Simultaneous Localization and Mapping for an Unmanned Aerial Vehicle Using Radar and Radio Transmitters

Examensarbete utfört i Reglerteknik vid Tekniska högskolan vid Linköpings universitet av

Alfred Dahlin

LiTH-ISY-EX–14/4794–SE
Linköping 2014
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Lokalisering och kartläggning för en UAV med hjälp av radar och radiosändare
Simultaneous Localization and Mapping for an Unmanned Aerial Vehicle Using Radar and Radio Transmitters

Alfred Dahlin

The Global Positioning System (GPS) is a cornerstone in Unmanned Aerial Vehicle (UAV) navigation and is by far the most common way to obtain the position of a UAV. However, since there are many scenarios in which GPS measurements might not be available, the possibility of estimating the UAV position without using the GPS would greatly improve the overall robustness of the navigation. This thesis studies the possibility of instead using Simultaneous Localisation and Mapping (SLAM) in order to estimate the position of a UAV using an Inertial Measurement Unit (IMU) and the direction towards ground based radio transmitters without prior knowledge of their position. Simulations using appropriately generated data provides a feasibility analysis which shows promising results for position errors for outdoor trajectories over large areas, however with some issues regarding overall offset. The method seems to have potential but further studies are required using the measurements from a live flight, in order to determine the true performance.
Abstract

The Global Positioning System (GPS) is a cornerstone in Unmanned Aerial Vehicle (UAV) navigation and is by far the most common way to obtain the position of a UAV. However, since there are many scenarios in which GPS measurements might not be available, the possibility of estimating the UAV position without using the GPS would greatly improve the overall robustness of the navigation. This thesis studies the possibility of instead using Simultaneous Localisation and Mapping (SLAM) in order to estimate the position of a UAV using an Inertial Measurement Unit (IMU) and the direction towards ground based radio transmitters without prior knowledge of their position. Simulations using appropriately generated data provides a feasibility analysis which shows promising results for position errors for outdoor trajectories over large areas, however with some issues regarding overall offset. The method seems to have potential but further studies are required using the measurements from a live flight, in order to determine the true performance.
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### Abbreviations

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<tr>
<td>AOA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>CRB</td>
<td>Cramér-Rao Bound</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<td>ID</td>
<td>Inverse Depth</td>
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<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>RMSE</td>
<td>Root Mean Squared Error</td>
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<tr>
<td>RSS</td>
<td>Received Signal Strength</td>
</tr>
<tr>
<td>SLAM</td>
<td>Simultaneous Localisation and Mapping</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
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### Notations

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<tr>
<td>$\phi$</td>
<td>Roll angle</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Yaw angle</td>
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<tr>
<td>$\rho$</td>
<td>Inverse depth</td>
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<tr>
<td>$\omega$</td>
<td>Angular velocity</td>
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<tr>
<td>$a$</td>
<td>Acceleration</td>
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<tr>
<td>$b$</td>
<td>Bias states</td>
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<tr>
<td>$d$</td>
<td>Landmark directional vector</td>
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<tr>
<td>$f$</td>
<td>Motion model</td>
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<td>$l$</td>
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<td>$m$</td>
<td>Landmark position</td>
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<tr>
<td>$p$</td>
<td>Position states</td>
</tr>
<tr>
<td>$P$</td>
<td>Covariance of state vector</td>
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<tr>
<td>$q$</td>
<td>Orientation quaternion</td>
</tr>
<tr>
<td>$R$</td>
<td>Rotational matrix</td>
</tr>
<tr>
<td>$R_{\text{AOA}}$</td>
<td>AOA measurement covariance matrix</td>
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<tr>
<td>$R_Z$</td>
<td>Landmark height variance</td>
</tr>
<tr>
<td>$R_P$</td>
<td>Inverse depth variance</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity states</td>
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<tr>
<td>$x$</td>
<td>State vector</td>
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Unmanned Aerial Vehicles (UAVs) have potential applications in a wide variety of fields. They have become an especially attractive alternative when it comes to reconnaissance and mapping in both military and civilian applications. The UAVs enable fast transportation and information gathering over large distances and areas, without human involvement. This results in higher efficiency of human resources, which then can be redistributed, and also enables safe operations in potentially hostile environments.

1.1 Motivation

The advantage of using a UAV is that it eliminates the need of a pilot or other person present in the area of operations. This does however mean that a large portion of the dynamic information processing of the UAV must be obtained through software and data rather than through human interpretation, which is especially important if the UAV is autonomous. This processing, including navigation and general decision making, is highly dependant on several changing properties of the UAV itself, such as position and orientation. In order to carry out any given operation the UAV needs to know its own location, preferably in relation to its starting position and destination.

The navigation and localisation of modern UAVs are almost exclusively based on measurements from the Global Positioning System (GPS), a dependency which spawns the problem upon which this thesis is based. That is, that for one reason or another, the GPS measurements might be unavailable which might occur due to passive obstructions, such as buildings and natural environment (e.g. weather interference), or due to active disruption of the signals themselves. Therefore the
possibility of using a position estimation that does not require a GPS is interesting in both civilian and military applications.

1.2 Purpose

As there are other methods of determining properties such as position, this GPS dependency seems avoidable and therefore unnecessary. The purpose of this thesis is to investigate the potential of combining the measurements from an Inertial Measurement Unit (IMU) with measurements of height and angle toward ground based radio transmitters in order to allow a UAV to estimate its own position and trajectory. This estimation is to be done without access to measurements from a GPS and without any prior knowledge or a predefined map containing information about transmitter placement.

1.3 Approach

The problem of determining position without access to a GPS is investigated by evaluating the feasibility of using a bearing-only Simultaneous Localisation and Mapping (SLAM) algorithm based on the Extended Kalman Filter (EKF). This filter will, based on measurements from an IMU, an altimeter and an antenna array, estimate the target states such as position and orientation as well as the position of the transmitters acting as landmarks. The measurements available are Angle of Arrival (AOA) toward the transmitters, height measurements as well as accelerometer and gyroscope measurements from the IMU.

This is performed in a simulated environment by generating the measurements from an arbitrary trajectory, due to the lack of available measurements containing the required data. The transmitters are placed randomly in an area around the trajectory and given random frequency properties based on a simple model. From these properties the UAV calculates its relative position continuously in flight while simultaneously estimating the landmark positions. The SLAM filter keeps track of all these properties, their uncertainties and the correlation between them in order to, through motion and measurement models, incorporate the IMU, AOA and height measurements into the estimations and predictions.

As these measurements must be generated, several models are required. These models describe radio wave propagation and the radio signal receivers as well as the IMU, where the models define the relation between a trajectory and the resulting accelerations and angular velocities.

Figure 1.1 illustrates the basic UAV environment with ground based radio transmitters to which the UAV antenna array measures direction. Based on this direction, the UAV creates an internal map of the transmitters, over the course of a flight, without prior knowledge of their position. This map is updated and refined continuously while the filter estimates the UAV position simultaneously while using the information in all measurements. The filter will then provide
estimates of both transmitter positions as well as UAV properties such as position, orientation and velocity over the course of a flight along with their corresponding confidence levels. This data can be used as base for navigation purposes or similar systems.

The thesis proceeds to evaluate this tracking performance for different transmitter densities and briefly compares two different landmark representations.

1.4 Limitations

As both time and resources were limited, some assumptions had to be made which affected the outcome of the thesis.

As all data is simulated, some approximations was made mainly with regard to sensor and noise models. A consequence of this is that the models used could not be validated against real data within the thesis.

Since there are algorithms available with high reliability for association when it comes to radio sources, the landmark association was assumed to be known.

There is no known map of the trajectory surroundings, such as a map of transmitters, available to use as a reference map. The landmarks were therefore placed randomly and the properties of the landmarks contained no information about where they were located. However, some assumptions were made, mainly that all landmarks were assumed stationary, with fixed frequencies and placed close to ground level.
Height was the only measurement available that could be related to the absolute position and since there was no prior knowledge, the absolute position of the UAV and the transmitters become unobservable in the horizontal plane. Therefore, the initial position and orientation was assumed known and only the relative position of the UAV was able to be estimated in this thesis.

1.5 Related Work

The essential development of the SLAM algorithm was surveyed in Durrant-Whyte and Bailey [2006] for the two most common versions, EKF SLAM and Fast SLAM, which is based on the particle filter which is described in Arulampalam et al. [2002]. Additional methods, such as the iterated filters described in Tully et al. [2008] or Bayesian formulation as in Durrant-Whyte et al. [2003], also exist.

One of the first successful attempts of performing SLAM using a single camera providing bearing only measurements and with very sparse prior knowledge was presented in Davison [2003]. Since then there has been steady progress within the field of SLAM and the number of applications has been increasing rapidly. Many indoor applications also utilise some sort of range measurement, for example by using a stereo camera or a separate range finder, as done in Karlsson and Bjärkefur [2010] or combining a stereo camera with an IMU as in Rydell and Emilsson [2012].

The vast majority of SLAM applications currently utilises visual features retrieved from mono or stereo cameras as landmarks. There are however a few cases where the SLAM is based on radio signals where the Received Signal Strength (RSS) is used, as done in Faragher et al. [2012] using smartphones, or range-only measurements which are calculated based on the RSS, using EKF SLAM as in Menegatti et al. [2009] or Fast SLAM as in Kuai et al. [2010].

There are several articles and reports covering UAV specific SLAM implementations. Many of these also focus on vision based bearing only SLAM or with IMU measurement support, as seen in Wang et al. [2013] and Fu et al. [2014].

This thesis instead chooses to evaluate the possibility of using bearing only measurements from ground based radio transmitters acting as landmarks in addition to the IMU measurement to obtain a dynamic base of navigation in the field of UAVs. The case that is considered in this thesis assumes no prior knowledge and will therefore focus on local position estimation.

BAE Systems has recently developed a robust navigation system, known as NAV-SOP, built around radio transmitters acting as ground based reference points [BAE Systems, 2014], similar to the way they are treated in this thesis. The main difference between this thesis and NAVSOP is that NAVSOP requires a predetermined map of the transmitters, a necessity which the SLAM algorithm used in this thesis is trying to eliminate.
There are several existing concepts which must be explained in order to provide the theoretical base that is needed before going into the specific implementation of the filter used.

This chapter contains general descriptions of the methods and algorithms used in this thesis. The main part of the chapter explains the ideas and algorithm behind the EKF and the SLAM extension which utilises the data from an IMU, the altitude as well as the AOA in order to obtain the relevant estimates. This is followed by a brief description of the quaternion which is used as a representation of the UAV orientation.

These descriptions and equations are then used in Chapter 3 where the specific implementations are defined.

Figure 2.1 provides an overview of the basic system structure and its main parts.

**Figure 2.1:** Block diagram of the general system set-up.
2.1 Angle of Arrival

Using an antenna array the direction towards radio transmitters can be measured. These angles often contain significant noise which is modelled as white noise with a correlation between azimuth and elevation errors. The noise is also generally bigger in elevation than in azimuth. There are several ways of modelling this noise using different transmitter, receiver and propagation models. In this thesis, the transmitters are assumed to be omnidirectional with a line of sight propagation model which assumes the signal power decreases squarely proportional to the distance and the emitted frequency. The receiver is modelled according to an elevation dependant antenna array with a maximum gain of 1. These models are further detailed in Section 3.1.2.

2.2 Altimeter

The altimeter is responsible for measuring the altitude of the UAV, most commonly by measuring atmospheric pressure through a barometer. Generally these barometric altimeters are affected by a fairly constant bias rather than higher frequency noise, as illustrated in Sabatini and Genovese [2013] covering barometric altimeters.

2.3 Inertial Measurement Unit

An IMU is a collective name for a system of sensors which contains an accelerometer measuring acceleration and a gyroscope measuring angular velocity. These devices are frequently used for navigation and estimation of location and attitude in everything from smartphones to vehicles. The acceleration and angular velocity is measured along three body fixed orthogonal axes which supplies a complete description of the changes in movement. These measurements can be integrated over a period of time in order to obtain an estimation of position, velocity and orientation.

The noise affecting all measurements from the IMU is a combination of a slowly changing bias, also known as brown or red noise containing mainly lower frequencies, and a completely uncorrelated noise, the common white noise or zero mean Gaussian noise with a constant frequency spectrum. Because of this noise and the fact that the integrals of the measurements are needed in order to calculate the estimates, significant error will accumulate in the results. This causes drift to occur over time and results in great inaccuracies of the estimations. It does, however, provide a good short term estimate of the motion where the error is manageable as the drift caused by the bias mainly affects the estimate over longer periods of time.

The model of the IMU was implemented as described in Section 3.1.4 according to this knowledge.
2.4 Estimation Theory

The filter selected for this application was based on the EKF SLAM algorithm. The EKF is used to describe the motion of the UAV over time through a number of states, mainly position, velocity and orientation which are estimated based on the sensor measurements. In this thesis, these properties are represented by Cartesian coordinates and velocities and the orientation is represented by a quaternion vector.

In the SLAM case used here, the filter tracks these states at the same time as it provides estimates of the surrounding environment through the position of landmarks. This way, a map of the environment is obtained at the same time as the relative properties of the UAV are estimated.

The implemented filter is based on the following theory, where the specific equations and models are defined in Section 3.2.

2.4.1 State-space Model

The tracking is built around the general discrete-time state space model

\[ x_k = f(x_{k-1}, u_{k-1}) + Bw_{k-1}, \]

\[ z_k = h(x_k) + v_k, \]

where (2.1a) describes the motion model and 2.1b describes the measurement model.

This nonlinear model illustrates how the states \( x_k \), containing information such as the position, velocity and orientation, change over time given previous states \( x_{k-1} \) and inputs \( u_{k-1} \). This is done in (2.1a) where the prediction of the states is calculated based on a model of the target motion. The measurement model in (2.1b) is then used in order to correct this prediction by relating the predicted states to the new measurements \( z_k \).

The \( B \)-matrix describes how the process noise is modelled and how it affects the states during the time update described in Section 2.4.2.

The noise itself is assumed to be a zero-mean Gaussian noise with covariance matrices \( Q \) for the noise of the motion model \( w \) and \( R \) for the measurement noise \( v \). These matrices can be used either as tuning parameters of the filter, or set to values corresponding to the expected noise levels.

2.4.2 Extended Kalman Filter

The EKF is the common nonlinear extension to the state estimator that is the Kalman filter. The Kalman filter optimises the state estimate by minimising the square of the error estimate thereby minimising the state covariance. The purpose of the EKF is to continuously estimate the system states using measurements, given the nonlinear motion and measurement models in (2.1). The filter linearisation is done by performing a Taylor expansion around the current state estimate. To do this the Jacobians of the motion and measurement equations with respect to the states are calculated and the resulting expressions are evaluated for the
current state estimate and used to perform the time and measurement updates.

**Time update:**

\[
\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1}),
\]

\[
P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + B_{k-1} Q_{k-1} B_{k-1}^T,
\]

where \( \hat{x}_{k|k-1} \) is the predicted states and \( P_{k|k-1} \) is the corresponding covariance.

The purpose of the time update is to predict the state and covariance based on the previous state and inputs using the motion model \( f(x, u) \). Due to the uncertainty in the motion model and input noise this update will increase the uncertainty of all states affected by it.

**Measurement update:**

\[
S_k = H_k P_{k|k-1} H_k^T + R_k,
\]

\[
K_k = P_{k|k-1} H_k^T S_k^{-1},
\]

\[
\epsilon_k = z_k - h(\hat{x}_{k|k-1}),
\]

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \epsilon_k,
\]

\[
P_{k|k} = (I - K_k H_k) P_{k|k-1},
\]

where \( \hat{x}_{k|k} \) is the states corrected with the latest measurements and \( P_{k|k} \) is the covariance of the updated estimates.

The measurement update is responsible for interpreting all of the measurement data received using the measurement model \( h(x) \) and update the states and covariance to better match this data. This is done by comparing the received measurements to predicted measurements and using the modelled uncertainty to correct the predictions from the time update.

These algorithms utilise the motion and measurement models’ Jacobians

\[
F_{k-1} = \frac{\partial f}{\partial x} \bigg|_{\hat{x}_{k-1|k-1}, u_{k-1}},
\]

\[
H_k = \frac{\partial h}{\partial x} \bigg|_{\hat{x}_{k|k-1}}.
\]

These equations illustrates the recursive algorithms of the time and measurement updates of the EKF respectively. See Ristic et al. [2004] or Gustafsson [2012] for more information.

### 2.4.3 Simultaneous Localisation and Mapping

The general idea behind EKF SLAM is to augment the regular EKF state vector \( x \), which generally contains information of the main target, in this case the UAV, with the states of so called landmarks. These landmarks can be any feature in the environment that can be recognised and measured, for example a visual feature in a camera image or the source of a radio signal. This enables the landmarks to
be estimated simultaneously with the target states as the EKF executes. By using
this algorithm a system can track its own position in relation to the surrounding
environment while a map of this environment is estimated and stored as a set of
landmarks. When these static landmarks are combined with the measurements
from an IMU the drift of the estimates caused by the IMU is reduced greatly.

The state vector is augmented with landmark properties \( l \) such as position co-
dordinates, directions or any property that can be related to the measurements
and used in order to obtain information about the rest of the states. This new
representation results in an augmented state vector and covariance matrix

\[
X = \begin{pmatrix} x \\ l \end{pmatrix}, \tag{2.6a}
\]

\[
P = \begin{pmatrix} P_{xx} & P_{xl} \\ P_{lx} & P_{ll} \end{pmatrix}, \tag{2.6b}
\]

where \( x \) are the states from the regular EKF and \( l \) are the states of the landmarks.
The resulting \( P \) is a bit more complicated as it is not only augmented with the
landmark uncertainty but also the cross-covariance between the added states and
the existent states, i.e. the correlation between the new landmark and all other
landmarks as well as the UAV-states [Durrant-Whyte and Bailey, 2006].

**Landmark Initialisation**

Every time a new landmark has been observed, a decision has to be made of
whether it should be added to the filter map or not. This is done differently de-
pending on application and design choices but may be done by simple gating. If
the decision is made that the newly observed landmark is to be added to the state
vector, it has to be initialised correctly by calculating not only the uncertainty of
the landmark itself, but also the correlation between the new landmark’s states
and all previous landmarks. The old state and covariance is augmented through

\[
X^+ = \begin{pmatrix} x \\ l \\ l_{new} \end{pmatrix}, \tag{2.7a}
\]

\[
P^+ = \begin{pmatrix} P_{xx} & P_{xl} & (G_x P_{xx})^T \\ P_{lx} & P_{ll} & (G_x P_{lx})^T \\ G_x P_{xx} & G_x P_{lx} & G_x P_{xx} G_x^T + G_z R_z G_z^T \end{pmatrix}, \tag{2.7b}
\]

with \( G_x \) and \( G_z \) being the Jacobians of the landmark equation \( g(x, z) \), correspond-
ing to the inverse of the measurement equation, i.e. the landmark states as a func-
tion of other states and the measurements. These Jacobians are calculated with
respect to the UAV states and the measurements respectively [Solà et al., 2005].

**Landmark Association**

In order to determine the source of a measurement, an association algorithm is
needed. The purpose of this algorithm is to make the connection between a mea-
surement and a landmark in the state vector. If there is no known landmark
likely to correspond to the measurement, a new landmark can be added to the map. This information was assumed known during this thesis and therefore no association algorithms were studied in depth.

2.5 Quaternion Representation

As an alternative to the traditionally common Euler representation of orientation using roll, pitch and yaw angles, the quaternion representation is instead defined by a vector containing four numbers describing an extension to the complex numbers. The elements in the four-dimensional quaternion vector are commonly referred to as 1, i, j and k, and the vector itself consist of the real numbers $q_0$, $q_1$, $q_2$, $q_3$ corresponding to the linear combination $q_0 + q_1i + q_2j + q_3k$. The multiplication between these elements is not commutative and defined from the basic rule

$$i^2 = j^2 = k^2 = ijk = -1.$$ (2.8a)

In order to correctly describe an orientation, which has three degrees of freedom, the quaternion must be of unit length.

The time derivative of the quaternion as a function of the quaternion orientation $q$ and the angular velocity $\omega$ is given by

$$\dot{q}(q, \omega) = \frac{1}{2} S(\omega)q = \frac{1}{2} \tilde{S}(q)\omega,$$ (2.9a)

where

$$S(\omega) = \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{pmatrix},$$ (2.9b)

$$\tilde{S}(q) = \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix},$$ (2.9c)

and the rotation matrix $R(q)$ is calculated according to

$$R(q) = \begin{pmatrix} 2(q_0^2 + q_1^2) - 1 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & 2(q_0^2 + q_2^2) - 1 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & 2(q_0^2 + q_3^2) - 1 \end{pmatrix}.$$ (2.9d)

The rotation matrix is an intuitive way to rotate vectors or translate points from one coordinate system to another by simple matrix multiplication. These spatial rotations can also be performed using the quaternion vector itself in a quaternion multiplication with the vector that is to be rotated.

Using this representation makes the description of the orientation more robust.
as it is not affected by Gimbal lock [Gustafsson, 2012].
First of all the data has to be generated in order to use for testing purposes which is done creating a trajectory and the corresponding measurement data that is to be used including the environment with transmitters. After this the SLAM filter itself can be implemented. The methods used are described in this chapter.

Measurements are generated using models of an IMU and an antenna array. These models are presented first in this chapter as most of the following sections are built around having access to this data. The EKF SLAM algorithm described in Section 2.4 is then further detailed along with its corresponding models. Finally two different state representations of landmarks are explained and briefly compared.

This chapter is mainly based on the theory in Chapter 2 and proceeds to explain relevant methods and choices behind the thesis work.

### 3.1 Data Generation

Since there is no real IMU, AOA or altimeter data available, this data is created manually. This is done by utilising the three dimensional position vector of an arbitrary flight path, either generated manually or calculated from the GPS coordinates of a test flight. The generated data includes the data from the IMU, the height, which was automatically obtained in the case of the GPS trajectory, as well as directional data to transmitters on the ground. These transmitters themselves have also been generated.
3.1 Transmitters

Radio transmitters placed close to ground level are simulated in order to have landmarks available for the SLAM filter. The transmitters were uniformly distributed with a set number of transmitters per square kilometre and with a random height uniformly distributed between 0 m and 100 m.

In addition to position, each transmitter also has a set of properties including a frequency $f_c (a \cdot 10^b$ where $a$ assumes a value between 1 and 10 with a uniform distribution and $b$ assumes either 8 or 9, with equal probability, resulting in frequencies between 100 MHz and 10 GHz), a bandwidth $f_b$ (approximated to $f_c/2000$) as well as an identification number in order to perform the given association in the filter.

3.1.2 Angle of Arrival

The AOA-data is generated by simply transforming the vector between the UAV and transmitters to a local coordinate system using the angles calculated in Section 3.1.4 and then converted into spherical coordinates. This is done for each transmitter with a received Signal to Noise Ratio (SNR) higher than 5 dB which equals a value of $\sim 3.16$. To model the antenna array, a scaling factor as a function of the elevation angle is added to the calculation of the SNR.

Sampling

The AOA is sampled at 10 Hz where every transmitter is detected at the same time. To reduce the effect of the noise of these measurements, the mean over one second was computed which enabled the use of consistency gating.

Noise Generation

The noise of the AOA-measurements is generated by applying a two dimensional Gaussian noise, for azimuth and elevation, adjusted with a covariance matrix based on the Cramér-Rao Bound (CRB). This matrix is calculated as the inverse of the array geometry dependant Fisher Information matrix which is a measurement of how much information the measurements contain with regard to the states, in this case the landmark position. The CRB can, in turn, be interpreted as the lower bound on the variance of any unbiased estimator, in this case in regards to position. This results in noise levels fairly close to those of real measurements.

The CRB is calculated as

$$\text{CRB} = G(B, z)^{-1} \left( \frac{2KN}{c^2} \sum_{i=1}^{I} \frac{\omega_i^2}{n_i} p_i \left( 1 - \frac{n_i}{p_iN + n_i} \right) \right)^{-1}$$

$$= G(B, z)^{-1} \left( \frac{T}{2\pi c^2} \left( \frac{(2\pi f_b)^3 + 12(2\pi f_c)^2(2\pi f_b)}{6} \frac{\text{SNR}^2}{\text{SNR} \cdot N + 1} \right) \right)^{-1}$$

(3.1)

where $N$ is the number of sensors, $K$ is the number of intervals the observation is partitioned into, $p_i$ and $n_i$ are the Fourier coefficients of the $I$-point discrete Fourier transform at frequency $\omega_i$ of the signal and the noise sources respectively.
3.1 Data Generation

\[ G(B, z) = J_z(u)^T B J_z(u), \]  
\( (3.2a) \)

where

\[ J_z(u) = \begin{pmatrix} \frac{\partial u}{\partial \phi} & \frac{\partial u}{\partial \theta} \end{pmatrix} = \begin{pmatrix} -\sin \phi \cos \theta & -\cos \phi \sin \theta \\ \cos \phi \cos \theta & -\sin \phi \cos \theta \\ 0 & \cos \theta \end{pmatrix} \]  
\( (3.2b) \)

is the Jacobian of the directional vector

\[ u = \begin{pmatrix} \cos \phi \cos \theta \\ \sin \phi \cos \theta \\ \sin \theta \end{pmatrix} \]  
\( (3.2c) \)

towards the observed landmark and

\[ B = \frac{1}{N} \sum_{i=1}^{N} (r_i - r_c)(r_i - r_c)^T, \]  
\( (3.2d) \)

\[ r_c = \frac{1}{N} \sum_{i=1}^{N} r_i, \]  
\( (3.2e) \)

with \( r_i \) as the Cartesian position of antenna \( i \) resulting in \( r_c \) being the center of the antenna array [Baysal and Moses, 2001].

The SNR is defined as the ratio between the received signal power and the noise power, i.e.

\[ \text{SNR} = \frac{P_s}{P_n}, \]  
\( (3.3a) \)

where \( P_s \) and \( P_n \) are the received signal and noise power respectively, calculated according to

\[ P_s = K_{ant}^2 P_e \left( \frac{c}{4\pi d F} \right)^2 \]  
\( (3.3b) \)

\[ P_n = 10^{\frac{S-30}{10}}. \]  
\( (3.3c) \)

\( K_{ant} \) is the modelled scaling factor based on elevation angle, \( P_e \) is the emitted signal power, which is assumed to be 1 W, \( d \) is the distance between UAV and transmitter, \( F \) is the frequency of the emitted signal, \( c \) is the propagation speed, in this case the speed of light, and \( S \) is the receiver sensitivity in dBm calculated in (3.3d). The scaling factor was based on the radiation pattern of the HL050S7 log-periodic directional antenna array by Rohde & Schwarz [2014] modified to fit the current application. The adjustments made to better suit this application is to slightly adjust the receivers angle of optimal reception, in this case directing it 10° downward and make it omnidirectional with respect to azimuth, resulting in the pattern shown in Figure 3.1. As illustrated in this figure, the antenna array is
assumed to have a maximum gain of 1.

\[ S = kTB + NF \quad (3.3d) \]

\( kTB \) is the internal thermal noise of the receiver and \( NF \) is the receiver system noise figure, which equals

\[ kTB = -114\text{dBm} + 10 \log(BW/1\text{MHz}) \quad (3.3e) \]
\[ NF = 7\text{dB} \quad (3.3f) \]

where \( BW \) is the transmitters bandwidth and \( T \) is the temperature in Kelvin (the standard temperature of 290 K is assumed, which is common practise in electronic warfare applications) [Adamy, 2009].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{radiation_pattern}
\caption{The interpretation of the radiation pattern as implemented in MATLAB, illustrated over the different elevation angles.}
\end{figure}

3.1.3 Altitude

The altimeter measurements are a slowly changing value and is therefore sampled at a fairly low frequency, in this case 1 Hz. As described briefly in Section 2.2, the altitude measurements are mainly characterised by a static error. However, the altitude measurements are the only measurements directly relatable to the absolute position estimates due to all other measurements being relative. This means that the bias error will act as a static displacement of all estimates where the bias is not observable. Because of this the noise is instead simply modelled as a white noise.
3.1.4 Inertial Measurements

In order to obtain the IMU data with an arbitrary sampling time, a cubic spline generation method is used for the conversion between a set of coordinates to a continuous trajectory. This simplifies both data extraction and the comparisons to a ground truth.

**Accelerometer**

The position vector is interpolated using a cubic spline with smoothing which is subsequently used to calculate both velocity and acceleration. The position spline is resampled at 10 Hz and a pink noise is added in each direction to simulate a more jagged and shaky trajectory. This pink noise, similar to brown noise, contains a larger amount of its energy in the lower frequencies compared to the higher frequencies, with a spectral density decreasing proportionally to the inverse of the frequency. Finally a noisy position spline is calculated based on these resulting points, using the same spline based smoothing as in the first case.

The resulting spline is used to obtain the velocity and acceleration by calculating its first and second order derivative. The resulting acceleration spline can then be sampled at an arbitrary sampling frequency.

**Gyroscope**

The calculation of the angular velocity is based on the accelerations, however, some assumptions has to be made since it can not be determined from the position alone.

First of all, the UAV is assumed to travel in the direction it was facing, i.e. the projection of the body fixed $x$-axis onto the global horizontal plane is parallel to the momentary velocity in the global horizontal plane.

This means that a rotation, $\psi_G$, around the global $z$-axis representing the height above ground, must be calculated.

This angle $\psi_G$ can then rotate the global acceleration, in order to line up the UAV with the velocity vector in the global $xy$-plane.

Secondly, the UAV is assumed to be able to rotate around any axis but only accelerate along its body fixed $z$-axis, thus simulating a simple helicopter model. The angular velocity is obtained by calculating the derivative of the angular pose, resulting in a finished model for generating the IMU measurements

\[
\begin{pmatrix}
\theta \\
-\phi
\end{pmatrix} = \frac{\arctan \left( \frac{\|a_x + a_y\|}{a_z + 9.82} \right)}{\|a_x + a_y\|} \cdot R^{L/G}(0, 0, \psi_G) \cdot \begin{pmatrix} a_x \\ a_y \end{pmatrix}.
\]

Equation (3.4) describes the assumed correlation between the accelerations $a_x$, $a_y$ and $a_z$ in a body fixed coordinate system and the roll and pitch angles in the same coordinate system. The matrix $R^{L/G}(\phi, \theta, \psi)$ represents the rotational matrix from global to local coordinates as a function of the roll $\phi$, the pitch $\theta$ and the yaw $\psi$. 
The time derivative of the angles $\phi$, $\theta$ and $\psi_G$ are then calculated and the roll and pitch, calculated according to (3.4) above, are used to rotate the global yaw angular velocity to a local coordinate system as described in (3.5).

\[
\begin{pmatrix}
\dot{\phi}_{\text{gyro}} \\
\dot{\theta}_{\text{gyro}} \\
\dot{\psi}_{\text{gyro}}
\end{pmatrix} = \begin{pmatrix}
\frac{d\phi}{dt} \\
\frac{d\theta}{dt} \\
0
\end{pmatrix} + R^{L/G}(\phi, \theta, 0) \cdot \begin{pmatrix}
0 \\
0 \\
\frac{d\psi_G}{dt}
\end{pmatrix}
\]  

(3.5)

This results in roll and pitch angles between $-\pi/2$ and $\pi/2$ which is reasonable considering the model in question.

**Sampling**

The IMU is sampled at 100 Hz for both the accelerometer and the gyroscope.

**Noise Generation**

As described in Section 2.3, the noise added to these signals is a combination of Brownian noise with an initial offset and white noise, which means that the signal has a slowly changing bias that can be estimated in order to obtain improved robustness. This is a common way to model noise in an IMU [Woodman, 2007].

The noise was generated by creating two vectors of normally distributed white noise, $n_{w,k}$ and $n_{b,k}$, with $\sigma_w$ as the white noise standard deviation and $\sigma_b$ as the Brownian noise standard deviation, or more specifically, the standard deviation of the white noise used to generate the Brownian noise. The complete noise term $e_k$ is then calculated according to

\[
e_k = \epsilon + n_{w,k} + \sum_{i=1}^{k} n_{b,i},
\]  

(3.6)

where the bias $\epsilon$ is implemented as the starting value of the noise and is generated by setting $\epsilon = 1000n_{b,1}$. The values for $\sigma_w$ ($\frac{0.013}{\sqrt{T_s}}$ m/s\(^2\) for the accelerometer and $\frac{0.013}{\sqrt{T_s}}$ rad/s for the gyroscope) and $\sigma_b$ ($\frac{1.5 \cdot 10^{-7}}{\sqrt{T_s}}$ m/s\(^2\) for the accelerometer and $\frac{8.7 \cdot 10^{-8}}{\sqrt{T_s}}$ rad/s for the gyroscope) are taken as an example from an arbitrary IMU in Woodman [2007].

**3.2 Simultaneous Localisation and Mapping**

The tracking algorithm used is based on the EKF SLAM as described in Section 2.4.3. In this section the models and methods implemented are described and put into context.
3.2 Simultaneous Localisation and Mapping

3.2.1 Models

To be able to estimate the states for the current application the time and measurement equations from (2.1) had to be specified. The measurements available are the AOA measurements, acceleration and angular velocity from the IMU and height. Both acceleration and angular velocity were treated as inputs, $u_{ACC}$ and $u_{GYR}$, while the other measurements were treated as outputs, $z_{AOA}$ and $z_{H}$. The motion model implemented is therefore a constant velocity model in the sense that the change in velocity, other than the measured acceleration, is modelled as Gaussian noise.

The same argument can be made for the orientation, which is represented using a quaternion vector, as described in Section 2.5. Since the orientation itself has three degrees of freedom and is represented by the four values of the quaternion vector there is a redundancy in the description. Therefore, in order to properly represent the orientation, the vector is assumed to be of unit length, i.e. $q^T q = 1$. Additionally, to avoid the ambiguities resulting from the fact that $q$ represents the same rotation as $-q$, the first element $q_0$ is assumed to always be positive. These properties are not ensured in the filter by itself and must therefore be maintained by normalising and multiplying the vector with the sign of $q_0$ after each update.

The states used to represent the UAV is position $p$, velocity $v$ and orientation $q$. To model the noise of the IMU a Brownian noise model is assumed. This results in both noise parameters directly affecting the velocity and quaternion states, $w_A$ and $w_w$, as well as random walk noise affecting the bias of the inputs, $w_{ACC}$ and $w_{GYR}$. To account for this bias, two more three dimensional states were added corresponding to the bias in accelerometer and gyroscope, $b_{ACC}$ and $b_{GYR}$. In addition to this there is measurement noise $v_{AZ,k}$ on the azimuth, $v_{EL,k}$ on the elevation and $v_{H,k}$ on the height measurement. The full model is described as

\[
x_k = \begin{pmatrix} p_k \\ v_k \\ q_k \\ b_{ACC,k} \\ b_{GYR,k} \\ l_k \end{pmatrix} = \begin{pmatrix} p_{k-1} + T v_{k-1} \\ v_{k-1} + T (R(q_{k-1})(u_{ACC,k-1} - b_{ACC,k-1}) - g) \\ (I + T S(u_{GYR,k-1} - b_{GYR,k-1}) q_{k-1}) q_{k-1} \\ b_{ACC,k-1} \\ b_{GYR,k-1} \\ l_{k-1} \end{pmatrix} + \begin{pmatrix} T^2 R(q_{k-1}) w_{A,k} \\ T R(q_{k-1}) w_{A,k} \\ \frac{T}{2} S(q_{k-1}) w_{w,k} \\ T w_{ACC,k} \\ T w_{GYR,k} \\ 0 \end{pmatrix},
\]

(3.7a)

\[
z_{AOA,k,i} = \begin{pmatrix} \arctan \frac{d_{y,k,i}}{d_{x,k,i}} \\ \arctan \frac{d_{z,k,i}}{\sqrt{d_{x,k,i}^2 + d_{y,k,i}^2}} \end{pmatrix} + \begin{pmatrix} v_{AZ,k} \\ v_{EL,k} \end{pmatrix}
\]

(3.7b)

\[
z_{H,k} = p_{Z,k} + v_{H,k},
\]

(3.7c)
where

\[
d_{k,i} = \begin{pmatrix} d_{x,k,i} \\ d_{y,k,i} \\ d_{z,k,i} \end{pmatrix} = R(q_k)^T (m_i - p_k).
\] (3.7d)

\(S(\omega)\) and \(\bar{S}(q)\) in the above equations are the matrices described in (2.9) and \(R(q)\) is the three dimensional rotational matrix, as a function of the quaternion vector \(q\), which represents a rotation from a local coordinate system described by \(q\) to a global coordinate system, also described in (2.9). The measurement \(z_{\text{AOA},k,i}\) represents the AOA-measurement at time \(k\) of landmark \(i\) and these measurements are stacked at every point in time to account for all landmarks at once. \(l_k\) is the part of the state vector containing all the landmark states, described in Section 3.2.2 below.

The time and measurement updates are performed as described in (2.2) and (2.3) using the equations in (3.7).

### 3.2.2 Landmarks

Since only bearing measurements are available, it is impossible to determine the position of a new landmark from only one observation. This, in combination with the fact that the distance to many of the landmarks is so large, makes an initial guess very inaccurate without prior knowledge. The one thing that may be assumed is that the landmarks are placed close to the ground, however due to the difficulty of expressing the uncertainty correlation between the \(x, y\) position and the landmark height at large distances, it is not as simple as initialising the landmark at a reasonable yet arbitrary height.

The initiation is performed as described in Section 2.4.3 and two main methods of solving the issue has been implemented, both of which are still utilising the fact that the approximate height is known.

#### Cartesian Representation

The landmark \(z\) coordinate, which represents the height of the landmark, is introduced as a pseudo measurement during the initialisation with variance \(R_z\) treated as a tuning parameter. This height is then utilised as a regular independent measurement in order to improve the initial guess and the Jacobian \(G_z\) from (2.7) is extended accordingly (see Tully et al. [2008] for an example of using range as a pseudo measurement). This results in each landmark being represented as

\[
l_i = m_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}, \quad (3.8a)
\]
3.2 Simultaneous Localisation and Mapping

and for the initiation use the augmented measurement equation

\[
z_{AOA,k,i} = \left( \arctan \frac{d_{y,k,i}}{d_{z,k,i}} \right) + \left( \begin{array}{c} v_{AZ,k} \\ v_{EL,k} \\ v_{H,k} \end{array} \right)
\]

(3.8b)

where

\[
d_{k,i} = \begin{pmatrix} d_{x,k,i} \\ d_{y,k,i} \\ d_{z,k,i} \end{pmatrix} = R(q_k)^T (m_i - p_k),
\]

(3.8c)

with \( m_i \) representing the global position of landmark \( i \) in Cartesian coordinates. This interpretation of the pseudo measurement also affects the Jacobians \( G_x \) and \( G_z \) of the measurement equation from (2.7). The landmark height uncertainty is in this case easily tunable by scaling the value of \( R_z \). The initial value of \( m_i \) corresponding to the landmark position is calculated as the point where the directional vector of the measurement is crossing the plane at the initial guess of the landmark height.

**Inverse Depth Representation**

The Inverse Depth (ID) representation represents each landmark as six states instead of the three representing the position. The purpose of using this representation is to make it easier to represent an unknown location where an infinite distance is still a possibility while staying closer to a Gaussian distribution with respect to the range uncertainty in the case of low observed parallax. The height of the landmark is still used as a simple way of obtaining a rough estimate of the distance. The distance is in turn used as a pseudo measurement much in the same way the height was used in the previous section and the Jacobian in the initiation from (2.7) is augmented accordingly. The new state representation for each landmark is

\[
l_i = \begin{pmatrix} x_i \\ y_i \\ z_i \\ \phi_i \\ \theta_i \\ \rho_i \end{pmatrix},
\]

(3.9a)

\[
m_i = \begin{pmatrix} x_i \\ y_i \\ z_i \\ \frac{1}{\rho_i} \begin{pmatrix} \cos \phi_i \cos \theta_i \\ \sin \phi_i \cos \theta_i \\ \sin \theta_i \end{pmatrix} \end{pmatrix},
\]

(3.9b)
and for the initiation use the augmented measurement equation

\[ z_{\text{AOA},k,i} = \begin{pmatrix} \arctan \frac{d_{x,k,i}}{d_{x,k,i}^2 + d_{y,k,i}^2} \\ \arctan \frac{d_{z,k,i}}{d_{x,k,i}^2 + d_{y,k,i}^2} \end{pmatrix} + \begin{pmatrix} \nu_{\text{AZ},k} \\ \nu_{\text{EL},k} \\ \nu_{\rho,k} \end{pmatrix} \] (3.9c)

where

\[ d_{k,i} = \begin{pmatrix} d_{x,k,i} \\ d_{y,k,i} \\ d_{z,k,i} \end{pmatrix} = R(q_k)^T (m_i - p_k). \] (3.9d)

The states \( x_i, y_i, z_i \) represents the position from where landmark \( i \) was first observed and \( \phi_i, \theta_i \) and \( \rho_i \) represents the angle towards as well as the inverse distance to the landmark. Again \( m_i \) is the position of the landmark in Cartesian coordinates and \( p_k \) is the UAV position.

The range uncertainty is set in such a way that infinity is a clear possibility, in this case such that the 95% confidence region spans \([2\rho_0, 0]\) which corresponds to a distance interval of \([d_0^2, \infty]\) where \( d_0 = \frac{1}{\rho_0} \) is the initial guess of the distance.

When sufficient parallax has been observed, the state representation can be converted into regular Cartesian coordinates as this representation is also close to linear when the range uncertainty decreases. This state transformation is not necessary as the ID representation still retains its linear properties even after high parallax has been observed, however, this transformation slightly reduces the computational cost. The new states are then simply calculated according to (3.9b) and substituted into the state vector and the new state covariance is calculated as

\[ P_{\text{new}} = JPJ^T, \] (3.10a)

where

\[ J = \begin{pmatrix} I & 0 & 0 \\ 0 & \frac{\partial m_i}{\partial l_i} & 0 \\ 0 & 0 & I \end{pmatrix}. \] (3.10b)

In order to determine if a conversion is suitable, the linearity of the Cartesian representation in the 95% confidence interval is evaluated for each landmark coded in ID. By examining the value of the index

\[ L = \begin{vmatrix} \frac{\partial^2 h}{\partial \rho^2} & \mu_\rho^2 \\ \frac{\partial h}{\partial \rho} & \mu_\rho \end{vmatrix} \] (3.11)

where \( h \) is the measurement equation, the linearity can be evaluated. When the function is perfectly linear, \( L \) will be equal to 0. The value of \( L \) then increases as the equation becomes more and more nonlinear. This means that \( L \) can be used
with a threshold to determine when to change representation. This threshold can be used to tune the filter, where a low value prioritises accuracy and a higher value prioritises speed.

When calculated for the Cartesian representation, \( L \) provides a hint of the performance of the filter. In the case of landmark \( i \), the linearity index is calculated as

\[
L_i = \frac{4\sigma_d}{d_i} \cos \alpha,
\]  

(3.12a)

with

\[
\sigma_d = \frac{\sigma_p}{\rho_i^*},
\]  

(3.12b)

\[
d_i = ||r_i||
\]  

(3.12c)

\[
\cos \alpha = \left( \begin{array}{c} \cos \phi_i \cos \theta_i \\ \sin \phi_i \cos \theta_i \\ \sin \theta_i \end{array} \right)^T r_i ||r_i||^{-1},
\]  

(3.12d)

where the vector

\[
r_i = \hat{m}_i - \hat{p}.
\]  

(3.12e)

See Civera et al. [2008] for more information.

### 3.2.3 Measurement gating

To eliminate some of the effects of the noise in the AOA-measurements, the mean over one second was calculated and used as measurement instead. This greatly reduced the effect of measurement outliers and enabled the use of gating based on the consistency of several successive measurements. The gating consists of trying to determine which measurements are good enough to use, for example with respect to noise levels, and to disregard the others. In this case, only measurements which are consistent over the one second period are allowed to affect the states, i.e. all individual measurements must pass a gating process to eliminate measurements with low SNR. If a single measurement from a landmark falls below the SNR-threshold, the entire second of measurements from that landmark is discarded. In addition to this, the overall variance over this second has to be below a certain value and the mean elevation angle has to be below zero for the measurements to be considered useful.

The resulting mean is calculated using a sliding window principle in order to keep the 10 Hz update frequency. This means that the average of measurements from the past second is calculated every time a new measurement is obtained. This mean can then be used as a more stable measurement.
In order to test the implementation described in Chapter 3, several different experiments using mainly the circular and the GPS trajectory were performed. These were performed using different numbers of transmitters for investigating robustness properties.

The results presented in this chapter are achieved for cases where the area extends 20 km from each maximum and minimum value of the true trajectory in the case of the GPS data and a simple 40 km by 40 km area is used with the circular trajectory. Within this area transmitters are randomly placed at a varying density as described in Section 3.1.1.

The data generated and the filter results are displayed in this section along with some simple feasibility tests used to determine whether the results are reasonable or not.

### 4.1 Models and measurements

Some simple attempts to validate the model for the data generation were performed as well as studies of the measurements to make sure they were reasonable.

The smoothed trajectory without any added noise was compared to the original data points and when sampled with the same frequency as the original data, the standard deviation of the positional error was found to be close to 1 m.
Figure 4.1: The roll angle and the local accelerations of the generated data where the acceleration in the body fixed xy-plane is very close to zero.

4.1.1 IMU

Figure 4.1 shows the roll angle and the acceleration when travelling in a circular motion with radius of 1 km at speeds from 50 m/s to 300 m/s and compares the results to the theoretical value of the total acceleration, calculated by using the Pythagorean theorem $a_{tot}^2 = g^2 + a_c^2$, using the gravitation and the circular acceleration $a_c = v^2/r$ with $v$ as the tangential velocity and $r$ as the radius. The speed used is not supposed to represent the speed of any real UAV but only to evaluate the model used to generate the data.

Figure 4.1 illustrates that the total acceleration is very close to the theoretical value, with a slight deviation at high velocities most likely due to the smoothing performed on the trajectory. Additionally, the roll angle can be seen possibly approaching $-\pi/2$ as the speed increases, which is the theoretical limit of the model. The fact that all acceleration is measured along the body fixed z-axis verifies that the generation tries to simulate a helicopter, of which a simple model can be said to have this dynamic.
The resulting trajectory and orientation estimates of the circular trajectory is illustrated in Figure 4.2. In this case, the circle radius is 1 km and the target is travelling at 30 m/s with noise added to the trajectory as described in Section 3.1.4. This figure serves to illustrate the result of a conversion from quaternion representation to Cartesian vectors and the change in orientation during a circular flight.

Despite the fact that it is using the data after estimation, as the original purpose of the graph was to visualise the quaternion representation, the results seem reasonable as it is slightly tilted inwards resulting in a centripetal force to achieve the circular motion.

The signals from the IMU are generated according to Section 3.1.4 and the results from the longer GPS trajectory are displayed with and without noise in Figure 4.3 and Figure 4.4 for the accelerometer and the gyroscope respectively.

4.1.2 AOA

The AOA measurements are calculated as described in Section 3.1.2 and the result is demonstrated through Figure 4.5. The AOA is represented in this plot by a directional vector towards a single, close by transmitter, where noise has been added to the measurement. From this figure, the measurements are easily veri-
Figure 4.3: The accelerometer signals from the GPS trajectory without and with measurement noise.
4.1 Models and measurements

![Graphs showing angular velocities without and with noise.](image)

(a) Angular velocity without noise.

(b) Angular velocity with noise.

Figure 4.4: The gyroscope signals from the GPS trajectory without and with measurement noise.
fied to be close to the truth.

The model used for generating the AOA is based on some approximations including an emitted signal power of 1 W and a spherical line of sight propagation assumption. The model based on CRB is arguably a good representation considering the noise levels discussed in Section 5.2.1.

Figure 4.6 illustrates the received signal power for the registered AOA measurements in a histogram, assuming an emitted signal power of 1 W.

### 4.1.3 Altitude

The altitude measurements were generated with a simple Gaussian noise added to the height from the trajectory, which is an approximation as the main noise component of altitude measurements is generally a bias. This will most likely not affect the results in this thesis but must be kept in mind for future applications.

### 4.2 State Estimation

The ID method of representing the landmarks was selected as the main focus when evaluating the results as this method is more accurate in theory. All results of the GPS-simulations had transmitters randomly placed in an area extended 20 km in each direction from the trajectory’s extreme points and in the case of the circular trajectory, the transmitter area was a fixed 40 km by 40 km area.

The initial states was set as true position, which was translated in order to start in origo with a height that was set to the first received value, and true orientation. The velocity and bias of the IMU was assumed unknown and initiated as 0. The covariance matrix was set as a diagonal matrix with non-zero variance in height and velocity which were set to $10^2$ and $30^2$ (in each direction) respectively. The initial bias uncertainty was set to the value corresponding to the variance of the starting bias, i.e. the square of 1000 times the brown noise standard deviation, as described in Section 3.1.4.

The AOA noise covariance used in the filter was scaled up by a factor 100 for both representations below, both in the filter and during the initialisation of the landmark, as a result of empirical tuning. In the same way, the noise used in the measurement equation was based on the IMU noise values in Section 3.1.4 and tuned up a bit to account for the model errors as well. However, this was tuned slightly differently for the two cases below as it was scaled up by a factor 2 for inverse depth representation and a factor 5 for the Cartesian representation, again from empirical tuning.

The need for this scaling is most likely due to uncertainty of the true measurement noise. This covariance is based on the measurements after noise has been added and therefore contains an uncertainty which is especially large for far away transmitters and transmitters with a low frequency.
Figure 4.5: The AOA measurements with measurement noise from a simulation of a circular trajectory.
Figure 4.6: Received signal power from the radio transmitters over the GPS trajectory.
4.2 State Estimation

4.2.1 Cartesian Representation

The initial height variance $R_Z$ was set to 1000 during the initialisation of these landmarks.

The results of a successful simulation using a transmitter density of 0.025 transmitters per square kilometre, i.e. transmitters were randomly placed until this density is achieved, are shown in Figure 4.7 with an absolute error according to Figure 4.10a over the course of the trajectory. This simulation was selected as the results were comparable to those of the ID representation demonstrated in Section 4.2.2, however, most simulations resulted in estimates significantly worse than the for the ID case.

4.2.2 Inverse Depth Representation

Results from a simulation using the same data as Section 4.2.1 are plotted in Figure 4.8 with the absolute positional error in Figure 4.10b. The simulation results using this representation are a lot more consistent compared to the Cartesian case. There are variations due to the amount of random variables in the data generation yet almost all tests resulted in decent trajectories. The results are much better than that of the Cartesian representation but some characteristics remain such as the bias as well as the underestimation of the positional uncertainty.

The covariance is illustrated by plotting the 95 % confidence regions of the beginning and the end of the trajectory as well as for a transmitter in Figure 4.9. The transmitter has been converted to Cartesian coordinates, meaning that the range uncertainty is low. The uncertainty is still largest in the range direction as can be seen by the alignment of the ellipse, as the trajectory is mainly centred around origo where it begins. The estimates converge fairly quickly at the beginning of the trajectory with reasonable uncertainty and towards the end there is an error in the form of an offset. The estimates of the transmitters suffer from the same offset issue as the UAV position however it relies more on the initial estimate of the position which could most likely be reduced by adding a small noise on the transmitter position in the time update of the filter.

The results of simulations using different transmitter densities are illustrated using the position error of the estimates in Figure 4.10. One simulation using a density of 0.01 transmitters per square kilometre was providing significantly worse estimations, otherwise the results are similar to the cases plotted. The final part of the trajectory is consistently providing larger errors for the UAV position estimate, however the result is significantly better for the ID representation compared to the Cartesian one.

Figure 4.11 plots the absolute positional error of a circular trajectory over the course of four circulations while travelling at 30 m/s. The error is significantly smaller when close to the starting location compared to the opposite side of the circle. The overall error decreases each new lap and more significant improvements can be seen in the beginning of the trajectory.
Figure 4.7: The estimation results using the Cartesian representation with a transmitter density of 0.025 transmitters per square kilometre.
4.2 State Estimation

Figure 4.8: The estimation results using the ID representation with a transmitter density of 0.025 transmitters per square kilometre.
Figure 4.9: The 95% confidence area of the trajectory and transmitters of an ID simulation.
4.2 State Estimation

Figure 4.10: The absolute error in position of simulations using different transmitter densities.

(a) Transmitter density of 0.025 using Cartesian representation.

(b) Transmitter density of 0.025 using ID representation.

(c) Transmitter density of 0.01 using ID representation.

(d) Transmitter density of 0.005 using ID representation.
Figure 4.11: The absolute error in position for a circular trajectory using ID representation.
As the results are obtained, these has to be analysed with respect to performance and feasibility. The analysis must be made while relating the results to the models used to obtain them.

This chapter contains a general discussion of the main issues encountered while using the proposed method and the reasons behind them, both in regards to the results and the methods used to generate them.

5.1 Results

The overall results are promising and if similar results can be produced on-line, the investigated methods may prove useful in many applications. Most of the methods and algorithms which have been implemented can be used for real flights but some issues such as association and noise modelling must be added or extended and are further discussed in Section 5.2.

The results are fairly reliable and of all the simulations performed only one was found to be unable to estimate the trajectory using the ID representation of the landmarks, which was for a simulation using lower transmitter density and was most likely caused by the lack of transmitters in the vicinity of the starting location. However only one, more complex trajectory was available which reduces the overall credibility of the results. The other tested trajectories, such as the circular and the sine trajectory, were affected by an incorrect scaling of the map and trajectory estimates compared to the ground truth. The Cartesian representation is a lot less reliable as it has a tendency to diverge after initialisation due to the noisy measurements.
5.1.1 Trajectory and Model

The results of the data generation are difficult to analyse as it is the ground truth itself that is created, or rather that which is used as the ground truth for the measurement generation and later used for estimate evaluation. Therefore no real comparisons can be made to analyse the performance of data generation, however, some points can still be discussed to analyse the validity of the generated data.

Transmitters

The transmitter bandwidth was assumed to be \( f_b = f_c / 2000 \), where \( f_c \) is the transmitter frequency, according to a rule of thumb. Whether the resulting frequency and bandwidth are realistic and correspond to a known application or not was ignored and no such transmitter models were used. While this could have been interesting from the application point of view, the results should not be affected as long as the frequency is within the detectable interval.

The limitation of assuming stationary transmitters should not cause future issues as long as the association and gating is robust. In this case, a simple motion model could be added to the landmarks. It might increase the overall filter sensitivity as if there are few landmarks the filter might have difficulties differentiating between UAV movement and transmitter movement.

Trajectory Smoothing

The smoothing is needed since the third derivative of the trajectory is calculated and even the smallest unevenness in the spline function will result in large jumps in the higher order derivatives. In the case of the GPS data set, this resulted in a major artefact which was not visible in the trajectory plots but very notable in the acceleration. The smoothing helped to greatly reduce this issue and using a smoothed trajectory will not pose any problems with regards to estimation accuracy since the measurements generated will be interpreted as the ground truth. It can even be beneficial since the measurement noise manually added can be known, unlike the measurement noise of the GPS which supplied the raw data.

Motion Model

The simple helicopter model used in the generation appears to be reasonable judging from Section 4.1. While this model is fairly basic, it provides a way to calculate the IMU signals from a series of coordinates and the results appear good.

5.1.2 Measurements

As seen in Section 4.1, the noise added to the IMU-measurements is quite significant, most noticeably for the angular velocity. The values for this noise was taken from Woodman [2007] who calculated these values from the data of a fairly cheap IMU. This means the IMU noise levels correspond to a cheap IMU from 2007, at the latest. The noise levels in a modern IMU therefore have the potential of being significantly lower than those used in this implementation. It is possible that this is not the most ideal way to model the noise of an IMU on a UAV and it may result
in poor filter performance in cases where less landmarks are observed, i.e. where the filter has to rely more heavily on inertial measurements.

The noise of the AOA measurements vary largely depending on the transmitter distance, frequency and whether elevation or azimuth is regarded. Elevation has a significantly larger error which results in even bigger difficulties estimating the position of far away landmarks, which were already sensitive due to the lack of range measurements. The values of the noise standard deviation do however seem reasonable in size although they become very large if the frequency is high and the landmark is far away. This causes the estimation of the noise to be inaccurate which is one of the reasons for why the gating is needed.

5.1.3 Estimations

The filter is able to estimate the position in most evaluated scenarios. Some issues appear for cases such as low flight due to the way the gating is performed or when large areas are covered in a short amount of time, when old landmarks are lost and new ones are not yet certain.

Because of the constant velocity motion model the filter will lose some accuracy during sharp turns, which are common in the GPS trajectory. While the measurements from the IMU somewhat reduce the error as they affect the prediction instantly, the measurements are noisy and can not completely predict the position during the time update.

Absolute Position and Scale

An issue intrinsic to the original problem is the lack of observability causing the global position to be impossible to estimate without prior knowledge. Assumptions regarding true initial position and orientation enables relative estimates to be obtainable and accurate in all aspects but the overall scale. This results in only accurate height estimation and errors in position that increases as the target moves further away from the initial position. This effect can be observed in several of the figures in Section 4.2, most noticeably in Figure 4.11.

The effects of this scaling issue are reduced by the IMU measurements which measure the acceleration and angular velocity relative to the world frame. Depending on the noise levels of these measurements, the scaling issue should be reduced during regular flight if the same landmarks are observed repeatedly. The effect is also reduced by the fact that an approximate height of the landmarks is known, however, this height is only used for initialisation purposes and since it is used with high uncertainties, this does not completely solve the issue. As the filter and the measurements are not ideal or without noise, some additional drift is to be expected.

Despite this, there is a possibility to relate the entire trajectory to the starting location given good enough initial states. This means that if the initial states are known, a good estimate of the true trajectory should be obtainable. Although this is theoretically possible, the filter has a tendency to drift slightly initially due to
the noise which, in rare cases, displaces the initial landmarks. This causes an offset in overall position and decreases the accuracy of the estimation.

**Estimation Error**

As seen in Figure 4.9 there is an error between some of the estimates and the ground truth considering the size of the covariance ellipse, i.e. the uncertainty of the estimation does not alone describe the true error. Within each simulation this error often is fairly consistent, suggesting an unobservable position offset in the horizontal plane. This might also be related to the fact that the scale is hard to determine but that the relative position estimate is more accurate, which explain why the error grows as the UAV travels further away from the starting point. It might also be contributed to drift that occurs during the longer simulations. Additionally, the EKF is known to underestimate the covariance of the estimates which might be part of the cause for the low covariance.

**Gating**

The current implementation of the gating is better at handling flight at a high altitude than at a low. This is a result of both the antenna array model illustrated in Figure 3.1, which slowly decreases the SNR as the elevation angle increases, and that the average of the elevation angle itself is increasing, which increases the possibility of being gated. This issue was somewhat lessened by the tuning of the gating thresholds but is still present.

**Landmark Density**

In order to obtain relevant estimates a certain number of landmarks are required, preferably with reliable observations at all times. This number is related to the preferred minimum number of landmarks that are observed at each point in time. Due to the gating this results in a requirement for a sufficient number of close by landmarks, although frequency is more important than proximity in most cases with regard to the effects on the SNR. For simple cases such as the circular trajectory in a small area, four detected transmitters are in some cases sufficient for a good result, but the number is harder to determine in large areas as more factors are contributing. As the landmarks get further away, the measurements of them become increasingly noisy which results in a need of more observable landmarks.

The simulations using the GPS-trajectory with the ID representation were utilising a few different densities of transmitters as described in Section 4.2. When using a density of 0.025 transmitters per square kilometre, 79 transmitters are placed, out of which around half in average were used at some point during the simulation. Most evaluated cases provided good results, however not identical since the initial noise conditions and transmitter placements affected the scaling of the estimation.

At densities of 0.01 transmitters per square kilometre, with 31 transmitters in total and again with around half providing sufficiently good measurements at some point, the results start to vary slightly. Some results are as good as estimations using higher densities, most likely due to the random properties of the noise and
transmitter placements, and some are significantly worse. This indicates that, at some point, more transmitters alone does not improve the results further, they only increase the possibility of receiving enough stable measurements to provide a good estimate.

In general, the fewer available landmarks there are, especially in the beginning, the more the filter has to rely on the inertial measurements. If by chance the UAV has very few landmarks available at the start, the estimates are not guaranteed to converge at all, regardless of the overall density. Some tests were done with a density of 0.005 transmitters per square kilometre which results in a mere 15 transmitters in total, with even fewer providing good enough measurements to be used, and the results are therefore not very robust. However, the filter is still able to provide accurate estimates in the simulations if the transmitters that are observed are sufficiently many and the measurements does not contain an excessive amount of noise.

As shown in Figure 4.10, the estimates slowly degrade with the reduction of the number of landmarks and the state uncertainties increase in response to this. While this is true, the capability of estimating the general appearance of the trajectory is still maintained to a greater extent than expected and several simulations with 8-9 registered transmitters resulted in decent estimates.

**Problematic Situations**

At the end of the GPS trajectory, the UAV is performing a lower flight that travels away from most of the known landmarks. This is seen in the figures in Section 4.2 and is the reason for the increase in error at the end of the trajectory in Figure 4.10. This increases the uncertainty in all cases, which is reasonable considering the issues discussed previously. During this period, several new landmarks are initiated and become important to the estimation. As the ID method is better able to model the uncertainty of very new landmarks, it is less sensitive yet still susceptible to this fast registration of new landmarks compared to the other initialisation methods tried. Cases where the UAV travels in these patterns from the start are hard for any current implementation to correctly estimate and the case of a sine shaped trajectory illustrates this by having a large error due to the scaling of the system not being easily observable. When the UAV is allowed to return to the starting point or travel in the proximity of it for an extended period of time, the filter is given a chance to reduce the effects of this issue, however they are still present.

Regardless of transmitter density, the performance of the filter depends largely on the estimations in the beginning of the trajectory. If the state estimates are poor during the first part of the trajectory, the filter will not converge towards the truth but rather towards an identical trajectory with an unknown offset which is not observable. This is most likely due to an initial misplacement of the landmarks which the filter can not correct. This will propagate through the trajectory to new landmarks which will be affected by a similar offset. Additionally, poor initial estimations also increases the effect of the overall scale issue, however this
will very slowly correct itself from the measurements of the IMU.

5.2 Method

While the filter works fairly well, the models are based on optimistic conditions and they only represent the results of a simulation. Everything regarding the data generation is an approximation or simplification and while exact models are extremely hard or impossible to determine, a more solid connection to reality would yield more trustworthy results.

The validity, as in how accurate these approximations are, is therefore hard to determine as the best way to investigate this would be to evaluate the same methods using true measurements. This should therefore be a future priority in order to determine if the results in this thesis are obtainable on-line.

5.2.1 Data Generation

The method for generating the data is simple and not necessarily a source of errors resulting from large approximations. What might cause future problems though, is the accuracy of the noise modelling, mainly in regards to the AOA.

When it comes to the noise added to the AOA one could argue that, since the noise is based on the CRB, it is an optimistic noise model. The CRB in this case specifies the lowest possible variance an estimation could achieve and might therefore be underestimating the measurement noise compared to real measurements. As mentioned in Section 5.1.2 however, the values seem reasonable where the azimuth standard deviation is mostly within 0.1° to 3° and for the elevation within 1° to 30°.

The antenna array model can be tuned a bit as the downward angle illustrated in Figure 3.1 might not be optimal at 10°. It worked sufficiently well during simulation and was considered of minor importance.

5.2.2 Filter

The two main SLAM versions considered for this application were the EKF SLAM and the Fast SLAM which is based on the Particle Filter instead (see Durrant-Whyte and Bailey [2006] or Gustafsson [2012] for more information and comparisons). These two candidates are the most basic and straightforward versions of SLAM and therefore a good place to start evaluating the performance. The reason EKF SLAM was chosen was a combination between it being the slightly simpler version and that it is able to handle loop closure in a better way. Loop closure refers to being able to recognise areas and landmarks which have been seen before and therefore reduce long term drift when returning to previously observed places. However, since the true association was assumed known, the filter recognises old landmarks automatically and a more advanced loop closure algorithm would prove redundant. The Fast SLAM does scale better in regards to number
of landmarks (although worse in regards to the state dimension) which was not considered reason enough to implement it over EKF SLAM.

The Cartesian representation had a tendency to let some estimates of landmarks drift off very far away for some simulations. This often negatively affected the estimation of the trajectory, yet not necessarily noticeably since the erroneous landmarks were accompanied by large uncertainties. This representation is a lot more susceptible to the scaling issue which is a result of the landmark initiation where an uncertain initial direction and height propagates in a bad way as the filter executes. This illustrates the issues of a nonlinear measurement equation combined with high noise levels. While the initial variance of the landmark height could be reduced in order to better incorporate the semi-unknown height, this restricted most landmarks to the initial height and in order to perform a fair comparison to the more robust ID method, the uncertainty was kept high.

The ID representation of the landmarks might not always result in better estimation or more accurate landmark positions but the uncertainty of the landmarks are represented in a far better and more linear way. The way of representing the range to the landmark with the inverted distance not only makes representing a very large uncertainty possible but it also enables the use of a more linear measurement equation when observed from a slightly different angle. This reduces the need for prior knowledge or a good initial guess and results in a more robust filter that dynamically determines whether to use a landmark for position and orientation estimation or more heavily as an orientation reference point. As for the Cartesian representation, landmarks sometimes drift far from their true location. This problem is, however, not as big as for the Cartesian representation as it is less frequent and the representation itself is more robust to errors in distance. The additional computational cost introduced by this representation, while not an important point for this thesis, is somewhat compensated for by transforming it to the regular Cartesian when sufficient parallax has been observed which happens only when the range uncertainty is small enough.

While the initial guess is fairly accurate in the cases evaluated in this thesis, this might not be the case when the terrain is varied which results in the conclusion that the ID representation is superior.

Models

Since the models behind the data was known, the filter could use some of these in order to obtain better estimates. The motion model, however, was chosen as a constant velocity model and did not incorporate any of the modelling responsible for generating the trajectory or IMU data in order to minimise the known factors. The IMU and height noise levels were assumed to be known along with the model for the AOA noise. Although the model was known, it was based on the received AOA which already contained an unknown noise level. Therefore the exact noise could not be accurately estimated and this is likely the reason that a mean over a fixed time had to be calculated, which gives a more accurate angle, in order to receive good results. Another possibility is that the measurements were simply
too noisy when used on their own.

**Gating**

The mean calculation leads to possible issues as the direction to each landmark has to be known in the world-fixed coordinate system. In order to get these values, the measurements has to be transformed using the orientation of the UAV each time a new measurement is received and then transformed back to local coordinates when the mean is calculated. This relies on the time update and the IMU measurements to be accurate enough between each measurement update in order to minimise the error in orientation, and even then the error in orientation risks propagating into the AOA. This is however a smaller problem than it might seem to be as the angles are transformed back after the mean is calculated, which removes any stationary error in orientation, and the fact that there is a gate on AOA variance during the period over which the mean is calculated, which eliminates most higher frequency errors in orientation. The method used here is suboptimal yet sufficient in the case of simulation.

Overall, the gating of the AOA is simple yet effective and fulfils its purpose of removing inconsistent measurements. The landmark needs to be observed at each point, have a sufficiently high SNR and the variance of the measurements has to be below the threshold. In addition to this the mean of the measurement has to be below the UAV. All of these criteria are reasonable and justified, the only issue lies in determining the threshold which was tuned to be a bit more forgiving after initial test performed poorly at the end of the GPS trajectory when the UAV started flying lower.

**Association**

In the current implementation the association of the landmarks is assumed to be known. The algorithms that exist are able to recognise the transmitters with good accuracy and combined with the kinematic constraints from the models of the filter, this accuracy may be improved even further. However, the association will always run the risk of having errors that has to be handled. This issue must be solved before a complete working solution can be found.

**Landmark Noise**

Currently, there is no noise on landmark position in the time update. An addition of a landmark noise might prove beneficial for a reduction of model errors, however, since the landmarks are assumed stationary, this should rather be done by increasing the existing process noise.
Overall the methods and algorithms used in this thesis show a potential of providing good results in the given application. As discussed in Section 5.1.3, the requirement for landmarks seem to be low enough to enable the methods to be used by a UAV in many areas, with urban areas providing large redundancy with regards to the number of observable landmarks. This enables the use of this system in areas where the landmarks are sparse as long as the observed transmitters are providing proper measurements.

Due to its robustness and to the arguments in Section 5.2.2, the Inverse Depth representation is superior compared to the Cartesian when no range measurements are available.

There are however extensions to the current implementation that must be considered before moving forward.

### 6.1 Future Improvements

First of all, the performance of this or a similar system has to be evaluated with data collected by real, imperfect sensors in order to evaluate the accuracy of the noise models used in this thesis. This will at least require a new evaluation of the tuning parameters. Additionally the noise models might have to be reformulated depending on how well the current ones describe the noise of real data. This includes a more thorough investigation of the filter tuning which was required in this implementation.

An association algorithm, preferably some batch association method, based on real measurements should be implemented as the robustness of the system is
heavily depending on the performance of the association.

Assuming stationary landmarks might be a bit optimistic when it comes to practical applications and moving transmitters would create a great deal of issues for the current implementation of the system. Adding movement models to the landmarks could be a solution to this, however the problem complexity does significantly increase.

The implementation of submaps for the landmarks would provide greater efficiency when dealing with many landmarks over large areas. This would improve the consistency of the map and reduce computation time at the cost of the correlation of some of the landmarks, which in most cases is negligible since the importance of the landmark correlation decreases with the distance between them. However, the performance of any loop closure attempts might suffer due to the loss of these correlations as the closure information acquired has to propagate through these correlations to reach all stored landmarks.

If access to velocity measurements or similar was provided, which is already available on some UAVs, the scaling issue could be reduced significantly. The estimates would then converge over time at a faster rate than when only using the acceleration as done in this thesis.

In order to use this method for navigation purposes, the absolute position must be available. For this to be possible, some prior knowledge has to be known, whether it is the true positions of some of the transmitters or the starting position and heading direction.

A possibility for cases where a GPS is available but unstable due to some sort of disturbances is to run the filter at a low frequency using the positions from the GPS when they are available. This would provide a rough map of transmitters which would greatly improve both the accuracy of the estimations as well as the robustness of the overall system.


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