Staff scheduling in elderly care -
A simulation study of trade-offs

Rebecka Håkansson

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Rebecka Håkansson

Optimeringslära, Linköpings Universitet

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Supervisor: Torbjörn Larsson,
Optimeringslära, Linköpings Universitet

Supervisor: Ann Bertilsson,
Schemagi AB

Examiner: Elina Rönnberg,
Optimeringslära, Linköpings Universitet

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Abstract

Numerous studies have been done in the area of nurse scheduling, since this is a complex area with a lot of aspects that has to be taken into account. An interesting but little studied subject is how the requirements for the scheduling affect the possibility to construct a feasible schedule, or how the requirements affect the quality of the schedule. Of special interest is the effect of the composition of the workforce and of the change in scheduling rules. What is missing is results showing which composition and changes that are possible, and if so what is needed to be able to follow through with them.

The changes tested in our simulation study are changes that is up for discussion at many wards in Sweden today, with topics such as split shifts and high part-time work percentages within the staff. In order to simulate various scheduling requirements and changes, an integer linear model for creating nurse schedules is developed. The results provide some insight into the dependence between scheduling requirements and the resulting schedules. In particular our simulation results indicate that there is an inherent conflict between high part-time work percentages and split or long work shifts. Our results can be used as a basis for future research on these topics in the area of nurse scheduling.

**Keywords**: Nurse scheduling, Nurse rostering, Causality relationships, Split shifts, Long shifts, Workforce composition, Integer linear programming.

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Chapter 1

Introduction

In this thesis we study staff scheduling in health and elderly care, which is one of the more complex scheduling problems. The thesis starts with this introduction and then a deeper study of nurse scheduling is presented in the following chapter. A simulation design will then be presented, followed by the simulation study and the results. The last chapter contains a discussion and conclusions from the results obtained in the simulation study.

The simulations are made to see how changes in workforce composition, for example to have all nurses working full-time, or changes in the rules, for example not allowing certain types of shifts, will affect the scheduling in terms of the use of, for example, overstaffing or undesired shifts.

1.1 Background

Scheduling in health and elderly care is a well renowned and well studied problem. Staff scheduling in general is a complex problem due to the numerous requirements and rules, and also because of the need to make the schedules acceptable for the staff. The rules include how the employees are allowed to work each day, week and year, what kind of breaks they are entitled to and what applies for the night rest. These rules are stated in the working hours act, see [1]. Making the schedules acceptable means fulfilling some quality aspects, e.g. having the unwanted shifts evenly distributed among the staff. Further, there is a staffing demand that has to be fulfilled during the day.

Scheduling in health and elderly care is one of the more complex staff scheduling problems. This is because the staff often have irregular working hours due to varying staffing demand during the day, and that there often is a need for staff around the clock. The scheduling is often done manually, sometimes with a little help of a computer. This is both time-consuming and difficult, having to make sure that the schedule fulfills all the necessary rules and demands as well as the quality aspects. There are however optimization tools that have been developed to create these schedules.

Even if scheduling in health and elderly care has been studied a lot, a shortcoming has been discovered. This shortcoming consists of studies that show how certain changes in for example the workforce composition or in the scheduling rules affect the scheduling in terms of, for example, having to use more nurses
than needed or using shifts that are unwanted.

As illustrated in Figure 1.1 the scheduling conditions can be seen as input to the scheduling and the schedule as the output. Is it possible to have certain conditions and still get a good schedule? A topic that is discussed a lot in Sweden is the possibility to offer all nurses to work full-time, in order to enable the nurses to support themselves economically. There are a few studies discussing such types of changes, but general results are missing. Examples of studies are a pilot study from Lidköping municipality [19] and a study from Linköping municipality [20]. These discuss the conditions and consequences of offering nurses full-time.

Another topic that is discussed a lot in health and elderly care is the possibility to abolish split shifts. These are shifts where a nurse works a shift in the morning, then has a longer unpaid break and again works a shift in the evening. The debate about the use of split shifts can be found for example in Östnytt, see [21]. Mårtensson and Wondmeneh, see [22], provide facts about split shifts and an insight into how people feel about working split shifts. Both the article from Östnytt and the study by Mårtensson and Wondmeneh show that the split shifts are unwanted. In the study by Mårtensson and Wondmeneh it is shown that the main reason for this is that the nurses are not able to do anything meaningful during the short break and that these shifts also affect their free time and social life.

Long shifts are also discussed. These shifts go from morning to evening without a break in the middle of the day. These shifts are sometimes mentioned when discussing high part-time work percentages and split shifts, where the long shifts maybe could replace the split shifts. The nurses will then work longer days, but then also get paid for the whole day instead of having the short unpaid break in the middle of the day.

Örebro municipality had a project regarding giving nurses the possibility to work full-time while also getting rid of split shifts, see [23]. The report present results of these two changes for different wards in Örebro. The focus in the report is on keeping balance in the economy, as well as fulfilling the needs of the patients and letting the nurses be involved in the scheduling.

The company Schemagi AB works with creating schedules for customers in the health care sector. The company would like to be able to show and illustrate for their customers the causality relationships when discussing trade-offs between different demands and wishes, like getting rid of split shifts and
having nurses working full-time. Although people working with scheduling often know intuitively what kind of changes that are possible to make they are missing general results to rely on or refer to. With such results they could not only conclude that something normally is or is not possible, but they can also show it.

1.2 Aim and goal

The aim of this thesis is to try to fill the knowledge gap described above by constructing an optimization model that can be used for creating schedules based on the changes that are of interest. These schedules will be evaluated in order to find out what the effects of a certain change are. The model will be solved by a structured optimization software. The optimization model will include some basic standard rules that normally applies when scheduling nurses, and then the changes of interest will be tested. These simulations will be based on real-life cases, regarding staffing demands as well as rules and quality aspects. In Figure 1.2 the general idea is presented and it includes the generic scheduling that everything is based on and three different parts that can be changed. The arrows show the idea of finding out how a change in one of the three parts could affect the other two. The simulations will be connected to workforce composition since, as mentioned before, working full- or part-time is discussed a lot in Sweden today. The use of split shifts will also have a big role in this thesis because of the discussion presented in the earlier section about the possibility to get rid of these types of shifts. The results will be illustrated in a simple and apprehensive way. The results can be used by Schemagi both internally and in meetings with their customers. They may also be used for extended research in this area.

1.3 Method

First a literature study is made to get to know more about workforce scheduling in general and especially in health and elderly care, and how these problems are solved with optimization methods. The following literature was handed out by the examiner to start with.


With the help of these studies and meetings with the supervisor at the university and at Schemagi a mathematical model for the generic scheduling is developed.
This contains all the necessary rules and quality aspects for scheduling staff in elderly care. The mathematical model is formulated as an integer linear program. The model is implemented in AMPL, see [6], which is a modelling language where the optimization model can be written in a natural way, see Lundgren et al. [5]. To solve the problem in AMPL the solver CPLEX is used, see [6].

For testing the generic scheduling a small test case is created with a small staffing demand, a few weeks and a few nurses. Based on the staffing demand a small number of shifts is created, and with the total number of shifts and working hours needed the nurses average part-time work percentages are calculated. Based on the generic scheduling a search for causality relationships for nurse scheduling is conducted. The cases and changes that are tested are provided by the supervisor from Schemagi, and these are based on real-life cases. As mentioned earlier, workforce composition and split shifts will be the focus of this thesis and the simulations are connected to these two aspects.

![Figure 1.2: General idea](image)

### 1.4 Topics covered

There are five chapters, including this introduction, and two appendices.

**Chapter 2:** Here the general concept of nurse scheduling is presented. This will explain how the scheduling process works.

**Chapter 3:** This chapter outlines how the generic scheduling is modelled and what kind of data that is used.

**Chapter 4:** The changes in workforce composition and scheduling rules that are simulated are specified in this chapter, including the results.

**Chapter 5:** Contains a discussion and conclusion from the results presented in Chapter 4.
1.4. Topics covered

Appendix A: Gives the full mathematical model of the generic scheduling.

Appendix B: Gives the changed mathematical model for the simulation study in Chapter 4.
Chapter 2

Nurse scheduling

Creating staff schedules in health an elderly care is a difficult task and not something that should be done last minute. For creating schedules there is a planning process that should be followed. This process is described in this chapter.

2.1 The planning process

The process for scheduling nurses can be divided into three different steps, which is illustrated in Figure 2.1. There is the long-term planning, the mid-term planning and the short-term planning. During the long-term planning the managers at the ward have to estimate what the staffing demand will be during all periods of the days and weeks. Based on this, decisions on how many nurses to be hired, what shifts each nurse can work, and so on, will be made. These estimates and decisions will form the basis for the whole staffing at the ward, and they are done for the first time when the ward is new and then updated regularly or if there are any major changes. Arthur and James [7], and Gosh and Cruz [8] delves further into the long-term planning. Arthur and James give an overview of current methods for determining nurse staffing. Gosh and Cruz consider the commonly used methods for nurse workforce planning and then discusses an alternative approach.

The next phase of the planning process is the mid-term planning. This is when each nurse should get a schedule where it is specified what shifts he or she should work each day during a period of 4 to 8 weeks, called a scheduling period. These schedules must ensure that the staffing demand given from the long-term planning is fulfilled as well as fulfilling certain rules and quality aspects. This phase is very well studied and a lot of different approaches to solve it have been suggested. Our work will consider the mid-term planning process, and is what is meant when the term scheduling is used in the remainder. More about this part of the planning process is found in Section 2.2.

The last phase is the short-term planning. The short-term planning means deciding how to tackle shortages of nurses when they occur due to a late change, such as a sick leave. The solution could be to use overtime, call in a substitute nurse or manage despite the shortage. Examples of approaches solving this problem is given by Moz and Pato [9] and Moz and Pato [10].
Chapter 2. Nurse scheduling

2.2 Mid-term planning

Schedules can have the property of being either cyclic or non-cyclic. A cyclic schedule means that a fixed schedule is repeated each scheduling period, while a non-cyclic scheduling means creating a new schedule each period. The cyclic scheduling has advantages such as the personnel knowing how they will work a long time in advance and having the work evenly distributed over time, but also disadvantages such as losing flexibility. A non-cyclic scheduling might be more time consuming, since it requires creating new schedules for every period, but it will be more flexible. This is for example mentioned in a study by Millard and Kiragu, see [11].

As mentioned before, scheduling nurses is challenging. The schedules have to follow the prevailing laws and regulations for staff scheduling, fulfill the quality aspects and of course also fulfill the staffing demand. The schedules have a large impact on the nurses personal lives, since they are working weekends, weekdays, days and nights. Trying to improve the working conditions, the use of preference scheduling has become popular, which means taking the staffs' requests and opinions into account. This, unfortunately, makes the scheduling even more complicated (as mentioned by Bard and Purnomo, see [13]). Taking this one step further, there is a scheduling type called self-scheduling. This is where the nurses themselves create the schedule for each period. Here each nurse can choose when he or she wants to work, and then they can trade shifts on their own, while making sure the staffing demand is fulfilled. This approach is studied by for example Bailyn et al. [14].

Both the usual scheduling and the preference scheduling have been studied a lot and tools have been invented that solve these problems with the help of optimization methods and computer programs. These tools take all the rules, demands and quality aspects into account and then create a schedule that is as favourable as possible. The approaches differ among these tools, see Chaeng et al. [15]. Two approaches mentioned are mathematical programming and heuristics. When using mathematical programming a function will be maximized or minimized while a number of constraints are satisfied. When working with scheduling these constraints are defined by rules, demands and quality aspects. What to maximize or minimize depends on what the goal of the scheduling is. One example is maximizing the fairness of the schedule as in Trilling et al. [16]. Using heuristic methods is very common, giving a good enough solution instead of an optimal solution (which often is very time consuming and often also unnecessary) and therefore save time while still creating a high quality schedule. For further reading about nurse scheduling, Burke et al. [17] and Ernst et
2.3 System

When scheduling nurses there is, as mentioned before, a lot of aspects to be taken into account. The workforce composition, the staffing demand, the rules and laws, the quality aspects, and so on, have to be considered to create a working schedule. Everything mentioned can be considered as inputs for the scheduling. These can be changed in many different ways, leading to changes in the output, which is the created schedule. Everything is connected, so that a change in the workforce might make it impossible to fulfill the rules, a change in the rules can make it impossible for the workforce to fulfill the demand, and so on. In our work all changes will be related to the workforce. This can both mean having a certain workforce and then test changing certain rules, but also changing the workforce to test certain rules. The rules can then be fixed or penalized. Fixed rules are hard, that is rules that have to be fulfilled. These are set in the mathematical model as constraints and can be seen in Appendix A. Penalized rules are soft. These rules do not have to be fulfilled, but as far as it is possible they should be fulfilled, and they are penalized in the objective function when they are not fulfilled. These types of rules are presented more in detail in Chapter 4.
Chapter 3

Simulation design

For the upcoming simulations a model that can create schedules based on certain rules has to be developed. At first this model will create schedules fulfilling some basic standard rules. The model generating schedules is then modified for the upcoming simulations. The focus is assisted living and elderly care where staffing profiles are used (that is when a demand is set for every half hour). The cases for the simulation study are based on real-life cases.

3.1 Generic scheduling

In this section the design of a generic scheduling model will be presented. The generic scheduling forms the basis for a simulation tool and consists of an optimization model formulated as an integer linear program (a special case of a mathematical program). The model is implemented in AMPL, and solved with CPLEX, see [6]. The full mathematical model for the generic scheduling can be found in Appendix A. The rules and quality aspects stated in this section come both from the working hours act, see [1], and from my supervisor at Schemagi. The company is well versed in scheduling in health and elderly care and knows what kind of rules and quality aspects that usually applies.

3.1.1 Staff and demand

In health care scheduling, a day can be divided in different ways. A common way is to divide the day into three parts, where each part is one shift. These are day, evening and night shifts, that each has its own staffing demand. Another way is to work with staffing profiles, which is common in elderly care and assisted living. This means dividing the day into much smaller parts, and have a staffing demand for every 15 minutes or every half hour. In our model, staffing profiles with half hour steps are used, though the parameters are easy to change if 15 minute steps is wanted instead. When using staffing profiles the shifts can vary in length, and this will make it possible to combine different shifts to fulfill the demand each half hour. If the demand is two nurses between 16.00 and 16.30 then for example one nurse working a shift between 12.00 and 17.00 and another nurse working a shift between 15.00 and 21.00 can cover that demand. Both cyclic and non-cyclic scheduling are usual, but we consider cyclic scheduling.
The number of hours each nurse can work has to be regulated in some way. Most health and elderly care units do not use only nurses working full-time, so the average time each nurse should work each week has to be calculated. In most of our experiments we assume that all nurses work the same part-time percentage. A full-time nurse works 37 hours a week and can work any kind of shift. To calculate the average time for each nurse the part-time percentage is multiplied by 37. Night shifts have been excluded since they are not always a need in elderly care. Night shifts would affect the full-time hours by making them lower.

The mathematical model will have a key variable making sure that all the demands and rules are fulfilled, formulated as follows.

\[ X_{i,s,t} = \begin{cases} 1 & \text{if nurse } i \text{ works shifts } s \text{ on day } t \\ 0 & \text{otherwise} \end{cases} \]

This variable, with the help of a parameter telling if a shift covers a certain half hour or not will make sure that the demand is fulfilled. The demand should be fulfilled by the nurses, but a variable "allowing" over- and understaffing is also added to the demand constraint, see A.2. This is of course not desirable to have, so the over- and understaffing will be penalized in the objective function. This is to make the schedules free of over- or understaffing, if this is possible. Understaffing will have a higher penalty than overstaffing, see A.1. This is because having too few nurses taking care of the patients is worse than the extra cost overstaffing entail. Both over- and understaffing penalties should be positive numbers in the objective function, so two functions that ensure this is added, see A.3 and A.4.

### 3.1.2 Scheduling rules and quality aspects

The simulation tool created has to comply with prevailing laws and regulations for staff scheduling. How they are interpreted in the mathematical model will be explained in this section. Some constraints listed are not laws but common rules when scheduling health and elderly care personnel. The schedules are cyclic, so many rules have to take into account that the schedule will be repeated, and that the rules have to apply when going from one scheduling period to the next.

At first each nurse may only work one shift each day, see A.5, and a maximum of five days in a row, see A.6. After that at least one day of rest has to follow. Because of the cyclic scheduling there has to be a check at the end of the scheduling period and the beginning of the next so that the rules for days in a row applies there as well, see A.7. At first a rule forbidding single days work was included. This means that a nurse is not allowed to be free one day, work the next and then get a free day again. This was then removed from the simulation study in order to ease the scheduling. This rule is used at some wards but it is not the most important one and not really necessary for what will be tested in the simulation study.

Each day has to contain at least a certain number of consecutive free hours and this rule is called day rest. When applying this rule a day can be defined in an arbitrary way as long as it consists of 24 consecutive hours. The time when the day begins and ends is called a day break (and when it is set it can not be changed for that scheduling period). So inside this day break there should
be at least the stated number of consecutive free hours. To give an example, figure 3.1 shows two days and two different cases with two shifts following each other. The straight line shows where the first day ends and the next day starts. The dotted line shows an example of a day break, here it is set at 8.00. Each square means one hour. Counting consecutive hours of rest between the dotted lines gives that there is 5 and 10 hours of consecutive rest in the first case and 5 and 11 hours of consecutive rest in the second case. The day rest is usually between 9 and 11 hours. If the day rest in this case is 11 hours, it is fulfilled in the second case but not in the first. The first case will thereby not be allowed when creating the schedule.

Figure 3.1: Example for day rest

The mathematical constraints for the day rest count the half hours between the end of the shift the first day and the end of the first day. If the nurse is free the first day the number of half hours between the day break and the end of the first day is counted instead. These half hours are added to the number of half hours between the beginning of the next day and the start of the shift the next day (or the day break depending on which comes first), see A.27. If a nurse is free the next day the half hours between the start of the day and the day break is counted.

We continue with the example of a day rest of 11 hours. If there is a shift the first day there is a check if this shift starts at least 11 hours after the day break or not, see A.29. Either the check has to say that there is no shift 11 hours after the day break or the earlier counted half hours has to be at least 22 (11 hours) to fulfill the day rest, see A.30 and A.31.

There is a special case in the mathematical model for the last and first day of a scheduling period. This is because of the cyclic property of the schedule, and it makes sure the day rest is fulfilled when going into the next scheduling period as well, see A.28.

Apart from the day rest there is a week rest that has to be fulfilled. This rule says that there should be at least one free day each week, that is also free of evening or split shifts the day before and free of day or split shifts the day after, see A.34 and A.37. This is a simplified version of the usual week rest which requires having 36 consecutive hours of rest at least once each week. The computational time with this simplification will be lower but the results will still be realistic.

To keep track of which days are free and which days are not, the following variable is introduced.

\[
F_{li} = \begin{cases} 
1 & \text{if nurse } i \text{ works day } l \\
0 & \text{otherwise}
\end{cases}
\]

This variable will for instance be used to make sure that there is at least a certain number of free days during the period. There should be 2.25 free days on average each week, so that the minimum number of free days during a period is 2.25 multiplied with the total number of weeks in the scheduling period, see
A.32. The number of free days is always rounded upwards to the nearest integer. As an example, a period of 5 weeks should have a minimum of 11.25 free days, and this is then rounded upwards to at least 12 free days during the period for each nurse. In the mathematical model the rounding up is noted "ceil", see A.33.

Working evenings and weekends is considered uncomfortable and is therefore regulated differently than the other shifts. There are rules for how many of these shifts that a nurse is allowed to work in a row, in a week and during a scheduling period, see A.16-A.22. Split shifts count as evening shifts and will thereby follow the same rules as evening shifts. As for the number of days in a row, weekends and evenings in a row have to have a check at the end of the scheduling period when going in to the next, because of the cyclic scheduling, so that the rule applies there as well. Working a weekend means working both Saturday and Sunday, see A.18. There is also a rule for how many days in a row over the weekend each nurse is allowed to work, see A.23-A.25. This number differs depending on the ward, but it is usually around four days. Last, but not least, the evenings have to be distributed evenly. The total number of evening and split shifts needed were calculated and distributed evenly over the staff, allowing a small difference of $\pm 1$, see A.38.

To keep track of how much time each nurse works over or under his or her average working hours, a time bank is created. The number of hours that differ between the calculated average working hours and the actual working hours are kept in this time bank, see A.8. Each week the differing hours will be added to the hours in the time bank from the week before. The status of the bank will be allowed to vary between -20 and 20 hours. The last time bank before the next period should however be zero. This is because the generic scheduling creates cyclic schedules. So if one nurse for example ends the scheduling period with a time bank status of -1 hour, that person will lack one hour of work every period summing up to a larger number as the time goes. With a non-cyclic scheduling that hour could be accounted for by having that person working one hour over his or her average working hours the next scheduling period. Some part-time percentages will make it impossible to force the last time bank to become zero. That is why a small variation in the last time bank status will be allowed in those cases. For the constraints for the last time bank, see A.11 and A.12.

Only following the rules for the time bank could make the distribution of working hours over the weeks uneven and make it possible to work a lot one week and very little the next, or vice versa. To maintain some kind of evenly distributed working hours each week, but still not demanding each nurse to work his or hers exact number of hours each week, which would make the schedule very inflexible, a span is created. This span allows each nurse to work, for example, between 60% and 140% of the average working hours each week, see A.13.

3.2 Summary of the model

The generic scheduling model has a staffing profile with a given demand for every half hour. The nurses should fulfill the demand, and both over- and understaffing should be avoided. Each nurse may at most work one shift each day and at most five days in a row. The model also makes sure that all daily
3.3 Data

The data used for the upcoming simulation study is presented in Table 3.1. These are taken from real-life cases and are typical for wards in Sweden. Some of the data are however set less hard than they usually are in real life, because of the otherwise long computation times. The upper and lower bounds on the working hours is set a bit higher and lower than usual. The maximum number of work days over a weekend is here set to five, but it is usually three to four days. This number will be set lower in a later case.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of weeks</td>
<td>6</td>
</tr>
<tr>
<td>Day break, half hour</td>
<td>16 (8.00)</td>
</tr>
<tr>
<td>Upper bound, % of working hours each week</td>
<td>160</td>
</tr>
<tr>
<td>Lower bound, % of working hours each week</td>
<td>40</td>
</tr>
<tr>
<td>Maximum number of weekends</td>
<td>3</td>
</tr>
<tr>
<td>Minimum number of weekends</td>
<td>3</td>
</tr>
<tr>
<td>Maximum weekends in a row</td>
<td>2</td>
</tr>
<tr>
<td>Maximum evenings in a row</td>
<td>2</td>
</tr>
<tr>
<td>Day rest, in half hours</td>
<td>20</td>
</tr>
<tr>
<td>Maximum number of days over weekend</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.1: Data used in the simulation study

Rules stating that single shifts are not allowed are common in practice. Single shifts mean working one shift while being free the day before and the day after. This will be allowed in our study, since it is not very important for
the results and it reduces the computational times. There are also workplace meetings where every nurse at the ward participates and those usually take place the same day each week. This makes the scheduling much harder and it is not very important for the results in our study. Such meetings are therefore not included in the model. All simplifications are made to ease the scheduling but they are made in such way that the results will still be useful.

All nurses can work all kinds of shifts and there are no special tasks that only some nurses can do. The shifts used are stated in Table 3.2. Two different staffing curves are used in the simulation study, and these are presented in Chapter 4. For the second staffing curve, all day, split and long shifts can start at 7.00 or 8.00. All shifts have breaks of 30 minutes, except for split and long shifts that have one hour breaks. Having this many shifts available can be seen as a relaxation, and not something all wards have (sometimes not wanted, sometimes not possible). It simplifies the scheduling quite a lot, but is not unrealistic.

<table>
<thead>
<tr>
<th>Type of shift</th>
<th>Start and end of shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>7.00 - 12.00</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 12.30</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 13.00</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 13.30</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 14.00</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 14.30</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 15.00</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 15.30</td>
</tr>
<tr>
<td>Day</td>
<td>7.00 - 16.00</td>
</tr>
<tr>
<td>Evening</td>
<td>12.00 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>12.30 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>13.00 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>13.30 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>14.00 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>14.30 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>15.00 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>15.30 - 21.00</td>
</tr>
<tr>
<td>Evening</td>
<td>16.00 - 21.00</td>
</tr>
<tr>
<td>Split</td>
<td>7.00-12.00 and 15.00-21.00</td>
</tr>
<tr>
<td>Split</td>
<td>7.00-12.00 and 15.30-21.00</td>
</tr>
<tr>
<td>Split</td>
<td>7.00-12.00 and 16.00-21.00</td>
</tr>
<tr>
<td>Split</td>
<td>7.00-12.30 and 15.30-21.00</td>
</tr>
<tr>
<td>Split</td>
<td>7.00-12.30 and 16.00-21.00</td>
</tr>
<tr>
<td>Split</td>
<td>7.00-13.00 and 16.00-21.00</td>
</tr>
<tr>
<td>Long</td>
<td>7.00 - 21.00</td>
</tr>
</tbody>
</table>

Table 3.2: Shifts used
Chapter 4

Simulation study and results

In this chapter the simulation studies are presented. These are related to the workforce composition in different ways, showing how it affect other parts of the scheduling. The demand for staffing is described by profiles during the day. Two different staffing profiles are used to show how a certain change in scheduling conditions can have different effects for different staffing profiles. The profiles used are defined in Table 4.1 and Table 4.2. These are taken from real-life cases, and are typical for assisted living and elderly care. They have the same number of total working hours. This is to make it easier to compare the results to each other. The first staffing profile is referred to as the first case and the second staffing profile is referred to as the second case.

<table>
<thead>
<tr>
<th>Day(s)</th>
<th>Demand 7.00-16.00</th>
<th>Demand 16.00-21.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>7 (7.00-15.30) and 6 (15.30-16.00)</td>
<td>4</td>
</tr>
<tr>
<td>Tuesday-Friday</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Saturday-Sunday</td>
<td>7 (7.00-13.00) and 3 (13.00-16.00)</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4.1: Staffing profile, first case

<table>
<thead>
<tr>
<th>Days</th>
<th>Demand 7.00-8.00</th>
<th>Demand 8.00-21.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday-Sunday</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4.2: Staffing profile, second case

The staffing profiles are illustrated in the following graphs.

Håkansson, 2015.
Chapter 4. Simulation study and results

Figure 4.1: Demand Monday, first case

Figure 4.2: Demand Tuesday-Friday, first case
Figure 4.3: Demand weekend, first case

Figure 4.4: Demand, second case
4.1 Penalizing split shifts

High part-time work percentages combined with the use of split shifts, or not, is a topic for debate in health and elderly care. The split shifts are very often unwanted by the staff, since they then work long days with an unpaid break of a few hours in the middle of the day. On the other hand part-time work percentages are preferred to be high, which is not always possible to achieve without the use of split shifts. The use of split shifts leads to a reduced need for nurses, by sometimes replacing two nurses with one. This leads to each nurse having more hours to work and thereby getting a higher part-time work percentage. To show how the use of split shifts changes depending on the workforce composition, the split shifts are penalized and different workforces are tested.

The penalization of the split shifts have different priorities. The most unwanted split shifts are the ones used Monday to Friday, and these will therefore be penalized the most in the objective function, second to over- and understaffing. The weekday split shifts are the easiest to get rid of, due to the fact that the staff works more weekdays than weekends. So having one person working a split shift instead of two persons working a day respective an evening shift is not always necessary during the weekdays. The next step is penalizing the use of two split shifts for each and one of the nurses each weekend, and that will be the second highest penalty in the objective function for the use of split shifts. The number of worked weekends should be kept low, and with the help of split shifts this is easier to accomplish. The goal is however to get rid of them, and the first step is therefore minimizing the use of two split shifts for each and one of the nurses each and one of the weekends. The last step is penalizing the total number of split shifts used during the weekends. These will have the lowest penalty and such occasions are the hardest to get rid of. The number of weekends allowed to be worked by each nurse is stated in the model, with the goal of requiring them to work only three out of six weekends. This means that the number of nurses has to be kept as low as possible during the weekends, but still covering the demand, which makes it difficult to avoid the split shifts.

The simulation starts off with a number of full-time nurses. This number is calculated from the total number of hours needed for the staffing curve considered. When the scheduling is done, the number of split shifts used is shown by the program and noted. The number of nurses is then increased by one, calculating new part-time work percentages and setting everyone on the average. The number of split shifts used is noted. This is then continued by increasing the number of nurses one at a time and calculating new part-time work percentages for every addition of nurses, always setting them at the average part-time percentage, until no split shifts are used. An illustration of this is found in Figure 4.5.

There is a total of 3108 hours that needs to be staffed for both staffing profiles. A full-time nurse works 37 hours each week, which gives 222 hours for the whole scheduling period. This gives that the number of full-time working nurses to cover these hours is 14. The first workforce to test is therefore 14 full-time nurses. These nurses will be able to fulfill the 3108 hours precisely, so that the last time bank is zero. The number of nurses is then increased to 15. A total of 3108 hours and 15 nurses gives 207.2 hours for each nurse, meaning a part-time percentage of 93.33%. The shifts and demands are set at half hours,
4.1. Penalizing split shifts

![Figure 4.5: Work flow](image)

Table 4.3: Workforces used in the simulation study

<table>
<thead>
<tr>
<th>Nurses</th>
<th>Average part-time percentages (%)</th>
<th>Last time bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>100,00</td>
<td>±0</td>
</tr>
<tr>
<td>15</td>
<td>93,33</td>
<td>±0,31</td>
</tr>
<tr>
<td>16</td>
<td>87,50</td>
<td>±0,25</td>
</tr>
<tr>
<td>17</td>
<td>82,35</td>
<td>±0,32</td>
</tr>
<tr>
<td>18</td>
<td>77,78</td>
<td>±0,33</td>
</tr>
<tr>
<td>19</td>
<td>73,68</td>
<td>±0,44</td>
</tr>
<tr>
<td>20</td>
<td>70,00</td>
<td>±0,4</td>
</tr>
<tr>
<td>21</td>
<td>66,67</td>
<td>±0,003</td>
</tr>
<tr>
<td>22</td>
<td>63,64</td>
<td>±0,29</td>
</tr>
<tr>
<td>23</td>
<td>60,87</td>
<td>±0,37</td>
</tr>
<tr>
<td>24</td>
<td>58,33</td>
<td>±0,493</td>
</tr>
</tbody>
</table>

and this means that they can not end with exactly 207.2 worked hours at the end of the period. So the last time bank is allowed to differ between +0.3 and −0.2 hours, so that it ends at either 207 or 207.5 hours. These calculations are made for every added nurse. The workforces tested are given in Table 4.3. The last time bank is set to the smallest number needed to make it possible to fulfill the exact working hours. It is however never allowed to be outside the span (−0.5, +0.5), because the last timebank should be kept as low as possible, as mentioned before, and it never needs to be outside this span to fulfill the hours.

In the second case, the same number of nurses as in the first studies are too few to enable them to work only three out of six weekends. The maximum number of weekends allowed to work is therefore increased by one, allowing a minimum of three and a maximum of four weekends worked. The scheduling of more than three weekends will be counted for each nurse, see equation B.1, and penalized in the objective function. This is because using more than three weekends is not desired and the number of nurses having to work more than three weekends should be kept low. This has lower priority than over- and understaffing but higher priority than getting rid of split shifts. For the full objective function, see equation B.4.
4.1.1 Results

The results obtained from the simulations are presented in figures 4.7 and 4.8. The thick lines in both pictures show the transition from using split shifts both weekdays and weekends to only using split shifts during the weekends. The thin line in figure 4.8 shows the transition from using at most four to using at most three working weekends for all nurses. For the second case, and with less than 18 nurses, it was not possible to use only three weekends for each nurse. How many nurses that work four weekends can be found in Table 4.4. For 22 and 24 nurses, respectively, in the two cases, the use of split shifts is zero. This means an average part-time work percentage of 63.64% and 58.33%, respectively.

When the number of nurses passes 16 in the first case and 18 in the second case the number of split shifts decreases by six for each added nurse. For these number of nurses the split shifts are only used during the weekends and every nurse works every second weekend. When a nurse is added, two split shifts (one during Saturday and one during Sunday) every second weekend can be abolished. This means getting rid of six split shifts for every added nurse. The split shifts are replaced by the nurse working the earlier split shift and the added nurse. This is illustrated in Figure 4.6. In Figure 4.8 the number of split shifts used for 17 nurses and 18 nurses are the same. This is due to the three nurses working four weekends just passing their extra weekend shifts to the added nurse.

<table>
<thead>
<tr>
<th>Nurses</th>
<th>Number of nurses working four weekends</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.4: How many nurses working four weekends in the second case
4.2 Penalizing long shifts

The split shifts have the advantage of being able to replace two nurses with one, which is an advantage that long shifts also have. These shifts go from morning to evening without a break in the middle. To see what their pros and cons are compared to the split shifts, when having different staffing profiles, the same simulations as for the split shifts are made. Here the split shifts are removed and replaced by long shifts, which are penalized in the same way and follow the
same rules as the split shifts. The workforces tested are the same as earlier. The new sets and variables and the new objective function are found in Appendix B, Section B.3.

4.2.1 Results

The results obtained are presented in Figure 4.9 and Figure 4.10. The thick lines in both pictures show the transition from using split shifts both weekdays and weekends to only using split shifts during the weekends. The thin line in Figure 4.9 shows the transition from using at most four to using at most three working weekends. For less than 16 nurses in the first case, it was not possible to use only three weekends for each nurse. How many nurses that work four weekends can be found in Table 4.5. In the first case the two transitions happen at the same time, and therefore the lines are placed together. For 22 and 24 nurses, respectively, the use of long shifts is zero. This translates into an average part-time work percentage of 63.64% and 58.33%, respectively.

As in the previous section for split shifts the number of long shifts eventually decreases by six for each added nurse. This is due to the same reasons as for the split shifts. This means that two nurses working long shifts can be replaced by these two nurses and the added nurse every second weekend. The illustration in Figure 4.6 can be used in this case as well. However the decrease of six long shifts when going from 15 to 16 nurses in the first case is due to abolishing six long shifts during the weekdays.

Figure 4.9: Results for the first staffing profile
4.3 Dispersion of part-time percentages

To see if the results obtained depend on the workforce diversity, the previous simulations of split shifts are tweaked a little bit. We saw that we can not have all the nurses working full-time and also avoid the split shifts. The question now is if it is possible to have at least some nurses on full-time. In the next experiment we put as many nurses as possible on full-time and the rest on 50%, while making sure they fulfill the total number of hours needed. The number of split shifts used will be penalized as in Section 4.1.

In the first case this experiment is made for 16 nurses. Here, in the first experiment, the number of split shifts during the weekdays is zero and every nurse works three weekends. In the second case this experiment is made for 18 nurses, for the same reasons as in the first case. In Table 4.6 the workforces for the two cases are presented.

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of nurses working 100%</th>
<th>Number of nurses working 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 4.6: Workforces with diverse part-time
4.3.1 Results

The number of split shifts used are in both cases the same as in the results for the penalized split shifts, see Section 4.1. According to Schemagi, this outcome is however not what could be expected in a real-life situation. The reason that it works in this case might be the large number of possible shifts, which facilitates the scheduling.

4.4 Rules for weekends

The rules for the weekends have not been set so tight in the earlier studies. For the next experiment the model used in the study of penalized split shifts also apply. The rules regarding the weekends are however tightened. The first rule added is that if a nurse works Friday evening, he or she should also work the weekend (Saturday and Sunday) in connection to that evening, see B.6. Another rule that is added is the inclusion of at least a long weekend off during the scheduling period. A long weekend off means a free Friday, Saturday and Sunday, or Saturday, Sunday and Monday, see B.7-B.13. The last rule considered is the maximum number of worked days over the weekend. The earlier allowed five days is now set to four. All these rules are common in the health care sector and are evaluated for the same cases and workforces as in section 4.1.

4.4.1 Results

The results regarding the number of split shifts used when the worked days over weekends is four, were exactly the same as in section 4.1 for penalized split shifts. The need for more than three working weekends for fewer nurses than 18 in the second case is not affected by this rule, so the results are the same there as well. It is thus possible to include rules closer to the ones in real-life, thereby getting a schedule with higher quality and still get the same results.

4.5 Letting all nurses work full-time

Going back to section 4.1, about penalized split shifts, the results showed that to be able to avoid the split shifts there is a need for 22 and 24 nurses, respectively, for the two cases. These had very low part-time work percentages. Setting these higher would lead to overstaffing, but then how much? Sometimes it is acceptable to have some overstaffing, so the next experiment is to see how much overstaffing there would be if giving all nurses full-time work. The number of shifts that should be used is set to the maximum, meaning that the number of days off has to be equal to the calculated number (earlier this was a greater than or equal to condition), see B.15. No split shifts are allowed, so there is no penalization of split shifts in the objective function, see B.14, and instead a constraint not allowing split shifts is added, see B.16. The results will show how much overstaffing there is on average each day.

The same experiment is done for 16 nurses in the first case and 18 nurses in the second case, to see how much the average overstaffing is in these "middle" cases. Split shifts during the weekends are allowed but penalized in the objective function, see B.17. Split shifts during Monday to Friday and the use of two split
4.5. Letting all nurses work full-time

Letting all nurses work full-time is not allowed, see B.19 and B.20. The number of shifts was set to the maximum as for 22 and 24 nurses, see B.18.

4.5.1 Results

The following figures show how much overstaffing there is each day when all nurses were given full-time. If there is no black area over the gray area it means that the demand is fulfilled exactly (no understaffing or overstaffing). The number of nurses working over the demand during the morning hours (7.00-12.00) is limited by the total number of shifts each nurse is allowed to work. The almost perfect staffing during the evening is due to the limited number of evenings each nurse is allowed to work. The evenings are distributed evenly over the staff but with a small allowed difference of $\pm 1$ evenings, and this is why there is sometimes a small amount of overstaffing during the evenings. During the weekends there are no overstaffing during the morning and the evening, and this is because the nurses are already working as many shifts as they can during the weekends. The overstaffing for the first hour for the second case is due to the possibility of starting a shift at 7.00 instead of 8.00.

The amount of overstaffing during 12.00-16.00 depends on how many of the morning and evening shifts that are long enough to cover parts of these hours. Shifts break (end or begin) at each half hour between 12.00-16.00, so a choice can be made by the model to use longer shifts if needed. In order to overstaff during the morning or the evening a nurse has to be added, because there is no shift that can be lengthened to cover the morning or the evening. Having the overstaffing during the morning, evening or distributed over them both does not matter (if the rule for how many evenings each nurse is allowed to work is changed). In Figure 4.11 and Figure 4.13, the small change in the demand during Monday is left out (only differing by one nurse during one half hour). For the cases with 16 and 18 nurses, the number of split shifts during the weekends is 36, as in Section 4.1.

For 22 and 24 nurses respectively the nurses works around 21 shifts each when working the average part-time work percentage. When working the maximum number of shifts, as they do in this simulation, they work 28 shifts each. So seven extra shifts are set out for each nurse, which leads to a total of 154 and 168 shifts respectively to add. This means around five extra shifts each weekday for both cases, because no shifts can be added to the weekends.

For 16 and 18 nurses respectively, the nurses works very close to their maximum number of shifts, which is 28. When the total number of shifts for each nurse is set to the exact maximum, it is very close to the original number of shifts. This is why the amount of overstaffing during the morning each weekday is very low.
Figure 4.11: Results for the first staffing profile Monday-Friday, 22 nurses

Figure 4.12: Results for the first staffing profile Saturday-Sunday, 22 nurses
4.5. Letting all nurses work full-time

Figure 4.13: Results for the first staffing profile Monday-Friday, 16 nurses

Figure 4.14: Results for the first staffing profile Saturday-Sunday, 16 nurses
Figure 4.15: Results for the second staffing profile Monday-Friday, 24 nurses

Figure 4.16: Results for the second staffing profile Saturday-Sunday, 24 nurses
4.5. Letting all nurses work full-time

Figure 4.17: Results for the second staffing profile Monday-Friday, 18 nurses

Figure 4.18: Results for the second staffing profile Saturday-Sunday, 18 nurses
Chapter 5

Summary and conclusions

Starting with the simulations of penalizing the split shifts, it could be seen that in order to avoid the split shifts the number of nurses has to be quite high in comparison to the least number of nurses needed to cover the total working hours. Their part-time work percentages also have to be very low, around 60%. This is the case even though the scheduling rules were not set as hard as they could have been. The use of split shifts enable the construction of schedules with less nurses with higher part-time work percentages (since every nurse can work more hours). In order to fulfill the demand exactly while avoiding the split shifts, more nurses working shorter shifts are needed, which leads to lower part-time work percentages.

Replacing the split shifts with long shifts gives about the same results in terms of number of used split and long shifts. The largest difference is in how many weekends each nurse has to work. When the staffing demand is higher during the day and lower during the evening (with a lower dip in the middle of the day during the weekends) and split shifts are used, the number of weekends used is three for all nurses and workforce compositions. When using long shifts some nurses have to work four weekends when the number of nurses is lower than 16 (and for more nurses the number of weekends worked is three).

With a more even staffing demand and long shifts the number of weekends each nurse has to work is three for all nurses and workforce compositions. When using split shifts some nurses have to work four weekends when the number of nurses is less than 18 (for more nurses the number of weekends worked is three). The long shifts and the split shifts are used to solve the same staffing problem, and that is why the number of nurses and part-time work percentages needed to get rid of these shifts are the same and the numbers of these shifts used for every workforce are mainly the same.

Both split and long shifts are used mostly during the weekends and they are strongly connected to how many weekends each nurse is allowed to work. This is so because of the number of weekends each nurse can work is considerably smaller than the number of weekdays. This leads to the conclusion that in the first case (when the demand is higher during the day and lower during the evening) the use of split shifts is a better alternative in the sense of weekends worked. With split shifts each nurse can work the desired three out of six weekends no matter which workforce composition that is used. For the second case (when the demand is more even during the day) it is the long shifts that
are better with respect to weekends worked. These conclusions are however only true when looking at the lower number of nurses. For the higher numbers the number of weekends used are the same in all cases.

In Section 4.3 an experiment with having as many nurses as possible on full-time and the rest on 50% led to the same result as the earlier simulations with having them all on the average part-time work percentage. This is as mentioned before not always reasonable, according to the experts at Schemagi, and the results obtained might be because of the use of many shifts, which introduces many degrees of freedom in the scheduling and makes it easier.

The earlier experiments included some simplifications of the scheduling rules, and in Section 4.4 some of the them were tightened. This leads to the same number of split shifts as in the earlier experiments. So the added rules did not make anything worse, but only better with respect to quality aspects. With these additional rules the nurses do not have to work as many days over the weekends and they have at least one long weekend off. So it was possible to get the same results with better schedules in these cases.

Lastly we tried to set 16 and 22 nurses for the first case and 18 and 24 nurses for the second case on full-time. This is to get a picture of how much overstaffing there is when having all nurses working full-time, both for cases where there are no split shifts and also when split shifts are used during the weekends. Here the results show that there is, in some cases, a need for overstaffing with up to 10 nurses in the middle of the day (12.00-16.00). This is nearly 167% of the staffing demand.

The overall conclusion of the experiments is that no matter if there are split shifts or long shifts that a ward wants to get rid of, there has to be more nurses available and at lower part-time work percentages, if there should be no over- or understaffing. Or the other way around, in order to let the nurses work full-time there is a need for split or long shifts. At least this is the case with the rules stated in Chapter 3 and the rules added from Section 4.4. To offer all the nurses that are needed full-time work without using split or long shifts, there is instead a need for a lot of overstaffing (and a little less if split shifts are allowed during weekends). If the overstaffing option is possible economically it might be possible to let the extra nurses work at other units of the ward, or in some other way redistribute the extra staff to utilize it in the best way. In order to avoid the split shifts with less nurses and higher part-time work percentages there is a need for the nurses to work more days, and more specifically more weekends. In our cases each nurse works three out of six weekends, which is quite usual. Having them work more weekends, for example four, is often not an option, which leads to the same conclusions as earlier.

Offering nurses full-time work and at the same time avoid uncomfortable shifts is, as mentioned earlier, debated. By using some fairly realistic cases, the simulations made in this thesis shows that there is a conflict between these two goals, and this is due to both the staffing demand and the scheduling rules.

The goal of this thesis was to be able to present more general causality relationships when discussing trade-offs between different demands and wishes, something that is missing today. The aim was for Schemagi to be able to use this work in meetings with their customers but also internally. This is now possible and the graphs make the effects of the trade-offs clear. The thesis was also supposed to be able to use in extended research in this area. This thesis has fairly realistic and general cases but still there are simplifications, such as
using many shifts (which might have affected the result in Section 4.5). So for further research there might be a possibility not to have these simplifications and of course extend the trade-offs tested. This thesis can be seen as a good start of filling the knowledge gap of how trade-offs between full-time working nurses and split or long shifts work.
Chapter 5. Summary and conclusions
Bibliography


Appendix A

Mathematical model

In this appendix the mathematical model for the generic scheduling is presented.

A.1 Sets and parameters

In the following table all sets and parameters are defined and described.

<table>
<thead>
<tr>
<th>notation</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>set of all nurses</td>
</tr>
<tr>
<td>$S^d$</td>
<td>set of all day shifts</td>
</tr>
<tr>
<td>$S^e$</td>
<td>set of all evening shifts</td>
</tr>
<tr>
<td>$S^s$</td>
<td>set of all split shifts</td>
</tr>
<tr>
<td>$S$</td>
<td>set of all shifts, $S^d \cup S^e \cup S^s$</td>
</tr>
<tr>
<td>$at_i$</td>
<td>average time nurse $i \in I$ should work each week</td>
</tr>
<tr>
<td>$ar$</td>
<td>number of free days each nurse should have during the period</td>
</tr>
<tr>
<td>$b_{sj}$</td>
<td>$= 1$ if shift $s \in S$ begins at half hour $j = 1, \ldots, 48$, and $0$ otherwise</td>
</tr>
<tr>
<td>$d_{jl}$</td>
<td>staffing demand half hour $j = 1, \ldots, 48$ on day $l = 1, \ldots, 7sw$</td>
</tr>
<tr>
<td>$db$</td>
<td>day break, given in half hours</td>
</tr>
<tr>
<td>$do$</td>
<td>maximum number of days allowed to work in a row over the weekend</td>
</tr>
<tr>
<td>$dr$</td>
<td>day rest, given in half hours</td>
</tr>
<tr>
<td>$e_{sj}$</td>
<td>$= 1$ if shift $s \in S$ ends at half hour $j = 1, \ldots, 48$, and $0$ otherwise</td>
</tr>
<tr>
<td>$er$</td>
<td>maximum number of evenings in a row</td>
</tr>
<tr>
<td>$h_{sj}$</td>
<td>$= 1$ if shift $s \in S$ covers half hour $j = 1, \ldots, 48$, and $0$ otherwise</td>
</tr>
<tr>
<td>$me$</td>
<td>maximum number of evenings per week</td>
</tr>
<tr>
<td>$minw$</td>
<td>minimum number of weekends per period</td>
</tr>
<tr>
<td>$mw$</td>
<td>maximum number of weekends per period</td>
</tr>
<tr>
<td>$nc$</td>
<td>percentage of a nurses total number of shifts that should be evenings</td>
</tr>
<tr>
<td>$p$</td>
<td>total number of nurses</td>
</tr>
<tr>
<td>$pt_i$</td>
<td>percentage of full-time nurse $i \in I$ works</td>
</tr>
<tr>
<td>$rd$</td>
<td>maximum number of shifts in a row</td>
</tr>
<tr>
<td>$sl_s$</td>
<td>length of shift $s \in S$</td>
</tr>
<tr>
<td>$ubt$</td>
<td>percentage of average time a nurse may work at the most in a week (upper bound)</td>
</tr>
<tr>
<td>$lbt$</td>
<td>the lowest percentage of average time a nurse has to work in a week (lower bound)</td>
</tr>
<tr>
<td>$wr$</td>
<td>maximum number of weekends in a row</td>
</tr>
</tbody>
</table>
A.2 Variables

The following section contains a table with the variables used in the mathematical model.

<table>
<thead>
<tr>
<th>notation</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{il}$</td>
<td>$\geq 0$, integer, auxiliary variable for week rest for nurse $i \in I$ day $l = 1, \ldots, 7sw - 2$</td>
</tr>
<tr>
<td>$D_i^l$</td>
<td>continuous, allowed difference between calculated number of evenings for nurse $i \in I$ and actual evenings for nurse $i \in I$</td>
</tr>
<tr>
<td>$DR_{il}$</td>
<td>$\geq 0$, integer, number of consecutive half hours of rest between shifts inside the day breaks, for nurse $i \in I$ day $l = 1, \ldots, 7sw - 2$</td>
</tr>
<tr>
<td>$DRH_{il}$</td>
<td>$= 1$ if the day rest is fulfilled for nurse $i \in I$ before a shift day $l = 1, \ldots, 7sw$, and 0 otherwise</td>
</tr>
<tr>
<td>$DRL_i$</td>
<td>$\geq 0$, integer, number of consecutive half hours of rest between the last days shift and the first days shift inside the day breaks for nurse $i$</td>
</tr>
<tr>
<td>$F_{il}$</td>
<td>$= 1$ if nurse $i \in I$ works day $l = 1, \ldots, 7sw$, and 0 otherwise</td>
</tr>
<tr>
<td>$O_{jl}$</td>
<td>$\geq 0$, integer, number of nurses working over the demand half hour $j = 1, \ldots, 48$ day $l = 1, \ldots, 7sw$</td>
</tr>
<tr>
<td>$OU_{jl}$</td>
<td>$\geq 0$, integer, the difference between nurses working and the demand half hour $j = 1, \ldots, 48$ day $l = 1, \ldots, 7sw$</td>
</tr>
<tr>
<td>$RW_{ij,l}$</td>
<td>$= 1$ if the week rest is fulfilled ($C_{il} = 0$) for nurse $i$ day 1, 0 otherwise ($C_{il} = 1, 2$ or $3$)</td>
</tr>
<tr>
<td>$TB_{iw}$</td>
<td>continuous, time bank status for nurse $i \in I$ week $w = 1, \ldots, sw$ (in hours)</td>
</tr>
<tr>
<td>$TB_e$</td>
<td>continuous, the last time bank status for nurse $i \in I$ (in hours)</td>
</tr>
<tr>
<td>$U_{jl}$</td>
<td>$\geq 0$, integer, number of nurses missing to fulfill demand half hour $j = 1, \ldots, 48$ day $l = 1, \ldots, 7sw$</td>
</tr>
<tr>
<td>$W_{iw}$</td>
<td>$= 1$ if nurse $i \in I$ works the weekend in week $w = 1, \ldots, sw$, and 0 otherwise</td>
</tr>
<tr>
<td>$X_{isl}$</td>
<td>$= 1$ if nurse $i \in I$ works shift $s \in S$ day $l = 1, \ldots, 7sw$, and 0 otherwise</td>
</tr>
</tbody>
</table>

A.3 Objective function

$$
\min \quad z = 10000 \times \sum_{j=1}^{48} \sum_{l=1}^{7sw} U_{jl} + 1000 \times \sum_{j=1}^{48} \sum_{l=1}^{7sw} O_{jl}
$$

A.4 Staffing demand

$$
\sum_{i \in I} \sum_{s \in S} h_{sj} \times X_{isl} = OU_{jl} + d_{jl}, \quad j = 1, \ldots, 48, \ l = 1, \ldots, 7sw
$$

$$
O_{jl} - OU_{jl} \geq 0, \quad j = 1, \ldots, 48, \ l = 1, \ldots, 7sw
$$

$$
U_{jl} + OU_{jl} \geq 0, \quad j = 1, \ldots, 48, \ l = 1, \ldots, 7sw
$$
A.5 Scheduling rules and quality aspects

\[ \sum_{s \in S} X_{isl} \leq 1, \quad i \in I, \; l = 1, \ldots, 7sw \]  \hfill (A.5)

\[ l + rd \sum_{l=1}^{l+rd} \sum_{s \in S} X_{isl,tl} \leq rd, \quad i \in I, \; l = 1, \ldots, 7sw - rd \]  \hfill (A.6)

\[ \sum_{l=1}^{7sw} \sum_{s \in S} X_{isl,tl} + \sum_{l=1}^{l-7sw+rd} \sum_{s \in S} X_{isl,tl} \leq rd, \quad i \in I, \; l = (7sw - rd + 1), \ldots, 7sw \]  \hfill (A.7)

\[ \sum_{k=1}^{7} \sum_{s \in S} X_{isl,k+7(w-1)} \cdot sl_s - at_i + TB_{i,w-1} = TB_{iw}, \quad i \in I, \; w = 1, \ldots, sw \]  \hfill (A.8)

\[ TB_{i0} := 0, \quad i \in I \]  \hfill (A.9)

\[ TB_{iw} \in [-20, 20], \quad i \in I, \; w = 1, \ldots, sw \]  \hfill (A.10)

\[ \sum_{k=1}^{7} \sum_{s \in S} X_{isl,k+7(sw-1)} \cdot sl_s - at_i + TB_{i,sw-1} = TB_{i}^c, \quad i \in I \]  \hfill (A.11)

\[ TB_{i}^c = 0, \quad i \in I \]  \hfill (A.12)

\[ \text{lbt} \cdot at_i \leq \sum_{k=1}^{7} \sum_{s \in S} X_{isl,k+7(w-1)} \cdot sl_s \leq \text{ubt} \cdot at_i, \quad i \in I, \; w = 1, \ldots, sw \]  \hfill (A.13)

\[ at_i := pt_i \cdot 37, \quad i \in I \]  \hfill (A.14)

\[ \sum_{l=1}^{l+6} \sum_{s \in S^c \cup S^s} X_{isl,tl} \leq me, \quad i \in I, \; l = 1, \ldots, 7sw - 6 \]  \hfill (A.15)

\[ \sum_{l=1}^{l+er} \sum_{s \in S^c \cup S^s} X_{isl,tl} \leq er, \quad i \in I, \; l = 1, \ldots, 7sw - er \]  \hfill (A.16)

\[ \sum_{l=1}^{7sw} \sum_{s \in S^c \cup S^s} X_{isl,tl} + \sum_{l=1}^{l-7sw+er} \sum_{s \in S^c \cup S^s} X_{isl,tl} \leq er, \quad i \in I, \; l = (7sw - er + 1), \ldots, 7sw \]  \hfill (A.17)

\[ W_{iw} = \sum_{s \in S} X_{isl,6+7(w-1)} = \sum_{s \in S} X_{isl,7+7(w-1)}, \quad i \in I, \; w = 1, \ldots, sw \]  \hfill (A.18)
$$\sum_{w=1}^{sw} W_{iw} \leq mw, \quad i \in I \tag{A.19}$$

$$\sum_{w=1}^{sw} W_{i,w} \geq minw \quad i \in I \tag{A.20}$$

$$\sum_{w=w-r}^{w+w'\leq sw} W_{i,w} \leq wr, \quad i \in I, \quad w = 1, \ldots, sw - w' \tag{A.21}$$

$$\sum_{w=w-r}^{w+w'\leq sw} W_{i,w} + \sum_{w=w'\geq sw} W_{i,w} \leq wr, \quad i \in I, \quad w = (sw - wr + 1), \ldots, sw \tag{A.22}$$

$$\sum_{l=1}^{l+do} \sum_{s \in S} X_{i,s,l+7(w-1)} \leq do, \quad i \in I, \quad w = 1, \ldots, sw - 1, \quad l = 7 - do, \ldots, 6 \tag{A.23}$$

$$\sum_{l=1}^{l+do} \sum_{s \in S} X_{i,s,l+7(sw-1)} \leq do, \quad i \in I \tag{A.24}$$

$$\sum_{l=1}^{l+do} \sum_{s \in S} X_{i,s,l+7(sw-1)} + \sum_{l=1}^{7 - do} \sum_{s \in S} X_{i,s,l} \leq do, \quad i \in I, \quad l = (7 - do + 1), \ldots, 6 \tag{A.25}$$

$$\sum_{s \in S} X_{i,s,l} = F_{il}, \quad i \in I \tag{A.26}$$

$$48 - \sum_{j=db+1}^{db+dr} \sum_{s \in S} j \cdot X_{isl} \cdot e_{sj} - \sum_{j=1}^{db} \sum_{s \in S} db \cdot X_{i,s,l+1} \cdot e_{sj} - db \cdot (1 - F_{il}) + db \cdot (1 - F_{i,l+1}) +$$

$$\sum_{j=1}^{db} \sum_{s \in S} (j - 1) \cdot X_{i,s,l+1} \cdot b_{sj} + \sum_{j=db+1}^{48} db \cdot X_{i,s,l} \cdot b_{sj} = DR_{il}, \quad i \in I, \quad l = 1, \ldots, 7sw - 1 \tag{A.27}$$

$$48 - \sum_{j=db+1}^{db+dr} \sum_{s \in S} j \cdot X_{i,s,7sw} \cdot e_{sj} - \sum_{j=1}^{db} \sum_{s \in S} db \cdot X_{i,s,7sw} \cdot e_{sj} - db \cdot (1 - F_{il}) + db \cdot (1 - F_{i,l+1}) +$$

$$\sum_{j=1}^{db} \sum_{s \in S} (j - 1) \cdot X_{i,s,1} \cdot b_{sj} + \sum_{j=db+1}^{48} db \cdot X_{i,s,1} \cdot b_{sj} = DRL_{i}, \quad i \in I, \quad l = 1, \ldots, 7sw - 1 \tag{A.28}$$
\[ \frac{DR_i}{dr} + DRH_{il} \geq 1, \quad i \in I, \quad l = 1, \ldots, 7sw - 1 \]  
(A.30)

\[ \frac{DRL_i}{dr} + DRH_{i,7sw} \geq 1, \quad i \in I \]  
(A.31)

\[ \sum_{l=1}^{7sw} (1 - F_{il}) \geq ar, \quad i \in I \]  
(A.32)

\[ ar := \text{ceil}(2.25 \times sw) \]  
(A.33)

\[ \sum_{s \in S_e \cup S_s} X_{isl} + F_{i,l+1} + \sum_{s \in S_d \cup S_s} X_{i,s,l+1} = C_{il}, \quad i \in I, \quad l = 1, \ldots, 7sw - 2 \]  
(A.34)

\[ (1 - RW_{il}) \geq \frac{C_{il}}{3}, \quad i \in I, \quad l = 1, \ldots, 7sw - 2 \]  
(A.35)

\[ (1 - RW_{il}) \leq C_{il}, \quad i \in I, \quad l = 1, \ldots, 7sw - 2 \]  
(A.36)

\[ \sum_{k=1}^{5} RW_{i,k+7(w-1)} \geq 1, \quad i \in I, \quad w = 1, \ldots, sw \]  
(A.37)

\[ \sum_{l=1}^{7sw} \sum_{s \in S_e \cup S_s} X_{isl} = D_i^e + \frac{\sum_{l=1}^{7sw} d_{35,l}}{p}, \quad i \in I, \quad l = 1, \ldots, 7sw \]  
(A.38)
Appendix B

Model, simulation studies

This appendix contains the changed objective functions and changed or added constraints, variables and parameters for the simulation studies.

B.1 General

The following variable and constraint were added when starting the simulation studies. In some cases it was sometimes necessary to work more than the desired weekends (three in the cases with scheduling periods of six weeks). A higher maximum number of weekends was therefore allowed.

\[
MTT_i \geq 0, \text{ integer, the number of weekends nurse } i \in I \text{ works over the desired three}
\]

\[
MTT_i = \sum_{w=1}^{sw} W_{iw} - 3 \quad i \in I
\]  

(B.1)

B.2 Split shifts

The following section contains the added variables, constraints and new objective function for the simulation study "Penalized split shifts".

B.2.1 New variables

<table>
<thead>
<tr>
<th>notation</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(TS_{iw})</td>
<td>= 1 if nurse (i \in I) works split shifts both Saturday and Sunday, and 0 otherwise</td>
</tr>
<tr>
<td>(TSH_{iw})</td>
<td>(\geq 0, \text{ integer, number of split shifts nurse } i \in I \text{ works Saturday and Sunday week } w = 1, \ldots, sw)</td>
</tr>
</tbody>
</table>

B.2.2 New constraints

\[
TSH_{iw} = \sum_{s \in S^w} \sum_{k=0}^{7} X_{i,s,k+7(w-1)}, \quad i \in I, \ w = 1, \ldots, sw
\]  

(B.2)
\[ TS_{iw} \geq TSH_{iw} - 1, \quad i \in I, \ w = 1, \ldots, sw \]  

(B.3)

### B.2.3 Objective function

\[ \min \ z = 10000 \sum_{j=1}^{7sw} U_{jl} + 1000 \sum_{j=1}^{7sw} O_{jl} + 500 \sum_{i \in I} MTT_i + \]

\[ 100 \sum_{w=1}^{sw} \sum_{k=1}^{5} \sum_{s \in S^l} X_{i,s,k+7(w-1)} + 10 \sum_{i \in I} \sum_{w=1}^{sw} TS_{iw} + \sum_{w=1}^{sw} \sum_{i \in I} \sum_{s \in S^l} \sum_{k=6}^{7} X_{i,s,k+7(w-1)} \]

(B.4)

### B.3 Long shifts

The following section contains the new sets, added variables and new objective function for the simulation study "Penalized long shifts". The constraints will look exactly the same as in the split shifts section with a replacement of the index "s" (split shift) with the index "l" (long shift).

#### B.3.1 New sets

<table>
<thead>
<tr>
<th>notation</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S^l )</td>
<td>set of all long shifts (replaces ( S^s ))</td>
</tr>
<tr>
<td>( S )</td>
<td>set of all shifts, ( S^l \cup S^d \cup S^e ) (replaces the earlier ( S ))</td>
</tr>
</tbody>
</table>

#### B.3.2 New variables

<table>
<thead>
<tr>
<th>notation</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TL_{iw} )</td>
<td>( = 1 ) if nurse ( i \in I ) works long shifts both Saturday and Sunday, and ( 0 ) otherwise (replaces ( TS_{iw} ))</td>
</tr>
<tr>
<td>( TLH_{iw} )</td>
<td>( \geq 0 ), integer, number of long shifts nurse ( i \in I ) works Saturday and Sunday week ( w = 1, \ldots, sw ) (replaces ( TSH_{iw} ))</td>
</tr>
</tbody>
</table>

#### B.3.3 Objective function

\[ \min \ z = 10000 \sum_{j=1}^{7sw} U_{jl} + 1000 \sum_{j=1}^{7sw} O_{jl} + 500 \sum_{i \in I} MTT_i + \]

\[ 100 \sum_{w=1}^{sw} \sum_{k=1}^{5} \sum_{s \in S^l} X_{i,s,k+7(w-1)} + 10 \sum_{i \in I} \sum_{w=1}^{sw} TL_{iw} + \sum_{w=1}^{sw} \sum_{i \in I} \sum_{s \in S^l} \sum_{k=6}^{7} X_{i,s,k+7(w-1)} \]

(B.5)

### B.4 Rules for weekends

The following section contains the added variables and constraints for the simulation study "Rules for weekends".
B.4.1 New variables

<table>
<thead>
<tr>
<th>notation</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(LW_{iw})</td>
<td>1 if nurse (i \in I) is free Friday, Saturday and Sunday or Saturday, Sunday and Monday, and 0 otherwise</td>
</tr>
<tr>
<td>(LWF_{iw})</td>
<td>1 if nurse (i \in I) is free Friday, Saturday and Sunday, and 0 otherwise</td>
</tr>
<tr>
<td>(LWM_{iw})</td>
<td>1 if nurse (i \in I) is free Saturday, Sunday and Monday, and 0 otherwise</td>
</tr>
</tbody>
</table>

B.4.2 New constraints

\[
W_{iw} \geq \sum_{s \in S^c \cup S^s} X_{i,s,5+7(w-1)} \quad i \in I, \ w = 1, \ldots , sw 
\]  \hspace{1cm} (B.6)

\[
1 - LWF_{iw} \geq \frac{W_{iw} + \sum_{s \in S^c \cup S^s} X_{i,s,5+7(w-1)}}{2} \quad i \in I, \ w = 1, \ldots , sw 
\]  \hspace{1cm} (B.7)

\[
1 - LWF_{iw} \leq W_{iw} + \sum_{s \in S^c \cup S^s} X_{i,s,5+7(w-1)} \quad i \in I, \ w = 1, \ldots , sw 
\]  \hspace{1cm} (B.8)

\[
1 - LWM_{iw} \geq \frac{W_{iw} + \sum_{s \in S^c \cup S^s} X_{i,s,1+7sw}}{2} \quad i \in I, \ w = 1, \ldots , sw - 1 
\]  \hspace{1cm} (B.9)

\[
1 - LWM_{iw} \leq W_{iw} + \sum_{s \in S^c \cup S^s} X_{i,s,1+7sw} \quad i \in I, \ w = 1, \ldots , sw - 1 
\]  \hspace{1cm} (B.10)

\[
1 - LWM_{isw} \geq \frac{W_{isw} + \sum_{s \in S^c \cup S^s} X_{i,s,1}}{2} \quad i \in I 
\]  \hspace{1cm} (B.11)

\[
1 - LWM_{isw} \leq W_{isw} + \sum_{s \in S^c \cup S^s} X_{i,s,1} \quad i \in I 
\]  \hspace{1cm} (B.12)

\[
\sum_{w=1}^{sw} (LWM_{iw} + LWF_{iw}) \geq 1, \quad i \in I 
\]  \hspace{1cm} (B.13)

B.5 Full time

The following objective functions and constraints were added in Section 4.5.

B.5.1 22 and 24 nurses

\[
\min z = 10000 \times \sum_{j=1}^{48} \sum_{l=1}^{7sw} U_{jl} + 1000 \times \sum_{j=1}^{48} \sum_{l=1}^{7sw} O_{jl} + 500 \times \sum_{i \in I} MTT_i 
\]  \hspace{1cm} (B.14)

\[
\sum_{l=1}^{7sw} (1 - F_{il}) = ar, \quad i \in I 
\]  \hspace{1cm} (B.15)

\[
\sum_{s \in S^c} X_{isl} = 0, \quad i \in I, \ l = 1, \ldots , 7sw 
\]  \hspace{1cm} (B.16)
B.5.2 16 and 18 nurses

\[
\begin{align*}
\min \quad z &= 10000 \sum_{j=1}^{48} \sum_{l=1}^{7} U_{jl} + 1000 \sum_{j=1}^{48} \sum_{l=1}^{7} O_{jl} + 500 \sum_{i \in I} MTT_i \\
&\quad + \sum_{w=1}^{sw} \sum_{i \in I} \sum_{s \in S^w} \sum_{k=6}^{7} X_{i,s,k} + 7(w-1) \\
&\quad + \sum_{l=1}^{7w} (1 - F_{il}) = ar, \quad i \in I \quad (B.18) \\
&\quad \sum_{s \in S^w} \sum_{k=6}^{7} X_{isk} = 0, \quad i \in I \quad (B.19) \\
&T S_{iw} = 0, \quad i \in I, \quad w = 1, \ldots, sw \quad (B.20)
\end{align*}
\]
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