$H_\infty$-Controller Design Methods Applied to One Joint of a Flexible Industrial Manipulator

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Abstract
Control of a flexible joint of an industrial manipulator using $\mathcal{H}_\infty$ design methods is presented. The considered design methods are i) mixed-$\mathcal{H}_\infty$ design, and ii) $\mathcal{H}_\infty$ loop shaping design. Two different controller configurations are examined: one uses only the actuator position, while the other uses the actuator position and the acceleration of end-effector. The four resulting controllers are compared to a standard PID controller where only the actuator position is measured. The choices of the weighting functions are discussed in details. For the loop shaping design method, the acceleration measurement is required to improve the performance compared to the PID controller. For the mixed-$\mathcal{H}_\infty$ method it is enough to have only the actuator position to get an improved performance. Model order reduction of the controllers is briefly discussed, which is important for implementation of the controllers in the robot control system.

Keywords: Robotics, Flexible, H-infinity control, Accelerometers
**Abstract:** Control of a flexible joint of an industrial manipulator using $H_\infty$ design methods is presented. The considered design methods are i) mixed-$H_\infty$ design, and ii) $H_\infty$ loop shaping design. Two different controller configurations are examined: one uses only the actuator position, while the other uses the actuator position and the acceleration of end-effector. The four resulting controllers are compared to a standard PID controller where only the actuator position is measured. The choices of the weighting functions are discussed in details. For the loop shaping design method, the acceleration measurement is required to improve the performance compared to the PID controller. For the mixed-$H_\infty$ method it is enough to have only the actuator position to get an improved performance. Model order reduction of the controllers is briefly discussed, which is important for implementation of the controllers in the robot control system.

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1. INTRODUCTION

The requirements for controllers in modern industrial manipulators are that they should provide high performance, at the same time, robustness to model uncertainty. In the typical standard control configuration the actuator positions are the only measurements used in the higher level control loop. At a lower level, in the drive system, the currents and voltages in the motor are measured to provide torque control for the motors. In this contribution different $H_\infty$-controller design schemes are compared when using two different sensor configurations. First, the standard case where only the position of the actuator rotation is used, and second a configuration where, in addition, the acceleration of the tool tip is measured. Two different $H_\infty$ methods are investigated: i) loop shaping [McFarlane and Glover, 1992], and ii) multi-$H_\infty$ design [mixedHinfDyn, 2013, Zavari et al., 2012].

Motivated by the conclusions from Sage et al. [1999] regarding the area of robust control applied to industrial manipulators, this contribution includes:

- results presented using realistic models,
- a comparison with a standard PID control structure,
- model reduction of the controllers to get a result that more easily can be implemented in practice.

The model used in this contribution represents one joint of a typical modern industrial robot [Moberg et al., 2009]. It is a physical model consisting of four masses, which should be compared to the typical two-mass model used in many previous contributions, see Sage et al. [1999] and the references therein. The joint model represents the first joint of a serial 6-DOF industrial manipulator, where the remaining five axes have been configured to minimise the couplings to the first axis. To handle changes in the configuration of the remaining axes, gain scheduling techniques can be used based on the results in this paper.

An important part of the design is the choice of the weighting functions, which is an essential task to get a satisfactory performance. In particular, the use of two measurements for control of one variable requires special treatment. The development of the weighting functions for the four controllers is briefly discussed, and provides a significant part of the contributions in the paper.

Controller synthesis using $H_\infty$ methods has been proposed in Song et al. [1992], Stout and Sawan [1992], where the complete non-linear robot model first is linearised using exact linearisation, second an $H_\infty$ controller is designed using the linearised model. The remaining non-linearities due to model errors are seen as uncertainties and/or disturbances. In both papers, the model is rigid and the $H_\infty$ controller, using only joint positions, is designed using the mixed-sensitivity method. In Sage et al. [1997] $H_\infty$ loop shaping with measurements of the actuator positions is applied to a robot. The authors use a flexible joint model which has been linearised. The linearised model makes it possible to use decentralised control, hence $H_\infty$ loop shaping is applied to n SISO-systems instead of the complete MIMO-system.

Explicit use of acceleration measurements for control in robotic applications has been reported in, for example, de Jager [1993], Dumetz et al. [2006], Kosuge et al.
For a Cartesian robot the joint acceleration is measured directly
by augmenting the original system

\[
F_l(P, K) = \begin{bmatrix}
W_u(s)G_{wu}(s) & -W_T(s)T(s) \\
-W_T(s)S(s) & W_S(s)
\end{bmatrix},
\]

where \( S(s) = (I + G(s)K(s))^{-1} \) is the sensitivity function, 
\( T(s) = I - S(s) \) is the complementary sensitivity function, 
and \( G_{wu}(s) = -K(s)(I + G(s)K(s))^{-1} \) is the transfer function from \( w \) to \( u \). The \( \mathcal{H}_\infty \) controller is then obtained
by minimising the \( \mathcal{H}_\infty \)-norm of the system \( F_l(P, K) \), i.e.,
minimise \( \gamma \) such that \( \|F_l(P, K)\|_\infty < \gamma \). Using (2) gives
\[
|W_u(i\omega)G_{wu}(i\omega)| < \gamma, \forall \omega,
\]
\[
|W_T(i\omega)T(i\omega)| < \gamma, \forall \omega,
\]
\[
|S(i\omega)S(i\omega)| < \gamma, \forall \omega.
\]

The transfer functions \( G_{wu}(s), S(s), \) and \( T(s) \) can now be shaped to satisfy the requirements by choosing the weights \( W_u(s), W_S(s), \) and \( W_T(s) \). The aim is to get a value of \( \gamma \) close to 1, which in general is a hard to achieve and it requires insight in the design method as well as the system dynamics. For more details about the design method, see e.g. Skogestad and Postlewaita [2005], Zhou et al. [1996].

The mixed-\( \mathcal{H}_\infty \) controller design [mixedHinfyn, 2013, Zavari a et al., 2012] is a modification of the standard \( \mathcal{H}_\infty \) design method. Instead of choosing the weights in (2) such that the norm of all weighted transfer functions satisfies (3), the modified method divides the problem into design constraints and design objectives. The controller can now be found as the solution to

\[
\text{Minimise} \quad \gamma
\]

subject to \( \|W_P(s)S(s)\|_\infty < \gamma \) (4b)
\( \|M_S(s)S(s)\|_\infty < 1 \) (4c)
\( \|W_u(s)G_{wu}(s)\|_\infty < 1 \) (4d)
\( \|W_T(s)T(s)\|_\infty < 1 \) (4e)

where \( \gamma \) not necessarily has to be close to 1. The method can be generalised to other control structures and in its general form formulated as a multi-objective optimisation problem. More details about the general form and how to solve the optimisation problem are presented in mixedHinfyn [2013], Zavari et al. [2012].

### 2.2 Loop Shaping using \( \mathcal{H}_\infty \) Synthesis

For loop shaping [McFarlane and Glover, 1992], the system \( G(s) \) is pre- and post-multiplied with weights \( W_1(s) \) and \( W_2(s) \), see Figure 1(b), such that the shaped system \( G_s(s) = W_2(s)G(s)W_1(s) \) has the desired properties. The controller \( K_s(s) \) is then obtained using the method described in Glover and McFarlane [1989] applied on the system \( G_s(s) \), giving the controller \( K_s(s) \). Finally, the controller \( K(s) \) is given by

\[
K(s) = W_1(s)K_s(s)W_2(s).
\]

Note that the structure in Figure 1(b) for loop shaping can be rewritten as a standard \( \mathcal{H}_\infty \) problem according to Figure 1(a), see Zhou et al. [1996] for details. It will be used in Section 6 for synthesis of low order controllers.

The MATLAB function ncfsyn, included in the Robust Control Toolbox, is used in this paper for synthesis of \( \mathcal{H}_\infty \) controllers using loop shaping.

### 3. Flexible Joint Model

The model considered in this paper is a four-mass benchmark model of a single flexible joint, see Figure 2, presented in Moberg et al. [2009]. The model corresponds to
In (6), \( \eta \) is the gear ratio and \( l_1, l_2, \) and \( l_3 \) are the respective link lengths.

Using Lagrange’s equation, the linearised flexible joint model can be described by a set of four ODES, which can be reformulated as a linear state space model according to

\[
\begin{align*}
\dot{x} &= Ax + Bu + B_w w, \quad (7a) \\
y &= Cx + Du + D_w w. \quad (7b)
\end{align*}
\]

where the state vector and disturbance vector are given by

\[
\begin{align*}
x &= \begin{pmatrix} q_m \ q_{a1} \ q_{a2} \ q_{a3} \ \dot{q}_m \ \dot{q}_{a1} \ \dot{q}_{a2} \ \dot{q}_{a3} \end{pmatrix}^T, \quad (8a) \\
w &= \begin{pmatrix} w_m \ w_p \end{pmatrix}^T. \quad (8b)
\end{align*}
\]

The linear state space model is used for design of the \( H_\infty \) controllers. Note that the matrices \( C, D, \) and \( D_w \) are different depending on which sensor configuration that is used, whereas the matrices \( A, B, \) and \( B_w \) stay the same.

4. DESIGN OF CONTROLLERS

In this section, four controllers based on the methods in Sections 2.1 and 2.2 are considered, using only the motor angle \( q_m \) or both \( q_m, \) and \( \dot{q}_m \) and the acceleration of the end-effector \( \ddot{P}. \) The controllers are

1. \( \mathcal{H}^{ls}_\infty(q_m): \) Loop shaping controller using \( q_m. \)
2. \( \mathcal{H}^{ls}_\infty(q_m, \ddot{P}): \) Loop shaping controller using \( q_m \) and \( \ddot{P}. \)
3. \( \mathcal{H}^m_\infty(q_m): \) Mixed-\( H_\infty \) controller using \( q_m. \)
4. \( \mathcal{H}^m_\infty(q_m, \ddot{P}): \) Mixed-\( H_\infty \) controller using \( q_m \) and \( \ddot{P}. \)

The four controllers are compared to a PID controller where only \( q_m \) is used. The PID controller is tuned to give the same performance as the best controller presented in Moberg et al. [2009].

To get high enough gain for low frequencies, without having the pole exactly in 0, the break-point of the magnitude function has to be very small, around \( 10^{-5} \) rad/s. From Figure 3 it can be seen that the main dynamics of the system is present in the frequency interval 30-110 rad/s.

4.1 Requirements

The controllers using \( H_\infty \) methods are designed to give better performance than the PID controller. In practice it means that the \( H_\infty \) controllers should attenuate the disturbances at least as much as the PID controller and the cut-off frequency should be approximately the same.

In Figure 3, the singular values of the systems from \( w \) to \( y = q_m \) and \( w \) to \( y = (\ddot{q}_m, \ddot{P})^T \) show that an integrator is present. It means that in order to attenuate the disturbances, it is required to have at least two integrators in the open loop \( GK. \) Since \( G \) already has one integrator, the other integrators have to be included in the controller \( K. \) For controllers 2 and 4, an integrator will be present if \( W_1 \) or \( W_2 \) include an integrator, recall (5). The requirements for controllers 1 and 3 become that \( |S(\omega)| \to 0 \) for \( \omega \to 0. \) Note that it is not possible to stabilise the plant \( P(s) \) with marginally stable weights. Instead the pole has to be moved into the left half plan a small distance.

4.2 Choice of Weights
4.2.1. \(H_\infty^{la}(q_m)\): Using only \(q_m\) as a measurement gives a \(siso\)-system, hence \(W_1\) and \(W_2\) are scalar transfer functions. For a linear \(siso\)-system it is possible to use one of \(W_1\) and \(W_2\) since the transfer functions commute with the system \(G(s)\). Therefore, \(W_1(s) = 1\) and \(W_2(s)\) is chosen such that the desired loop shape is obtained. First of all, it is necessary to have an integrator as discussed above. Having a pure integrator will lead to that the phase margin will be decreased, a zero in \(s = -10\) is therefore added in order not to change the loop gain for frequencies above 10 rad/s. Next, the gain is increased to get the desired cut-off frequency. The result using the weight is that the loop shape has peaks above 30 rad/s. To reduce the magnitude of the peaks a modified elliptic filter\(^2\)
\[
H(s) = \frac{0.5227s^2 + 3.266s + 1406}{s^2 + 5.808s + 2324} \tag{9}
\]
is introduced in \(W_2\). The weights are finally given as
\[
W_1(s) = 1, \quad W_2(s) = \frac{s + 10}{s} H(s). \tag{10}
\]
Using \text{ncfsyn} a controller of order 13 is obtained.

4.2.2. \(H_\infty^{ls}(q_m, \hat{P})\): Adding an extra measurement signal in terms of the acceleration of the end-effector gives a system with one input and two outputs. For stability reasons, it is not possible to have an integrator in both control channels. Therefore, the integrator is placed in the channel for \(q_m\), since the accelerometer measurement has low frequency noise, such as drift. For the same reason as for the other controller, a zero in \(s = -3\) is introduced. The transfer function from input torque to acceleration of the end-effector has a high gain in the frequency range of interest. To decrease the gain such that it is comparable with the motor angle measurement, a low pass filter is added in the acceleration channel. The final weights are
\[
W_1(s) = 50, \quad W_2(s) = \text{diag} \left( \frac{s + 3}{s}, \frac{0.2}{(s+5)^2} \right), \tag{11}
\]
giving a controller of order 13. Introducing an elliptic filter to attenuate the peaks in the open loop did not give the same results as for the \(H_\infty^{la}(q_m)\)-controller. Instead of improving the loop gain, the elliptic filter made it worse.

4.2.3. \(H_\infty^{m}(q_m)\): For this controller, four different weights have to be chosen, recall (4). The weight \(M_S\) should limit the peak of \(S\) and is therefore chosen to be a constant\(^3\). The peak of \(G_{uu}\) is also important to reduce in order to keep the control signal bounded, especially for high frequencies. A constant value of \(\theta_u\) is therefore also chosen. In the spirit of try simplest thing first, the weight \(W_T\) is also chosen to be a constant

In order to attenuate the disturbances it is, as was mentioned above, necessary to have at least one integrator in the controller. Forcing \(S\) to 0 is the same as letting \(W_P\) approach \(\infty\) when \(\omega \to 0\). To get a proper inverse, a zero is also included in the weight. Since a pure integrator is not used, the slope of the weight has to be higher than 20 dB per decade frequency, in order to force \(S\) to be low enough. This was accomplished by taking the weight to the power of 3 (2 was not enough). The numerical values of the weights are chosen as
\[
W_u = 10^{-50/20}, \quad W_T = 10^{-10/20}, \quad M_S = 10^{-10/20}, \quad W_P = \left( \frac{s + 100.1}{s + 0.1} \right)^3. \tag{12a}
\]
The constant weights in the form \(10^{-\alpha/20}\) can be interpreted as a maximum value, for the corresponding transfer function, of \(\alpha\) dB. The resulting controller is of order 10.

4.2.4. \(H_\infty^{m}(q_m, \hat{P})\): Like for the controller \(H_\infty^{la}(q_m, \hat{P})\), designing the weights for the mixed-\(H_\infty\) method becomes somewhat more involved with two measurements and one control signal. The aim is to attenuate the disturbances influence on the end-effector position. A variant is to find a rough estimate of the end-effector position and then choosing the weights from that. A straightforward estimate of \(\hat{P}\) using \(\hat{P}\) is
\[
\hat{P} = \frac{1}{s} \hat{\theta}. \tag{13}
\]
Due to low frequency drift and bias in an accelerometer, this estimate is only useful for high frequencies. A high pass filter is therefore used according to
\[
\hat{P}_{\text{high}} = c_2 \frac{s^2}{(p+s)^2} \frac{1}{s^2} \hat{\theta} = c_2 \frac{1}{(p+s)^2} \hat{\theta} \tag{14}
\]
where \(c_2\) and \(p\) are constants to choose. Another straightforward estimate of \(\hat{P}\) is to use the motor angle \(q_m\) according to \(\hat{P} = \hat{l}q_m\), where \(\hat{l}\) is the length of the arm. Compared to the estimated position using the acceleration, this new estimate is only valid for low frequencies. Using a low pass filter gives an estimate for low frequencies. It is important that the two estimates do not overlap each other, hence the low pass filter is chosen as the complementarity to the previous used high pass filter. The low frequency estimate is now given by
\[
\hat{P}_{\text{low}} = c_1 \left( 1 - \frac{s^2}{(p+s)^2} \right) \hat{l}q_m = c_1 \frac{2s+p}{(p+s)^2} \hat{l}q_m \tag{15}
\]
where \(c_1\) is a design variable. The final estimate of \(\hat{P}\) is the sum of the two estimates above, hence
\[
\hat{P} = \frac{c_1 \frac{2s+p}{(p+s)^2} \hat{l} + c_2 \frac{1}{(p+s)^2} q_m}{\varphi} \tag{16}
\]
Using the weights
\[
M_S = \tilde{M}_S W, \quad W_P = \tilde{W}_P W, \quad W_T = \tilde{W}_T W, \tag{17}
\]
where \(M_S, \tilde{W}_P, \) and \(\tilde{W}_T\) can be designed in a similar way as in Section 4.2.3, makes it possible to use more than one output together with one input. The last weight \(W_u\) can be chosen similar as in Section 4.2.3. The numerical values of the weights are
\[
W_u = 10^{-40/20}, \quad W = \left( \frac{30s + 75}{(s + 5)^2} \right) \frac{0.1}{(s + 5)^2}, \tag{18a}
\]
\[
\tilde{M}_S = 10^{-2/20}, \quad \tilde{W}_P = \left( \frac{s + 80}{s + 0.15} \right)^3 \tag{18b}
\]
and it turns out that \(W_T\) is not needed for the performance. Using these weights results in a controller of order 13.
controller. The robot joint model is implemented in continuous time whereas the controllers operate in discrete time. The continuous-time controllers developed in Section 4, are therefore discretized using Tustin’s formula. The measurements are affected by a time delay of one sample as well as zero mean normal distributed measurement noise. The sample time is $T_s = 0.5\, \text{ms}$.

The system is excited by a disturbance signal $w$ containing steps and chirp signals on both motor and end-effector. The performance is evaluated using a performance index, which is a weighted sum of different properties in the simulated end-effector position and motor torque. The reader is referred to Moberg et al. [2009] for complete details about the disturbance signals and the performance index.

Figure 5 shows how the motor torque differs between the five controllers. In the upper diagram it can be see that $H_{\infty}^{l}(q_m)$ gives higher torques than the PID and the $H_{\infty}^{m}(q_m)$ controllers. The PID gives higher torque noise during steady state due to the gain of the controller for high frequencies, recall Figure 4. In the lower diagram in Figure 5 it is shown that the controllers $H_{\infty}^{l}(q_m, \ddot{P})$ and $H_{\infty}^{m}(q_m, \ddot{P})$ give similar torque signals, and lower compared to the PID controller. A low torque signal is preferred to reduce the energy consumption and to decrease the wear in the motor and gear.

The end-effector position is presented in Figure 6. In the top graph it is seen that $H_{\infty}^{l}(q_m)$ gives, compared to the PID, higher oscillations during the time intervals 10-15 s and 37-42 s, which corresponds to a chirp disturbance at the end-effector. For step disturbances and chirp disturbances on the motor (time intervals 16-21 s and 43-58 s) $H_{\infty}^{l}(q_m)$ and the PID are more similar. The controller $H_{\infty}^{m}(q_m)$ is better than the other two controllers in the simulation. The bottom graph shows that $H_{\infty}^{m}(q_m, \ddot{P})$ can handle the chirp disturbances on the motor (time intervals 16-21 s and 43-58 s) and step disturbances very good. For a chirp disturbance on the end-effector, the two $H_{\infty}$ controllers give similar results. For step disturbances, the controller $H_{\infty}^{m}(q_m, \ddot{P})$ gives lower peaks than the PID controller, however the settling time is longer.

The performance index for the five controllers is presented in Table 1. It shows, as discussed above, that $H_{\infty}^{l}(q_m)$ and the PID controller behave similar and that $H_{\infty}^{l}(q_m, \ddot{P})$ and $H_{\infty}^{m}(q_m)$ give similar behaviour. The $H_{\infty}^{m}(q_m, \ddot{P})$-controller gives the best result.

The steady state error of approximately 2 mm after 25 s is a result of a constant torque disturbance on the end-effector. The size of the error will depend on the size of the disturbance and the stiffness of the joint. The motor position, which is measured, is controlled to zero for all five controllers.

5. SIMULATION RESULTS

The five controllers are evaluated using a simulation model. The simulation model consists of the flexible joint model described in Section 3, a measurement system, and a

![Loop gain $|KG|$ and controller gain $|K|$ for the five controllers.](image)

![Controller gain $|K|$](image)

**Table 1. Performance index for the five controllers, where lower value is better.**

<table>
<thead>
<tr>
<th>PID</th>
<th>$H_{\infty}^{l}(q_m)$</th>
<th>$H_{\infty}^{l}(q_m, \ddot{P})$</th>
<th>$H_{\infty}^{m}(q_m)$</th>
<th>$H_{\infty}^{m}(q_m, \ddot{P})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>55.7</td>
<td>55.4</td>
<td>45.8</td>
<td>42.4</td>
</tr>
</tbody>
</table>
Fig. 5. Applied motor torque from the five controllers. The top graph shows the controllers using only \( q_m \). The bottom graph shows the PID and the controllers using \( q_m \) and \( \ddot{P} \).

6. LOW ORDER SYNTHESIS

For implementation of the controller in the robot control system it is important to have a low order controller. A controller in state space form requires \( O(n_x^2) \) operations to calculate the control signal, where \( n_x \) is the dimension of the state vector in the controller.

The low order controllers are here obtained using the MATLAB function \textit{hinstruct}, which is included in Robust Control Toolbox and it is based on techniques from Apkarian and Noll [2006].

To find controllers with low orders using \textit{hinstruct} requires a model description, including the weights, in the form of (1). This structure is already used for the controllers \( H_{\infty}^m(q_m) \) and \( H_{\infty}^m(q_m, \ddot{P}) \), hence it is straightforward to synthesize low order controllers using the weights presented in Sections 4.2.3 and 4.2.4. For the loop shaping design method, the structure in Figure 1(b) can be rewritten in the form of (1) including the weights \( W_1(s) \) and \( W_2(s) \), explained in e.g. Zhou et al. [1996]. Using the rewritten structure, the low order controllers based on \( H_{\infty}^m(q_m) \) and \( H_{\infty}^m(q_m, \ddot{P}) \) can be obtained using the weights from Sections 4.2.1 and 4.2.2.

Table 2 shows the lowest order for the respective controllers, that can be achieved without changing the closed-loop performance too much. The table also shows the performance index obtained when the controllers are used in the simulation environment. The orders can be reduced by a factor of two to three but the performance of the reduced order controllers is worse than the full order controllers. Since the controller based on loop shaping with only \( q_m \) as measurement has the same performance for the full order controller as the PID controller, the low order controller gives a worse performance than the PID controller. The other full order controllers are much better than the PID controller and afford to get a reduced performance for the low order controllers without getting worse than the PID controller.

Finally, note that the controllers only represents local minima solutions, hence rerunning \textit{hinstruct} with other initial values can give a better, or worse, controller. To handle this, several initial values have been used in \textit{hinstruct}.

7. CONCLUSIONS AND FUTURE WORK

Four different \( H_{\infty} \) controllers for a flexible joint of an industrial manipulator are designed using mixed-\( H_{\infty} \) controller design and the loop shaping method. The model, on which the controllers are based, is a four-mass model.
As input, the controllers use either only the motor angle or both the motor angle and the acceleration of the end-effector. Tuning of the controllers requires understanding of both the synthesis method and how the system behaves. For example, the measurements for the mixed-$H_{\infty}$ controller are first pre-filtered to give an estimate of the tool position. The weighting functions for the resulting SISO system, from input torque to the estimated tool position, are then chosen similar to the case where only the motor position is used.

The controllers are compared to a PID controller and it is shown that if only the motor angle is measured it is much better to use the mixed-$H_{\infty}$ design method compared to loop shaping. If instead the end-effector acceleration is added then the performance is improved significantly for both methods. The steady state error for the end-effector position is unaffected since the accelerometer does not provide low frequency measurements. Using a low order controller synthesis method, it is possible to reduce the order of the controllers by a factor of two to three but at the same time a decrease in the performance index of 10–30% can be observed.

Investigation of robustness for stability with respect to model errors is one of several future directions of research. The mixed-$H_{\infty}$ method has an advantage compared to the loop shaping method since a model of the error is possible to incorporate in the augmented plant $P(s)$.

Another continuation is to investigate the improvement for other types of sensors. One possibility is to have an encoder measuring the position directly after the gearbox, i.e., $q_m$. This will improve the stiffness of the system, although it will not eliminate the stationary error for the end-effector position. The ultimate solution is to measure the end-effector position, but for practical reasons this is in general not possible, instead the end-effector position can be estimated, as described in Axelsson [2012], Axelsson et al. [2012], Chen and Tomizuka [2013], and used in the feedback loop.

Extending the system to several joints giving a non-linear model, which has to be linearised in several points, is also a future problem to investigate. A controller, using the results from this paper, is designed in each point and for example gain scheduling can be used when the robot moves between different points.

### References


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