MODELING FOR OPTIMAL CONTROL: A VALIDATED DIESEL-ELECTRIC POWERTRAIN MODEL

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ABSTRACT

An optimal control ready model of a diesel-electric powertrain is developed, validated and provided to the research community. The aim of the model is to facilitate studies of the transient control of diesel-electric powertrains and also to provide a model for developers of optimization tools. The resulting model is a four state three control mean value engine model that captures the significant nonlinearity of the diesel engine, while still being continuously differentiable.

Keywords: Modeling, Optimal Control, Diesel engine, Diesel-Electric

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>ω</td>
<td>Rotational speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>N</td>
<td>Rotational speed</td>
<td>rpm</td>
</tr>
<tr>
<td>m</td>
<td>Massflow</td>
<td>kg/s</td>
</tr>
<tr>
<td>p</td>
<td>Power</td>
<td>W</td>
</tr>
<tr>
<td>M</td>
<td>Torque</td>
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<tr>
<td>Π</td>
<td>Pressure ratio</td>
<td>m³压</td>
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<td>Efficiency</td>
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</tr>
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<td>Area</td>
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</tr>
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<td>Head parameter</td>
<td>-</td>
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<tr>
<td>Φ</td>
<td>Flow parameter</td>
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<tr>
<td>γ</td>
<td>Specific heat capacity ratio</td>
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<tr>
<td>c_p</td>
<td>Specific heat capacity constant pressure</td>
<td>J/(kg ⋅ K)</td>
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<tr>
<td>c_v</td>
<td>Specific heat capacity constant volume</td>
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<td>R</td>
<td>Gas Constant</td>
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<td>(A/F)_0</td>
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<tr>
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<td>J</td>
<td>Inertia</td>
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<td>BSR</td>
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<td>R</td>
<td>Radius</td>
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<td>λ</td>
<td>Air/fuel equivalence ratio</td>
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</tr>
<tr>
<td>θ</td>
<td>Fuel/air equivalence ratio</td>
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</table>

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Table 1: Symbols used

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
<th>Index</th>
<th>Description</th>
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<tbody>
<tr>
<td>amb</td>
<td>Ambient</td>
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<td>Compressor</td>
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<td>01</td>
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<td>tc</td>
<td>Turbocharger</td>
<td>ref</td>
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Table 2: Subscripts used

results to be relevant, higher demands are set on model quality. This relates both to differentiability of the model, for efficient solution processes of the optimal control problem, and also its extrapolation properties since the obtained solutions are often on the border to or outside the nominal operating region. This paper presents the modelling and final model of a diesel-electric powertrain to be used in the study of transient operation. This optimal control ready model will also be made available to the research community to further encourage optimal control studies.

The resulting model is a four state, three control, mean value engine model (MVEM) that consists of 10 submodels that are all continuously differentiable, and suitable for automatic differentiation, in the region of interest in order to enable the nonlinear program solvers to use higher order search methods.
The contributions of the paper are three-fold: 1) A methodology how to model and parametrize a model of a diesel-electric powertrain is presented. The measurements are conducted without a dynamometer, the only requirements are a diesel-electric powertrain and sensors. 2) A model structure and modeling approach with provided equations, enabling researchers to adjust the parameters of the model to represent their own powertrain. 3) It also provides researchers without engine models or data a relevant and validated open source model on which control design or optimization can be performed.

MODEL STRUCTURE

The aim of the model is control systems design and optimization. This imposes the requirement that the model has to be detailed, but at the same time computationally fast. This leads to a 0-D or MVEM approach. Within MVEM there are two different approaches, one is black box modelling or standard system identification techniques, another is physical modelling where the engine is described using standard physical relations. Due to that one of the model aims is optimization and the solution of optimization problems often are on the border to or outside the nominal operating region the physical modeling approach is selected for its extrapolation properties. For more information about engine modelling as well as the state of the art of engine models the reader is referred to [1, 2].

MODELING

The measured and modeled engine-generator combination (GenSet) consists of a generator mounted on the output shaft of a medium-duty tier 3 diesel-engine. The engine is equipped with a charge air cooled wastegated turbocharger. The states of the developed MVEM are engine speed, \( \omega_{ice} \), inlet manifold pressure, \( p_{im} \), exhaust manifold pressure, \( p_{em} \), turbocharger speed, \( \omega_c \). The controls are injected fuel mass, \( u_f \), wastegate position, \( u_{wg} \), and generator power, \( P_{gen} \).

The submodels are models for compressor massflow and power, engine out and exhaust manifold temperatures, cylinder massflow, turbine massflow and power, wastegate massflow, engine torque and generator power, with connections between the compo-
and the manifolds are modelled using the standard isothermal model [4]

\[
\frac{dP_{im}}{dt} = \frac{R_a T_{im}}{V_{im}} (m_c - m_{ac})
\]

(4)

\[
\frac{dP_{em}}{dt} = \frac{R_a T_{em}}{V_{em}} (m_{ac} + m_f - m_i - m_{wg})
\]

(5)

where in the tuning the measured intake manifold temperature, \(T_{im}\) is used but in the final model the intercooler is assumed to be ideal, i.e. no pressure loss and \(T_{im}\) constant. The dynamic models have four tuning parameters, \(J_{GenSet}, J_{tc}, V_{im}\) and \(V_{em}\).

**Compressor**

The compressor model consists of two sub-models, one for the massflow and one for efficiency. In order to avoid problems for low turbocharger speeds and transients with pressure ratios \(\Pi_c < 1\) a variation of the physically motivated \(\Psi\Phi\) model in [1] is used. The idea is that \(\Psi\) approaches a maximum at zero flow and that the maximum flow in the region of interest is quadratic in \(\omega_c\).

**Massflow model**

The pressure quotient over the compressor:

\[
\Pi_c = \frac{p_{02}}{p_{01}}
\]

(6)

Pressure ratio for zero flow:

\[
\Pi_{c, max} = \left( \frac{\omega_{c,max}^2 R_c^2 \Psi_{max}}{2 c_{p,a} T_{01}} + 1 \right)^{\frac{\gamma_a}{\gamma-1}}
\]

(7)

Corrected and normalized turbocharger speed:

\[
\omega_{c, corr,norm} = \frac{\omega_c}{15000\sqrt{T_{01}/T_{ref}}}
\]

(8)

Maximum corrected massflow:

\[
\dot{m}_{c,corr,max} = c_{m_{c,1}} \omega_{c,corr,norm}^2 + c_{m_{c,2}} \omega_{c,corr,norm} + c_{m_{c,3}}
\]

(9)

Corrected massflow:

\[
\dot{m}_{c,corr} = \dot{m}_{c,corr,max} \sqrt{1 - \left( \frac{\Pi_c}{\Pi_{c, max}} \right)^2}
\]

(10)

The massflow is then given by:

\[
\dot{m}_c = \frac{\dot{m}_{c,corr} p_{01}/p_{ref}}{\sqrt{T_{01}/T_{ref}}}
\]

(11)

The surge-line is modeled using the lowest massflows for each speedline from the compressor map and is well described by the linear relationship:

\[
\Pi_{c, surge} = c_{m_{c, surge,1}} \dot{m}_{c,corr} + c_{m_{c, surge,2}}
\]

(12)
In an optimization context surge is undesirable why this is implemented as a constraint according to:

$$\Pi_c \leq \Pi_{c,\text{surge}}$$  \hspace{1cm} (13)

**Efficiency model**

The efficiency of the compressor is modeled using a quadratic form in the flow parameter $\Phi$ and speed $\omega_c$ following [1]. The dimensionless flow parameter is defined as:

$$\Phi = \frac{m_c R_g T_0}{\omega_c 8 R_c^3 P_{01}}$$  \hspace{1cm} (14)

Deviation from optimal flow and speed:

$$d\Phi = \Phi - \Phi_{opt}$$  \hspace{1cm} (15)
$$d\omega = \omega_{c,\text{corr, norm}} - \omega_{opt}$$  \hspace{1cm} (16)

The compressor efficiency is given by:

$$\eta_c = \eta_{c,\text{max}} - \left[ d\Phi \right]^T \begin{bmatrix} Q_1 & Q_3 & Q_3 \\ Q_3 & Q_3 & Q_3 \end{bmatrix} \left[ d\Phi \right]$$  \hspace{1cm} (17)

The consumed power is calculated as the power from consumed in an isentropic process divided by the efficiency:

$$P_c = \frac{m_c c_{p,a} T_0}{\eta_c} \left( \Pi_{c,\text{opt}}^{\frac{\gamma_c-1}{\gamma_c}} - 1 \right)$$  \hspace{1cm} (18)

**Initialization**

The compressor has 10 tuning parameters, $\Psi_{\text{max}}$, $c_{m_{1,\ldots,3}}$, $\Phi_{\text{opt}}$, $\eta_{c,\text{max}}$ and $\omega_{\text{opt}}$, $Q_{1-3}$. The model is first fitted to the compressor map then to the stationary measurements, using data set A, but then $m_c$ is measured and $\eta_c$ and $P_c$ are calculated according to:

$$\eta_c = \frac{T_0(\Pi_{c,\text{opt}}^{1/\gamma_c} - 1)}{T_{02} - T_{01}}$$  \hspace{1cm} (19)
$$P_c = m_c c_{p,a} (T_{02} - T_{01})$$  \hspace{1cm} (20)

The results are mean/max absolute errors of [2.4/8.2, 2.3/23.2, 1.4/7.8] \% for $[m_c, \eta_c, P_c]$ respectively.

**Cylinder Gas Flow**

The cylinder gas flow models are models for the air and fuel flow in to the cylinder. The airflow model is a model for the volumetric efficiency of the engine. The model used is the same as in [5] according to:

$$\eta_{vol} = c_{vol,1} \sqrt{P_{lim} + c_{vol,2} \sqrt{\omega_{ice}}} + c_{vol,3}$$  \hspace{1cm} (21)
$$m_{ac} = \frac{\eta_{ac} \omega_c V_d}{4 \pi R_g T_{in}}$$  \hspace{1cm} (22)

The control signal $u_f$ is injected fuel mass in mg per cycle and cylinder and the total fuel flow is thus given by:

$$m_f = \frac{10^{-6}}{4 \pi} u_f \omega_c \eta_{cyl}$$  \hspace{1cm} (23)

The air-fuel equivalence ratio $\lambda$ is computed using:

$$\lambda = \frac{m_{ac}}{m_f (A/F)_s}$$  \hspace{1cm} (24)

In diesel engines a lower limit on $\lambda$ is usually used in order to reduce smoke. However in fuel cut, i.e. $u_f = 0$, $\lambda = \infty$ which is undesirable in optimization. Instead the fuel-air equivalence ratio $\phi$ is used and the lower limit on $\lambda$ can be expressed as:

$$\phi = \frac{m_f}{m_{ac}} (A/F)_s$$  \hspace{1cm} (25)
$$0 \leq \phi \leq \frac{1}{\lambda_{\text{min}}}$$  \hspace{1cm} (26)

**Initialization**

The tuning parameters of the gas flow models are $c_{vol,1-3}$. The model is initialized using all stationary measurements, i.e. set A using that at stationary conditions $m_{ac} = m_c$. The volumetric efficiency model corresponds well to measurements with a mean/max absolute relative error of [0.9/3.7] \%.

**Engine torque and generator**

The engine torque is not measured so the tuning of the torque models have to rely on the DC-power out from the power electronics. Then there are actually three efficiencies that should be modeled, the power electronics, the generator, and the engine efficiencies. In Fig. 3-left the total efficiency of the powertrain is shown, with the maximum power line. First the engine torque model is tuned. In the tuning the engine torque is calculated using the stationary efficiency map of the generator, provided by the manufacturer. The efficiency of the power electronics is lumped with the generator efficiency and is here assumed to be 0.98. Then the generator model
is tuned, first using the stationary map and then measurements but with the torque calculated using the efficiency map.

**Engine torque model**

In Fig. 3-right the efficiency of the engine is shown, with $M_{\text{ice}}$ calculated using the generators efficiency map and 2% losses in the power electronics assumed. The engine torque is modeled using three components, see \[4\], i.e. friction torque, $M_{fric}$, pumping torque $M_{pump}$ and gross indicated torque, $M_{ig}$. The torque consumption of the high pressure pump is not modeled on it’s own, but lumped in to the following models. The net torque of the engine can then be computed.

$$M_{\text{ice}} = M_{ig} - M_{fric} - M_{pump}$$

The pumping torque is proportional to the pressure quotient over the cylinder:

$$M_{pump} = \frac{V_d}{4\pi} (p_{em} - p_{in})$$

The friction torque is modeled as a quadratic shape in engine speed:

$$M_{fric} = \frac{V_d}{4\pi} 10^5 (c_{fr1} \omega_{ice}^2 + c_{fr2} \omega_{ice} + c_{fr3})$$

The indicated gross torque is proportional to the fuel energy:

$$M_{ig} = \frac{u_f 10^{-6} n_{CY} qHV \eta_{ig}}{4\pi}$$

Where the indicated gross efficiency is defined as:

$$\eta_{ig} = \eta_{ig,t}(1 - \frac{1}{r_{\text{cyt}}})$$

The torque model in (27)-(31) is fairly common, and if $\eta_{ig,t}$ is implemented as a constant maximum brake torque (MBT)-timing is assumed. A typical internal combustion engine normally has an efficiency ”island” located near the maximum torque line where its peak efficiency is obtained, see \[1, 2, 3\]. However looking at Fig. 3-right this is clearly not the case. Therefore the model is provided with two different torque models, seen in Fig. 4.

Torque model 1 (TM1) is used in the model tuning and validation and is designed to capture the nonlinear nature seen in Fig. 3. TM1 consists of two second order polynomials and a switching function:

$$\eta_{ig,t} = M_{f,1} + g_f (M_{f,2} - M_{f,1})$$

$$g_f = \frac{1 + \tanh(0.1(\omega_{ice} - 1500\pi/30))}{2}$$

$$M_{f,1} = c_{M_{f,1},1} \omega_{ice}^2 + c_{M_{f,1},2} \omega_{ice}$$

$$M_{f,2} = c_{M_{f,2},1} \omega_{ice}^2 + c_{M_{f,2},2} \omega_{ice} + c_{M_{f,2},3}$$

Torque model 2 (TM2) is designed and provided to represent a ”typical” engine with an efficiency island, to be used for optimal control studies, and is thus not used in the tuning or validation. TM2 is quadratic in $\frac{uf}{\omega_{ice}}$ and expressed as

$$\eta_{ig,t} = \eta_{ig,\text{ch}} + c_{uf,1} (\frac{uf}{\omega_{ice}})^2 + c_{uf,2} \frac{uf}{\omega_{ice}}$$

The maximum power line is implemented as a limit on the net power of the engine, $P_{\text{ice}} = P_{\text{ice,\text{max}}}$. which is well approximated by two quadratic functions and a maximum value:

$$P_{\text{ice}} \leq P_{\text{ice,max}}$$

$$P_{\text{ice}} \leq c_{P_1} \omega_{ice}^2 + c_{P_2} \omega_{ice} + c_{P_3}$$

$$P_{\text{ice}} \leq c_{P_1} \omega_{ice}^2 + c_{P_2} \omega_{ice} + c_{P_3}$$

**Initialization**

The two torque models have eight and six tuning parameters respectively. The tuning parameters are $c_{fr1}$,$c_{fr2}$, and $c_{M_{f,1},1-2}$, $c_{M_{f,2},1-3}$, or $\eta_{ig,\text{ch}}$ and $c_{uf,1-2}$ The models are fitted using set C. For (32) it is rather straight forward. For model (36) the ”island” is not
visible in the measured data, therefore the parameters of $\eta_{ig, ch}$ are manually tuned and the $M_{fric}$ model is tuned assuming MBT-timing. The mean/max absolute relative errors of TM1 are [2.2/10.9] %.

**Generator model**

Looking at Fig. 5 a reasonable first approximation of the relationship between mechanical and electrical power of the generator is two affine functions, something normally denoted willans line, [6], where the slope of the line depends on whether the generator is in generator or motor mode.

$$P_{mech}^+ = e_{gen,1} P_{gen} + P_{gen,0}, \quad \text{if } P_{gen} \geq 0 \quad (40)$$

$$P_{mech}^- = e_{gen,2} P_{gen} + P_{gen,0}, \quad \text{if } P_{gen} < 0 \quad (41)$$

This model is not continuously differentiable so therefore to smoothen it out a switching function is used. The model is then given by:

$$P_{mech} = P_{mech}^- + \frac{1 + \tanh(0.005 P_{gen})}{2} (P_{mech}^+ - P_{mech}^-) \quad (42)$$

$e_{gen,1-2}$ are seen to have a quadratic dependency on $\omega_{ice}$, a reasonable addition to the willans line is thus to model $e_{gen,1-2}$ as:

$$e_{gen,x} = e_{gen,x-1} \omega_{ice}^2 + e_{gen,x-2} \omega_{ice} + e_{gen,x-3} \quad (43)$$

which constitutes the full model.

**Initialization**

The generator model has seven tuning parameters, $P_{gen,0}$ and $e_{gen,1/2,1-3}$. The model is first fitted to the generator map and secondly to measurement data, using set C. The mean/max absolute relative errors of the generator model are [0.7/2.5] %.

**Exhaust temperature**

The cylinder out temperature model is based on ideal the Seiliger cycle and is a version of the model found in [5]. The model consists of the pressure quotient over the cylinder:

$$\Pi_e = \frac{P_{en}}{P_{im}} \quad (44)$$

The specific charge:

$$q_{in} = \frac{m_f qHV}{m_f + m_{ac}} (1 - x_r) \quad (45)$$

The combustion pressure quotient:

$$x_p = \frac{P_{3}}{P_{2}} = 1 + \frac{q_{in} x_{cv}}{c_{v,a} T_1 r_c^{\gamma - 1}} \quad (46)$$

The combustion volume quotient:

$$x_v = \frac{v_3}{v_2} = 1 + \frac{q_{in} (1 - x_{cv})}{c_{p,a} (q_{in x_{cv}} - c_{v,a} + T_1 r_c^{\gamma - 1})} \quad (47)$$

The residual gas fraction:

$$x_r = \frac{\Pi_e^{1/\gamma_e} x_p^{1/\gamma_e}}{r_c x_v} \quad (48)$$

Temperature after intake stroke:

$$T_1 = x_r T_{eo} + (1 - x_r) T_{im} \quad (49)$$

The engine out temperature:

$$T_{eo} = \eta_e \Pi_e^{1 - 1/\gamma_e} r_c^{1-x_r} x_p^{1/\gamma_e - 1} \left( q_{in} \left( \frac{1-x_{cv}}{c_{p,a}} + \frac{x_{cv}}{c_{v,a}} \right) + T_1 r_c^{\gamma - 1} \right) \quad (50)$$
To account for the cooling in the pipes the model from [7] is used, where $V_{pipe}$ is the total pipe volume:

$$T_{em} = T_{amb} + (T_{eo} - T_{amb})e^{-\frac{h_{ew}V_{pipe}}{(m_f + m_{ac})c_{p,e}}}$$  \hfill (51)

The model equations described in (45)-(50) are non-linear and depend on each other and need to be solved using fixed point iterations. In [5] it is shown that it suffices with one iteration to get good accuracy if the iterations are initialized using the solution from last time step. In an optimization context remembering the solution from last time step is difficult and also using a model that uses an unknown number of iterates is undesirable. However the loss in model precision of assuming no residual gas, i.e. $x_e = 0$, is negligible therefore this is assumed. Further, the addition of heat loss in the pipe through (51) drives $x_{eo}$ to zero. The reduced model is then given by:

$$q_{in} = \frac{m_f q_{HV}}{m_f + m_{ac}}$$  \hfill (52)

$$T_{eo} = \eta_e \Pi^{1 - \gamma_e} r^{1 - \gamma_e} \left( \frac{q_{in}}{c_{p,a}} + T_{in}^{\gamma_e - 1} \right)$$  \hfill (53)

$$T_{em} = T_{amb} + (T_{eo} - T_{amb})e^{-\frac{h_{ew}V_{pipe}}{(m_f + m_{ac})c_{p,e}}}$$  \hfill (54)

**Initialization**

The used temperature model has two tuning parameters, $\eta_{isc}$ and $h_{tot}$. The first step of the initialization assumes that there is no heat loss in the manifold before the sensors. Then the complete model is fitted using the results from $T_{em} = T_{eo}$. The nominal set is used in the fitting. The mean/max absolute relative error of the temperature model is $[1.9/5.4]$ % and the error increase from assuming $x_e = 0$ is $[0.014/0.06]$ %/o.

**Turbine and Wastegate**

Since the massflow is not measured on the exhaust side, the models for wastegate and turbine have to be fitted together.

$$\Pi_t = \frac{p_{es}}{p_{em}}$$  \hfill (55)

**Turbine**

The massflow is modeled with the standard restriction model, using that half the expansion occurs in

$$\begin{align*}
\Pi_t &= \max(\sqrt{\Pi_t}, \left( \frac{2}{\gamma_e + 1} \right)^{\frac{\gamma_e}{\gamma_e - 1}}) \\
\Psi_t(\Pi_t^*) &= \sqrt{\frac{2\gamma_e}{\gamma_e - 1} \left( (\Pi_t^*)^{\gamma_e/2} - (\Pi_t^*)^{\gamma_e/2 + 1} \right)} \\
m_t &= \frac{p_{em}}{\sqrt{R_t T_{em}}} \Psi_t \pi_{t,eff} \\
BSR &= \frac{R_t \omega_c}{\sqrt{2c_{p,e} T_{em} (1 - \Pi_t^{\gamma_e})}} \\
\eta_{lm} &= \eta_{lm,max} - c_m (BSR - BSR_{opt})^2
\end{align*}$$

Due to uncertainty of the behaviour outside the mapped region, and to avoid problems with negative turbine efficiency, a reasonable constraint is to restrict $BSR$ to the maximum and minimum values provided in the map, i.e. $BSR_{min} \leq BSR \leq BSR_{max}$.

**Wastegate**

The wastegate massflow is modeled with the standard restriction model and an effective area that changes linearly in $u_{wg}$.

$$\begin{align*}
\Pi_{wg}^* &= \max(\sqrt{\Pi_{wg}}, \left( \frac{2}{\gamma_e + 1} \right)^{\frac{\gamma_e}{\gamma_e - 1}}) \\
\Psi_{wg} &= \sqrt{\frac{2\gamma_e}{\gamma_e - 1} \left( (\Pi_{wg}^*)^{\gamma_e/2} - (\Pi_{wg}^*)^{\gamma_e/2 + 1} \right)} \\
m_{wg} &= \frac{p_{em}}{\sqrt{R_t T_{em}}} \Psi_{wg} u_{wg} \pi_{wg,eff}
\end{align*}$$
Initialization

The initialization uses data set C. The massflow models need to be fitted together and the turbine efficiency cannot be calculated from measurements since none of the massflows are measured. Looking at the nominal data set the quadratic shape in BSR is not observed since the measurements are rather constant in BSR, see Fig. 6. Since this shape is nonexistent in the measurements the efficiency model of the turbine is locked to the map fit since otherwise it would converge to an arbitrary shape trying to capture as much as the cloud nature of the measured data as possible. One could consider adding pulse compensation factors for the massflow and efficiency but the resulting improvements are small.

The massflow models are fitted together using \( \dot{m}_{ac} + \dot{m}_f = \dot{m}_i + \dot{m}_{wg} = \dot{m}_{exh} \). Friction losses according to \( P_c = P_i \eta_m - w_{frie} \omega^2_t \) can be added, however the parameter \( w_{frie} \) becomes small in the optimization.

The final turbine and wastegate models have three tuning parameters, \( A_t, \eta_{lm,max} \) and \( A_{wg,eff} \). The results are mean/max relative errors of [2.3/5.4, 4.7/17.0] % for \( \dot{m}_{exh}, P_i \eta_{lm} \) respectively.

Exhaust flow models

Using the standard restriction model a max-expression is necessary under the square root to keep the flow real, representing choking which occurs at \( \Pi_t^{-1} \approx [3.3, 1.8] \) for the turbine and wastegate. However such expressions are undesirable when using optimization tools. Instead the following expressions are used:

\[
\psi_t = c_{t,1} \sqrt{1 - \Pi_t^{c_{t,2}}} \tag{65}
\]

\[
\psi_{wg} = c_{wg,1} \sqrt{1 - \Pi_t^{c_{wg,2}}} \tag{66}
\]

The flow models are fitted to produce the same flow profile as the standard restriction models in (57), (63), where \( c_{t,1-2} \) and \( c_{wg,1-2} \) are tuning parameters.

Dynamic models

So far the models are tuned using stationary measurements. The next step is to tune the parameters of the dynamic models in (2)-(5). Since torque is not measured \( J_{GenSet} \) is fixed to it’s real value and only \( V_{im}, V_{em} \) and \( J_{ic} \) are tuned. Since torque and eventual torque errors might lead to engine stalling the torque model is inverted to track the real engine speed trajectory. This will lead to that there will be almost no errors in engine speed. To fit the dynamic models data set D-I are used but only the transients in the measurements, plus a couple of seconds before and after. As in [5] the transient is also normalized to 0-1 so that the stationary point has no effect on the dynamics.

Full models

The full models are tuned using both dynamic and stationary measurements, using a similar cost function as in [5]. If the same cost function is used the model will not be able to reach the same maximum torque as the real engine for low engine speeds without \( \lambda \) being excessively low. Therefore to ensure that the model is able to span the entire operating range of the engine an addition is made. The model is simulated with \( \lambda = \lambda_{min} \) for \( N_{ice} = 800 \) rpm and the models maximum torque is added to the cost function according to:

\[
V_{M_{\max}} = w_{M_{\max}} \left( \frac{M_{ice,max,mod}(800rpm)}{M_{ice,max,meas}(800rpm)} - 1 \right) \tag{67}
\]

(67) assumes that the engine is smoke-limited at 800 rpm and maximum torque and thus tries to force the max torque of the model to coincide with that of the real engine, where \( w_{M_{\max}} \) is a weighting parameter.

To ensure reasonable behaviour also when the generator is in motoring mode this side is fitted using the efficiency map from the manufacturer with an assumed power electronics efficiency of 98%. For the stationary tuning set C is used and for the dynamics sets D-I are used. The full cost function is given by:

\[
V_{tot}(\theta) = \frac{1}{y_{dyn} M_{dyn}} \sum_{k=1}^{M_{dyn}} \sum_{y_a=1}^{y_{dyn}} \sum_{l=1}^{N_{dyn}} \left( \frac{e_{rel,dyn}(l)}{N_{dyn}} \right)^2
\]

\[
+ \frac{1}{y_{stat} N_{stat}} \sum_{y_a=1}^{y_{stat}} \sum_{m=1}^{N_{stat}} \left( e_{rel,stat}(m) \right)^2 \tag{68}
\]

\[
+ \frac{V_{M_{\max}}^2}{N_{data}}
\]

where \( y \) is the number of outputs, \( M \) the number of datasets and \( N \) the number of operating points in each dataset.

The models are also, as in [5], validated using only dynamic measurements and in particular all load transients, i.e. set \( J_{0.1}, 1, 2^2, N_{0.1}, 1, 2 \).
Table 4: Mean relative errors of the complete model. Bold marks variables used in the tuning and T, V, are the errors relative tuning and validation sets respectively.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{icc}$</th>
<th>$p_{im}$</th>
<th>$p_{em}$</th>
<th>$\omega_{tc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyn.</td>
<td>0.0</td>
<td>0.0</td>
<td>2.8</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>2.9</td>
<td>2.9</td>
<td>2.9</td>
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<table>
<thead>
<tr>
<th></th>
<th>$m_c$</th>
<th>$P_c$</th>
<th>$m_{ac}$</th>
<th>$T_{cm}$</th>
<th>$m_{exh}$</th>
<th>$P_t$</th>
<th>$P_{mech}$</th>
<th>$P_{mech}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stat.</td>
<td>2.5</td>
<td>1.8</td>
<td>2.5</td>
<td>2.4</td>
<td>3.3</td>
<td>5.4</td>
<td>3.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

RESULTS

The resulting fit to both tuning data and validation data is shown in Table 4. The variables used in the tuning are written in bold in the resulting tables. Table 4 shows that the model is a good mathematical representation of the measured system with state errors less than 3% and stationary errors in the same range. In Fig. 7 the state trajectories of the model are compared to measurements. There it is also seen that the agreement is good.

The pressure dynamics, and in particular the exhaust pressure, are faster than the speed dynamics therefore the resulting model is moderately stiff. This is also seen when selecting ode-solvers. In matlab ode23t and ode15s are twice as fast as the standard ode45 when simulating the model. When the states are normalized with their maximum values the relative and absolute tolerances $[1e-4, 1e-7]$ are found to be good trade-offs between accuracy and performance.

CONCLUSION

In this paper a validated optimization ready model of a diesel-electric powertrain is presented. The resulting model is a four state-three control mean value engine model, available for download in the LiU-D-El-package from [10]. The model is able to capture the highly nonlinear nature of the turbocharger diesel engine, and is at the same time continuously differentiable in the region of interest, to comply with optimal control software. The model is provided with two torque models to be used for optimal control studies. The first model, called $MVEM_0$, with a torque model representing the actual engine, as well as a model with a more general torque model aimed to represent a typical engine, called $MVEM_2$. Both $MVEM_0$ and $MVEM_2$ are included in the LiU-D-El-package together with a small example that can be downloaded fully parametrized from [10] implemented in matlab.

REFERENCES


APPENDIX
DATA USED

There are a total of 192 stationary points measured. Of those 192, 53 are with the wastegate locked in a fixed position. Since injection timing is not measured those points are only used when fitting the gas flow models since there are some questions about what the engine control unit does when the wastegate control is altered. Nominal refers to unaltered wastegate, see Table 5

The dynamic data set consists of 21 measurements. The first six, D-I, are engine speed transients with constant (as close as the generator control can track) generator power and a sequence of steps in reference speed that the engine speed controller tries to track, see Table 6.

The last 15 sets are with constant reference speed, and different load steps, see Table 7. As with the speed transients the ECU controls the engine speed and the generator acts as a disturbance. The load transients are conducted at different engine speeds and then a programmed sequence of 23 power steps is performed with varying rise time, or rate at which the power changes. The first five, $J_{0,1} - N_{0,1}$ are with a ramp duration of 0.1s and the other are with 1s and 2s respectively. The total length of each set is approximately 300s.

Table 5: Stationary Data

<table>
<thead>
<tr>
<th>Data Set</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delimiter</td>
<td>all</td>
<td>nominal</td>
<td>nominal &amp; $P_{gen} &gt; 0$</td>
</tr>
<tr>
<td>Nr. of points</td>
<td>192</td>
<td>139</td>
<td>127</td>
</tr>
</tbody>
</table>

Table 6: Speed transients

<table>
<thead>
<tr>
<th>Data Set</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{gen}$ [kW]</td>
<td>30</td>
<td>60</td>
<td>90</td>
<td>130</td>
<td>160</td>
<td>180</td>
</tr>
<tr>
<td>Nr. of steps</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>21</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 7: Load transients

<table>
<thead>
<tr>
<th>Data Set</th>
<th>$J_{0,1,1,2}$</th>
<th>$K_{0,1,1,2}$</th>
<th>$L_{0,1,1,2}$</th>
<th>$M_{0,1,1,2}$</th>
<th>$N_{0,1,1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed [rpm]</td>
<td>1100</td>
<td>1500</td>
<td>1800</td>
<td>2000</td>
<td>2200</td>
</tr>
<tr>
<td>Nr. of steps</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
</tr>
</tbody>
</table>