Joint User Selection and Beamforming Schemes for Inter-Operator Spectrum Sharing

Johannes Lindblom, Erik G. Larsson and Eleftherios Karipidis

Linköping University Post Print

N.B.: When citing this work, cite the original article.

Original Publication:

Johannes Lindblom, Erik G. Larsson and Eleftherios Karipidis, Joint User Selection and Beamforming Schemes for Inter-Operator Spectrum Sharing.

Manuscript.

Postprint available at: Linköping University Electronic Press
http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-100813
JOINT USER SELECTION AND BEAMFORMING SCHEMES FOR INTER-OPERATOR SPECTRUM SHARING

Johannes Lindblom*, Erik G. Larsson*, and Eleftherios Karipidis†

*Communication Systems Division, Department of Electrical Engineering (ISY), Linköping University, SE-581 83 Linköping, Sweden. {lindblom,erik.larsson}@isy.liu.se
†Ericsson Research, SE-164 80 Stockholm, Sweden. eleftherios.karipidis@ericsson.com

ABSTRACT
We consider the downlink of an inter-operator spectrum sharing scenario where two operators share the same piece of spectrum and use it simultaneously. A base station of one operator cooperates with a base station of the other operator in order perform joint user selection and beamforming using a central unit. Optimal scheduling, in the sense of maximizing sum-rate or proportional fairness, is in many cases impractical due to high computational complexity. Therefore, we propose a heuristic algorithm that schedules users based on simple beamforming techniques. Once the users are scheduled, we compute the optimal beamforming vectors for them. This method still performs an exhaustive user search. Therefore, we also propose a greedy user selection scheme. From numerical evaluations, we notice that these schemes perform close to the optimal selection. Also, we use our proposed methods to identify when spectrum sharing provides extra gains over the non-sharing scenario.

Index Terms—Beamforming, spectrum sharing, user selection.

1. INTRODUCTION
We study the joint user selection and beamforming problem in the downlink of a cellular network where two operators share the spectrum. The concept of spectrum sharing was proposed to overcome the spectrum scarcity in wireless networks, e.g., in [1]. Recently, in [2], it was shown that one of the most promising spectrum sharing techniques is when the users can switch to another operator than their contracted service provider on transmission time interval. We assume that two operators share a piece of spectrum and use it simultaneously. Hence, they cause interference to each others’ users. We model this scenario as two overlapping cellular networks as illustrated in Fig. 1. We assume that the base stations (BSs) are equipped with multiple antennas and serve \( K \) mobile stations (MSs) (or users) each. A BS of the first operator, we denote it BS\(_1\), coordinates its operation with the closest BS of the second operator, i.e., BS\(_2\), but they do not cooperate with the other BSs. Since we assume frequency reuse one, this is a worst-case interference scenario.

For the described setup, we consider proportional fair scheduling (PFS), which is implemented by performing weighted sum-rate (WSR) maximization. It is known that this problem is NP-hard even for the single-antenna case [3]. Therefore, in practical systems, suboptimal iterative distributed methods for the WSR problem are preferable, e.g., in [4]. However, these kinds of algorithms, apart from BS coordination, need feedback channels from MSs to BSs which are used in each iteration and consume resources. For the single-cell scenario, i.e., the multiple-input single-output (MISO) broadcast channel (BC), the user selection problem is well-studied, e.g., see [5]. The advantage is that there is no need to exchange information between BSs. In that case heuristic user selection algorithms are used only to reduce the computational complexity. Therefore, it is not straightforward to extend the methods developed for the MISO BC to the multi-cell setup.

In this initial study, we restrict to \( n_t = 2 \) BS antennas. In the spectrum sharing scenario we use one spatial degree-of-freedom (DoF) to serve one MS and the other DoF to suppress interference to the other BS’s MS. This setup is modeled by the two-user MISO interference channel (IC) for which we use our method [6] to efficiently and optimally find the maximum WSR. For the non-sharing scenario, modeled by the two-user MISO BC, we use the DoFs to serve two users. For this case we find the maximum WSR using the BC-multiple-access channel (MAC) duality [7, Ch.10]. Using these methods, we can efficiently validate the proposed algorithms with the globally optimal solutions.

We propose to use a two-phase algorithm. First, the algorithm selects users based on simple beamforming vectors such as zero-forcing (ZF) and maximum signal-to-leakage-plus-noise ratio (MSLNR) beamforming. The rate pairs associated with these beamforming vectors give an indication of the shape of the achievable rate region and, hence, the optimal WSR. The larger the WSR is the ZF or MSLNR beamforming vectors, the larger we expect the optimal WSR to be. Second, the algorithm chooses the pair of MSs that yield the largest WSR and performs optimal beamforming to improve the WSR. Since the computational complexity still is quite high, we propose a greedy user selection algorithm. First,
the algorithm selects the MS that achieves the largest single-user rate. Second, from the MSs of the other operator it selects the MS that achieves the largest WSR for the two MSs using either ZF or MSLNR beamforming. Recently, in [8], a method similar to ours was proposed. However, this is a decentralized method which involves more MS to BS communication than our method. We evaluate the proposed algorithms by comparing them to the cases of WSR-optimal beamforming for the selection of the MSs, uncoordinated selection and beamforming, and no spectrum sharing. The sum-rate degradation of the suboptimal algorithms is small and we point out scenarios where there are potential spectrum sharing gains.

2. SYSTEM MODEL

We focus on the two cooperating BSs and let MS$_k$ denote the $k$th MS of BS$_j$. We assume that the $K$ MSs per BS are selected from a larger set by an upper-layer application. The system occupies $N$ frequency-flat subchannels, where each subchannel is either a time or a frequency resource. Assuming that the scheduling algorithm selected the MS pair $(k_1, k_2)$, the received signal at MS$_{k_1}$ is

$$\text{y}_{k_1} = h_{1k_1}^H \text{w}_1 \text{s}_1 + h_{2k_1}^H \text{w}_2 \text{s}_2 + e_{k_1} + u_{k_1}. \quad (1)$$

In (1), $h_{jk} \in \mathbb{C}^n$ is the (conjugated) channel vector between BS$_j$ and MS$_k$, $\text{w}_j$ is the beamforming vector of the signal $s_j \sim \mathcal{CN}(0, 1)$ intended for MS$_j$ scheduled for BS$_j$. The transmit power per scheduled MS and subchannel is constrained as $\|\text{w}_j\|^2 \leq P$. We model the temporal noise at MS$_{k_1}$ by $e_{k_1} \sim \mathcal{CN}(0, \sigma^2_{k_1})$ and the interference from uncoordinated BSs by $u_{k_1} \sim \mathcal{CN}(0, I_{11})$. Note that for ease of exposition, we skip the subchannel index in (1). For the MS pair $(k_1, k_2)$ we have the situation modeled via the MISO IC and the achievable rate of MS$_{k_1}$, in bits per channel use (bpcu), assuming Gaussian coding and treating interference as noise, is

$$R_{k_1}(\text{w}_1, \text{w}_2) = \log_2 \left(1 + \frac{|h_{1k_1}^H \text{w}_1|^2}{h_{2k_1}^H \text{w}_2^2 + \sigma_{k_1}^2 + \|\text{s}_{k_2}\|^2} \right). \quad (2)$$

We model the channel vectors as

$$h_{jk} = \sqrt{\beta_{jk}} \tilde{h}_{jk}, \quad i, j = 1, 2, \quad k \in \{1, \ldots, K\}. \quad (3)$$

In (3), $\tilde{h}_{jk} \sim \mathcal{CN}(0, I)$ models the small-scale (fast) fading. We assume that the realizations of $h_{jk}$ are independent both over the users and the subchannels. By $\beta_{jk} \triangleq z_{jk} / \alpha_{jk}$, we model the effects of shadowing and path-loss. The log-normal random variable $z_{jk}$ models the shadow fading between BS$_j$ and MS$_k$, whereas $\alpha_{jk}$ is the normalized distance between BS$_j$ and MS$_k$, and $\alpha$ is the path-loss exponent. The path-loss and shadowing are the same for all subchannels and transmit antennas. We assume that each BS has perfect knowledge of the channels to the MSs both in its cell and the coordinated one.

We model the channels between the uncoordinated BSs and the coordinated MSs as in (3) and assume that the BSs use random beamforming vectors. The elements of these vectors are i.i.d. complex Gaussian and normalized to the desired transmit power. This assumption makes sense since the WSR optimal beamforming vectors are linear functions of the i.i.d. complex Gaussian channel vectors, see [6] and references therein. Moreover, we assume that the uncoordinated transmit with power $\rho P$. The activity factor $\rho$ models that a BS is active with probability $\rho$. For the case without and with full uncoordinated interference, we have $\rho = 0$ and $\rho = 1$, respectively.

3. SCHEDULING ALGORITHMS

We assume that PFS is performed. The reason is that we by using PFS can guarantee that all users get some service. A maximum sum-rate scheduler would select the strong users close to the BSs and ignore the remote users. The objective of the PFS is to maximize $\prod_{j=1}^K \prod_{k=1}^{K_j} R_{k_j}$ with $R_{k_j} \triangleq \mathbb{E} \{ R_{k_j} \}$, where the expectation is over the small-scale fading. This optimization is too complex and requires non-causal channel knowledge. To overcome that, we adapt the standard approximation, e.g., see [9]. On each subchannel $n$, we maximize

$$\sum_{j=1}^K \sum_{k_j=1}^{K_j} R_{k_j}/n_{k_j,n}, \quad \text{where} \quad n_{k_j,n} = 1 \quad \text{if} \quad k_j \neq k \quad \text{and} \quad n_{k_j,n} = 1 - 1/n_c \quad \text{if} \quad k_j = k. \quad (4)$$

$$T_{k_j,n+1} = (1 - 1/n_c) T_{k_j,n} + R_{k_j,n}/n_c. \quad \quad (5)$$

is a moving average of the throughput of MS$_{k_j}$ with $n_c$ being the time constant of the scheduler. The larger $n_c$, the longer will the scheduler wait for a MS to go from a bad to a good fading state. Since, we independently select one pair of MSs per subchannel, we formulate the WSR maximization problem

$$\max_{k_1, k_2} \left\{ \left( \frac{R_{k_1}(\text{w}_1, \text{w}_2)}{T_{k_1,n}} + \frac{R_{k_2}(\text{w}_1, \text{w}_2)}{T_{k_2,n}} \right) \right\}. \quad (6)$$

We obtain the optimal solution to (6) by solving the inner maximization for each MS pair $(k_1, k_2)$. The inner WSR maximization problem is NP-hard in general [3], but for the two-user scenario we find the solution really fast using our method in [6]. However, that method requires sampling of the boundary of the rate region. To avoid that, we propose that the first algorithm uses simple beamforming vectors to lower bound the optimal value of the inner maximization. The outer maximization is a combinatorial problem with complexity $O(K^2)$, which might be bothersome for large $K$. Therefore, we propose a greedy selection algorithm with complexity $O(K)$.

Algorithm 1 (exhaustive): The inner problem of (6) is approximately solved by using simple beamforming strategies with good performance, e.g. ZF or MSLNR beamforming vectors. For the pair $(k_1, k_2)$, the ZF beamforming vector is given as [10]

$$\text{w}_{k_1, k_2}^{\text{ZF}} = \arg \max_{\|\text{w}_1\| \leq 1} |h_{1k_1}^H \text{w}_1|^2 = \sqrt{P} \frac{\text{h}_{1k_1}}{\|\text{h}_{1k_1}\|}. \quad (7)$$

where $\text{h}_{1k_1}$ is the $k_1$th MS of BS$_1$ and $\|\cdot\|$ is the vector norm. The MSLNR beamforming vector is given as [11]

$$\text{w}_{k_1, k_2}^{\text{MSLR}} = \sqrt{P} \arg \max_{\|\text{w}_1\| \leq 1} |h_{1k_1}^H \text{w}_1|^2 / (\|\text{s}_{k_2}\|^2 + \|\text{s}_{k_1}\|^2) = \sqrt{P} \frac{\text{C}_1 \text{h}_{1k_1}}{\|\text{C}_1 \text{h}_{1k_1}\|}, \quad (8)$$

where $\text{C}_1 \triangleq I - P \text{h}_{1k_2} \text{h}_{1k_2}^H / (P \|\text{h}_{1k_2}\|^2 + \|\text{s}_{k_2}\|^2 + \|\text{s}_{k_1}\|^2)$. In general, ZF beamforming is suboptimal in the sense that the rate point lies inside or on the boundary of the achievable rate region. On the other hand, at high signal-to-noise ratio (SNR) it converges to the maximum sum-rate point. The MSLNR beamforming point lies on the outer boundary of rate region [11]. For the MS pair that achieves the highest WSR using either ZF or MSLNR beamforming, we compute the maximum WSR beamforming vectors using our method in [6]. As we will see in Sec. 4, the choice of ZF beamforming vectors leads to slightly lower rates than MSLNR. This exhaustive search algorithm is summarized in Tab. 1.

Algorithm 2 (greedy): To reduce the complexity we propose a greedy selection algorithm. First, it finds the MS that maximizes $\sum_{j=1}^K R_{k_j}/n_{k_j,n}$, then the MS that maximizes $\sum_{j=1}^K R_{k_j}/n_{k_j,n}$ among the remaining MSs. This is repeated until all MSs are scheduled.
In this section, we provide various numerical results in order to evaluate the performance of the proposed algorithms and to identify potential gains of performing inter-operator spectrum sharing. The parameters which are common for all simulations are given in Tab. 3. It should be noted that one Monte-Carlo (MC) run is one drop of MS positions. The MSs are uniformly placed in the cell outside the inner circle of normalized radius \( r_I \). The choice of \( N_s = 64 \) is comparable to the number of resource blocks on a 10 MHz band in LTE. The choice of \( n_c = 10 \) is quite small. On the other hand, it more or less corresponds to the number of resource blocks on a 10 MHz band in LTE. The solid and dashed curves are for \( K = 2 \) and \( K = 8 \) users, respectively. For the non-sharing scenario, each operator uses \( N_s/2 \) subchannels each.

For the sharing scenarios we also compare with the cases of uncoordinated scheduling but coordinated beamforming, i.e., the BSs select MSs separately but they cooperate in the beamforming design, and the case of no coordination at all, i.e., each BS uses MRT to its best MS, ignoring the interference.

In Figs. 3 and 4, we fix the distance between the cooperating BSs and vary the SNR. For uncoordinated interference in Fig. 3, we notice a gain of about 10% for the spectrum sharing case for \( K = 8 \). For \( K = 2 \) there is even a small loss by doing spectrum sharing. For the interference free scenario in Fig. 4, we observe a spectrum sharing gain of 10-20% for both \( K = 2 \) and \( K = 8 \).

In Figs. 5 and 6, we fix the cell-edge SNR and vary the BS distance. For uncoordinated interference in Fig. 5, we observe that spectrum sharing is only feasible if \( d_{BS} \) is small. The reason might be that the uncoordinated BSs come closer to the MSs when the \( d_{BS} \) increases. For the interference free scenario in Fig. 6, the spectrum sharing gain is 10-20% for both \( K = 2 \) and \( K = 8 \). The performances, except for that of no coordination, do not change much.

For all the cases in Figs. 3–6, we see that there are gains of 5-10% by coordinating both selection and beamforming. On the other hand this loss might vanish if we also consider the overhead of coordination. Clearly, if one considers to do spectrum sharing, the BSs should at least coordinate the beamforming design.

In Fig. 7, we use the exhaustive search algorithm to study multiuser diversity gains. We plot the median rate and compare it to the non-sharing case. We consider the cases of no uncoordinated interference (\( \rho = 0 \)), full interference (\( \rho = 1 \)), and low activity (\( \rho = 0.3 \)). Except for the cases of uncoordinated interference with a handful of MSs, we have spectrum sharing gains. Even at low activity, the performance is substantially decreased.

### 5. CONCLUDING REMARKS

We proposed two heuristic joint user selection and beamforming algorithms to mix an inter-operator spectrum sharing setup. We observed that both the exhaustive search and the greedy selection algorithms have only a small (approximately 1%) rate-loss compared to optimal selection and beamforming. It is hard to draw general conclusions on the potential of spectrum sharing in the way we treat it. For the case of only a few MSs per BS and uncoordinated interference should not be performed. On the other hand, for a large number of users, there are some potential gains.

Extensions of this work can be made in several directions. We should relax the restriction of only one scheduled MS per BS and extend the number of cooperating BSs, since larger cooperation clusters might yield higher spectrum sharing gains. Also, it will be important to study the problem of acquiring channel knowledge. Basically, the larger coordination cluster we have, the more orthogonal pilots we need, which will compete for signal space with the payload.

### 4. SIMULATION RESULTS

In Figs. 2–7, we use the proposed algorithms to identify potential spectrum sharing gains. We plot the average rate that the BSs achieve with an outage probability of at most 10% versus either the distance \( d_{BS} \) between the BSs or the cell-edge SNR. The solid and dashed curves are for \( K = 2 \) and \( K = 8 \) users, respectively. For the non-sharing scenario, each operator uses \( N_s/2 \) subchannels each.

For the sharing scenarios we also compare with the cases of uncoordinated scheduling but coordinated beamforming, i.e., the BSs select MSs with and without uncoordinated interference. The sum-rate loss of the proposed schemes is approximately 2% for exhaustive search with ZF and slightly less for both the exhaustive and greedy search algorithms with MSLNR.

In Figs. 3–6, we use the proposed algorithms to identify potential spectrum sharing gains. We plot the average rate that the BSs achieve with an outage probability of at most 10% versus either the distance \( d_{BS} \) between the BSs or the cell-edge SNR. The solid and dashed curves are for \( K = 2 \) and \( K = 8 \) users, respectively. For the non-sharing scenario, each operator uses \( N_s/2 \) subchannels each.

For the sharing scenarios we also compare with the cases of uncoordinated scheduling but coordinated beamforming, i.e., the BSs select MSs with and without uncoordinated interference. The sum-rate loss of the proposed schemes is approximately 2% for exhaustive search with ZF and slightly less for both the exhaustive and greedy search algorithms with MSLNR.

### Table 1. Description of Algorithm 1 (exhaustive).

| 1: **Input:** | \( P, h_{jk}, \sigma_{jk}^2, \) and \( s_{jk} \) for all \( j \) and \( k_j \) |
| 2: Initialize \( T_{k_j, 0} = 1 \) for \( j = 1, 2, \) and \( k_j = 1, \ldots, K \) |
| 3: for \( n = 1 : N_s \) |
| 4: Find the pair \((k_1, k_2)\) that maximizes the weighted sum-rate for either ZF or MSLNR beamforming vectors.
| 5: For the pair \((k_1, k_2)\), compute optimal max WSR beamforming vectors using [6].
| 6: Update the throughput \( T_{k_j, n} \) according to (5). Non-scheduled users are updated with \( R_{k_j, n} = 0 \). |
| 7: **end** |

### Table 2. Description of Algorithm 2 (greedy).

| 1: **Input:** | \( P, h_{jk}, \sigma_{jk}^2, \) and \( s_{jk} \) for all \( j \) and \( k_j \) |
| 2: Initialize \( T_{k_j, 0} = 1 \) for \( j = 1, 2, \) and \( k_j = 1, \ldots, K \) |
| 3: for \( n = 1 : N_s \) |
| 4: Find user that maximizes \( \log_2(1 + P \| h_{jk} \|^2) / T_{k_j} \). |
| 5: Find user of operator \( j \neq j \) such that WSR is maximized using either ZF or MSLNR beamforming vectors. |
| 6: For the pair \((k_1, k_2)\), compute optimal max WSR beamforming vectors using [6]. |
| 7: Update the throughput \( T_{k_j, n} \) according to (5). Non-scheduled users are updated with \( R_{k_j, n} = 0 \). |
| 8: **end** |

### Table 3. Parameters which are common in all simulations.

| Number of subchannels | \( N_s = 64 \) |
| Number of Monte-Carlo runs | \( N_m = 10000 \) |
| Normalized cell-radius | \( r_O = 1 \) |
| Normalized inner cell-radius | \( r_I = 0.1 \) |
| BS distance within an operator | \( \sqrt{3} \) |
| Path-loss exponent | \( \alpha = 3.5 \) |
| Shadowing | log-normal with std 4 dB |
| Small-scale (fast) fading | i.i.d. Rayleigh |
| Time-constant of PFS | \( n_c = 10 \) |
Fig. 2. Evaluation of user selection algorithms using maximum sum-rate scheduling with cell-edge SNR=10 dB and $d_{BS} = 0.5$.

Fig. 3. 10% outage-rate versus cell-edge SNR with uncoordinated interference for $d_{BS} = 0.5$.

Fig. 4. 10% outage-rate versus cell-edge SNR without uncoordinated interference for $d_{BS} = 0.5$.

Fig. 5. 10% outage-rate versus BS distance with uncoordinated interference for cell-edge SNR=10 dB.

Fig. 6. 10% outage-rate versus BS distance without uncoordinated interference for cell-edge SNR=10 dB.

Fig. 7. Illustration of multi-user diversity for PFS with cell-edge SNR=10 dB and $d_{BS} = 0.5$. 
6. REFERENCES


