Energy-time entanglement, elements of reality, and local realism

Jonathan Jogenfors and Jan-Åke Larsson

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Jonathan Jogenfors

E-mail: jonathan.fors@liu.se
Institutionen för systemteknik, Linköpings Universitet, 581 83 Linköping, SWEDEN

Jan-Åke Larsson

E-mail: jan-ake.larsson@liu.se
Institutionen för systemteknik, Linköpings Universitet, 581 83 Linköping, SWEDEN

Abstract. The Franson interferometer, proposed in 1989 [J. D. Franson, *Phys. Rev. Lett.* **62**:2205–2208 (1989)], beautifully shows the counter-intuitive nature of light. The quantum description predicts sinusoidal interference for specific outcomes of the experiment, and these predictions can be verified in experiment. In the spirit of Einstein, Podolsky, and Rosen it is possible to ask if the quantum-mechanical description (of this setup) can be considered complete. This question will be answered in detail in this paper, by delineating the quite complicated relation between energy-time entanglement experiments and Einstein-Podolsky-Rosen (EPR) elements of reality. The mentioned sinusoidal interference pattern is the same as that giving a violation in the usual Bell experiment. Even so, depending on the precise requirements made on the local realist model, this can imply a) no violation, b) smaller violation than usual, or c) full violation of the appropriate statistical bound. Alternatives include a) using only the measurement outcomes as EPR elements of reality, b) using the emission time as EPR element of reality, c) using path realism, or d) using a modified setup. This paper discusses the nature of these alternatives and how to choose between them. The subtleties of this discussion needs to be taken into account when designing and setting up experiments intended to test local realism. Furthermore, these considerations are also important for quantum communication, for example in Bell-inequality-based quantum cryptography, especially when aiming for device independence.

1. Introduction

In 1989 a new interferometric setup was proposed by J. D. Franson [1]. The main intent was to test the possibility of local realist models as a possible description, more complete than quantum mechanics. The sinusoidal interference obtained from the experiment when restricting to coincident events is larger than the bound from given by the Bell inequality [2]. But the selection of coincident events at the two sites introduces postselection into the data analysis. This need for postselection has been under discussion for some time [3–8], and this paper is intended to review the discussion and to provide some insight into the matter at hand. We will see that, depending on what is required from the tested model class, the appropriate inequality changes so that the same experimental outcomes in some cases do violate the Bell inequality as usual, and in some cases do not. Interestingly, the class of models that uses EPR elements of reality (and nothing more) falls between these two, and violates the Bell inequality at a lesser degree than in other setups.

The paper is organized as follows: the rest of the Introduction is devoted to background, in a level of detail that enables an in-depth discussion in what follows. Section 2 introduces the Franson interferometer and discusses effects of postselection. In section 3 the usual Bell inequality is re-established by adding path realism to the model class. Section 4 concentrates on using only EPR elements of reality, giving a weaker inequality but nonetheless a violation of local realism, and section 5 contains a few examples of modified experimental setups and their properties.

A central concept in this analysis is "EPR elements of reality" as proposed by Einstein, Podolsky, and Rosen (EPR) in 1935 [9]. The concept is well-known, but a brief repetition is in place. The setting is as follows: consider a (small) physical system on which we intend to measure position Q or momentum P. The physical measurement devices associated with these measurements are mutually exclusive, and furthermore, the quantum-mechanical description for this physical system tells us that the measurements Q and P do not commute. The standard way to interpret this is that the system does not possess the properties of position or momentum, only probabilities are possible to obtain from quantum mechanics.

What EPR ask in their paper is whether the quantum-mechanical description can be considered complete, or if it is possible to argue for another, more complete description. They use a combined system of two subsystems A and B of the above type in a combined state, so that measurement of the sum of the positions gives $Q_A+Q_B=0$ and measurement of the momentum difference gives $P_A-P_B=0$. These two combinations are measurable at the same time even though the individual positions and momenta are not, which means that it is possible to produce a joint state with these properties. Letting the two subsystems separate, usually very far, they consider individual measurement of position or momentum. The system is such that the position sum and momentum difference is preserved under the separation process, which means that if the position of one subsystem has been measured, the position of the remote subsystem can be predicted.

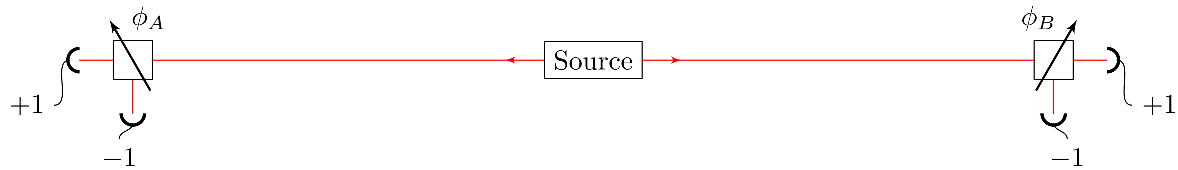

Figure 1. The EPR-Bohm-Bell setup. The two systems are spin-1/2 systems, and the local measurements are made along a direction in space ϕ_A or ϕ_B, respectively. The source is such that if the directions $\phi_A = \phi_B$, the outcomes $A + B = 0$ with probability one.

Therefore, EPR argue, the position of the remote subsystem must exist as a property of the subsystem. EPR write:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with a probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Likewise, if the momentum of one subsystem has been measured, the momentum of the remote subsystem can be predicted. In this case, the momentum of the remote subsystem must exist as a property of the subsystem. EPR continue to argue that *both* position and momentum must simultaneously exist as properties of the remote subsystem, otherwise

> ...the reality of P and Q depend[s] upon the process of measurement carried out on the first system, which does not disturb the second [remote] system in any way. No reasonable definition of reality could be expected to permit this.

The above definition of an "EPR element of reality" is the philosophical motivation for considering properties of a system as existing, independent of measurement. The possibility of remote prediction ("without in any way disturbing a system") enables the notion of EPR element of reality, and we will use this notion below to motivate existence of properties of the involved physical systems.

A measurement setup more suited to experiment was proposed by Bohm in the fifties [10] (see figure 1), and uses a system combined of two spin-1/2 subsystems in a total spin 0 state, so that the spins $A + B = 0$ when measured along equal directions $\phi_A = \phi_B$. The subsystems are allowed to separate and a spin measurement is made on one of the subsystems. The choice of measurement directions is a continuous choice instead of the dichotomic choice in the original EPR setup. When a measurement has been made on one subsystem, the result can be used to predict the result of a measurement on the remote subsystem along the same direction. And because the reality of the spin measurement result in the remote system cannot depend on the local choice, the spin along *any* direction is an EPR element of reality.

This was used in the celebrated Bell paper [2] where a statistical test was devised in the form of an inequality that must be fulfilled by any mathematical model that is realist and local. Here, realism is motivated by the spin being an EPR element of reality, and locality by the finite speed of light, or more specifically, because local measurement is made "without in any way disturbing" the remote system (the formulation below is adapted from [11]).

Theorem 1. *A local realist model has the following two properties:*

Realism. *Measurement outcomes can be described by two families of random variables. A is the outcome for site 1 with local setting* ϕ_A *and B the outcome for site 2 with local setting* ϕ_B*:*

$$A(\phi_A, \phi_B, \lambda) \text{ and } B(\phi_A, \phi_B, \lambda)$$

where the absolute values of the outcomes are bounded by 1. The dependence on the hidden variable λ *is usually suppressed in the notation*

Locality. *Outcomes do not depend on the remote settings*

$$A(\phi_A, \phi_B, \lambda) = A(\phi_A, \lambda), \quad B(\phi_A, \phi_B, \lambda) = B(\phi_B, \lambda).$$

By writing $A_i = A(\phi_{A,i})$ *and* $B_j = B(\phi_{B,j})$ *the outcomes from a local realist model obey*

$$\left|E\Big(A_1B_1\Big) + E\Big(A_1B_2\Big)\right| + \left|E\Big(A_2B_1\Big) - E\Big(A_2B_2\Big)\right| \le 2. \tag{1}$$

Inequality (1) is violated by the predictions of quantum mechanics, for instance measurement on a state with total spin zero gives the correlation

$$\Big\langle A(\phi_A)B(\phi_B)\Big\rangle = -\cos(\phi_A - \phi_B) \tag{2}$$

with $\phi_A - \phi_B$ being the angle between the two directions ϕ_A and ϕ_B. Choosing the four directions $\pi/4$ apart in a plane in the order b_1, a_1, b_2 and a_2 one obtains

$$\left|\Big\langle A_1B_1\Big\rangle + \Big\langle A_1B_2\Big\rangle\right| + \left|\Big\langle A_2B_1\Big\rangle - \Big\langle A_2B_2\Big\rangle\right| = 2\sqrt{2}, \tag{3}$$

and this violates inequality (1). Therefore, the quantum-mechanical predictions cannot be obtained from a local realist model.

There is one problem that is present in most experiments done to test this inequality: inefficient detectors. This problem was first noticed by Pearle in 1970 [12] but has been treated in many subsequent papers. The problem is that only pairs with coincident detections are obtained, which is a subset of all pairs emitted by the source. The correlation obtained from experiment is conditioned on coincident detection of two particles, one at each side. Taking this into account, the theorem needs to be modified as follows (adapted from [11]).

Theorem 2. *A local realist model with inefficiency has the following three properties:*

Realism. *Outcomes are given by random variables on subsets of detection (in a probability space* $(\Lambda, \mathcal{F}, P)$*),*

$$A(\phi_A, \phi_B, \lambda) \text{ on } \Lambda_{A,\phi_A,\phi_B} \quad \text{and} \quad B(\phi_A, \phi_B, \lambda) \text{ on } \Lambda_{B,\phi_A,\phi_B}.$$

Locality. *Outcomes and detections do not depend on the remote settings,*

$$\begin{aligned} A(\phi_A,\phi_B) &= A(\phi_A) \quad \text{on} \quad \Lambda_{A,\phi_A,\phi_B} = \Lambda_{A,\phi_A} \\ B(\phi_A,\phi_B) &= B(\phi_B) \quad \text{on} \quad \Lambda_{B,\phi_A,\phi_B} = \Lambda_{B,\phi_B}. \end{aligned}$$

Efficiency. *There is a lower bound to the efficiencies,*

$$\eta = \min_{\substack{\text{settings}\\ \text{local sites}}} P\Big(\text{coincidence}\Big|\text{local detection}\Big).$$

The outcomes from a local realist model with inefficiency obey

$$\begin{aligned} &\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big) + E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big| \\ &+\Big|E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big) - E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big)\Big| \le \frac{4}{\eta} - 2. \end{aligned} \tag{4}$$

The effect is that the inequality is weakened by inefficient detectors, and is no longer violated by quantum mechanics at $\eta = 2\sqrt{2} - 2 \approx 82.83\%$. We will see effects and extensions of this below.

2. The Franson interferometer

In 1989 a new experimental setup was proposed by Franson [1] (see figure 2). The setup uses a source that emits time-correlated photons at unknown moments in time, and two analysis stations consisting of unbalanced Mach-Zehnder interferometers. The analysis stations should have a path difference that is large enough to prohibit first-order interference. Therefore, the probability is equal for a photon to emerge from each port of the final beamsplitter. The path differences of the analysis stations should be as equal as possible; the path-difference difference (repetition intended) should be so small that the events of both photons "taking the long path" and both photons "taking the short path" are indistinguishable. Quotation marks are used here to remind the reader that photons are not particles, but quantum objects and as such, do not take a specific path. Given this indistinguability, since the emission time is unknown (and is a quantum variable in second quantization), there can be interference between two possibilities at the second beamsplitter.

There will be no interference if one photon "takes the long path" and the other "takes the short", because then the emission time can be calculated easily as the early detection time minus the short path length divided by c. When this happens the emission time will become known. When both photons "take the same path", the emission time remains unknown, and this is what enables the interference. The interference is not visible as a change in output intensity, as in first-order interference, but instead in correlation of the outputs. Given coincident detection, if the total phase modulation $\phi_A + \phi_B = 0$, then a photon emerging in the $+1$ channel on the left is always accompanied by a photon emerging in the $+1$ channel on the right, and the same for the -1 channels. Therefore,

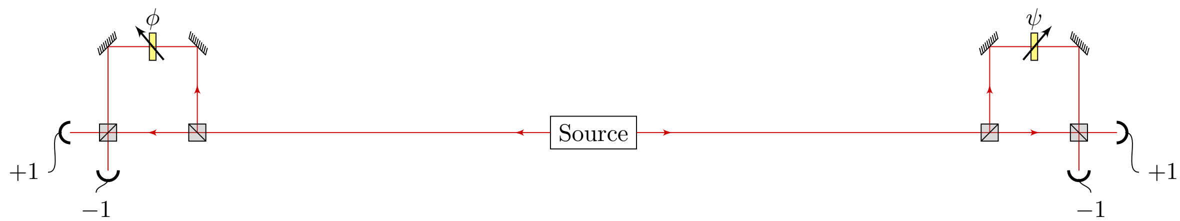

Figure 2. The Franson setup. The source sends out time-correlated photons at unknown moments in time. These travel through analysis stations consisting of unbalanced (but equal) Mach-Zehnder interferometers with variable phase modulators ϕ_A and ϕ_B. If the detections are coincident and $\phi_A + \phi_B = 0$, then $A = B$ with probability one.

when a measurement has been made at one analysis station the result can be used to predict what port the photon will emerge from at the remote analysis station whenever $\phi_A + \phi_B = 0$. Since the local phase modulation can be chosen freely, the output port along *any* direction is an EPR element of reality, when coincidence occurs. As a function of the total phase modulation, we have

$$\Big\langle A(\phi_A)B(\phi_B)\Big|\text{coinc. for } A(\phi_A) \text{ and } B(\phi_B)\Big\rangle = \cos(\phi_A + \phi_B). \tag{5}$$

Not the similarity to equation (2). It is now simple to obtain

$$\begin{split}\Big|\Big\langle A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big\rangle + \Big\langle A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big\rangle\Big| \\ + \Big|\Big\langle A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big\rangle - \Big\langle A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big\rangle\Big| = 2\sqrt{2},\end{split} \tag{6}$$

which exceeds the bound in inequality (1). Does this mean that the Franson interferometer violates local realism?

The answer is no. The problem is that there exists a local realist model that gives the exact same predictions as quantum mechanics [4] (see figure 3). Since the model fulfulls the requirements of Theorem 1, it is strange that it seems to violate inequality (1). But it only *seems* to give a violation. The model does not violate the inequality; it is true that the model gives the correlation

$$E\Big(A(\phi_A)B(\phi_B)\Big|\text{coinc. for } A(\phi_A) \text{ and } B(\phi_B)\Big) = \cos(\phi_A + \phi_B), \tag{7}$$

but this is a *conditional* expectation of the type used in Theorem 2. We have stumbled upon a loophole that arises from the fact that we need to postselect coincident events, which means that we have to discard 50% of the events right away. This "postselection loophole" gives us a case that is similar to the detection loophole discussed in Theorem 2. More accurately, we the following theorem from [13] can be established:

Theorem 3. *A local realist model with delays has the properties "Realism" and "Locality" from Theorem 2 together with the following property:*

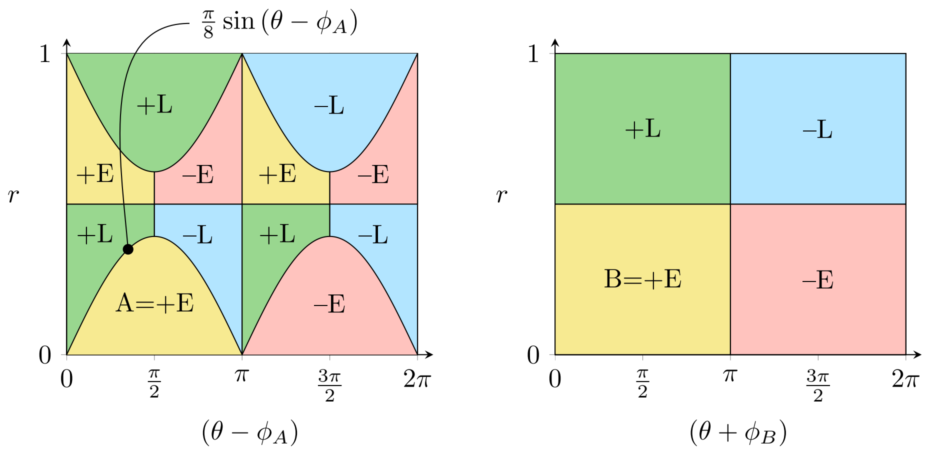

Figure 3. The local hidden variable model of Aerts et al. [4]. The hidden variable is a pair of numbers $\lambda = (\theta, r)$ evenly distributed over the rectangle $\Lambda = \{(\theta, r) : 0 \leq \theta < 2\pi, 0 \leq r < 1\}$. The outcomes are determined by the above graphs where detections are "early" (E) or "late" (L). This model reproduces the quantum predictions for the Franson interferometric setup, including those for the coincident detections.

Delays. *Detections occur after local realist time delays,*

$$T_A(\phi_A) : \Lambda \mapsto \mathbb{R} \quad \text{and} \quad T_B(\phi_B) : \Lambda \mapsto \mathbb{R},$$

and usage of a coincidence window gives an apparent efficiency of

$$\eta = \min_{\substack{\text{settings} \\ \text{local sites}}} P\Big(\text{coincidence}\Big|\text{local detection}\Big).$$

The outcomes from a local realist model with delays obey

$$\begin{aligned} &\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big) + E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big| \\ &\quad + \Big|E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big) - E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big)\Big| \leq \frac{6}{\eta} - 4. \end{aligned} \tag{8}$$

In the Franson setup the delays are not continuously distributed but rather limited to two values: "Early" and "Late". Coincidence only occurs when these timeslots equal between the analysis stations, corresponding to a coincidence window smaller than the delay. In inequality (8) the bound for quantum-mechanical violation is $\eta = 3 - 3/\sqrt{2} \approx 87.87\%$, which is higher than the standard bound. The apparent efficiency that arises in the Franson interferometer due to postselection is 50% even with ideal detectors. It is therefore possible for a local realist model (like the one in figure 3) to violate inequality (8).

At this point we want to re-establish a violation. To do this we must first understand why Theorems 1 to 3 fail to provide a usable Bell test. What is the basic reason for the existence of the model in figure 3? It is tempting to blame only the postselection, but this is not the whole story. It should be noted that the theorems treat the individual sites as black boxes; feed them a setting and they give an outcome in the late or early timeslot with a value of $+1$ or -1, much like the setup of figure 1. We will see that taking more properties of the setup into account will enable a violation, and the remainder of this paper will discuss the possible ways to do this.

3. Path Realism

The key ingredient of the local realist model in figure 3 is that the timeslot in which detection occurs at an analysis station depends on the relation between the hidden variables (θ, r) and the local setting (ϕ_A or ϕ_B) at the analysis station. To avoid this, one possibility is to have the "path taken by the photon" as a realist property [5, 8], in essence requiring particle-like properties of the photons. In this case, a photon will encounter the first beamsplitter before the variable phase modulation, so that the "decision" to "take the long path or the short path" must be independent of the phase modulation setting of the analysis station—the *local* measurement chosen—in contrast to a standard Bell experiment where only independence from the *remote* measurement choice is required.

It is certainly possible to list path realism as an expected model property and test it. It is however important to note that, in this setup, path realism is very different from measurement-outcome realism. The outcomes are EPR elements of reality because they can be remotely predicted, and by locality they can be argued to exist independently of what remote measurement was made. On the other hand, the path taken is not an EPR element of reality because it cannot be remotely predicted. There is no measurement at one site that enables a path prediction for the remote site in the setup of figure 2.

One may attempt to argue for path realism by bringing in properties from classical physics into the picture [5], which would make models like that in figure 3 inconsistent. However, the question at hand is not if an experiment like the above can be described within classical physics; there is no doubt that this is not possible. Instead, the question is whether the quantum-mechanical model can be considered complete. It is entirely possible that our classical intuition fails us, while quantum mechanics still can be completed. The discussion on EPR-Bell arguments is an attempt to find precisely what minimal requirements are needed to give a contradiction with quantum predictions. This question cannot be answered if the model is required to obey some complicated requirements from classical physics.

Another argument for path realism [8] would be to appeal to local prediction rather than prediction from the remote site as EPR do. By measuring in one path and finding the photon there it is possible to predict that it is not present in the other. Locality and spacelike separation between the paths could then be used as support for path realism, and one could even attempt to extend the notion of reality [8] by changing "without in any way disturbing a system" into "spacelike separation". This is a large modification of EPR elements of reality because EPR requires that a photon—when detected in one path—is a different undisturbed system when predicted not to be present in the other path, which is clearly not the case. It is true that finding the photon in one position enables a prediction that it is not anywhere else. But this cannot be used as evidence of an underlying realist model. Prediction of properties of *one system* immediately after measurement merely suggests that the measurement is repeatable.

Furthermore, it is central in the EPR reasoning that the system that we predict properties for is unaffected by the measurement used for predictions. EPR could choose to remotely predict position or momentum and therefore conclude the simultaneous reality of both. However, photon path is only available through a measurement in the local analysis station, and such a measurement prevents the remote prediction of the analysis station output, since the interference is destroyed in the process. This means that we cannot conclude in the same way that path and analysis station output both simultaneously are realist properties. Even though it is claimed in Refs. [5, 8] that "path taken" must be a realist property independently of whether the path measurement is performed or not, this is clearly not supported by EPR-style reasoning.

If we choose to use path realism we must remember that path realism on its own does not prohibit interference. First order interference is not prohibited, since the model could randomly select a path for the photon and send an "empty wave" through the other path, that registers any phase shift in that path. The phase shift difference can be used to determine through which output port to emit the photon from the second beamsplitter. Second order interference in the present setup is also not directly prohibited by path realism itself, because the same mechanism would work.

However, large second order interference will be prohibited by the combination of path realism and local realist analysis station output. These two together will enable Theorem 1 separately both for Late-Late coincidences (both photons "taking the long path") and Early-Early coincidences (both photons "taking the short path"), because the existence of a delay cannot depend on the local choice of phase modulator settings. This re-establishes the bound on the whole set of coincidences [5].

Theorem 4. *A local realist model with path realism has the properties "Realism" and "Locality" from Theorem 1, and*

Path realism. *The path taken is a realist property, and is setting-independent.*

The outcomes from a local realist model with path realism obey

$$\begin{aligned}\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big) + E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_1\Big)\Big| \\ + \Big|E\Big(A_2B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big) - E\Big(A_2B_2\Big|\text{coinc. for } A_1 \text{ and } B_1\Big)\Big| \le 2.\end{aligned} \tag{9}$$

Of course, path realism requires intimate knowledge of the internals of the analysis stations, for example, that there really are two distinct paths used. This means that path realism makes it difficult to argue for device independence since the requirements used involve low-level properties of the devices. Also, an experiment meant to test the inequality needs to ensure that the "decision" to "take the long or the short path" really is independent of the phase modulator setting. This would be accomplished by space-like separation of the choice of phase modulation and the "photon's path choice", that is, the event of "the photon passing the beamsplitter". Since the experimenters do not know the detection moments in advance, this must be done by switching the settings fast enough to ensure that a new random setting is chosen in-between these two events. This is in contrast to a Bell experiment where the setting choices only need to be chosen at spacelike separation from the emission and each other, which is a much less demanding task.

Using path realism does have benefits, since postselection has no negative effects on the bound. The appropriate inequality has the standard bound as in inequality (9) and is violated by the quantum prediction in equation (6). There is no local realist model with path realism that gives the probabilities predicted by quantum mechanics for the Franson interferometer.

4. Local realism only

It is now interesting to ask whether it is possible to obtain a violation using *only* EPR elements of reality. Obviously, local realist measurement outcomes is not enough. It *is* possible to establish another EPR element of reality in this setup, but it is slightly different than the path realism used above. By replacing one of the analysis stations with a single detector it is possible to measure the source emission time. This enables prediction of the emission moment for the remote photon, without in any way disturbing it. Therefore, *the moment of emission is an EPR element of reality*, which means that a local realist model must be similar to the one in figure 3 in that it needs to specify whether a detection is "early" or "late". This is not the same as specifying the path because it does not require particle-like behaviour of the quantum object (the photon "takes a path"). The only requirement is that the detection occurs in one of the two timeslots, treating the analysis station like a black box just as it is in the original Bohm-Bell setup. The important observation here is that the detection moment also is an element of reality and must be present in any local realist model.

In this situation, the settings ϕ_A and ϕ_B can still influence whether a delay occurs or not. The event that needs to be spacelike separated from the setting choices is no longer "passing the first beamsplitter", a point in time which here is EPR element of reality, but the event of detection. This is because the detection is the event that gets delayed (or not) in the local realist model. The problem is that detection takes place after the photon has (or could have) passed the phase modulation, so it seems impossible to have spacelike separation between setting choice and detection event.

Fortunately, there are two possible detection events, one early and one late. This means that it is possible to have two different settings for the two detection events: one at the "early-setting readoff" event for the early detection, and one at the "late-setting readoff" event for the late detection (see figure 4A). The choice of the early setting cannot be spacelike separated from any of the possible detection events since they are both inside (or on) the forward light cone with respect to it. The choice of the late setting, however, can be spacelike separated or even inside the forward light cone of

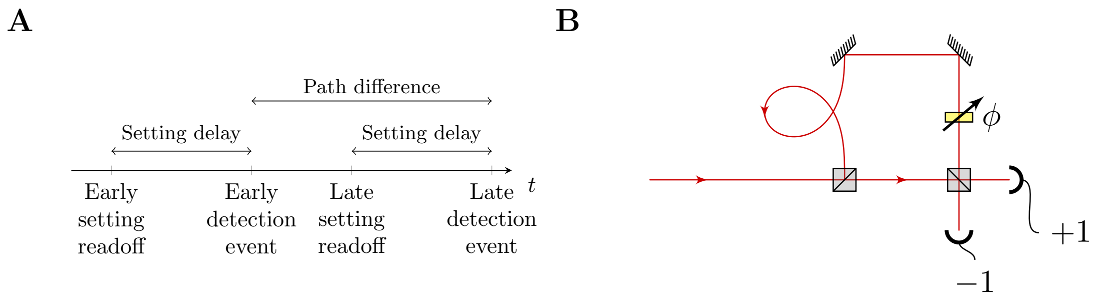

Figure 4. **A:** Two settings influence the outcome, one for the early detection, and one for the late detection. The setting delay is the time that it takes for the information to travel from the phase modulator to the detectors, via the light path. **B:** To ensure the events occur in the order indicated in a), three things can be done: a) making the path length difference large, b) placing the phase modulator late in the analysis station, and c) placing the detectors close to the second beamsplitter.

the (possible) early detection event. To make this possible, one needs to make the path difference of the analysis station longer than the distance from the phase modulator to the detectors. In other words, one needs to make the path difference large, place the phase modulator as late as possible in the analysis station and have the detectors close to the second beamsplitter (see figure 4B).

When this is the done, the early setting can still be used by a (hypothetical) model to delay the detection. But as soon as a delay has occured the setting cannot cause the delay to be undone as this would violate causality. The early detection event would already have taken place when the late-setting choice is made. So while the early coincidences only are bounded by the trivial bound 4 in Theorem 1 applies for the late coincidences. We therefore have the following theorem [4]:

Theorem 5. *A local realist model with long realist delays has the properties "Realism" and "Locality" from Theorem 1, and*

Long realist time delays. *The delay is a realist property as in Theorem 3 and is long enough to ensure that the setting relevant for the late detection cannot undo the delay.*

The outcomes from a local realist model with long realist delays obey

$$\begin{aligned}&\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big) + E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big|\\&\quad + \Big|E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big) - E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big)\Big| \le 3.\end{aligned} \tag{10}$$

The bound in inequality (10) is 3. This stems from the fact that early and late coincidences are equally probable which lets us take the mean value of the trivial bounds 4 (early coincidences) and 2 (late coincidences). Unfortunately, this is larger than the maximal quantum prediction $2\sqrt{2}$. To establish a better bound we need to use so-called "chained" Bell inequalities [12, 14] with more terms, and this gives the following theorem [4].

Theorem 6. *The outcomes from a local realist model with long realist delays (as specified in Theorem 5) obey*

$$\begin{aligned}&\Big|E\Big(A_1B_3\Big|\text{coinc. for } A_1 \text{ and } B_3\Big) + E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big|\\ &\quad+\Big|E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big) + E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big)\Big|\\ &\quad+\Big|E\Big(A_3B_1\Big|\text{coinc. for } A_3 \text{ and } B_1\Big) - E\Big(A_3B_3\Big|\text{coinc. for } A_3 \text{ and } B_3\Big)\Big| \le 5.\end{aligned} \tag{11}$$

Again the bound is the mean value of the trivial bound 6 for the early coincidences and the chained-Bell bound 4 for the late coincidences. While the postselection does have an effect on the bound, this case allows quantum mechanics to give a violation; choosing the six directions $\pi/6$ apart in a plane in the order b_3, a_1, b_2, a_2, b_1, a_3, will yield the quantum prediction

$$\begin{aligned}&\Big|\Big\langle A_1B_3\Big|\text{coinc. for } A_1 \text{ and } B_3\Big\rangle + \Big\langle A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big\rangle\Big|\\ &\quad+\Big|\Big\langle A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big\rangle + \Big\langle A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big\rangle\Big|\\ &\quad+\Big|\Big\langle A_3B_1\Big|\text{coinc. for } A_3 \text{ and } B_1\Big\rangle - \Big\langle A_3B_3\Big|\text{coinc. for } A_3 \text{ and } B_3\Big\rangle\Big|\\ &\quad= 6\cos\tfrac{\pi}{6} \approx 5.196,\end{aligned} \tag{12}$$

There is no local realist model with setting-independent delays that gives this value and we therefore have established a test for the Franson interferometer that is free of the postselection loophole. Performing this experiment is however more difficult than the one in Theorem 1 since the value in equation (12) is so close to the bound in inequality (11). The visibility needs to be as high as 96.2% which is demanding. By adding even more terms to inequality (11) it is possible to slightly decrease this requirement to 94.6%. This minima is found when using ten terms, increasing the number of terms even more only increases the critical visibility. Table 1 lists visibility requirements for different number of terms.

When only using local realism it becomes easier to argue for device independence since one does not rely on internal properties of the analysis stations. It is, of course, still important to verify that the delays do occur at equal probability, but it is no longer needed to verify the existence of two distinct paths. Similarly as before, an experiment meant to test the inequality needs to ensure that the possible early detection event really is independent of the late phase delay choice. And again, since the experimenters do not know the detection moments in advance, this must be done by switching the settings fast enough to ensure that a new random setting is chosen in-between the early detection event and the late-setting readoff. This is similarly difficult to the corresponding requirement when using path realism (see above), but this is the price to pay for only using EPR elements of reality in the derivation of the bound. One should also note that the early-setting choice and the late-setting choice needs to be independent, which would be achieved with a good source of randomness. For the purists [15] the late-setting and

Number of terms	Emission-time realism bound	Quantum prediction	Critical visibility
4	3	$4\cos\frac{\pi}{4} \approx 2.828$	$> 100\%$
6	5	$6\cos\frac{\pi}{6} \approx 5.196$	96.23%
8	7	$8\cos\frac{\pi}{8} \approx 7.391$	94.71%
10	9	$10\cos\frac{\pi}{10} \approx 9.511$	94.63%
12	11	$12\cos\frac{\pi}{12} \approx 11.59$	94.90%
$2N \geq 14$	$2N-1$	$2N\cos\frac{\pi}{2N}$	incr. with N

Table 1. Critical visibilities for violation of chained Bell inequalities that the outcomes from a local realist model with long realist delays (as specified in Theorem 3) must obey.

early-setting choices need to be spacelike separated, something that can be achieved with independent sources of randomness suitably arranged around the analysis stations.

We have found that when establishing the time of emission as an EPR element of reality, postselection still has negative effects, but only on half of the selected subensemble. This modifies the relevant Bell inequalities, but some are still violated by the quantum prediction in equation (5). There is no local realist model with realist emission time that gives the probabilities predicted by quantum mechanics for the Franson interferometer.

5. Modified setups

Another approach to reach a violation of local realism from the quantum-mechanical predictions is to modify the setup. A number of alternatives exist, and we will here briefly go through three of these alternatives.

The first modified setup was proposed by Strekalov et al. in 1996 [16] (see figure 5). This setup uses a polarization-entangled source and three polarizing beamsplitters at each site. The interference occurs at the third and last polarizing beamsplitter. In this setup, the path taken *is* an EPR element of reality because a polarization measurement (in place of one analysis station) can be used to remotely predict which path the remote photon is going to travel through. But this is not needed; because of the polarization entanglement the photon pairs always end up in the same timeslot, giving coincident detection. There is no postselection, and we can in fact use Theorem 1 (modulo other experimental problems). This is good, because the experimental realization proposed in [16] does not use separate paths, but instead uses a single birefringent optical element at each site to implement the whole analysis station, so an argument based on a realist path cannot be used. But as we have seen, local realism can be violated in this setup even when the paths coincide.

The second alternative, sometimes called "time-bin entanglement", uses a source proposed in Brendel et al. in 1999 [17] and switched mirrors in place of the first beamsplitters [18]. The source uses a pulsed pump, an unbalanced interferometer, and

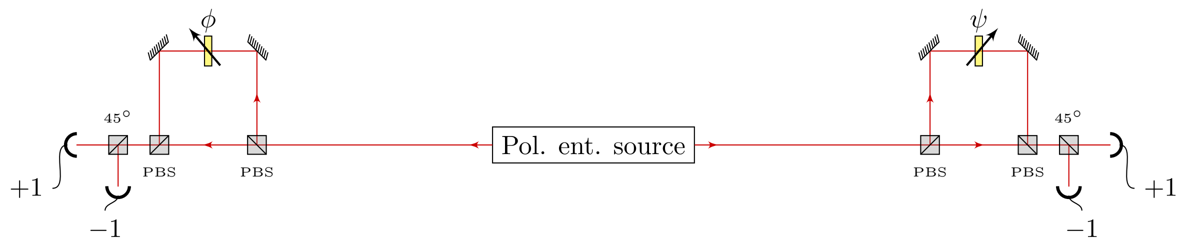

Figure 5. Using a polarization-entangled source. The time-correlated photons are still sent out at unknown moments in time, but are now also polarization-entangled. The beamsplitters used are polarizing beamsplitters (PBS), and the interference occurs at the third PBS at each site, because this is oriented $\pi/4$ in relation to the other two. In this setup there is no postselection, so that Theorem 1 can be used directly, and the bound is violated by the quantum prediction.

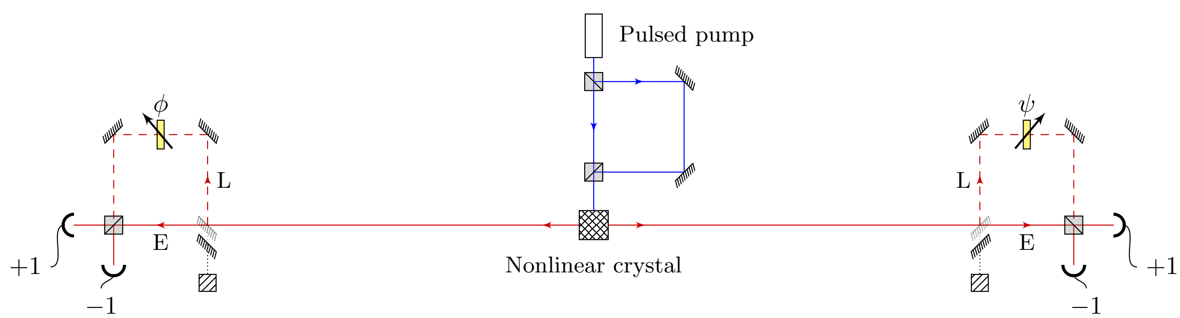

Figure 6. Time-bin entanglement. The mirrors are synchronized with the source so that photons produced by the early part of the pulse would be delayed in the analysis station, while photons produced by the late part of the pulse would not be delayed. In this setup there is no postselection, so that Theorem 1 can be used directly, and the bound is violated by the quantum prediction. If we instead replace the controlled mirrors with passive beamsplitters we have phase-time encoding which gives us Theorem 5.

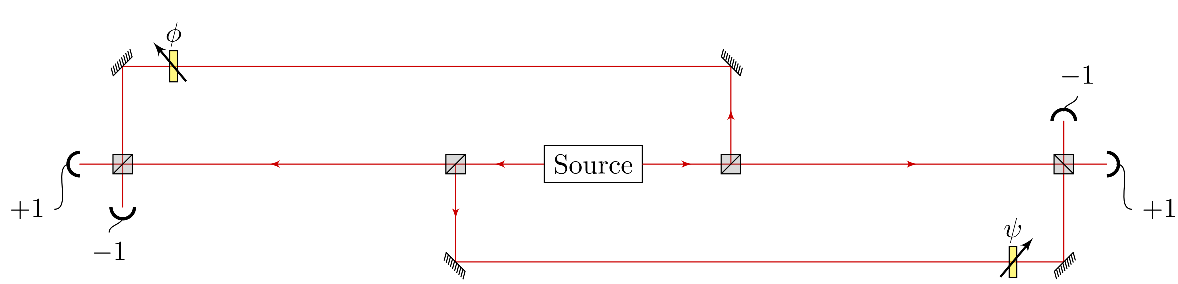

Figure 7. Hugging interferometers. This setup has many of the properties of the original Franson setup. The interferometers are very large and this becomes experimentally challenging. Here, the path taken by each photon is an EPR element of reality so that Theorem 1 can be used, and the bound is violated by the quantum prediction.

a nonlinear device that creates photon pairs. The active mirrors are pushed in and pulled out of the photon path in sync with the source (see figure 6). In this setup, the path taken is also an EPR element of reality, because a measurement of the time of emission (in place of one analysis station) can be used to remotely predict which path the remote photon is going to travel through. Again, this is not needed; because of the synchronization of the mirror positions the photon pair is detected in the same timeslot and every pair gives coincident detections. There is no postselection, and Theorem 1 (modulo other experimental problems) can be used to rule out local realist models.

It is possible to modify the time-bin-entanglement setup further by using passive switching. Gisin et al. refers to this setup as “phase-time encoding” [19]. In this method, the movable mirrors are replaced by beam splitters. We can remove one of the analysis stations and measure the emission time from the source, which becomes an EPR element of reality. However, in contrast to time-bin-entanglement it is not possible to predict the path taken and we get a situation very similar to the Franson interferometer. In fact, with passive switching we recover the weaker Theorem 5 and the only difference to the Franson setup is the fact that phase-time encoding uses a pumped source.

The third alternative in this list was proposed by Cabello et al. in 2009 [20] and uses interferometers in a “hugging” configuration as seen in figure 7. In this setup, postselection is performed because the two photons both may end up at the same site, at both input ports of the beamsplitter at that site but in different timeslots. The events that give coincident detection at both sites make up 50% of the total. But also here the path is an EPR element of reality, because a local path measurement (done by removing the final beamsplitter) can be used to remotely predict which path the remote photon is going to emerge from. Since the path is an EPR element of reality, we can use Theorem 4 (modulo other experimental problems), and the quantum prediction will violate the bound. There is no local realist model (with realist path) that gives the quantum predictions of this setup, and because of this, the hugging interferometer setup is sometimes called a setup with “genuine energy-time entanglement”. Lima et al. [21] and Cuevas et al. [22] have both demonstrated violations of Bell’s inequality using hugging interferometers, the latter was performed with a fibre length of 1 km.

6. Conclusions

This paper has discussed tests of local realism using energy-time entanglement. Even though most of the proposed experiments use postselection, we have seen that certain types of models can be ruled out. But we have also seen that the tests are subtle, because depending on the precise requirements made on the local realist model, the same experimental data can imply a) no violation, b) smaller violation than usual, or c) full violation of the appropriate statistical bound.

Using only that the measurement outcomes are EPR elements of reality is not enough to yield a violation [4]: the appropriate Bell inequality would be trivial and reads

$$\begin{aligned}&\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big)+E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big|\\&\quad+\Big|E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big)-E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big)\Big|\le 4.\end{aligned} \tag{13}$$

Adding that the moment of emission is an EPR element of reality enables a violation, although a weaker violation than that of the usual Bell setup [4]: there is a violation using the chained equation (12) with six terms while the appropriate four-term Bell inequality would read

$$\begin{aligned}&\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big)+E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big|\\&\quad+\Big|E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big)-E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big)\Big|\le 3.\end{aligned} \tag{14}$$

Requiring that the model also uses a realist path will enable a violation, equally large as that from a standard Bell test [5, 8]. Note that, in the original Franson setup, the path is not an EPR element of reality; it cannot be predicted without disturbing the system. But path realism can still be listed as an expected model property, and tested in experiment; one motive for requiring this particle-like behaviour from the model would be correspondence to properties from classical physics. The violation would be equally strong as that of the usual Bell setup: the appropriate Bell inequality has the usual bound

$$\begin{aligned}&\Big|E\Big(A_1B_1\Big|\text{coinc. for } A_1 \text{ and } B_1\Big)+E\Big(A_1B_2\Big|\text{coinc. for } A_1 \text{ and } B_2\Big)\Big|\\&\quad+\Big|E\Big(A_2B_1\Big|\text{coinc. for } A_2 \text{ and } B_1\Big)-E\Big(A_2B_2\Big|\text{coinc. for } A_2 \text{ and } B_2\Big)\Big|\le 2.\end{aligned} \tag{15}$$

A final alternative is to modify the setup to ensure that the path taken is an EPR element of reality [16–18, 20]. This enables a violation that is equally strong as that of the usual Bell setup: the appropriate Bell inequality is inequality (15).

These considerations must also be taken into account in quantum communication. For example, in Bell-inequality based quantum cryptography [23], the inequality is used as test of security. The original Bell inequality (bounded by 2) is only available as a security test if path realism can be used, and this is only possible when a) there really are distinct paths within the analysis stations, and b) the attacker is unable to control which path the "photon will take". This is highly device-dependent; when aiming for device-independent security, the security test should not rely on the internal structure of the analyzing stations. Thus, path realism should not be used. The users must rely on the black-box formulation obtained when using emission time as an EPR element of reality. In this case, the original four-correlation Bell inequality cannot be used as test of security (see above), and using the chained inequalities, the security margin will be smaller than from the standard Bell test, since the critical visibility is higher than in this test.

Energy-time entanglement obtained with the Franson interferometer and its variants is a subtle way to test local realistic models, with or without added properties. It will be used in many future experiments intended to violate local realism, but in performing them it is important to be aware of exactly what is tested in these experiments, and the size of the violation. Even with the subtleties associated with this interferometer, or more correctly, because of these subtleties, the interferometer will continue to be an important tool to extend our knowledge on the foundations of quantum mechanics.

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