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Tire Radii Estimation Using a Marginalized Particle Filter

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Abstract—In this work measurements of individual wheel speeds and absolute position from a global positioning system (GPS) are used for high-precision estimation of vehicle tire radii. The radii deviation from its nominal value is modeled as a Gaussian random variable and included as noise components in a simple vehicle motion model. The novelty lies in a Bayesian approach to estimate online both the state vector and the parameters representing the process noise statistics using a marginalized particle filter. Field tests show that the absolute radius can be estimated with sub-millimeter accuracy. The approach is tested in accordance with the ECE R-64 regulation on a large data set (22 tests, using two vehicles and 12 different tire sets), where tire deflations are detected successfully, with high robustness, i.e., no false alarms. The proposed marginalized particle filter approach outperforms common Kalman filter based methods used for joint state and parameter estimation when compared with respect to accuracy and robustness.

Index Terms—marginalized particle filter, tire radius, conjugate prior, noise parameter estimation.

I. INTRODUCTION

TIRE pressure monitoring has become an integral part of today's automotive active safety concept [1]. With the announcement of the US standard (FMVSS 138) and the European standard (ECE R-64), vehicle manufacturer must provide a robust solution to early detect tire pressure loss. A direct way to measure the tire pressure is to equip the wheel with a pressure sensor and transmit the information wireless to the vehicle body. In the harsh environment that the tires are exposed to, e.g., water, salt, ice and heat, the sensors are error-prone. Furthermore, the direct system with pressure sensors in each wheel, wirelessly communicating with the body, is expensive and the existence of sensors complicates the wheel changes. Therefore, indirect solutions have been introduced on the market lately, and are today implemented in more than 5 million passenger cars. In this paper, an indirect approach is presented where the tire radius is estimated simultaneously with the vehicle trajectory. The estimation is based on the relation between a reduction in tire radius and a reduction tire pressure.

The indirect approach presented in [2] is only based on the wheel speed sensors and it is shown how a tire pressure

loss in one wheel leads to a relative radii error between the wheels. An advantage is that the wheel speed sensors are an important part of standard ABS (anti-lock braking system) and that all cars are therefore already equipped with these sensors. In later publications GPS measurements have also been included to improve the radius estimation, which even make estimating of the absolute radius of one tire possible. The effective tire radius is estimated using a simple least-squares regression technique in [3]. A nonlinear observer approach to estimate the tire radius is presented in [4], and a second order sliding mode observer is used to estimate the tire radius in [5] and [6]. A simultaneous maximum likelihood calibration and sensor position and orientation estimation approach for mobile robots is presented in [7], where among other parameters the wheel radii are estimated. An observer based fault detection algorithm, which detects tire radius errors using yaw rate measurement and a bicycle model of the vehicle, is described in [8]. An extended Kalman filter based approach is presented in [9], where the tire radius is estimated via vertical chassis accelerations.

In the present contribution the difference between the static radius and the rolling radius of the tire is modeled as a Gaussian random variable, where both the mean and the covariance are unknown and time varying. Each wheel is treated separately, not relative to each other, hence it is possible to detect tire pressure loss in all four wheels using the proposed method. The vehicle dynamics and the measurements are described by a general state space model and the noise statistics are treated as the unknown parameters of the model. Hence a joint estimation of the state vector and the unknown model parameters based on available measurements is required. Such a problem is hard to treat as both the state estimation and the parameter estimation stages effects the performance of the other. The model used in this paper is nonlinear with biased and unknown noise, which requires approximate estimation algorithms. The *particle filter* (PF), [10]–[12], provides one generic approach to nonlinear non-Gaussian filtering. Probably the most common way to handle a joint parameter and state estimation problem is by augmenting the state vector with the unknown parameters and redefine the problem as a filtering problem, see e.g., [13]–[15]. A state augmentation approach has some major disadvantages as it requires artificial dynamics for the static parameters and it leads also to an increase in the state dimension which is not preferable for particle filters. This is particularly important to stress in automotive applications, where the computational cost must be kept low. In this work, an efficient Bayesian method is proposed for approximating the joint density of the unknown parameters and the state

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based on the particle filters and the marginalization concept, introduced in [16]. The statistics of the posterior distribution of the unknown noise parameters are propagated recursively conditional on the nonlinear states of the particle filter. An earlier version of this work was presented in [17], and the current, significantly extended version, includes comparisons with other methods, a more comprehensive evaluation based on a large number of data sets and an algorithm to detect tire deflation.

The proposed method is implemented and tested on real vehicle data utilizing several different tires. The state augmentation technique is also implemented and tested, using both the PF and the *extended Kalman filter* (EKF) variants.

The paper is outlined as follows. The problem is formulated together with the vehicle model and with the description of the sensors in Section II. The marginalized particle filter approach is described in Section III. Results based on real data collected with a production type passenger car are presented in Section IV. The work is concluded in Section V.

II. MODEL

The aim of this work is to jointly estimate the vehicle trajectory and the unknown tire radii errors based on the available measurements in a Bayesian framework. The proposed estimation approach is model based and a general state space model is given by

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) + g(\mathbf{x}_k, \mathbf{u}_k)\mathbf{w}_k, \quad (1a)$$

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{e}_k. \quad (1b)$$

where $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the deterministic measurement signal, $\mathbf{y}$ is the stochastic measurements signal, $\mathbf{w}$ is the process noise and $\mathbf{e}$ is the measurement noise. Further, k is the discrete time counter.

It is assumed that the unknown wheel radii affect the expected vehicle state trajectory through the wheel speed sensors, since a wheel with reduced radius must turn faster to travel the same distance. Hence, measurement of wheel revolutions from the wheel speed sensors and the trajectory from the GPS are used in the proposed approach. The angular velocities ω of the wheels are considered as deterministic measurement signals $\mathbf{u}$ under the assumption that the measurement noise of these signals are negligible. The GPS positions are considered as measurement signals $\mathbf{y}$.

The state vector is defined as

$$\mathbf{x} = [\mathbf{x} \quad \mathbf{y} \quad \psi]^T, \quad (2)$$

where $\mathbf{x}$, $\mathbf{y}$ is the planar position and ψ is the heading angle of the vehicle. The tire radii deviations from the nominal values are considered as time varying parameters and later in this section it is shown how the parameters are integrated in the state space model (1).

To begin with, the discrete time motion model (1a) is derived. The tire radii are primarily estimated during normal and non-extreme driving on paved roads, hence a very simple motion model is sufficient to describe the vehicle motion,

which is also beneficial from a computational point of view. The model is given by

$$\mathbf{x}_{k+1} = \mathbf{x}_k + T\mathbf{v}_k \cos \psi_k \quad (3a)$$

$$\mathbf{y}_{k+1} = \mathbf{y}_k + T\mathbf{v}_k \sin \psi_k \quad (3b)$$

$$\psi_{k+1} = \psi_k + T\dot{\psi}_k, \quad (3c)$$

where $\mathbf{v}_k$ is the vehicle longitudinal velocity, $\dot{\psi}_k$ is the yaw rate of the vehicle and T is the sampling time. A more advanced vehicle model would be the bicycle model, also denoted single track model, see e.g., [18], [19]. It describes the vehicle motion better and more accurate but it has the disadvantages that it both requires more parameters, such as cornering stiffness which must be identified online, and it has a higher state dimension.

For a wheel, there exist primarily three different definitions of the wheel radius, see e.g., [19]. The *static tire radius* is the distance between the center of the wheel and the ground. The *rolling radius* is the radius of a free rolling tire and it is defined as the linear speed of the tire center divided by the angular speed of the tire. Finally, the *effective rolling radius* is the radius of a driven wheel, which is smaller than the rolling radius of a free rolling wheel because of slip. In this work a front wheel driven vehicle is assumed and only the angular velocity measurements from the free rolling rear wheels are used to avoid slip estimation. The angular velocities of the rear wheels ω_i , $i = 3, 4$ can be converted to virtual measurements of the absolute longitudinal velocity and yaw rate as described in [20], [21], according to

$$\mathbf{v}^{\text{virt}} = \frac{\omega_3 r + \omega_4 r}{2} \quad (4a)$$

$$\dot{\psi}^{\text{virt}} = \frac{\omega_4 r - \omega_3 r}{l_T}, \quad (4b)$$

where r is the static wheel radius and l_T is the wheel track; see Fig. 1 for the notation. The static radius, given by the tire manufacturer, differs from the rolling radius, which in reality is unknown and needs to be estimated on the run. The *wheel radius errors*, are the important parameters to be estimated, and they are defined as the difference between the rolling radius and the static radius of the rear left and right wheels $\delta_3 \triangleq r_3 - r$ and $\delta_4 \triangleq r_4 - r$, respectively. In this paper, we have simplified the model, studying only the rear axle. The model can be extended with the front axle radii if a complete slip model is introduced. The rolling tire radii are

$$r_3 = r + \delta_3 \quad (5a)$$

$$r_4 = r + \delta_4. \quad (5b)$$

Multiplying r_3 with ω_3 and r_4 with ω_4 in equations (4) to derive the longitudinal velocity $\mathbf{v}$ and yaw rate $\dot{\psi}$ results in

$$\mathbf{v} = \frac{\omega_3 r_3 + \omega_4 r_4}{2} = \mathbf{v}^{\text{virt}} + \frac{\omega_3 \delta_3}{2} + \frac{\omega_4 \delta_4}{2} \quad (6a)$$

$$\dot{\psi} = \frac{\omega_4 r_4 - \omega_3 r_3}{l_T} = \dot{\psi}^{\text{virt}} + \frac{\omega_4 \delta_4}{l_T} - \frac{\omega_3 \delta_3}{l_T}. \quad (6b)$$

Fig. 1. The front wheels are denoted 1 and 2, and the rear wheels of the vehicle are denoted 3 and 4. Further the yaw angle is defined around the center of gravity, denoted $\otimes$.

Substituting these into the motion model (3) gives

$$\mathbf{x}_{k+1} = \mathbf{x}_k + T \left(\mathbf{v}_k^{\text{virt}} + \frac{\omega_{3,k} \delta_{3,k}}{2} + \frac{\omega_{4,k} \delta_{4,k}}{2} \right) \cos \psi_k \quad (7a)$$

$$y_{k+1} = y_k + T \left(\mathbf{v}_k^{\text{virt}} + \frac{\omega_{3,k} \delta_{3,k}}{2} + \frac{\omega_{4,k} \delta_{4,k}}{2} \right) \sin \psi_k \quad (7b)$$

$$\psi_{k+1} = \psi_k + T \left(\dot{\psi}_k^{\text{virt}} + \frac{\omega_{4,k} \delta_{4,k}}{l_T} - \frac{\omega_{3,k} \delta_{3,k}}{l_T} \right), \quad (7c)$$

where the input vector $\mathbf{u}$ only consists of the wheel speeds, i.e.,

$$\mathbf{u}_k = [\omega_{3,k} \quad \omega_{4,k}]^T \quad (8)$$

since $\mathbf{v}_k^{\text{virt}}$ and $\dot{\psi}_k^{\text{virt}}$ are functions of the wheel speeds.

At this point, the observability of the parameters should be analyzed. Consider a simple example, where the numbers are taken from our test vehicle shown in Fig. 2. The rear left tire radius is reduced by 1 mm, i.e., $\delta_3 = -0.001$, and without loss of generality the radius of the rear right wheel is kept without radius reduction, i.e., $\delta_4 = 0$. The nominal tire radius is 0.27 m, the wheel track is $l_T = 1.52$ m and the wheel revolutions are $\omega_3 = 76$ rad/s and $\omega_4 = 75.8$ rad/s. The vehicle velocity according to (6a) is then

$$v = 0.27 \frac{76 + 75.8}{2} - 0.001 \frac{76}{2} = \underbrace{20.5}_{\text{virtual}} - \underbrace{0.038}_{\text{error}}, \quad (9a)$$

i.e., the wheel radii error is approximately 0.2% of the virtual velocity. Now consider the yaw rate according to (6b)

$$\dot{\psi} = 0.27 \frac{75.8 - 76}{1.52} + 0.001 \frac{76}{1.52} = \underbrace{-0.036}_{\text{virtual}} + \underbrace{0.05}_{\text{error}} \quad (9b)$$

where the wheel radii error is approximately 140% of the virtual yaw rate. Note that the sum of errors is considered in (9a) and that the difference is considered in (9b). This example shows that it is much easier to estimate the tire radii difference between the left and the right wheel than to estimate the sum and thereby the absolute value of them.

The time varying parameters δ_3 and δ_4 are integrated in the motion model (7). One common approach in joint state and parameter estimation is to augment the state vector with the unknown parameters [15]. In such case, the augmented state vector is defined as

$$\mathbf{x}^a = [x \quad y \quad \psi \quad \delta_3 \quad \delta_4]^T, \quad (10)$$

and the state space model can be written in the general form (1) with the components according to

$$f(\mathbf{x}_k^a, \mathbf{u}_k) = \begin{bmatrix} x_k + T \left(\mathbf{v}_k^{\text{virt}} + \frac{\omega_{3,k} \delta_{3,k}}{2} + \frac{\omega_{4,k} \delta_{4,k}}{2} \right) \cos \psi_k \\ y_k + T \left(\mathbf{v}_k^{\text{virt}} + \frac{\omega_{3,k} \delta_{3,k}}{2} + \frac{\omega_{4,k} \delta_{4,k}}{2} \right) \sin \psi_k \\ \psi_k + T \left(\dot{\psi}_k^{\text{virt}} - \frac{\omega_{3,k} \delta_{3,k}}{l_T} + \frac{\omega_{4,k} \delta_{4,k}}{l_T} \right) \\ \delta_{3,k} \\ \delta_{4,k} \end{bmatrix} \quad (11a)$$

$$g(\mathbf{x}_k^a, \mathbf{u}_k) = \begin{bmatrix} \frac{T \omega_{3,k} \cos \psi_k}{2} & \frac{T \omega_{4,k} \cos \psi_k}{2} & 0 & 0 \\ \frac{T \omega_{3,k} \sin \psi_k}{2} & \frac{T \omega_{4,k} \sin \psi_k}{2} & 0 & 0 \\ -\frac{T \omega_{3,k}}{l_T} & \frac{T \omega_{4,k}}{l_T} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (11b)$$

The process noise $\mathbf{w}_k$ is zero mean Gaussian noise. i.e.,

$$\mathbf{w} = \mathcal{N}(0, Q). \quad (11c)$$

and it will influence the parameters δ_3 and δ_4 directly and the position and orientation of the vehicle through the noise model (11b), as if the noise was associated with the parameters. The presented rather simple vehicle model is accurate enough for tire radii estimation when the vehicle is not maneuvering or accelerating too much. Hence, in order to keep the model simple, estimation disable criteria, for instance dynamic driving, braking and velocity constraints are introduced.

The augmented state vector (10) can be estimated using e.g., an EKF. The observability can also be analyzed by deriving the observability Gramian, [22]. Since the model (11) is nonlinear and discrete it must be linearized to derive the Gramian. The Gramian is non-singular for the present system, hence weakly observable.

Another solution would be to use a particle filter (PF) instead of an EKF to estimate the states. An advantage with the PF is that the state space easily can be constrained, to not allow obvious estimation errors, such as a non negative radius error caused by tire deflation. However, the augmented state vector with 5 states is too large to obtain good results without the need to use a large number of particles. A marginalized approach has recently been published in [16], which makes it possible to keep a state vector of size 3, obtaining good results with a low number of particles and keeping the computational cost low. The marginalized PF approach is summarized in Section III. Using that approach, the motion model (7) can be written in the form (1a), where the functions f and g are identified as

$$f(\mathbf{x}_k, \mathbf{u}_k) = \begin{bmatrix} x_k + T \mathbf{v}_k^{\text{virt}} \cos \psi_k \\ y_k + T \mathbf{v}_k^{\text{virt}} \sin \psi_k \\ \psi_k + T \dot{\psi}_k^{\text{virt}} \end{bmatrix} \quad (12a)$$

$$g(\mathbf{x}_k, \mathbf{u}_k) = \begin{bmatrix} \frac{T \omega_{3,k} \cos \psi_k}{2} & \frac{T \omega_{4,k} \cos \psi_k}{2} \\ \frac{T \omega_{3,k} \sin \psi_k}{2} & \frac{T \omega_{4,k} \sin \psi_k}{2} \\ -\frac{T \omega_{3,k}}{l_T} & \frac{T \omega_{4,k}}{l_T} \end{bmatrix} \quad (12b)$$

and where the process noise term $\mathbf{w}_k$ contains the time varying

parameters and is given as,

$$\mathbf{w}_k = \begin{bmatrix} \delta_{3,k} \\ \delta_{4,k} \end{bmatrix} \sim \mathcal{N}(\mu_k, \Sigma_k) = \mathcal{N}\left(\begin{bmatrix} \mu_{3,k} \\ \mu_{4,k} \end{bmatrix}, \begin{bmatrix} \sigma_{3,k}^2 & 0 \\ 0 & \sigma_{4,k}^2 \end{bmatrix}\right). \quad (12c)$$

The process noise, or the tire radius error, is described by two parameters, the mean value μ and the covariance Σ for the left and the right wheel. Intuitively the mean value corresponds to the slow time variations from the nominal tire radius, while the variance corresponds to the fast variations due to tire vibrations. One interpretation is that the mean value μ models the change in the wheel radii due to abrasion, tire pressure changes and effects of cornering, and the covariance Σ can account for the vibrations arising from an uneven road surface. An interesting special case is when $\sigma_3 = \sigma_4$, which represents homogeneous road conditions, in comparison with split road surface when $\sigma_3 \neq \sigma_4$.

The measurement model (1b) defines the relation between the GPS position measurements and the state variables as follows.

$$\mathbf{y}_k = [\mathbf{x}_k^{\text{GPS}} \quad \mathbf{y}_k^{\text{GPS}}]^T \quad (13a)$$

$$\mathbf{h}(\mathbf{x}_k) = \begin{bmatrix} I & 0_{2 \times (n_x - 2)} \end{bmatrix} \mathbf{x}_k \quad (13b)$$

where $0_{2 \times (n_x - 2)}$ is a zero matrix, where $n_x = 3$ for the regular state space model and $n_x = 5$ for the augmented state space model. The measurement noise is assumed to be Gaussian with zero mean and known constant covariance R , i.e.,

$$\mathbf{e}_k = [\mathbf{e}_x \quad \mathbf{e}_y]^T \sim \mathcal{N}(0, R), \quad R = \sigma_{\text{GPS}}^2 I_2 \quad (13c)$$

where σ_{GPS} is the standard deviation of the GPS measurements. Other sensor measurements are also plausible to include in the measurement vector, at the cost of a more complex model. For instance, using a yaw rate gyro to measure the third state, also requires modeling the drifting offset in the sensor. The steering angle can also be converted to yaw rate, but it suffers from dynamic lag and other dynamic states of the vehicle.

III. MARGINALIZED PARTICLE FILTER FOR PARAMETER ESTIMATION

This section focuses on the evaluation of the joint density $p(\mathbf{x}_k, \boldsymbol{\theta}_k | \mathbf{y}_{1:k})$ of the state variable $\mathbf{x}_k$ defined in (2) and the parameters $\boldsymbol{\theta}_k$, that are the radii error biases and the covariances according to,

$$\boldsymbol{\theta}_k \triangleq \{\mu_{3,k}, \mu_{4,k}, \sigma_{3,k}, \sigma_{4,k}\}, \quad (14)$$

conditioned on all measurements $\mathbf{y}_{1:k}$ from time 1 to k . In order to simplify the calculations, the target density is decomposed into conditional densities as follows

$$p(\mathbf{x}_{0:k}, \boldsymbol{\theta}_k | \mathbf{y}_{1:k}) = p(\boldsymbol{\theta}_k | \mathbf{x}_{0:k}, \mathbf{y}_{1:k}) p(\mathbf{x}_{0:k} | \mathbf{y}_{1:k}). \quad (15)$$

The resulting two densities are estimated recursively. The implementation of the estimation algorithm is described in three steps. In Section III-A we define the approximations made to derive the state trajectory $p(\mathbf{x}_{0:k} | \mathbf{y}_{1:k})$ and in Section III-B we describe the estimation of the sufficient statistics

of the parameter distribution $p(\boldsymbol{\theta}_k | \mathbf{x}_{0:k}, \mathbf{y}_{1:k})$. The predicted trajectory is derived conditioned on the parameter estimates and the estimation of the joint density and the marginal density of the states $p(\mathbf{x}_{0:k} | \mathbf{y}_{1:k})$ is finalized in Section III-C.

A. State Trajectory

The posterior distribution of state trajectory $p(\mathbf{x}_{0:k} | \mathbf{y}_{1:k})$ is approximated by an empirical density of N particles and their weights as follows:

$$p(\mathbf{x}_{0:k} | \mathbf{y}_{1:k}) \approx \sum_{i=1}^N w_k^{(i)} \delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^{(i)}) \quad (16)$$

where $\mathbf{x}_{0:k}^{(i)}$ is a trajectory sample and $w_k^{(i)}$ is its related weight. The approximate state trajectory distribution (16) is propagated with a particle filter, using the sequential importance sampling (SIS) scheme [23]. In this scheme, at any time k , first the samples, which are also denoted by particles, are generated from a proposal distribution $q(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k})$ by using the particles from time $k-1$. The sampling step is followed by the update step in which the weights are updated according to

$$w_k^{(i)} \propto \frac{p(\mathbf{y}_k | \mathbf{x}_k^{(i)}) p(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k-1})}{q(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k})} w_{k-1}^{(i)}. \quad (17)$$

where $p(\mathbf{y}_k | \mathbf{x}_k^{(i)})$ is the likelihood function. Furthermore, if the proposal distribution $q(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k})$ is chosen as the prediction distribution $p(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k-1})$, then the weight update reduces to,

$$w_k^{(i)} = w_{k-1}^{(i)} p(\mathbf{y}_k | \mathbf{x}_k^{(i)}). \quad (18)$$

The prediction and update steps of the parameter estimation are described in the next section and the joint recursive estimation of both states and estimation are stringed together in Section III-C.

B. Parameter Estimation

In the factorization of the joint density given in equation (15), the distribution of the unknown parameters (which corresponds to the first term) is computed conditional on the realization of the state trajectory and the measurements. For a given sample $\mathbf{x}_{0:k}^{(i)}$ the motion model (1a) may be rewritten according to

$$\mathbf{w}_k^{(i)} = g^{-\dagger}(\mathbf{x}_{k-1}^{(i)}, \mathbf{u}_{k-1})(\mathbf{x}_k^{(i)} - f(\mathbf{x}_{k-1}^{(i)}, \mathbf{u}_{k-1})), \quad (19)$$

where $\dagger$ is the pseudo inverse. Hence, for a specific realization $\mathbf{x}_{0:k}^{(i)}$ of the trajectory, the posterior density of the parameters can also be written conditional on the realization of the noise terms according to

$$p(\boldsymbol{\theta}_k | \mathbf{x}_{0:k}^{(i)}, \mathbf{y}_{1:k}) = p(\boldsymbol{\theta}_k | \mathbf{w}_{0:k}^{(i)}). \quad (20)$$

Furthermore, this posterior can be decomposed into a likelihood function and a prior according to Bayes rule

$$p(\boldsymbol{\theta}_k | \mathbf{w}_{0:k}) \propto p(\mathbf{w}_k | \boldsymbol{\theta}_k) p(\boldsymbol{\theta}_k | \mathbf{w}_{0:k-1}). \quad (21)$$

The likelihood function $p(\mathbf{w}_k | \boldsymbol{\theta}_k)$ is assumed to be multivariate Gaussian, as previously mentioned in (12c), and since

the mean μ_k and the covariance Σ_k are considered unknown parameters θ_k , a Normal-inverse-Wishart distribution defines the conjugate prior¹ $p(\theta_k | \mathbf{w}_{0:k-1})$. Normal-inverse-Wishart distribution defines a hierarchical Bayesian model given as

$$\mathbf{w}_k | \mu_k, \Sigma_k \sim \mathcal{N}(\mu_k, \Sigma_k) \quad (22a)$$

$$\mu_k | \Sigma_k \sim \mathcal{N}(\hat{\mu}_{k|k}, \gamma_{k|k} \Sigma_{k|k}) \quad (22b)$$

$$\Sigma_k \sim \text{iW}(\nu_{k|k}, \Lambda_{k|k}) \quad (22c)$$

$$\propto |\Sigma_k|^{-\frac{1}{2}(\nu+d+1)} \exp\left(-\frac{1}{2} \text{Tr}(\Lambda_{k|k} \Sigma_k^{-1})\right) \quad (22d)$$

where $\text{iW}(\cdot)$ denotes the inverse Wishart distribution and d denotes the dimension of the noise vector $\mathbf{w}_k$. The statistics $S_{\mathbf{w},k} \triangleq \{\gamma_k, \hat{\mu}_k, \nu_k, \Lambda_k\}$ can according to [24], [25] be recursively updated as follows. The measurement update is

$$\gamma_{k|k} = \frac{\gamma_{k|k-1}}{1 + \gamma_{k|k-1}} \quad (23a)$$

$$\hat{\mu}_{k|k} = \hat{\mu}_{k|k-1} + \gamma_{k|k}(\mathbf{w}_k - \hat{\mu}_{k|k-1}) \quad (23b)$$

$$\nu_{k|k} = \nu_{k|k-1} + 1 \quad (23c)$$

$$\Lambda_{k|k} = \Lambda_{k|k-1} + \frac{1}{1 + \gamma_{k|k-1}} (\hat{\mu}_{k|k-1} - \mathbf{w}_k)(\hat{\mu}_{k|k-1} - \mathbf{w}_k)^T \quad (23d)$$

where the statistics of the predictive distributions are given by the time update step according to

$$\gamma_{k|k-1} = \frac{1}{\lambda} \gamma_{k-1|k-1} \quad (24a)$$

$$\hat{\mu}_{k|k-1} = \hat{\mu}_{k-1|k-1} \quad (24b)$$

$$\nu_{k|k-1} = \lambda \nu_{k-1|k-1} \quad (24c)$$

$$\Lambda_{k|k-1} = \lambda \Lambda_{k-1|k-1}. \quad (24d)$$

The scalar real number $0 \leq \lambda \leq 1$ is the forgetting factor. The forgetting factor here helps in the estimation of the dynamic variables. The statistics relies on roughly the measurements within the last $h = \frac{1}{1-\lambda}$ frames/time instances. That allows the algorithm to adapt the changes in the noise statistics in time. Such an approach is appropriate when the unknown parameters are slowly varying, and the underlying parameter evolution is unknown.

C. Noise Marginalization

In this section the two densities in (15) will be unified into one iterative estimation scheme. Consider first the state prediction and note that by using (19) it may be rewritten using the lemma on transformation of variables in probability density functions according to

$$p(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k-1}) \propto p(\mathbf{w}(\mathbf{x}_k) | S_{\mathbf{w},k}^{(i)}) \quad (25)$$

Using the marginalization concept it may also be rewritten as

$$\begin{aligned} p(\mathbf{x}_k | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k-1}) \\ = \int p(\mathbf{x}_k | \mathbf{x}_{k-1}^{(i)}, \theta) p(\theta | \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k-1}) d\theta. \end{aligned} \quad (26)$$

¹A family of prior distributions is conjugate to a particular likelihood function if the posterior distribution belongs to the same family as the prior.

One important advantage of using the conjugate priors reveals itself here as it is possible to integrate out the unknown noise parameters as they follow Normal-inverse-Wishart distribution. The integrand in (26) is the product of a Gaussian distribution and a NiW distribution and the result of the integral is a Student-t distribution which can be evaluated analytically.

Now, by combining (25) and (26), the predictive distribution of $\mathbf{w}_k$ becomes a multivariate Student-t density, according to

$$p(\mathbf{w}_k | S_{\mathbf{w},k}^{(i)}) = \text{St}(\hat{\mu}_{k|k-1}^{(i)}, \Lambda_{k|k-1}^{(i)}, \nu_{k|k-1}^{(i)} - d + 1) \quad (27)$$

with $\nu_{k|k-1}^{(i)} - d + 1$ degrees of freedom. The Student-t distribution is located at $\hat{\mu}_{k|k-1}^{(i)}$ with variance

$$\text{Var}(\mathbf{w}_k) = \frac{1 + \gamma_{k|k-1}}{\nu_{k|k-1} - d - 1} \Lambda_{k|k-1}, \quad (28)$$

where the relevant statistics are given in (24).

In the implementation, the noise is first sampled from (27) and used in (1a) in order to create samples $\mathbf{x}_k^{(i)}$. The samples from (27) can be used directly in the statistics update (23). The pseudo code of the algorithm used in the simulations is given in Table I.

In the proposed method, each particle i keeps its own estimate for the parameters $\theta^{(i)}$ of the unknown process noise. In the importance sampling step, the particles use their own posterior distribution of the unknown parameters. The weight update of the particles is performed according to the measurement likelihood. The particles are keeping the unknown parameters, and those which best explain the observed measurement sequence will survive in time.

The marginal posterior density of the unknown parameters can be computed by integrating out the states in the joint density

$$\begin{aligned} p(\theta | \mathbf{y}_{1:k}) &= \int p(\theta | \mathbf{x}_{0:k}, \mathbf{y}_{1:k}) p(\mathbf{x}_{0:k} | \mathbf{y}_{1:k}) d\mathbf{x}_{0:k} \\ &\approx \sum_{i=1}^N \omega_k^{(i)} p(\theta | \mathbf{x}_{0:k}^{(i)}, \mathbf{y}_{1:k}). \end{aligned} \quad (29)$$

Then the estimate of the unknown parameters could be computed according to a chosen criterion. As an example, according to the minimum mean square error (MMSE) criterion, the noise mean and variance estimates at time k are computed as

$$\hat{\mu}_k = \sum_{i=1}^N \omega_k^{(i)} \hat{\mu}_k^{(i)} \quad (30a)$$

$$\hat{\Sigma}_k = \sum_{i=1}^N \omega_k^{(i)} \left(\frac{\nu_k^{(i)} - d + 1}{\nu_k^{(i)} - d - 1} \Lambda_k^{(i)} + (\mu_k^{(i)} - \hat{\mu}_k)(\mu_k^{(i)} - \hat{\mu}_k)^T \right), \quad (30b)$$

where the weights are inherited from the particles.

IV. EXPERIMENTAL RESULTS

The presented method is evaluated on a number of real data sets collected with standard passenger cars under normal driving conditions. First a detailed analysis of one data set is presented in Section IV-A, followed by a statistical evaluation

TABLE I
PSEUDO CODE OF THE ALGORITHM

Initialization:
1: **for** each particle $i = 1, \dots, N$ **do**
2: Sample $\mathbf{w}_0^{(i)}$ from (27)
3: Compute $\mathbf{x}_0^{(i)}$ from (1a)
4: Set initial weights $\omega_0^{(i)} = \frac{1}{N}$
5: Set initial noise statistics $S_{\mathbf{w},0}^{(i)}$ corresponding to each particle
6: **end for**
Iterations:
7: **for** $k = 1, 2, \dots$ **do**
8: **for** For each particle $i = 1, \dots, N$ **do**
9: Predict noise statistics $S_{\mathbf{w},k|k-1}^{(i)}$ using (24)
10: Sample $\mathbf{w}_{k|k-1}^{(i)}$ from (27)
11: Compute $\mathbf{x}_{k|k-1}^{(i)}$ from (1a)
12: update the weights:

$$\tilde{w}_k^{(i)} = w_{k-1}^{(i)} p(\mathbf{y}_k | \mathbf{x}_k^{(i)})$$

13: Update noise statistics $S_{\mathbf{w},k}^{(i)}$ using (23)
14: **end for**
15: Normalize weights, $w_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{i=1}^N \tilde{w}_k^{(i)}}$.
16: Resample the particles.
17: Compute state estimate $\hat{\mathbf{x}} = \sum_{i=1}^N w_k^{(i)} \mathbf{x}_k^{(i)}$
18: Compute the parameter estimates using (30)
19: **end for**

of 22 data sets in Section IV-B (including two different vehicles and 12 different tire sets). In all cases both the detection of a tire deflation is shown as well as the robustness, i.e., that no false alarm is triggered when no deflation occurs over a significant timespan. The proposed marginalized particle filter (MPF) is compared with the extended Kalman filter's (EKF) estimates of the augmented state (10), which is denoted AUG-KF in this section.

A. Detailed Analysis of One Data Set

In the experiments, the measurements were collected with a passenger car equipped with standard vehicle sensors, such as the ABS wheel speed sensors, and a GPS receiver, see Fig. 2. The sampling time of the wheel speed sensors is 0.1 s and the sampling time of the GPS receiver is 0.25 s, i.e., the prediction is performed more often than the measurement update. The standard deviation of the GPS measurements is set to 20 m, however in reality it is probably much smaller.

In regions where the car moves at low velocities (less than $\gamma = 11$ m/s), the steering wheel angle measurement was utilized in order to avoid quantization problems of the wheel cogs in the third row of the motion model (7c), according to

$$\psi_{k+1} = \begin{cases} \psi_k + T(\dot{\psi}_k^{\text{virt}} + \frac{\omega_4 \delta_4}{l_T} - \frac{\omega_3 \delta_3}{l_T}) & \text{if } v > \gamma \\ \psi_k + T\delta_F(v_k^{\text{virt}} + \frac{\omega_3 \delta_3}{2} + \frac{\omega_4 \delta_4}{2})/l_b & \text{if } v < \gamma \end{cases} \quad (31a)$$

The filter is shut off completely at very low velocities (less than 7 m/s).

The GPS measurements of the 12 km test circuit is shown as a red solid line in Fig. 3. It is overlayed by the estimated trajectory, which is black-white dashed. The photo is a very accurate flight photo (obtained from the Swedish mapping,

Fig. 2. The test vehicle of Linköping University is logging standard CAN data. The vehicle is in addition equipped with a GPS receiver.

cadastral and land registration authority), which can be used as ground truth to visualize the quality of the trajectory estimate. The circuit took about 18 min to drive and it starts and ends in urban area of Linköping, in the upper right corner in Fig. 3. The test vehicle is driving clockwise, first on a small rural road, and then on the left side of the figure entering a straight national highway, before driving back to urban area on the top of the figure. The test was performed two times, first with balanced tire pressure and thereafter with unbalanced tire pressure, where the pressure of the rear left tire was released to be approximately 50% of the right tire pressure.

For the first circuit the pressure of the rear wheel tires was adjusted to be equal 2.8 bar on both tires. The estimated mean parameter for the left and the right wheel are shown in the Fig. 4a and 4b, respectively. The MPF method, shown as a black solid line, performs well in estimating the mean values μ_3 and μ_4 , and it is clearly visible that the mean values are similar and close to zero. The augmented model with EKF (AUG-KF) performs less well; it takes longer time to reach its end-value and is then moving around zero.

For the second circuit the pressure of the rear left tire was released to 1.5 bar. Comparing Fig. 4c with Fig. 4d it is visible that the pressure reduction leads to a smaller μ_3 than μ_4 value. Note that both methods reach the same end-value, approximately 1.5 mm, but the proposed MPF is much faster and more stable than the AUG-KF method.

B. Statistical Analysis Based on Multiple Data Sets

When evaluating pressure loss due to puncture or diffusion this is usually carried out using step deflations. Hence, we have a controlled way of evaluating performance for a calibration phase where the nominal wheel radius can be estimated followed by a detection phase. In order to evaluate the performance there are two standardized test procedures, FMVSS138 NHTSA for the US market and ECE R-64 for the European market. In the experiments presented in this paper we use 22 ECE R-64 test cases, using two different vehicles

Fig. 4. Tire radius error of the left and right rear wheels. Fig. (a) and (b) show the situation with balanced wheel pressure and as expected the radius error is similar and close to zero. Fig. (c) and (d) show the situation with unbalanced wheel pressure, and the radius error of the left wheel in Fig. (c) is larger than the error of the right wheel in (d). The black solid line is the MPF estimate and the gray dashed line is the AUG-KF estimate.

from the same platform and 12 different tire sets. In this paper we focus on one-wheel puncture deflations on the non-driven wheels. The presented filters and approach can be applied to multiple wheel deflations and four-wheel driven vehicles if the model is extended and an appropriate slip compensation is included.

The standardized ECE R-64 test procedure constitutes of a calibration phase, in which the car is driving in the interval 40 – 120 km/h for minimum 20 min, followed by a step deflation of 20% from the pressure measured at deflation time. During the calibration and deflation, driving time is only considered when driving within the speed interval, excluding braking or extreme dynamic driving.

The data collected consists of GPS longitudinal and lateral information with a sampling rate of 0.5 Hz. In addition wheel speed and yaw rate measurements as well as brake information are available in 10 Hz. Note that we have focused on the more difficult problem in the current publication, where only position measurements are available from the GPS. If GPS Doppler velocity is measured the performance can be enhanced, but this signal is not always available on the CAN bus.

The ECE R-64 deflation scenario data is used to evaluate

the performance of the radius estimator and the detector. In order to ensure sufficient robustness of the system, analysis is also performed on non-deflated robustness data. In this paper this is carried out re-using the same data and splitting the non-deflated data file in two. Hence the first part is used for calibration and the second part for detection. The goal is to have fast detection when a deflation is present and minimizing the risk of false alarms when robustness data is used.

The detector used in this paper is a standard CUSUM detector, [26], with some extensions, see Table II, where estimated radius error $\mu_{i,k}$ for each wheel i and time k is compared against its calibrated value. The implemented CUSUM uses a drift parameter ν and an adaptive threshold ρ_{th} . To make it more robust against outliers in the input signal ($\mu_{i,k}$), is formed by low pass filtering the difference in radius compared to calibration and apply a simple rate limiter. In the evaluation the following nominal parameters are used: $\nu = 0.4 \cdot 10^{-3}$ and $\rho_{th} = 9 \cdot 10^{-3}$.

1) *MPF – Performance evaluation:* An example of the radius estimation and radius uncertainty estimation is depicted in Fig. 5 for a step deflation. As seen, the deflation is easily detected. In order to analyze the performance, the detection

Fig. 3. The red line is GPS position measurements and the black-white dashed line is the estimated driven trajectory. The experiment starts and ends at a roundabout in the upper right corner. The resolution of this photo is one square meter per pixel. (©Lantmäteriet Medgivande I2011/1405, reprinted with permission)

TABLE II
CUSUM PSEUDO CODE FOR TIRE DEFLATION ALARM.

Require: estimated tire radius error $\mu_{i,k}$ for each wheel i at time k

- 1: Initialization: $g_{i,k} = 0$ for each wheel i
- 2: **for** each wheel i and time k **do**
- 3: $\bar{\mu}_{i,k} = LP^{\text{rate limit}}(\mu_{i,k})$
- 4: $g_{i,k} = g_{i,k-1} + \bar{\mu}_{i,k} - \nu$
- 5: $g_{i,k} = 0$ if $g_{i,k} < 0$.
- 6: Issue an alarm if $g_{i,k} > \rho_{th}$
- 7: **end for**

times are analyzed using a performance evaluation of 22 one-wheel deflations. The MPF was able to detect 21 of the deflations using a CUSUM detector, considering $\mu_{i,k}$, i.e., the calibrated radius versus the current estimated radius, as a difference signal for each wheel. The test case with a missed detection was very close to trigger an alarm. In Fig. 6 the detection time histogram for the MPF is depicted and as seen most of the detections are below 10 minutes which complies with the requirements for a one-wheel deflation, according to ECE R-64. Note, that if the aim only was to detect a one-wheel deflation, the fastest method would be to compare the wheel speeds between all wheels of the vehicle, now every wheel is treated separately. Using the presented method it is also possible to detect diffusion (a slow reduction of air pressure) up to four wheels, which allows a detection time of 60 minutes according to the specification. A histogram plot of deflation

Fig. 5. An example of radius and radius uncertainty estimation using the MPF. First the calibration part is depicted followed by the step deflation part. As seen the change in radius is essential.

Fig. 6. The figure shows the detection histogram for the MPF. One missed detection not shown.

estimates Δ is depicted in Fig. 8. The detection and robustness statistics are summarized in Table III.

2) *AUG-KF – Performance evaluation:* The AUG-KF performance is significantly worse than the MPF, only detecting 16 of the 22 deflations correctly. The detection time histogram is presented for the AUG-KF in Fig. 7 and the detection and robustness statistics are summarized in Table III. The better performance of the MPF can be explained by the particle filter's ability to restrict parameter estimates to only allow decrease of tire radius after calibration and that it can handle nonlinearities optimally. Furthermore, the MPF method distinguishes better between the parameters (wheel radii) and the position state variables. The disable criteria, the motion model, the noise variables and the CUSUM parameters are equal, or in the cases where the model structure not comprises the same parameter set as similar as possible, for the two compared methods.

Fig. 7. The figure shows the detection histogram for the AUG-KF. Six missed detections not shown.

Fig. 8. The figure shows a histogram of estimated wheel radius differences $\mu_{i,k}$ due to tire deflation. Tires from different manufacturers were used, which can explain the spread.

V. CONCLUSION

In this study, we address the problem of joint estimation of unknown tire radii and the trajectory of a four-wheeled vehicle based on GPS and wheel angular velocity measurements. The problem is defined in a Bayesian framework and an efficient method that utilizes marginalized particle filters is proposed in order to accomplish the difficult task of joint parameter and state estimation. The algorithm is tested on real data experiments performed in accordance with the regulation 64 of the United Nations Economic Commission for Europe (ECE R-64). The results show that it is possible to estimate the separate

tire radius within sub-millimeter accuracy.

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TABLE III

DETECTION AND FALSE ALARM STATISTICS FOR MPF AND AUG-KF USING 22 TESTS.

	Detection [%]	False Alarm [%]
PF	95.5	0.0
EKF	72.7	0.0

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