{S\,model}$. Roy and Oberkampf (2011) and Coleman and Steele (2009) agree that the objective of model validation is to estimate the model form uncertainty.

2 UQ During System Development

In large part, model validation commonly involves comparison of simulation results and measurement data. The added value of UQ changes with the system development phase and the availability of measurement data for model validation purposes.

Figure 2-1: Typical A/C system development phases.

In early phases, i.e. conceptual and preliminary design, measurement data are scarce and UQ is the main means to gain an understanding of the credibility of a simulation model. As system development continues, the availability of measurement data increases – starting with measurement data for separate equipment or smaller subsystems, on to test rig data, and in later phases flight test data.

Even though later phases enable model validation against measurement data, it is often difficult to obtain satisfactory coverage of the *validation domain*, or *domain of applicability*. See e.g. Hemez et al. (2010) for an illustrative description of a model with a domain of applicability spanned by two parameters. In such cases UQ is a useful support also in later development phases.

It should be noted that for some models, to actually define the validation domain might be a problem in itself. Another problem is how to visualize the validation domain, as well as the coverage and the goodness-of-fit throughout the validation domain when the validation domain is spanned by multiple parameters. This is the case for the Environmental Control System (ECS) used as an industrial application example.

3 Method for UQ

Roy and Oberkampf (2011) describe a comprehensive framework for UQ as a six-step procedure. The framework is described in detail and includes an application example, which makes it suitable as reference process in this paper. Briefly described, the six steps are as follows:

1. *Identify all sources of uncertainty:* Locate uncertainties in parameters, inputs, and boundary conditions.
2. *Characterize uncertainties:* Classify the identified uncertainties as either aleatory, epistemic, or a mixture. Assign precise DPFs for the aleatory uncertainties, intervals for the epistemic uncertainties, and imprecise PDFs for the mixed type of uncertainties. *Imprecise* here refers to when the parameters of the PDF, for example the mean and standard deviation, are given as intervals or PDFs themselves.
3. *Estimate uncertainty due to numerical approximations:* Use for example post-processing based methods such as Richardson extrapolation to estimate numerical uncertainty of all model outputs.
4. *Propagate input uncertainties through the model:* As aleatory and epistemic uncertainties are two different types, it is recommended to use Probability Bounds Analysis (PBA) to treat them independently. Briefly described, the interval of the epistemic input is divided into a number of sub-intervals, and for each sub-interval of each epistemic input a complete propagation of aleatory inputs is carried out, typically with Monte Carlo sampling based techniques. The result is one Cumulative Distribution Function (CDF) for each epistemic sub-interval. The complete set of CDFs for a model output is referred to as a probability box, or a *p*-box.
5. *Estimate model form uncertainty:* Use a well-defined validation metric to compare model output with measurement data. In this framework, the measurement data for a specific system characteristic as well as the corresponding model output are described as two CDFs.

6. *Determine total uncertainty in the model outputs:* To the *p*-box generated from the uncertainty propagation, add the model form uncertainty and the numerical uncertainty as epistemic uncertainties.

4 Industrial Application Example

A detailed Modelica model of the Gripen C/D Environmental Control System (ECS) is used as an application example of realistic industrial size. The main purpose of the ECS is to provide sufficient cooling of the avionics equipment, as well as tempering and pressurizing the cabin. In addition to this, essential tasks are to enable pressurization of the fuel system and anti-g system, and to provide conditioned air to the On-Board Oxygen Generating System (OBOGS), which provides breathing air to the pilots. Briefly, this is achieved by means of a bootstrap configuration using engine bleed air which is decreased in pressure and temperature and dried prior to distribution. The main H/W components in the ECS are heat exchangers, compressor, turbine, water separator, pipes, and control valves. The ECS S/W, which is physically located in the General systems Electronic Control Unit (GECU), controls and monitors pressure, temperature, and flow levels in various parts of the system.

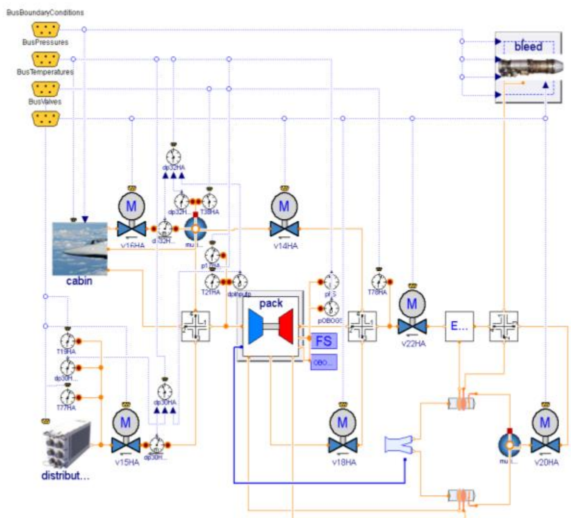

Figure 4-1: A graphical overview of the ECS H/W model.

The model is mainly used for:

- Concept studies
- Detailed equipment specifications
- Control system design
- Performance analysis
- Incident analyses
- System safety analyses
- Computation of loads

The model is developed in the modeling and simulation tool Dymola using the modeling language Modelica (Dymola 2013, Modelica 2013). A system equipment component library for aircrafts is used. The component parameters are based on information from drawings, vendor sheets, bench test measurement data, and experience. The control code integrated in the Modelica model is C-code automatically generated from a Simulink control code specification.

5 Effort Estimations of UQ

The intention of this section is to make an estimation of the effort required to carry out (as far as possible) each step in the methodology described in section 3 on the ECS model. The following sub-sections describe each step from a practical viewpoint including estimations of work effort.

Figure 5-1 shows a typical layout of a closed-loop model of a specific aircraft vehicle system. System S/W and ECU H/W denote system-specific software and hardware placed in an Electronic Control Unit, and are typically modeled in Simulink. System H/W is a model of the physical system, typically developed in Modelica using Dymola. BC denotes boundary conditions in terms of for example flight case and climate profile, and the gray-dashed arrows indicate communication with other systems.

The UQ dealt with in this paper is performed on the red-dashed part of the closed-loop model. That is, the models of ECS S/W and GECU H/W are seen as specifications (with possible errors or "bugs") rather than descriptive models subject to uncertainty (Andersson 2009). However, to obtain interpretable results, the full closed-loop model is used when propagating uncertainties during the UQ.

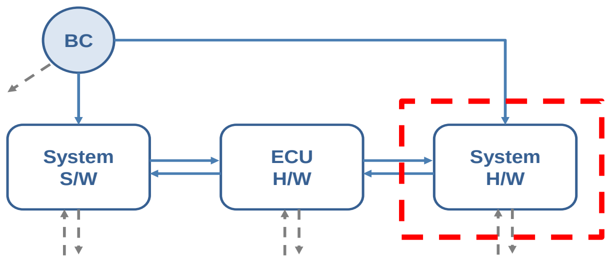

Figure 5-1: Typical layout of an aircraft vehicle system closed-loop model. UQ delimited to ECS H/W model.

A prerequisite for the UQ is that the current development phase is preliminary design or early detailed design. In this phase, there is typically rig test data available to some extent, while flight test data is unavailable or very limited.

Another prerequisite is that model verification is already performed as a continuous part of the model development, i.e. the effort estimation of the UQ does not include typical verification tasks. This means that all nominal parameter values are verified against supporting documentation, which is also referred to and available. Without this verification having been performed and documented, the step “Identify all sources of uncertainty” will be much more time-consuming than assumed below.

Informal model validation against available test rig data is assumed to be performed. The purpose of the informal model validation is to check that nominal simulation results are reasonable. The purpose of the following UQ is to increase the understanding of the uncertainties embedded in the model, and how these affect simulation results. In other words, the purpose of the UQ is to go from nominal simulation results to simulation results with an estimated uncertainty.

The UQ performer is assumed to be a person in close relationship with the people who performed all the above mentioned requisite activities but the UQ performer did not actually participate. This person has adequate knowledge of tools and some knowledge of the component library. His or her ECS system knowledge and professional skills are assumed to be on a senior level although he or she has not performed any UQ analysis on a large system model. The following sub-sections follow the structure of section 3.

5.1 Identify all sources of uncertainty

By categorizing all model parameters and inputs as either certain or uncertain the workload downstream the process can be reduced. Which of the parameters and inputs are certain or uncertain, respectively, is preferably and pragmatically decided through experience. Concerning parameters, this work is straightforward, e.g. a pipe diameter is easy to find and has a high manufacturing accuracy compared to a pump performance that can differ between pumps and as a function of wear. The categorization of inputs is trickier. For example, an ECS model may have engine bleed temperature and pressure as input via an engine model. If the simulation results are presented in the context of engine bleed data, the engine bleed data can obviously be categorized as certain inputs. On the other hand, if the engine bleed data is a consequence of flight case specified by inputs such as Mach, altitude, and thrust to an engine model and presented in the context of the flight case, the uncertainties in the engine model must be considered and the engine bleed data should consequently be treated as uncertain inputs.

The estimation of work effort required for the UQ is based on the following categorization of parameters and input signals. The list includes number of parameters or input signals representative of the ECS H/W model described in section 4.

A) Geometry parameters considered certain (e.g. pipe diameter): 92
B) Geometry parameter considered uncertain (e.g. hose length or pipe surface roughness): 76
C) Physical parameters considered certain (e.g. pressure loss coefficient of simple pipe bend): 31

D) Physical parameters considered uncertain (e.g. participating thermal mass or pressure loss coefficient of complex geometry): 196
E) Input signal considered certain (e.g. one-seater or two-seater): 7
F) Input signal considered uncertain (e.g. engine bleed pressure): 4

Naturally, the effort of identifying and categorizing parameters and input signals is dependent on the model's size. Currently, Dymola provides information about the total number of parameters in a model, but does not provide support for categorization. This step therefore requires some manual work possibly supported by scripting. For the ECS H/W model, the effort involved in this step is estimated to be 16 hours.

5.2 Characterize uncertainties

This step concerns characterization of the uncertainties of parameters and input signals in categories B), D), and F) from the previous section. Categorization begins with defining parameters and inputs as either epistemic, aleatory, or a mixture of the two. Supporting information like data sheets, drawings, CAD-models, or bench test data is used to assign intervals, PDFs or imprecise PDFs.

A rough, experience-based estimate of the time needed to characterize each parameter type is given below. Fully representative estimates cannot be obtained without performing the actual characterization, which is beyond the scope of this paper. The estimates are however believed to give a reasonably good indication of the total work effort required.

Category	**Number**	**Time/par. [hours]**	**Time total [hours]**	**Comment**
B) Geometry parameter considered uncertain	76	6	456	4 hours for obtaining necessary information, e.g. from installation drawing, CAD-model, or repeated measurements on several equipment individuals. 2 hours for characterization of interval or PDF.
D) Physical parameter considered uncertain				
D-1) Information available	117	10	1170	8 hours for obtaining necessary information, e.g. from existing bench test measurement data or assessment using basic physics. 2 hours for characterization of interval or PDF.
D-2) New bench testing or CFD simulations required	79	22	1738	80 hours for conducting new bench tests or new CFD-simulations. However, significant synergies are obtained by for example coordination of physical testing. The estimation is therefore reduced to 20 hours. 2 hours for characterization of interval or PDF.
F) Input signal considered uncertain	4	8	32	6 hours for obtaining necessary information from e.g. model description of connected models or assessment by applicable domain expert. 2 hours for characterization of interval or PDF.

Table 5-2: Typical layout of an aircraft vehicle system closed-loop model. UQ delimited to ECS H/W model.

To simplify the estimation, it is assumed that uncertainties in geometry parameters are related to tolerances in production and assembly and thus dominated by a stochastic behavior. The geometry parameters are therefore treated as aleatory and are assigned PDFs. It is assumed that uncertainties in physical parameters and input signals are mainly due to limited knowledge, and are therefore treated as epistemic and assigned intervals. In reality, most uncertainties in the model are a mix of aleatory and epistemic behavior and according to section 3 should be assigned imprecise PDFs. However, this implies additional complexity to an already complicated UQ and is not deemed a feasible approach for the type of model used in this paper. All in all, we have 76 aleatory uncertainties and 200 epistemic uncertainties.

Since an input can differ from one simulation to another (unlike the model parameters), its accuracy may also differ. As with the engine bleed pressure which varies with boundary conditions in terms of altitude, Mach, Power Lever Angle (PLA), and climate profile, the *uncertainty* of the engine bleed pressure may also vary. To ease the workload in this phase, a reasonable simplification is to assume that the input uncertainties are constant rather than functions of inputs/boundary conditions.

5.3 Estimate uncertainty due to numerical approximations

As long as the solver tolerances are sufficiently strictly chosen, the effect of numerical approximations are normally marginal for this type of simulation model and are therefore typically ignored. This is also what has been done for the ECS H/W model; an assessment using different solvers (Dassl and Radau) and varying tolerances ($1\cdot10^{-6}$ to $1\cdot10^{-4}$) shows maximum deviations in the investigated temperature, pressure, and mass flow levels of 0.04°C, 0.39 kPa, and 0.002 kg/s, respectively. This indicates that the numerical errors are insignificant compared to other simulation result uncertainties for the model under study. In addition to comparing a set of model outputs for varying solver tolerances, a good indicator in this type of model is to check that conservation of mass and energy is fulfilled for a set of connection points or branches in the model.

For the ECS model, the effort involved in this step is estimated to be 4 hours.

5.4 Propagate input uncertainties through the model

The uncertainty propagation methods applicable for the type of model used are Monte Carlo based sampling techniques. To decrease the number of simulation runs needed, Latin Hypercube Sampling (LHS) is used instead of brute force Monte Carlo sampling.

Due to the physical relations embedded in the model, it may well be the case that uncertainties in the simulation results vary throughout the validation domain. Due to the multidimensionality of the validation domain of the ECS H/W model, complete coverage is practically impossible. In this effort estimation it is assumed that *one* compressed flight mission covering the main parts of the flight envelope (Mach, altitude) for *one* climate profile with *one* fixed relative air humidity is analyzed. Later on, only the one-seater A/C configuration is considered. The flight mission is constructed in such a way that uncertainties at both steady-state levels and transients are considered. Steady-state levels are obtained by keeping Mach and altitude constant during a specified time interval, and transients are obtained by positive and negative changes in Mach, altitude, and Power Lever Angle (PLA). The compressed flight mission used in the uncertainty propagation is 5 minutes real-time, which for the model under study means approximately 8 minutes execution time.

Purely epistemic uncertainties have no structure over the uncertainty interval, i.e. there are no PDFs, and the only values considered known are the limits of the interval. In line with this, Roy and Oberkampf (2011) propose LHS with at least three samples for each epistemic uncertainty. In addition to this, for *each* combination of epistemic uncertainties, the aleatory uncertainties are sampled using a number of Monte Carlo samples. This implies nested sampling, with an outer LHS loop for the epistemic uncertainties and an inner Monte Carlo loop for the aleatory uncertainties. For m epistemic uncertainties with l LHS intervals and s inner loop samples, the total number of samples n is computed as:

$$n = m^l \cdot s \qquad (5.1)$$

With, say, as few as 100 inner loop samples, the total number of samples for the ECS model would be $8\cdot10^8$. That is, $8\cdot10^8$ simulations of 8 minutes each, which certainly confirms the conclusion drawn by Roy and Oberkampf

(2011) that *"for more than a handful of epistemic uncertainties, the total number of samples required for convergence becomes extraordinarily large, and other approaches should be considered"*.

An alternative approach is to lump aleatory and epistemic uncertainties together in one single sampling scheme, preferably using a sampling technique more efficient than brute force Monte Carlo, e.g. LHS. This requires deviations from the above methodology in terms of also assigning PDFs to the epistemic uncertainties. Since the true nature of the parameters earlier classified as epistemic probably is a mix of aleatory and epistemic, assigning uniform PDFs may be seen as a middle road.

As described in Carlsson et al. (2012), LHS using 250 intervals proved to provide reasonable output distributions for a simulation model with 10 uniformly uncertain inputs, i.e. 25 simulation runs per parameter. Using this number for the ECS model results in $297 \cdot 25 = 7425$ simulation runs. This number is not scientifically justified but may still provide practical guidance as to what sample size to expect. With 8 minutes per simulation run this implies 990 hours (41 days) execution time (using one CPU). The actual work effort involved in this step is estimated to be 40 hours, consisting mainly of setting up a framework for LHS and results post-processing.

5.5 Estimate model form uncertainty

Two major contributors to model form uncertainty are simplifications in the model and in the components' underlying equations. The two contributions are much the same thing but on different scales. For example, the component's equations cannot represent the domain of interest, e.g. can handle turbulent but not laminar flow, or may be used in the domain of interest but with low accuracy. On model level, the components' equations correspond to system functions/parts not modeled or, for example, lumped together.

Due to the incompleteness of the measurement data, consisting of poor coverage in the flight envelope, few tests in the same test point, tests performed by a system rig or bench test and sometimes with immature prototype system equipment, a compilation of CDFs according to step 5 in section 3 is not feasible. What can be done is the following.

First of all, the question *"Has the components' equations capability to describe the physical phenomena of interest?"* must be asked. For example, if a valve component lacks an equation for chocked condition, then it is not of any interest to quantify the model form accuracy for simulation results including large pressure ratio between valve inlet and outlet. The components used for the ECS model are a mix of standard Modelica components and a commercial component library developed for aircraft fluid systems. The standard Dymola components are assumed not to contribute to the model form accuracy since they are often of a mathematical and logical nature, e.g. tables, gains, and switches without "physical behavior" equations. In our case, the commercial component library documentation will support the phase "Estimate model form uncertainty". What then remains is to analyze the underlying physical equations in the components concerning their contribution to the model form, e.g. to what extent heat transfer coefficients are described and in which regions they are valid.

Circumstances that may complicate the analysis are whether some components' code is IPR-protected and therefore not accessible. Component equations can also be difficult to find if inheritance technique has been used intensively for the component structure. The ECS model contains 16 different component types assumed to be relevant for this step, sensors and mathematical components excluded. For each component it is assumed that 3 days are needed to find documentation, read, understand model code, evaluate by simulating components, and finally document the result. For the ECS model, the effort is estimated to be $16 \cdot 8 \cdot 3 = 384$ hours, which is rounded down to 350 hours since some synergies are assumed to exist.

The lack of suitable measurement data in early phases makes it hard to estimate the uncertainty originating from model simplifications. A manageable strategy for the ECS model is to estimate the uncertainty with the help of experience based on other more mature models. For the ECS model, the effort is estimated to be 40 hours to read model documentation and compare it to the real system layout, and another 40 hours for comparison, via documentation and interviews, with other models; 40 + 40 = 80 hours.

For the ECS model, the effort involved in this step is estimated to be 350 + 80 = 430 hours. This type of analysis and experience can at best lead to reasoning that the model form uncertainty is probably lower/higher relative to the effects of parameters and inputs.

5.6 Determine total uncertainty in the model outputs

Once the earlier steps have been carried out, this final step mostly involves compilation of results, visualization, and documentation. For the ECS model, the work effort involved in the final step is estimated to be 40 hours.

5.7 Total workload for the UQ framework

The estimations in steps 1 to 6 are summarized in the table below.

Step	Engineering workload [hours]	Machine execution time [hours]
1) Identification	16	N/A
2) Characterization	3396	N/A
3) Numerical approx.	4	N/A
4) Propagation	40	990
5) Model form	430	N/A
6) Total uncertainty	40	N/A
Total:	**3926**	**990**

Table 5-3: Summary of estimated work effort.

As can be seen from the table, a comprehensive UQ of a detailed simulation model is time-consuming and human-/skill-intensive. Naturally, there is an uncertainty in the estimation but the workload is not believed to be overestimated. The total workload is primarily dominated by the step *characterize uncertainties*, followed by *estimate model form uncertainty*.

The uncertainty propagation is demanding when it comes to computational resources. The time required can however be shortened by distributing a number of batch simulations over several machines. That is, the 990 hours of execution time for the uncertainty propagation can be significantly reduced.

6 Simplifications Enabling UQ of Large A/C System Simulation Models

With time and budget constraints typical for A/C development programs, the figures presented in section 5.7 are not acceptable. That is, for a UQ to be carried out, the workload has to be significantly reduced. If not, the resources are surely seen to be better spent on activities other than UQ. Also, if numbers of this magnitude are included in the estimation of model development time, there is a risk that needed early sponsors of the model will never spend resources on model-based system development and consequently no model will be developed.

As a comparison, the total time for development, verification, and steady-state validation of the ECS model was approximately 2000 hours. Adding around 4000 hours for UQ alone cannot be considered realistic. Furthermore, the UQ should be updated when new measurement data is available or if the model or the component library is updated. Therefore, the intention of this section is to discuss simplifications, compromises, and methods to ease the UQ workload.

The estimation performed is representative of a comprehensive UQ. However, a number of simplifications are already performed:

1) Geometry parameters and physical parameters are treated as purely aleatory and purely epistemic, respectively. The more realistic mix of aleatory and epistemic behavior is not considered.
2) Uncertainties in inputs and parameters are treated as constant PDFs or intervals,

and not as functions of other inputs or boundary conditions.
3) Minimalistic assessment of uncertainty due to numerical approximations.
4) Uncertainty propagation is performed using single loop LHS instead of nested sampling with strict separation of aleatory and epistemic uncertainties.
5) Model form uncertainty is assessed by analysis of underlying equations rather than extensive validation against measurement data.

In addition to the above, two main measures for reducing the workload are to reduce the number of parameters considered uncertain and to simplify the characterization of uncertainties. These two measures are treated in the subsections below.

6.1 Reducing the number of uncertain parameters

This fundamental measure reduces the workload in both characterization and propagation, and may be done using local sensitivity analysis and/or experience. As an example, for the ECS model experience tells us that production tolerances causing aleatory uncertainty in geometry parameters such as pipe diameter can be safely ignored. Uncertainties in for example efficiency characteristics of compressor, turbine, and heat exchangers are significantly larger. Regarding local sensitivity analysis and how it may be applied in the context of aircraft vehicle system models, see Steinkellner (2011).

An alternative method for reducing the number of parameters considered uncertain is *output uncertainty* (Carlsson 2013). Briefly described, each component in a model, e.g. a compressor, a pipe, or a heat exchanger, is typically validated against some available data. The result from this low-level validation is information concerning component-level uncertainty. In the output uncertainty method, this information is integrated directly into the components. The uncertainties in a component's original parameters are aggregated and expressed in a smaller set of uncertainty parameters. In this way, the level of abstraction is raised from uncertainty of component input parameters to uncertainty of component output characteristics. It should be noted that the output uncertainty for each component may include not only parameter uncertainty but also estimations of model form uncertainty.

6.2 Simplify characterization of uncertainties

As seen in section 5, the characterization of uncertainties is a main contributor to the total UQ workload. Therefore, in addition to reducing the number of uncertain parameters, it is beneficial to simplify the characterization itself as far as possible. An approach that is reasonable for the ECS model is to abandon the use of PDFs of several different types and instead consistently use uniform distributions throughout the UQ – or even simpler, only use intervals with upper and lower bounds. Naturally, while this approach results in a less precise UQ, it is more worthwhile to make an approximate UQ than no UQ at all.

7 Conclusions

Uncertainty Quantification (UQ) is commonly seen as a value adding activity that increases the credibility of a simulation model. However, analysis of industrial applicability and estimations of the UQ workload are rare. This work has shown that a comprehensive but not excessive UQ of a large aircraft system simulation model may be very time-consuming. The UQ workload of the ECS model is estimated to be twice the time for the model development, verification, and steady-state validation.

As a comparison, our gut feeling of what a model sponsor may consider affordable in terms of UQ workload is, say, 10% relative to the sum of model development, verification, and steady-state validation. Furthermore, UQ does not have all the answers. Questions such as *"How uncertain is UQ?"* can always be asked. Will the system design be better and sounder and will we become wiser with much more UQ data? It should not be forgotten that even with poor or unknown accuracy of the simulation results, models are useful for many activities during the development phase.

To find a way forward, simplifications, compromises, and methods to ease the UQ workload have been discussed in this paper. Without adding too much uncertainty to the UQ, a number of simplifications are available. The most significant measures proposed in this paper concerns reducing the number of uncertain parameters, simplifying the characterization of uncertainties, and simplifying the uncertainty propagation. All in all, these simplifications make UQ of large aircraft system simulation models more feasible.

Despite the proposed simplifications, a comprehensive UQ will remain a significant part of the model development budget. However, if the UQ implies that a physical system-level test rig is no longer necessary or can be significantly simplified, a large UQ workload can motivated.

Finally, the NASA Standard for Models and Simulations describes factors that affect the credibility of the results, i.e. how useful they are (NASA 2008). In this case, "Result uncertainty" is only one of eight factors. "M&S management" and "People qualification" may be mentioned as two other factors. In other words, although a UQ is carried out in detail with high quality, it is still a fact that if the simulations that will be a basis for design decisions have been carried out with poor configuration management of the model and its inputs and by inexperienced personnel, the risk of large uncertainties and even errors in simulation results is high. A balance of available funds between the various factors that affect the credibility of the simulation result is desirable.

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