Information Entropy in the Design Process

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Abstract: In this paper the design process is viewed as a process of increasing the information of the product/system. Therefore it is natural to investigate the design process from an information theoretical point of view. The design information entropy is introduced as a state that reflects both complexity and refinement, and it is argued that it can be useful as some measure of design effort and design quality. The concept of design information entropy also provides a sound base for defining creativity as the process of selecting areas for expanding the design space in useful direction, “to think outside the box”, while the automated activity of design optimization is focused, so far, on concept refinement, within a confined design space. In this paper the theory is illustrated on the conceptual design of an unmanned aircraft, going through concept generation, concept selection, and parameter optimization.

Keywords: information entropy, design complexity, product platform

1 Introduction

During the process of design, information is gradually increased as the design progress, and the uncertainty of the design is reduced. Every design decision reduces the uncertainty of the design, as well as parameter calculations do. It could be argued that this is the central aspect of design. Design in general is about increasing the information of the product/system. This can be viewed as a learning process, as described in [2]. Therefore design theory should really be a theory of design information. Although there is a rich literature regarding design process, there has been little or no effort to describe the generation of information in quantitative terms.

The classical information theory of communication was founded in 1948 by C.E. Shannon with his paper “A Mathematical Theory of Communication” [1]. Subsequently it has been recognized that information is a key property of design, and for describing and analyzing the design process. The notion of information theory in design has been introduced by several authors. Notables are N.P. Suh[Error! Reference source not found.,3], Kahn and Angeles [8] and Frey and Jahingir [6]. The two first are discussed later in the text. Frey and Jahingir deals with the transformation of information content in design parameters to information content in the functional characteristics, so is Bras and Mistree [10] where robustness is defined from maximizing the signal to noise ratio,

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which also deals with the relation between design parameters and functional characteristics. Information theory was also used to analyze design optimization in Krus and Andersson [11]. Information theory has also been used to define complexity in software, and notably by Bansiya et al [9] to describe complexity in object oriented systems, which is very close to general design.

The notion of information is used in the second axiom of N.P. Suh's Axiomatic design, which states that the information content in a design should be minimized [Error! Reference source not found.].

2 The characteristics of design information entropy

2.1 Design Information

In a product, the design information x is transformed into functional attributes y

$$y = f(x) \quad (1)$$

The design information could be the bit string in a CAD-file, and/or the set of parameters in a design that at some stage are free. It could also be the genome in an organism. The design information is the code of the design space D ; every bit-combination represents a unique design in the design space. At this stage it is not necessary to separate design parameters from system architecture, it is all included in x . If only little information is present, only parts of the design space can be excluded, the design can be any of several unique designs. This introduces a presence of uncertainty, since the design is not precisely known. An important aspect of design information is that it can only be defined relative to a design space. The design space need not, however, be static, but can be expanded if found necessary. This is also the case in the genome in biological systems, where, different organism has different sizes of the genome. The definition of design information used here is therefore:

Design information is the information needed to define a design, relative to a design space, to within a certain precision.

As a consequence, information is the inverse of uncertainty since lower precision results in less design information needed to describe a design.

2.2 Design Space

In order to generate a concept, a design space has to be established first. The design space contains all the possible designs. A LegoTM set is an example of a design space. A large number (although finite) number of designs can be build from a particular set. Another design space is represented by all the different Lego pieces. Different finite design spaces are then represented by the different number of pieces allowed in the design i.e 1,2,...n pieces.

Figure 1. Design in a design space of two lego bricks.

The design space of a set of Lego bricks represents all (discrete) combinations of arranging these bricks, n_{Dstate} . With a set of only two bricks with four knobs on each there are 51 discrete possible arrangements (two of these represents picking only one brick and one state is to pick no one).

The 51 different configuration (states) means that the amount of information needed to specify a particular design is:

$$I_x = \log_2 n_{Dstate} = \log_2 51 = 5.7\text{bits} \quad (2)$$

Another example is the design space provided by all standard components, or a product platform i.e a car platform that is used to generate different cars in a product family. Design space generation is also made in parameterization of models such as CAD models or simulation models. By coupling parameters to each other to reflect different constraints in the design, a smaller more efficient design space can be produced, where waste in the form of unfeasible designs is minimized. This means that less information is needed to arrive at a particular design from the design space.

2.3 Design Information Entropy

The definition of Information entropy for the discrete case is defined by Shannon [1]

$$H_d = \sum_{i=1}^n p_i \log_2 p_i \quad (3)$$

Here the system can be in n different states with probabilities p_i for each of them.

A more general definition than the information entropy for the discrete case is the differential information entropy for continuous signals, defined by Shannon [1] as:

$$H_c = - \int_{-\infty}^{\infty} p(x) \log_2(p(x)) dx \quad (4)$$

This gives a measure of the average information content of a variable x . Here $p(x)$ is the probability density function. One problem with this expression is that it does not make sense unless x is dimensionless, since the probability density function has the unit of the inverse of x . If not, the differential entropy of the probability density function $p(x)$ needs to be related to another distribution $m(x)$. The result is called the Kullback-Leibler divergence [5] from the distribution $m(x)$. This is the relative entropy, and it is defined as:

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$$H_{rel} = \int_{-\infty}^{\infty} p(x) \log_2 \left(\frac{p(x)}{m(x)} \right) dx \quad (5)$$

This is the difference in entropy between having information that a random variable is within $m(x)$, and knowing that it is within the distribution $p(x)$. Furthermore, it represents a measure of information in bits. It can also be generalized to any dimensionality.

$$H_{rel} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p(x_1 \dots x_n) \log_2 \left(\frac{p(x_1 \dots x_n)}{m(x_1 \dots x_n)} \right) dx_1 \cdots dx_n \quad (6)$$

A rectangular distribution of $m(x)$ in the bounded interval $x \in [x_{\min}, x_{\max}]$, with $x_R = x_{\max} - x_{\min}$, would mean that the distribution $m(x)$ of the *design space* is a space of equal possibilities, where no particular region can be considered more likely than another a priori. Other distributions can also be considered but they can always be mapped on a rectangular distribution by transforming the design space, which can be very useful (this also includes infinite distributions), for i.e. design optimization. Equation (5) can then be rewritten as:

$$I_x = H_{rel}(x) = \int_{x_{\min}}^{x_{\max}} p(x) \log_2 (p(x)x_R) dx \quad (7)$$

The letter I is here used here to indicate relative information entropy related to a rectangular distribution, and it has the unit *bits*. For the multidimensional case it becomes:

$$I_x = \int_{x_{1,\min}}^{x_{1,\max}} \cdots \int_{x_{n,\min}}^{x_{n,\max}} p(x_1 \dots x_n) \log_2 (p(x_1, \dots, x_n)x_{R1} \cdots x_{Rn}) dx_1 \cdots dx_n \quad (8)$$

This can also be written in a more compact form as:

$$I_x = \int_D p(\mathbf{x}) \log_2 (p(\mathbf{x})S) d\mathbf{x} \quad (9)$$

where D is the design space. I_x is defined as the *design information entropy* where the design $\mathbf{x}$ is defined within the design space D . S is the size of the design space and is defined as:

$$S = \int_D \mathbf{x} d\mathbf{x} \quad (10)$$

If the range of one variable is divided into equal parts Δx that have the same probability, the probability density distribution will be:

$$\begin{aligned} p(x) &= \frac{x_R}{\Delta x} : x \in [x_{0,\min}, x_{0,\max}] \\ p(x) &= 0 : x \notin [x_{0,\min}, x_{0,\max}] \end{aligned} \quad (11)$$

where:

$$x_R = x_{0,\max} - x_{0,\min} \quad (12)$$

This yields the information content (in bits) for that variable as:

$$\begin{aligned} I_x &= \int_{-x_{\min}}^{x_{\max}} \frac{x_R}{\Delta x} \log_2 \left(\frac{x_R}{\Delta x} \right) dx = \\ &= \log_2 \frac{x_R}{\Delta x} = \log_2 \frac{1}{\delta_x} = -\log_2 \delta_x \end{aligned} \quad (13)$$

where Δx is the uncertainty of the variable, and x_R its design range. δ_x is introduced as the relative uncertainty in parameters. The same expression holds if the probability distribution is normally distributed. In that cases

$$\delta_x = \frac{2\sigma_x}{x_R} \quad (14)$$

Here σ_x is the standard deviation in x . In the following text it is assumed that the uncertainty can be described by δ_x . If the legoTM example is expanded with an axis, the position of the inserted axis represents a continuous variables x . The information entropy associated with that, is dependent on the accuracy Δx with which it is specified, and the number of discrete positions (three) where it can be placed.

$$I_x = \log_2 n'_{States} + \log_2 \frac{x_R}{\Delta x} \quad (15)$$

Figure 2. Design with both discrete and continuous variables.

The axis can be in three positions (adding three discrete states) and if the position of the axis within one hole is specified within 10% the total information entropy is:

$$I_x = \log_2 (51 + 3) + \log_2 \frac{1}{0.1} = 8.2 \text{bits} \quad (16)$$

The concept of design information entropy hence provides a framework for defining design information in very general terms. This is the information that causes the uncertainty of the design to be shrunk from an initial state high uncertainty, to another state with less uncertainty.

3 Design information entropy in the design process

3.1 Design space generation

In information theoretical terms the design space corresponds to the reference distribution $m(x)$ of the Kullback-Leibler divergence, or in this case the design space D_0 , with the size S_0 , against which the design information entropy is defined. Within the design space concepts s are generated. In general only part of the design space falls within the constraints of the design D_c , with the size S_c , finally the refined final design represents only a small fraction of a concept s_δ .

Figure 3. Subspaces of the design space.

3.2 Concept generation

In concept generation, the design space is limited by selecting n_0 subsets of possible concepts for further analysis. This represents an increase in information. If the design space distribution $m(x)$ is rectangular and uniform over the design space D_0 and each concept i occupies the region with the size S_i the information generated in this phase is:

$$I_I = -\log_2 \frac{\sum_{i=1}^{n_0} S_i}{S_0} \quad (17)$$

In general the amount of information generated in this stage is quite large. It involves selecting a few concepts from a highly dimensional multi modal design space.

3.3 Concept screening

In this phase concepts that have no or little possibility of being successful are eliminated. If the concepts are reduced from n_0 to n concepts in this phase, the information added is:

$$I_H = -\log_2 \frac{\sum_{i=1}^n s_{j(i)}}{\sum_{i=1}^{n_0} s_i} \quad (18)$$

For the special case that all regions are of the same size it becomes:

$$I_H = -\log_2 \frac{n}{n_0} \quad (19)$$

3.4 Aircraft design example

Aircraft design is a good example of a complex product that involves all the phases illustrated in **Error! Reference source not found..**

Figure 4. Some UAV configurations.

Design space generation constitute the process of collecting possible elements needed for the design in response to functional requirements. Looking at existing design there is a variety in concepts that can be dissected into components to recreate a design space from where they all can be derived. From the example in Figure 4, there are various arrangements for wing and tail arrangements, engine location, etc.

There is a tail at the end of the fuselage, or at a twin boom arrangements, there are inverted butterfly tail and conventional tail. The engines can be mounted front or rear. The horizontal stabilizer can be mounted aft or forward (canard configuration) and

With these as example a design space can be defined some design elements can be identified.

- The horizontal stabilization front, aft or integrated in the main wing. In the aft configuration it can also be integrated with the vertical stabilizer in a butterfly tail.
- Vertical stabilization can be central or at wing tip, or integrated with horizontal configuration. It can be upwards or downwards.
- The tail arrangement can be located on a single fuselage or on twin boom arrangements.
- Although these example all have pusher prop (in order to have a clear front view for sensors), a prop in the front is of course also possible. The fuselage could be a single or with a twin boom aft section.

One popular tool for concept generation is the morphological matrix (or table). It was introduced in [10]. Here a table is set up where, for each function, a list of alternative solution elements is presented. A specific concept is obtained by selecting one solution element for each function. The morphological matrix represents a tool to display a design space of possibilities. Table 1 shows a morphological matrix for aircraft configuration.

Design elements	Alternative solutions				
Horizontal stabilization	Front (canard)	Aft	Aft fin integrated	Wing integrated	
Vertical stabilization	Central	Wing tip	Integrated	Upper	Lower
Tail mount	Single fuselage	Twin boom			
Propulsion	Tractor	Pusher			

Table 1. Morphological matrix for aircraft.

The total number of possibilities, n_s , is in the general case:

$$n_s = \prod_{i=1}^{n_f} n_{m,i} \quad (20)$$

where n_s is the number of functions. In the example this becomes functions:

$$n_s = 4 \times 5 \times 2 \times 2 = 80 \quad (21)$$

This represents information entropy of

$$I_w = I_c = -\log_2 \frac{s_c}{s} = -\log_2 \frac{1}{80} = 6.32 \text{ bit} \quad (22)$$

That means that selecting one of the configurations in the design space represents 6.32 bits of information. An interesting property of the information entropy is that it is roughly proportional to the number of design elements.

3.5 Concept optimization and selection

For making concept selection, it is really necessary to do an optimization of each concept left, to investigate its properties at the optimal parameter set. The optimization process then represents the contraction of the domain for each concept down to a domain of specified tolerance s_δ . The increase in information for optimizing all concepts and selecting one concept, k , is represented by:

$$I_{III} = -\log_2 \frac{s_{\delta k}}{\sum_{i=1}^n s_{j(i)}} \quad (23)$$

An initial optimization of aircraft concept can be done using handbook formula based on physics and statistics from existing designs as in [13], especially if the concept is similar to an existing product.

For the aircraft example typical design parameters would be; wing span, root cord, tapering, thickness, and sweep, structural weight, fuel weight, engine size, wing position, span of horizontal tail, cruise speed.

Other parameters could be established using simple design rules, e.g. the vertical fin. Here the Complex-RF method [11] is used for the optimization. It is a method that is

interesting because it is easy to estimate the information entropy as the optimization progress as a function of the spread of parameter set, as the optimization start with a spread the same size as the original design space (given by the constraints of the parameters). In this example eleven parameters was used as the accumulation of information entropy for one particular concept is shown in Figure 5. The information entropy is estimated as:

$$\hat{I}_x = -n \log_2 \left(\max(\delta_{x,i}) \right) \quad (24)$$

Where $\delta_{x,i}$ is the relative uncertainty in the i :th parameter. It is defined as:

$$\delta_{x,i} = \frac{x_{i,\max} - x_{i,\min}}{x_{0,i,\max} - x_{0,i,\min}} \quad (25)$$

Here the denominator represents the original design space for the optimization. At about 1200 evaluations there is a reduction in estimated entropy as the solution moves away from a false optimum. In the end the entropy has increased about 75 bits.

Figure 5. Accumulation of information as a function of number of objective function evaluations

4 Discussion

The design information entropy should be seen as a measure of the design space that has been under consideration during the design process. As such it provides some measure of the effort that has been going into the design.

To be effective it is desirable to have a small design space, but that still contain sufficiently good designs. A hallmark of a good design space is therefore that it is easy to assemble viable designs from a limited set of design elements, where there are ready to use sub systems and components that can be combined into new products e.g. like in a Lego set, or a good product platform. It can also be applied to parameterization of a design. In a good parameterization, all parameter combinations should yield viable designs, or at least possible geometries. In fact the fraction of viable parameter combinations can be seen as a quality of a parameterization [14]. This means that a smaller design space needs to be searched during design optimization.

The concept of design information entropy also provides a sound base for defining creativity as the process of selecting areas for expanding the design space, “to think outside the box”, it can also be the process of navigation in highly dimensional multimodal design spaces for concept generation, which is simplified by a limited design space, with few unviable designs.

5 Conclusions

A formal theory of design should be based on the generation and transformation of information during the design process, and information theory provides a set of tools that can be used in this context. In this paper it is demonstrated that introducing design information entropy as a state, can be used for quantitative description for various aspects in the design process, both regarding structural information regarding architecture and connectivity, as well as for parameter values, both discrete and continuous. It is consistent with the view that the design process is a learning process i.e. where information is gained as uncertainty is reduced.

It is also clear that the design of the design space as such, is critical to promote creativity by making viable design alternatives clear to the designer, not obstructed among noise of a sea of unviable designs.

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