Exploring systematic lesson variation

A teaching method in mathematics

Laurence Russell

Nationella forskarskolan i naturvetenskapernas,
teknikens och matematikens didaktik

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SAMMANFATTNING

Den här licentiatavhandlingen redogör för tillämpningen av en specifik undervisningsmetod i matematik i grundskolans senare del. Den belyser hur undervisning i matematik har sett ut över tiden, vad som begränsar dess utformning och huruvida matematikundervisningens utformning ger elever möjlighet att öva olika matematiska förmågor. I syfte att främja elevernas positiva upplevelser av matematikundervisningen och därigenom stimulera deras motivation till att lära sig matematik har jag utformat, låtit tillämpa och utvärderat en egenkonstruerad undervisningsmodell: *Systematiskt varierade lektioner* (SLV).

Grundskolans styrdokument har förändrats i takt med samhällsförändringar och deras krav på kunskaper. Läroplanerna har ändrats och skrivits om i frågor om mål och kunskapskrav men också beträffande styrning av undervisningens utformning. Hur den enskilde läraren utformar och applicerar sin undervisning i matematik är numera förbehållet hen själv och med denna frihet följer också ett ansvar att utforma undervisningen så att den tillgodoser elevernas möjlighet att öva de förmågor som styrdokumenten nämner.

En stor del av grundskolans matematikundervisning är centrerad kring någon form av lärobok i matematik. Läroboken utgör ett nödvändigt men också begränsande inslag. Den ger en tydlig struktur och en tillsynes god överblick över de centrala delarnas innehåll. Samtidigt är matematikboken så normsättande för undervisningens utformning att den gör alternativa undervisningsmetoder svåra att implementera. Den är konserverande för matematikämnet generellt, undervisningens utformning i allmänhet och också för både elevernas och lärarnas synsätt på matematikundervisningen. Lärobokens dominans medför att variationen i matematikundervisningen ofta blir begränsad till variationen inom läromedlet och hur väl denna variation tillvaratas av läraren.

Olika undervisningsmetoder ger olika förutsättningar att uppnå kunskapskraven. Forskning om undervisningens utformning och utformningens resultat visar på positiva effekter av en varierad undervisning och också en begränsning av kunskaper och färdigheter vid ensidig metodanvändning. Användning av en enskild undervisningsmetod räcker inte till för att bereda eleven möjlighet att öva olika förmågor. En blandning av

undervisningsmetoder, anpassade för att tillgodose elevens möjligheter att öva förmågorna är nödvändig.

Den konstaterade försämringen av elevernas kunskaper och förmågor i matematik är inte entydigt förklarad, men kan grunda sig i minskad motivation och lust till lärande, snarare än minskad förmåga att lära sig. Att identifiera och tillämpa matematikundervisning som vidrör den affektiva domänen, som stimulerar, erbjuder lustfyllda utmaningar och arbetssätt, kan medföra en ökad motivation till att lära sig. Vidare framstår det klart att den traditionella, lärobokstyrda undervisningsmetoden, så som den används, inte tillgodoser elevers möjlighet att uppöva de förmågor som efterfrågas.

Undervisningsmodellen *Systematiskt varierade lektioner* (SLV) är utformad för att systematisera variationen i matematikundervisningen. Avsikten i systematiken är att eleven ska känna igen sig i lektionsutformningen och dess arbetsmetoder, om än med ett varierat innehåll. Systematiken syftar också till att normalisera variationen och därigenom undvika onödigt stresspåslag hos eleven och också minimera tidsåtgången till indelning, instruerande och iordningställande. Interventionens lektionsserie omfattar tre lektioner per vecka som upprepas varje vecka under en fyraveckors- period. Lektion ett omfattar en traditionell textboksspecifik lektion. Läraren inleder med förklaringar och exempel för att därefter fasa över lektionen till enskilt arbete med uppgifter i matematikboken. Lektion två omfattar par- och gruppvisa arbeten med problemlösningssuppgifter. Eleverna instrueras om uppgiftens syfte och mål och, i förekommande fall, om vilka redskap som finns att tillgå samt hur de skall användas. I lektion tre i serien redovisar eleverna resultat och lösningsmetoder av/från uppgiften i lektion två. Läraren organiserar redovisningarna och leder diskussionerna om resultaten och lösningsmetoderna. Läraren tillhandahåller nödvändiga tekniska hjälpmedel för redovisningarna.

Den systematiskt varierade lektionsmodellen (SLV) tillämpades i två klasser i årskurs nio. Under den inledande fyraveckorsperioden, med *algebra* som centralt innehåll, tillämpades den systematiskt varierade lektionsmodellen i den ena klassen samtidigt som det i den andra klassen tillämpades en traditionell, textboksbaserad undervisning. Den traditionella, textboksbaserade undervisningen, benämnd CM, var utformad så att den skulle representera den vanligast förekommande undervisningsmetoden i de senare årskursernas matematikundervisning. Den traditionella undervisningen (CM) bestod av en lärarledd introduktion i inledningen av den första av de tre

lektionerna under veckan. Läraren förklarade och gav exempel på lösningar till olika uppgifter. Därefter bedrevs enskilt elevarbete i respektive häfte resterande delen av lektionen och under de återstående lektionerna. Läraren hjälpte eleverna efter behov. Efter den första fyraveckorsperioden skiftade respektive klass undervisningsmodell och studerade delar ur det centrala innehållet i *samband och förändring*. Före och efter respektive undervisningsmetods tillämpning besvarade eleverna en enkät om upplevelser av och förhållningssätt till sin matematikundervisning. Enkätfrågorna är utformade för att härleda förändringar beträffande elevernas specifika upplevelser av och förhållningssätt till de tillämpade undervisningsmetoderna. Därutöver genomfördes innehållsspecifika kunskapstester både före och efter respektive metods tillämpning.

Resultaten visar ringa förändringar i uppfattningar om innehållsspecifikt intresse, men ett ökat antal elever som upplever stimulerande lärandesituationer i samband med den systematiskt varierade lektionsmetoden. Båda klasserna visar en ökning av matematiska kunskaper i respektive del av de centrala innehållen, oavsett vilken undervisningsmetod som tillämpas. Beträffande forskningsmålen om påverkan i den affektiva domänen visar resultaten att i såväl klass A som klass B ökar antal elever som ser fram emot sina matematiklektioner under den systematiskt varierade lektionsmetoden. Antalet elever som är nöjda med lektionernas utformning under SLV ökar markant i klass A. Vidare ökar antalet elever som uttrycker att de lär sig under de flesta eller alla lektioner i klass B. En majoritet av eleverna uttrycker att de uppskattar SLV:s specifika inslag såsom *tycker om att arbeta med praktiska, problemlösande uppgifter* och *tycker om att arbeta med andra* och dessa förhållningssätt förändras minimalt. Under tillämpningen av den traditionella undervisningsmetoden ökar antalet elever som uttrycker missnöje över metodens utformning markant i klass B. Därutöver ökar antalet elever markant i klass B som upplever att de inte lär sig bra till följd av den traditionella metoden.

Då respektive klass har avsevärda skillnader i tidigare upplevelser av matematikundervisningen och förhållningssätt till matematikämnet och därutöver har olika förkunskaper i de olika centrala innehållen måste resultaten betraktas utifrån dessa olika förutsättningar och som klassspecifika. Klass A har på det hela taget en positiv utveckling avseende sin kunskapsutveckling, intresse för och upplevelse av matematikundervisningen, oavsett undervisningsmetod. Klass B uppvisar ett tydligt missnöje med den

traditionella metoden, om än med fortsatt positiv kunskapsutveckling. Resultaten antyder påverkan i den affektiva domänen då upplevelser av förnöjsamhet, stimulerande lärande-situationer och känsla av lärande tilltar under den systematiskt varierade lektionsmodellen (SLV) och avtar i motsvarande grad under den traditionella undervisningen (CM).

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If I could do this again, there are many things I would do differently.

I have done my best with what was given me; designed, tested and evaluated the teaching method I decided to call *systematically varied lessons*, theorized and contextualized its belonging in current mathematics teaching and finally managed to produce this licentiate thesis. Even if this is a small-scale study, it can, hopefully, contribute to the development of mathematics teaching. I am fortunate to have been a participant in the Swedish National Graduate School in Science, Mathematics and Technology Education Research, FontD, and for that I am grateful.

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Thank you Anton, Simon, Moa and Rose. Being the beautiful, happy people you are, you have all given me the strength to achieve beyond what I thought was possible.

*These are the days of the endless summer
These are the days, the time is now
There is no past, there's only future
There's only here, there's only now*

Van Morrison

ABSTRACT

The study presented in this licentiate thesis explores the student perception of *systematically varied lessons*, a specific lesson design and exploratory method of teaching mathematics in comprehensive school.

Two 9th grade classes are taught in the curriculum defined mathematical core content sections of algebra and mathematical relationships and rate of change but by two different methods. One class is taught using a conventional textbook approach and the other with a series of systematically varied lessons; the first lesson containing instructions, lectures and individual textbook work, the second lesson containing exploratory, problem-solving worksheet tasks, some with manipulative materials, and with the third lesson containing presentation, reasoning and defending results and solutions from the second lesson. This is repeated over a four week period with three lessons a week. The classes then switch teaching methods for a second defined mathematical core content section. Both before and after applying the different teaching methods both classes perform tests for evaluation of content specific mathematical skills och answer questionnaires regarding their perception of learning. Comparisons of perceptions of the teaching methods and mathematical skills development are then evaluated. Results show minor changes of how the students perceive *interest*, *enjoyment* and *anxiety*, but an increased number of students that perceive *stimulating learning settings* during the *systematically varied lesson* method. Both classes show development of mathematical skills regardless of teaching methods. Results suggest influence in affective domains, as experiencing contentment, stimulating learning settings and sense of learning increase during the *systematically varied lessons* method, and decrease correspondingly during the conventional method.

The teaching methods, in the context of stimulating the desire to learn, and finding settings to stimulate learning, and thereby affecting attitudes, are discussed and conceptualized.

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1. INTRODUCTION

1.1 A STUDY OF APPLYING A SPECIFIC TEACHING METHOD AND LESSON DESIGN IN COMPREHENSIVE SCHOOL MATHEMATICS EDUCATION

This licentiate thesis is about educational engineering of teaching mathematics. Engineering in the sense of pursuing and identifying teaching methods and classroom settings that promote enjoyment and learning mathematics in the comprehensive school of today. The denomination “today” implies the complex and demanding teaching settings associated with education in our fast-changing society, where teaching methods triggering affective domains, stimulating interest and thereby motivating learning seem all the more necessary. Identifying teaching methods that can capture pupils’ interest and enjoyment seem essential for providing settings for sustainable learning and maintaining motivation to learn.

I intend to present an overall description of mathematics education in the Swedish comprehensive school, its assets, limitations, results, and implications of standard settings of teaching. I also intend to elaborate what tends to frame these settings and the norms and heritage that further establish standards of teaching and learning mathematics. Thereafter I will describe the journey towards the experimental teaching approach of *systematically varied lessons*, its design, application and results of the method’s short-term implementation. Finally I will elaborate on possible paths for the future, based on results and implications from the study and from other evaluations and surveys regarding areas of interest in the educational engineering of teaching mathematics.

1.2 BEHIND THE IDEA OF A SYSTEMATICALLY VARIED LESSON APPROACH TO TEACHING AND LEARNING MATHEMATICS

It must be emphasized that the idea, design and to some extent application of the systematically varied lesson approach to mathematics education is (to my knowledge) entirely a product of my own experience and my ideas about how to go about teaching mathematics. The theoretical framing of the systematically varied lesson design is multi-layered and located both within affective and educational engineering domains as well as norms associated with teaching and learning mathematics and the mathematical classroom settings.

What lies in the mathematics textbooks is what's necessary to know. The textbook approach is the only way to go about learning what is necessary; to even attempt to deviate from its content is a waste of important time! I dare say that this statement is the common stand-point of most students in senior comprehensive school. But we must consider that this attitude towards learning mathematics has not emerged without reason. Undoubtedly, the textbook driven approach to teaching and learning mathematics seems the easiest way to satisfy framing and visualizing “what needs to be done to obtain necessary knowledge”. In addition, it also facilitates planning and assessment for the teacher (Johansson, 2006), but does it meet the demands of stimulating and maintaining interest, experiencing enjoyment and developing abilities in line with curriculum (Lgr 11)? I often confront the academic questions “what is necessary mathematical knowledge?” and “what mathematical abilities are necessary in the future?”. These questions are academic in the sense that I am a mathematics teacher and not in a position to deviate from curriculum content or predict necessary future knowledge. As a teacher, I have to relate to the curriculum at hand and teach accordingly. But at the same, time I know that these questions are also asked by the current senior comprehensive students and this in turn implies individual motivational contradictions. Based on the national results in mathematics in the PISA (Skolverket, 2013) and TIMSS (Skolverket, 2008) surveys showing decreasing mathematical knowledge, one can assume that the students of today lack motivation to learn rather than lack the capability to learn. My assumption is that lack of interest and motivation and not lack of capability is the cause of the decrease in mathematical ability and knowledge. In turn, this assumption inspired me to search for teaching

methods that enable enjoyment, arouse curiosity, inspire interest, affect attitudes, and strengthen beliefs in capability. Being privileged to have experience of several years of teaching mathematics in upper comprehensive school, I am well acquainted with classroom settings. These years of experience have strengthened my belief in the necessity of regularity in lesson design, but also a varied approach to maintain interest, enjoyment and problem-solving curiosity. But only having taught mathematics in later years of comprehensive school (8th and 9th grade), I have had to adapt to the regularity and inherited norms of the students prior seven years of mathematics education. Somewhere along the trail of school years' "mostly using the textbook" has been the normalized and accustomized and dealing with a sustainable different approach in later years of comprehensive school is difficult. A systemized varied teaching approach is probably more favourably introduced in early school grades and maintained for the remainder of comprehensive school.

My point is systemizing variation in the sense that the structure of the lessons is well known, and the methods of working and learning are likewise familiar, but the problem-solving and collaborative contents vary as well as conversational aspects in reasoning and presenting solutions. The systemization serves the purpose of customizing variation, minimizing time-consuming instructional start and stop sections as well as normalizing the varied approach among the students. My experience of attempts to vary teaching and learning methods (without systemization) is that deviation from the students having access to and working in their textbooks, mostly causes frustration and restlessness. The extent of textbook dominance and reluctance to listen to other explanations or try other methods of approach is so far advanced that attempts render in sub-groups or individuals working in their textbooks regardless of the intended approach and problems presented.

The self-composed *systematically varied lessons* method emerged from several years of experience teaching mathematics in upper compulsory school. The regular design of, and approach to, mathematics education is framed by the mathematics textbook and the level of variation in practice is limited to the level of variation provided by the textbook. In addition to the textbook framing the content for the teacher, students' have, over the years of compulsory school, been indoctrinated in relying on the textbook providing necessary knowledge. Teachers' attempts to apply alternative methods often turn in to "happenings" and many students consider these deviations a waste

Introduction

of valuable time better spent on textbook tasks. While the textbook frames teaching, the teacher has to tackle aspects of design to counteract the results of the PISA (Skolverket, 2013) and TIMSS (Skolverket, 2008) surveys indicating a decrease in knowledge. It was obvious that the students were not less provided with able to learn but seemed to lack interest and motivation. It appeared necessary to at least try to develop a design that could address aspects of motivation and enjoyment in the affective domain by alternating methods of teaching. Students recognizing routines in lesson procedures, even if tasks and content vary, has many advantages, and this is why systemically repeating the lesson design is central to the SLV method. Furthermore, the textbook framing, or at least inherited possibilities of deviating from textbook tasks, leave national aims and ability goals less accessible. A varied approach is necessary to provide a number of abilities, which will be shown further on. The *systematically varied lessons* method consists of repeating a three lesson series. The first lesson contains individual work on tasks with standard textbook features. This individual work is preceded by a lecture and explanatory introduction. The second lesson contains collaborative problem-solving tasks worked on in pairs or small groups and the third lesson in the series contains presenting, defending and discussing results and methods of solutions of the tasks from the second lesson. The reason for maintaining a textbook task lesson is to provide both recognition of “standard settings” and also general procedural and calculating skills. The second lesson’s explorative and collaborative approach intends to generate new, enjoyable experiences of approaches to learning and tackling mathematical tasks. The third, the presenting and defending results and methods lesson, aims to develop communicative and reasoning abilities.

1.3 RESEARCH AIMS AND QUESTIONS

The presented research aims are to apply and evaluate the lesson design of *systematically varied lessons*, a teaching method consisting of a systematically repeated lesson design using a problem-solving/exploratory/collaborative approach to the content, and to some extent the use of practical elements. Perceptions of the *systematically varied lessons* method will be compared with perceptions of a textbook based teaching approach.

- How do the students perceive the *systematic varied lesson* design?
- To what extent does the lesson design, compared to a traditional textbook approach, create appreciated learning settings?
- Does the *systematic varied lesson* design affect motivation, enjoyment, knowledge achievement, and the desire to learn mathematics?

2. SWEDISH COMPULSORY SCHOOL MATHEMATICS EDUCATION

2.1 NATIONAL GUIDELINES AND THEIR DEVELOPMENT

As an introduction to this chapter, it seems appropriate to define the frames of the arena of compulsory school mathematics in Sweden. The last three curricula in compulsory school mathematics in Sweden (Lgr 80, Lpo 94, and Lgr 11) have changed in terms of what is considered necessary mathematical knowledge, mathematical abilities and also how to go about teaching these skills. Compared with later curricula, curricula prior to Lgr 94 were far more detailed in mathematical content and specifications of how the teacher was supposed to go about teaching. Curriculum Lgr 80 started to develop, and curriculum Lpo 94 further developed a transition from focus on calculation skills and procedural knowledge towards a deeper mathematical thinking and reasoning. The teacher's responsibility for choice of content and freedom in teaching approach increased considerably with these steps towards decentralized governing. The most significant changes in mathematics education between Lgr 80 and Lpo 94 are changes

- *From calculation of algorithms to understanding of numbers and comprehensive calculation skills.*
- *From rule based calculation skills and rule-driven problem-solving to the development of students thinking and reasoning in mathematics, to discover, explore and consolidate in meaningful contexts.*
- *From mathematics as a formal controlling tool to mathematics for reflection, communication and problem solving in a democratic society.*

(Skolverket, 1997, p. 41)

Furthermore, Lpo 94 transfers descriptions of learning goals to *goals to be aimed for* and *goals to be attained*, which in some sense implies the ambition of not being content with the lowest levels of attainment goals but always striving for and stimulating challenge and development of knowledge towards goals to be aimed for. The latest curriculum, Lgr 11, further develops the transition from focus on calculation skills and procedural knowledge towards a deeper mathematical thinking and reasoning by introduction of abilities or skills that the students are to be provided opportunities to develop: problem-solving skills, mathematical conceptual skills, abilities to choose methods, calculation ability, reasoning ability and mathematical communicative skills. These abilities combined form comprehensive mathematical competencies considered necessary in our present and future society. The guidelines in the latest two curricula (Lpo 94, Lgr11) are the same, namely that students should be given the opportunity to practice mathematics so that many different skills can be developed. The teacher's task is then to plan and implement activities that provide opportunities to develop these competencies (Engvall, 2013). The main arguments for mathematics education in compulsory school are still that of mathematical skills and abilities being necessary competencies in our everyday lives in a democratic society (Kilborn & Löwing, 2002; Pettersson, 2003; Wyndhamn, 1997). Intending to reform curricula from isolated ideas of topics, concepts and knowledge, different countries have conducted development of guidelines somewhat influenced by each other. Engvall (2013) compiles frameworks of recent mathematical competencies (Kilpatrick, Swafford, & Findell, 2001; Nationellt centrum för matematikutbildning, 2001; Niss, 2003) and the latest curriculum's, Lgr 11 (Skolverket, 2011b) abilities. This compilation somewhat intertwines the latest decades educational development of knowledge aims in mathematics: *Adding It Up* (Kilpatrick et al., 2001) regarding mathematical proficiency in the American curriculum domain, the Danish KOM project (Niss, 2003) with the purpose of reforming the Danish curriculum from a syllabus in traditional sense to a description of "mathematical competency" and "Hög tid för matematik" (Nationellt centrum för matematikutbildning, 2001) regarding identifying aspects of multifaceted mathematical knowledge in the Swedish compulsory school setting.

Swedish compulsory school mathematics education

Frameworks for recent curricula development of mathematical competencies

Adding It Up (2001)	“Hög tid för matematik” (2001)	KOM project (2003)	Curriculum Lgr 11 (2011)
Strategic competence	Strategic competence	Problem-processing skills	Problem-solving ability
Instructional routines	Instructional routines	Thinking mathematically	Conceptual ability
Procedure competence	Mastering procedures	Handling mathematical symbols and formalisms	Choice of method and calculating ability
Adaptive reasoning	Ability to reason mathematically	Reasoning mathematically	Ability to reason mathematically
	Communicative ability	Communicating in, with and about mathematics	Ability to communicate mathematically
Productive disposition	Productive disposition		Interest in mathematics and beliefs in ability to use mathematics in different situations.
	Holistic perspective		Identify the context and relevance of mathematics

Table 1. Frameworks for recent curricula development of mathematical competencies (Interpretation from Engvall, 2013, p. 46)

2.2 STANDARD SETTINGS IN COMPULSORY SCHOOL MATHEMATICS EDUCATION

In the aftermath of new curricula and attempts to adjust teaching methods appropriately, there have been several reports and surveys of what curriculum objectives have been applied and their impact on mathematics education. In the mathematics delegations report (Matematikdelegationen, 2004), the delegation concludes that the mathematics curriculum's (Lpo 94) emphasis on development of argumentation and communication skills has not yet had an impact on teaching. Furthermore, the report showed descriptions of educational settings that may not always provide for students' development of competencies in line with the curriculum objectives. The national quality review done by the Swedish National Agency for Education in 2003 (Skolverket, 2003) took a closer look at different educational and teaching settings in the years 2001 and 2002. The inspectors' interviews and observations of classroom practice showed that there was an educational model which dominates the teaching of mathematics. This dominant model was most evident in the 7th-9th grade, but was also common in the 5th grade. The model consisted of lectures and explanations, students' individual work in the textbook and written tests after finishing each section. Intentional student collaboration was relatively uncommon and conversation between teachers and students about mathematical problems and possible solution strategies or explorative work in mathematics were likewise uncommon.

Planned collaboration between individuals or work in groups does not occur very often, which means communication and interaction between students is of limited extent. It is a form of teaching that contains only minor elements of variation in both content and approach (Skolverket, 2003, p. 20).

But in this review the inspectors also found examples of teaching situations where they met many committed and interested students. These situations were characterized by variation in both content and working method and left room for both students' and teachers' involvement and activity. Students worked individually but also in different group configurations and with different approaches to tasks. These students were very positive and perceived collaboration as clarification and openings for learning opportunities (Skolverket, 2003).

Swedish compulsory school mathematics education

The dominance, uniformity, and framing of mathematics education by textbooks is also reinforced and further described in Johansson (2006), *Teaching mathematics with textbooks*. In addition to mentioning the advantages of mathematical textbooks simplifying the daily work of the teachers, she also states the disadvantages; that textbooks somewhat reduce both dimensions of the freedom and responsibility of the teachers and that they also are tools with constraints and weaknesses.

In Sweden, we have a deep-rooted tradition of using textbooks in school mathematics. Many students, parents, and even colleagues expect that a teacher would use a textbook in order to ensure that the students are offered all parts of mathematics that are necessary for the next level in the school system (Johansson, 2006, p. 29).

This statement not only explains the cultural heritage of textbook dominance in both school environment and homes, but also the unspoken framing and limitation of variation in mathematics education. The variation in approach being confined to the diversity of the textbook and having little to do with educational skills the teacher might possess or wish to develop. It is quite possible that these observations are some of the reasons behind the national “break out” and initiated government funding of mathematical initiatives to decrease textbook use and increase educational variation in mathematics in compulsory schools in Sweden (Matematikdelegationen, 2004; Nationellt centrum för matematikutbildning, 2001).

Results from TIMSS (Trends in International Mathematics and Science Study) further confirm Swedish textbook dominance as above stated (Johansson, 2006; Skolverket, 2003, 2012b), where findings showed that Swedish mathematics teaching is, compared with the average EU / OECD country, more textbook-driven and more time in class is devoted to students’ independent work (Skolverket, 2008). The results of the latest survey in 2011 indicates a continued decrease in pupils’ (8th grade) mathematical ability and a continued, above average use of textbooks in Sweden, compared to other EU/OECD countries (Skolverket, 2012b). The results from both the 2003 and 2012 PISA survey show continued decrease in mathematics ability among Swedish 15-year-olds.

2.3 THE LATEST NATIONAL GOVERNMENT FUNDED INITIATIVE IN MATHEMATICS EDUCATION

Based on the results of the TIMSS 2007 survey (Skolverket, 2008) and the national evaluation of compulsory school (Skolverket, 2003), the Swedish government initiated a three-year development project named *matematik-satsningen*. Between 2009 and 2011 municipalities and independent schools were given the opportunity to apply for funding toward development and improvement of mathematics education in the compulsory school system. This was a welcomed initiative, which ended up with 12,000 teachers and 200,000 students in some way participating in different kinds of efforts during the project period. Summarizing the main motives of municipalities and independent schools applying for funds, two main themes emerged;

- Exploratory and practical mathematics
- Using and developing technical aids (ICT) in mathematics education

The main purpose of the applications for *exploratory and practical mathematics* was to reduce the mathematics textbooks' influence on teaching approaches and develop a more practical, problem-solving and explorative approach. The production and development of so-called mathematical workshops (*matematikverkstäder*), which were supposed to provide access to a balanced practical and theoretical approach, was frequent among application headings. The main reasons for requests for funding to develop and increase the use of technical aids were making lesson settings more interesting and teaching more adapted to what students experience as everyday technical aids. Common words headings of applications were: *Computers*, *Interactive Tablets* (handheld writing boards) and *Classroom Whiteboards*, such as "*Smartboard, Clever Boards and Active Boards*". Significant for both objectives is the urge to improve, adjust, and develop mathematics education; one perspective moving towards a less dominant textbook setting and more varied teaching approach, which implies dissatisfaction with opportunities to vary one's teaching in a textbook dominant setting, and the other perspective moving towards bringing the classroom up to date regarding the use of ICT equipment and technical tools (Skolverket, 2012a).

2.4 QUALITY REVIEWS AND EVALUATION OF INITIATIVES IN MATHEMATICS EDUCATION

The Swedish Schools Inspectorate's quality review (Skolinspektionen, 2009) comments that lack of variation in mathematics education is quite common and implications of these findings have stimulated the Swedish mathematics teachers' community to broaden perspectives and search for alternative, stimulating teaching methods with a more balanced approach between using textbooks and other more diverse activities. This quest and implementations of the government funded municipal initiatives towards a more varied teaching approach are well documented (Rystedt & Trygg, 2010; Skolverket, 2011a, 2012a), and results and recommendations have, to some extent, had impact on classroom settings.

Commissioned by the Swedish Agency for Education (Skolverket), Madeleine Löwing, Marie Fredriksson, and Eva Färjsjö conducted an evaluation of "Matematiksatsningen" 2009-2011 (Skolverket, 2012a). Regarding the primary goal of developing a more varied approach in teaching, reducing the influence of mathematics textbooks and individual work, the initiative was very successful. But when they analyzed what mathematics the students learned from this shift in direction of teaching approach, they noticed that the learning outcome was not favourable. They report that most of the schools used much of their grants to purchase exploratory/manipulative materials and equipment. This led to teachers focusing on how to practically work with the materials and exercises, neglecting (forgetting) the actual mathematical learning objectives that were to be met using the materials. Most effort was put into identifying appropriate materials and experiments and very little effort into developing professional teaching skills in line with the new teaching approach (Skolverket, 2011a). The shift towards increasing the use of manipulative materials and exploratory mathematics educational settings has, in many ways, proved fruitful (Moyer, 2001; Skolverket, 2011a), but has also faced doors of criticism, both from professionals and the public, who believe that lessons and education are losing their/its mathematical content and focus on actual learning objectives (Rystedt & Trygg, 2010).

2.5 EXPLORATORY, COLLABORATIVE WORK APPROACH, PROBLEM-SOLVING TEACHING SETTINGS AND THE USE OF MANIPULATIVE MATERIALS.

The use of manipulative and physical objects has increased in recent years in Sweden, and this increase has been evaluated by Rystedt & Trygg (Rystedt & Trygg, 2010) at the National Centre of Mathematics Education in Sweden (NCM). They noted that students tend to experience laboratory and manipulative mathematics as a kind of happening or “fun mathematics” and something separate from “real mathematics”, i.e. traditional textbook studies preceded at best by a lecture or instructions and examples from the teacher. These observations correspond with conclusions drawn in Moyer’s American study (Moyer, 2001) where “fun mathematics” was also considered separate from “real mathematics”. This tendency may depend on lack of systematic or regular deviation from the use of textbooks resulting in minor educational variation and students feeling unprepared and unfamiliar with the new settings. We need to bridge the gap between “fun mathematics” and “real mathematics” and create regularity in deviation. As stated by Johansson (2006), what is problematic are the high anxiety levels and restlessness; a result of students being forced to leave the path predetermined by the textbooks.

The use of manipulative materials is widely accepted and frequent in early school years, where there is a greater focus on spatial development, desire to learn and elements of play (Kilpatrick et al., 2001; Rystedt & Trygg, 2010). As mathematics becomes more abstract and the use of textbooks becomes more dominant in later years of elementary school (7th to 9th grade), the diversity in teaching methods decreases (Johansson, 2006), and, likewise, the use of manipulatives and out-of-textbook settings. Analysis of the different municipal mathematics education initiatives between 2009 and 2012 shows that laboratory exercises have great potential as effective educational tools, but one has to take into consideration the tendencies so far. A common reason for schools to apply for funding for a development project was that the teachers were striving for variety in teaching approach and a desire to leave the textbook and its controlling influence on teaching. This is fully consistent with current mathematics educational development, but one consequence has been that the mathematical content and students’ understanding of the content is often lost when working with manipulatives (Skolverket, 2011a). Efforts so

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far have produced a shift from the textbook controlling to manipulative materials controlling instead, with the effect of losing track of the intended mathematical content and learning objectives.

3. THEORETICAL CONSIDERATIONS

With the intention of introducing a more varied approach in teaching mathematics, it was necessary to immerse into areas of research comprising a number of topics. The overall aim of influence in the affective domains suggested looking further into motivational and affective research in mathematics teaching and learning. Furthermore, there were issues of lesson and instructional design to be considered. After consultation with my supervisors an action research approach (Robson, 2002) was decided on, using both quantitative and qualitative methods. A mathematical, core-content specific, comparative teaching-method intervention using pre and post testing of students' perceptions of the teaching methods was initiated.

3.1 MOTIVATIONAL ASPECTS AND INSTRUCTIONAL DESIGN

Inspired by the work of researchers in the areas of instructional design, classroom-based research and educational engineering, and research on affective and motivational aspects of mathematics education, the idea of a teaching approach intervention took form in the spring of 2012. Cobb, Confrey, Lehrer & Schauble (2003) and Cobb, Stephan, McClain & Gravemeijer (2011) have conducted research regarding the evolution of classroom mathematical practices and their implications. This research in classroom mathematical practices has contributed to the development of empirically grounded analyses of the learning-teaching process as constituted in the mathematical classroom. Cobb et al. theorize classroom-based design research and have prepared guidelines and frameworks for analysis of such activities. M Artigue's didactical engineering is foremost object specific and in line with that of Brousseau's *Theory of didactical situations in mathematics* (Brousseau, Balacheff, Cooper, Sutherland, & Warfield, 1997) but all the same useful in the sense of highlighting the teacher's role in the design and approach to teaching, and not the modelling of teaching approach centred on the student's relations to the knowledge (Artigue, 1994, 2009). The theory of didactical situations in mathematics (TDS) has the didactic postulate of *each item of knowledge must originate from the adaptation to a specific situation* (Brousseau et al., 1997, p. 23). The teacher refraining from interfering and indicating what knowledge is targeted once the task or problem is identified creates a situation where learning occurs. Even if specific mathematical

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knowledge is targeted, the approach to creating the situation for learning and the interaction is essential. The teacher does not convey knowledge but provides for the devolution of good problems that create learning opportunities. Learning is viewed as a process of adaptation to situations where learning takes place (Artigue, 2009).

Hannula's research (Hannula, 2006b), highlights, among other issues, how motivation is regulated, and emphasizes difficulties in motivational measurement, as motivation cannot be directly observed.

It is observable only as it manifests itself in affect and cognition, for example as beliefs, values and emotional reactions (Hannula, 2006b, p. 165)

In Hannula's earlier research, emphasis was placed on understanding the dynamic aspects of affect: how domains of affect influence learning processes and how experiences in the mathematical classroom influence the development of affect.

Figure 1 Sub domains of affect (McLeod, 1992)

McLeod's (1992) initial three sub-domains of the affective domain; *emotions*, *attitudes* and *beliefs*, provide the basis for my assumption that increasing beliefs, achievements, and satisfaction in learning mathematics by identifying and developing a teaching approach primarily focused on the domains of *emotions*, *feelings*, *moods* and *enjoyment*. *Emotions* are local and contextually embedded (DeBellis & Goldin, 2006), which suggests a thorough effort and care for lesson design that is aimed at these aspects. The hypothesis

is that positive experiences in the emotional area of the affective domain changes the somewhat moderately stable sub-domain *attitude* and this in turn has an impact on the sub-domain of *beliefs* which are highly cognitive and stable. *Attitudes* being interwoven in both affect and cognition interact and provide a bridge between emotions and beliefs (DeBellis & Goldin, 2006).

3.2 MOTIVATIONAL ASPECTS AND THE ISSUES OF LEARNING

There is much more, but the key ingredients of what it means to be strategic in teaching and learning relates to teachers finding ways to engage and motivate students..... (Hattie, 2013, p. 161)

Learning occurs through interaction (Säljö, 2005), in a sense being in some way exposed to sensory input and thereafter processing the experience and what it might imply is the foundation of knowledge development. The diversity in motivation, individual ability and necessity of applying obtained knowledge is far too great to unfold in this study, but nevertheless, to some extent, worth elaborating.

What motivates learning? Interest, necessity and being worthwhile (gain) are the main reasons for us to pursuit learning. Besides these basic motives, desire, curiosity, and maybe even frustration due to lack of ability, can also represent foundations of motivations to learn. The two different types of motivation, *intrinsic* and *extrinsic*, are well discussed and argued in the academic field of education (Fennema & Sherman, 1976; Hannula, 2006a, 2006b; Leder, Pehkonen, & Törner, 2002; Middleton & Spanias, 1999; Pintrich & Schunk, 1996), and these motivational issues of learning are highly relevant in school settings. Developed from branches of psychology, educational psychology has taken shape and formed central parts of educational philosophies (Ernest, 1991; Piaget & Inhelder, 1969; Vygotskij, 1978). Intrinsic motivation is the desire or urge of the student to undertake learning for the sake of learning. Intrinsically motivated pupils enjoy engaging in academic tasks and benefit from the pleasure of reinforcing self-images. Students who are mainly intrinsically motivated focus on learning goals such as understanding and mastering mathematical concepts (Ames & Archer, 1988). Extrinsically motivated students engage in academic tasks to obtain some kind of favourable judgment leading to reward or avoiding punishment.

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Earning good grades or acceptance and confirmation from peers, the teacher or parents/guardians are recognizable examples of rewarding performance goals (Ames, 1992). Due to intrinsic motivation being superior extrinsic motivation in academic areas (Middleton & Spanias, 1999), instructional design and development of teaching settings should focus on finding the approaches, tasks, and classroom settings providing opportunities for the student to experience surroundings stimulating the development of intrinsic motivation. Strategies for addressing aspects of intrinsic motivation may be providing opportunities to interact with peers, to respond actively rather than merely listening and working with problem-solving brainteasers (Kilpatrick et al., 2001). Key factors are possibilities of success, moderately challenging tasks, and inquiry-based classrooms. The teacher chooses tasks that deliver an appropriate level of difficulty and, at the same time, allow a high degree of success. This requires that the teachers know their students, from aspects of both ability and motivation, which corresponds well with the findings of Hattie (2013), regarding contributions from the teacher and the necessity of knowing and having trustworthy relationships with the pupils.

When students see themselves as capable of doing well in mathematics they tend to value mathematics more than those with the opposite view (Middleton & Spanias, 1999). Moreover, providing situations to generate personal solution strategies together with a favourable affective climate, positively influences both students intrinsic motivation and learning orientation in mathematics (Perry, Turner, & Meyer, 2006; Wæge & Pantziara, 2013). In line with the motivational aspects of confidence, the national quality review *Lusten att lära; med focus på matematik* (Skolverket, 2003), emphasizes the necessity of confidence in personal ability. Students' beliefs in their own ability to learn mathematics appears to be a key factor in the desire to learn. Good self-confidence tends to improve performance beyond what one "objectively" learns, and correspondingly, a poor self-esteem may decrease performance. A moderately elevated self-confidence is good for performance. Students' attitude to themselves and to their performance thus has a significant impact on how they tackle school tasks.

3.3 SOCIAL NORMS, SOCIO-MATHEMATICAL NORMS, CLASSROOM INTERACTION AND THE PRESERVATION OF MATHEMATICS TEACHING PRACTICES

Analyzing and trying to identify the forces that influence both student and teacher behaviour led to the development of the socio-didactical tetrahedron (Rezat & Sträßer, 2012), which visualizes and frames the complexity and diversity of didactical situations in mathematics and the influence of mediating variables (Hiebert & Grouws, 2007). Conventions and norms about being both teacher and student interact with the original context of the didactical triangle (artefacts-student-mathematics). These interacting elements and social norms create greater inertia to change the practice and norms in the classroom.

Figure 2 Socio-didactical tetrahedron (Rezat & Sträßer, 2012, p. 648)

Analysis of social norms serves to probe the classroom participation structure. General social norms characterize the classroom community and documents regularities in classroom activities, jointly established by the students and the teacher(s). These social norms and branches of teacher-student behaviour maintain frames of interaction and work processes in the mathematical classroom (Cobb et al., 2011). Socio-mathematical norms further deepen

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general social norms in the sense of identifying specific mathematical aspects of students' activities (Yackel & Cobb, 1996). Defining acceptable mathematical explanations, justification of methods of solutions, and mathematical efficiency are all components of socio-mathematical norms.

These “rules of engagement” may well normalize more or less favourable settings and teaching approaches. The mathematical and social interaction between students, and between students and their teachers, play central roles in framing mathematical instruction and designing lessons in mathematics. Balacheff (1990) redefines Brousseau's (1984) didactical contract to include a broader definition of social interaction, embracing norms in the classroom atmosphere:

The rules of social interaction in the mathematics classroom include such issues as the legitimacy of the problem, its connection with the current classroom activity, and the responsibilities of both the teacher and pupils with respect to what constitutes a solution or to what is true. We call this set of rules a didactical contract. (p.260)

But it seems appropriate to refine the extent of the *didactical contract* to a metaphor for a set of rules of social and mathematical interaction in a particular classroom, as Wedege et al. (Wedege, Skott, Henningsen, & Waege, 2006) use it:

The didactical contract, then, in our terminology constitutes the rules of the game in that classroom, rules that on the one hand frame the practices that emerge and on the other are regenerated and transformed by those very same practices (p. 41)

Regardless of the framing and expression of *the didactical contract*, the choice of tasks and teaching approach, and thereby initial rules of the contract, is shaped by choices made by the teacher. The development and adjustment of the contract and “rules of engagement” appear in students' adaptation to the settings; but changing the contract and norms is concentrated to teachers' ability to adjust the approach accordingly. The manner of conversation, instructional approach and feedback that the teacher applies when teaching and interacting with students may well contribute to the enculturation of what Lave (1988) calls *folk beliefs about mathematics*. These beliefs include the conviction that it is prohibited to use any method other than the standard

procedures taught in school. The only acceptable way to solve mathematical tasks, in any situation, is the use of taught procedures. An incautious and insensitive discourse can reinforce the perception of methods used to solve mathematical problems as being a set of fixed and objective rules and thereby contribute to the inheritance of unfavourable social and socio-mathematical norms, both in the mathematical classroom and the social environment adjacent to and interacting with it. The mathematical textbook, as argued by Johansson (2006), also contributes to inheriting beliefs of classroom mathematics and thereby serves to preserve the common view of the textbook as necessary and therefore norm-setting.

Figure 3 Layers of norms

Rezat (Rezat, 2012) investigated students' use of mathematics textbooks, and findings suggest that changing norms, attitudes, and accepting an alternative approach is difficult. The teacher follows a discovery learning

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approach based on a textbook not used at the school in question, worksheets were handed out and the students worked in small groups. The tasks were supposed to guide the students to discover how to multiply decimals in different ways. Because the students had access to their textbooks while working with the worksheet problems, they tended to search for solutions and verifications in the textbook, and not go about attempting to discover rules and methods of solution by themselves.

Summarizing the notion of norms in the mathematical classroom one could draw the less affective layers of the classroom environment for mathematical learning as interacting parts boiling down to didactical contracts in some sense framing conditions for teaching and learning as shown in figure 3.

3.4 DIFFERENT TEACHING APPROACHES AND THEIR IMPLICATIONS

Hiebert & Grouws (2007) claim that analyzing teaching approach and learning outcome is complex due to the numerous variables and limited possibilities of isolating interacting factors. The conceptual challenges of investigating “what works” in the teaching and learning approaches of mathematics are at least threefold:

- *Different teaching approaches might be effective for different learning goals.*

Learning goals differ and there is no reason to believe that a single teaching method is the most effective for all types of students.

- *Teaching is a system of interacting features.*
Independent measurement of effects of features of teaching on learning outcome cannot be done because of the interaction of other features within the teaching/learning setting.

The effects of a particular feature on students learning likely depends on its interactions with other features in the system (p.374)

- *There is an influence of mediating variables.*
A number of contextual conditions influence the level and nature of learning. Situations outside the classroom such as parent or guardian support, out of school activities together with inside school settings,

such as interpretation of teachers' instructions, students' prior knowledge and attentiveness during class all influence the conditions for learning.

These conceptual challenges translate into the paradox of evaluating a specific lesson design and teaching method searching for whether it proves favourable for the students and therefore affecting knowledge achievement, motivation and desire to learn mathematics (research questions). Considering the above named challenges the implementation of the lesson design study has to be seen from the perspective of being in specific settings, with specific content, different limitations and with different preconditions. However difficult it is to ascertain the effects of the teaching design, individual student perception of the different approaches is possible and can provide us with viable data to reason about causality.

Environments for developing different mathematical competences and settings for stimulating learning should have a symbiotic relationship. Even if the mathematical textbook writers strive to meet demands on a diversity of exercises and activities, it is still necessary to provide a variety of approaches to accommodate different learning opportunities and develop mathematical abilities in different contexts.

Our definition of teaching methods or approaches and how work in the classroom can be conducted is expressed by Samuelsson (2007), as:

What the teacher does when he or she teaches, what the students do when they go about learning and what tools and devices are used in these enterprises. (p.252)

For the purpose of designing an adequate teaching approach for the *systematically varied lesson method* (SLV), it was necessary to identify desirable teaching methods that provide access to multidimensional learning settings, students' methods of engagement, and also what opportunities selection of these teaching characteristics implies for students' possibility to experience appreciated learning settings. Bearing in mind the goal of stimulating in the affective domain (McLeod, 1992), triggering emotions, feelings, and moods, the three-lesson design series needed to include elements of methods appealing to different types of motivation.

Ernest's (1991) five teaching ideologies are based on the assumption that of teachers' epistemological beliefs are the foundation for which teaching

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approach will be applied in the teaching of mathematics. Case (1996) stresses that these different ideologies have to complement each other, because separately they emphasize different forms of knowledge. If school mathematics education is supposed to develop different kinds of abilities (as stated in the national curricula), then it seems reasonable to approach teaching using a variety of methods. Different methods provide different opportunities to practice and develop different abilities (Samuelsson, 2007). Engvall's (2013) chart of connections between Ernest's (1991) educational ideologies, teaching approaches, students' learning approaches and accessible knowledge, which was inspired by Samuelsson (2007), frame my methodological starting point regarding the design of the SLV lesson series.

Ernest's first two ideologies, *industrial trainer* and *technological pragmatist*, and their implications for methods of teaching and learning, only provide access to knowledge of facts and mastering procedures. These two methods of teaching and learning are not appropriate in the sense of providing for all curricula-stated competencies and abilities and are therefore excluded from further discussion. In some contexts these methods are appropriate, but all too limiting for development of knowledge and abilities in line with curriculum goals (Skolverket, 2011b), and to satisfy the motivational necessities of today's compulsory school settings.

Ideologies, Methods of teaching and learning, Accessible knowledge, Competencies and Abilities

Ideology (Ernest, 1991)	<i>Old humanist</i>	<i>Progressive educator</i>	<i>Public educator</i>
Teaching methods (Engvall, 2013; Marton & Booth, 2000)	Lectures, Visually demonstrates and explains concept.	Arranges teaching environments with great variety of resources, encourages, guides, facilitates and listens	Provides authentic and practical teaching materials, challenges students by questioning, discusses and reasons. Gives responsibilities, analyses and thereafter changes
Student learning methods (Carlgren & Marton, 2000; Engvall, 2013)	Listens, Exerts theoretically.	Examines, discovers, discusses and suggests solutions. Collaborates and openly presents results	Takes responsibility, develops skills, cooperates and reasons.
Accessible knowledge (Carlgren & Marton, 2000; Engvall, 2013)	Facts, procedures and understanding concept	Desire and interest, Efficacy , conceptual understanding, problem-solving ability, communicative skills social skills, productive attitude	Conceptual understanding, skills of communication and reasoning, efficacy, productive attitude and holistic perspective.
Competencies Kompetenser (NCM, 2001)	Understanding concept and mastery of procedure	Productive attitude, Communication skills, strategic and reasoning competence	Holistic, productive, Communication skill and reasoning.
Abilities (Förmågor, Lgr 11)	Method and calculating ability, Under- standing concept	Problem-solving ability, method and calculating ability, under-standing concept and reasoning ability	Interest, efficacy, identifying mathematical context and relevance and reasoning ability

Table 2. Accessible competencies and abilities via methods

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As Samuelsson (2007) points out

*If the students are to become mathematically competent
then different teaching approaches should be used in
mathematics education in school (p.257)*

If education in mathematics is supposed to develop different kinds of competencies, as stated in the curricula, it seems reasonable that using different teaching approaches is an appropriate way to satisfy these demands. In addition to the individual skills and method of the teacher, one has to consider creating an approach and surroundings that provide opportunities to learn and which enforces development of intrinsic motivation. If design and tasks are carefully chosen, and are able to intrinsically motivate, a number of pedagogical desirable behaviours that emerge. Increased time-on-task, more elaborative processing, greater creativity, and persistence in methods of confronting failure, seem to appear and develop when exposed to intrinsically motivating tasks (Lepper, 1988). Students' perceptions of their competence in mathematics is also related to intrinsic motivation (Gottfried, 1985), which further enforces carefully choosing tasks that are moderately challenging and allow success.

Based on Cottons (1998), findings from observing English classrooms, Skovsmose (2001) classifies traditional mathematics education as being part of the *exercise paradigm*, with lessons being textbook-driven and two-parted. The traditional approach is where the lessons are divided into two undetermined sections; one part of the lesson the teacher presents mathematical ideas or techniques, and the other part with the students work with selected exercises. Skovsmose's elaboration of *Milieus of learning* contrasts the *exercise paradigm* with what he calls *landscapes of investigation* which can support investigative work approaches, inviting students to formulate and develop "what if" questions, collaborate, and look and argue for explanations and solutions in a problem-solving setting.

Information

	Tradition of exercises	Landscapes of investigation
References to pure mathematics	(1)	(2)
References to a semi-reality	(3)	(4)
Real-life references	(5)	(6)

Table 3. Milieus of learning (Skovsmose, 2001, p. 126)

Skovsmose claims that a large part of mathematics education moves between milieus (1) and (3), which in turn provides for the exercise paradigm framing traditional mathematics education. These conclusions are well in line with textbook question design and the common teaching approach (Löwing, 2004; Skolverket, 2003). Boaler and Greeno's chapter in *Multiple perspectives on mathematics teaching and learning* (Boaler & Greeno, 2000) again emphasizes the dimension of variation in teaching methods. Both traditional teaching methods and more reform-orientated teaching approaches regard the activity of learning mathematics as a vehicle for acquiring mathematics knowledge. Boaler's findings (Boaler, 1998b, 2000a) support more recent theories where practices of learning mathematics define the knowledge that is produced. Mathematical knowledge is co-constituted by learning practices and therefore differentially useful in real-world situations. Students developed forms of knowledge and beliefs as a response to different teaching methods (p.172). Thus the teaching method strongly influences knowledge outcome.

Recommendations from the Swedish National Agency for Education (Skolverket) regarding how teachers should go about managing their teaching clearly points out the necessity of "mixing it up" regarding teaching approaches:

In managing their teaching the teacher should structure and control the processes involved with education and create a balanced approach between lecturing, collaborative discussions, individual work and collaboration among the students, thereby creating good conditions for each student to learn. (Skolverket, 2014, p. 17)

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With this recommendation in mind, a reminder of the dominant, main teaching method in mathematics seems appropriate (previously mentioned in Chapter 1):

The common model consisted of lectures and explanations, student's individual work in the textbook and written tests after finishing each section. Planned student collaboration was relatively uncommon and conversation between teachers and students about mathematical problems and possible solution strategies or explorative work in mathematics was likewise just as uncommon. (Skolverket, 2003, p. 20)

The recommendation is in line with desirable classroom conditions, but the everyday activities of the teacher, and school environmental interactions, often frame limitations for fulfilling these recommendations and the teachers' expectations of creating good conditions for learning. However these limitations may well provide inspiration and fuel for developing teaching methods and educational settings that serve to systemize variation in approach and thereby relieve unnecessary burdens and optimize teaching/learning situations.

Summarizing (reviewing) the field of mathematical teaching practices, two overall general descriptions of teaching methods appear; the mainly textbook-orientated, which is solitaire in its nature, and the more varied, open, collaborative and problem-solving approach, which invites discussion and cooperation. Applying only one or the other method over a period of time would imply learning outcome of a specific nature. In turn, this would certainly mean leaving out some desirable capabilities, and therefore lead to gaps in necessary knowledge. There are numerous studies of teaching methods and their outcome and, even though no evidence of any method being generally superior in impact or effect, there are implications that the different methods are favourable in different settings and for different learning goals (Boaler, 1998b, 2000b, 2001, 2013; Boaler & Greeno, 2000; Samuelsson, 2007, 2008). Boaler has, on several occasions, studied the effects of different teaching methods and their outcomes (Boaler, 1998a, 2001, 2002) and where students learned mathematics by working through textbook exercises, she concludes that these students performed well in similar textbook situations, but found it difficult to use mathematics in open, applied or discussion-based situations. The students who had learned mathematics through open projects and a variety of approaches developed more flexible forms of knowledge that

were useful in a range of different situations, including conceptual examination questions and authentic assessments. The assumption of students adopting specialized skills through specific teaching approaches is not surprising. What seems obvious is that some teaching methods do not provide for development of abilities to adapt and cope with various mathematical problems in different contexts. The most dominant model of mathematics education, the textbook driven model (Skolverket, 2003), seems a less profitable educational setting in terms of curriculum goals. The nature of textbook questions may well provide opportunities to learn methods of calculating and solving routine tasks, but they are limited in providing for the possibility to learn such abilities as discussing and following mathematical reasoning, defending choice and standpoint in methods and solutions and applying mathematics in everyday situations (future, post comprehensive school setting). Bauersfeld (1992) points out missing opportunities when using mathematical textbooks due to the lack of inviting or encouraging discussion among students. The teacher could make up for this lack by asking open ended questions about solutions to the textbook tasks, aiming to provoke the students to think about how others have reasoned and tackled different challenges, including those who had the wrong answers. As Engvall (2013) points out, if teaching primarily consists of individual work in the textbook then this teaching approach can contribute to the students perceiving procedural knowledge as equivalent to mathematical knowledge. If teaching can provide opportunities for collaborative work, enforcing mathematical reasoning and discussion, then mathematical knowledge is shown to be much more extensive than procedural methods. Variety, flexibility and avoiding monotony in teaching is important to maintain the desire to learn, and the method of teaching needs to alternate to meet the students' different learning styles and their preferred ways of both learning and interacting in the classroom environment (Hannula, 2006b).

These aspects of necessity in variation address content and relevant teaching approaches as well as the use of different teaching appliances. Samuelsson (2008) studied the effect of three different teaching methods;

- *Traditional*: the teacher explain methods and procedures preceding textbook work
- *Independent*: textbook work without teacher introduction
- *problem solving*: introduction to different problems and methods, working and discussing in groups

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Even though no significant differences were found in academic achievement between the methods, the problem-solving method showed positive effects on students' internal motivation. Stedöy (2006) lists components of beliefs about the teaching and learning of mathematics. She states that difference in mathematical tasks and preferred learning styles of individuals demands a variety in classroom organization. Teaching, using the opportunity to stimulate several senses, increases the possibility of grasping the individual learning style and facilitating understanding. A pluralistic approach to the teaching of mathematics that respects different teaching styles is to prefer. Bentley's (2003) survey study "Mathematics Teachers and Their Teaching" implies the necessity of a repertoire of teaching approaches to meet future demands. Different learning styles and prior knowledge has to be taken into account when individualizing education. A crucial issue affecting student achievement seems to be whether teachers use their professionalism fully. Using adequate teaching approaches and adapting instructional materials is essential. Professional content knowledge is one side of teacher professionalism, but without social skills, the sense for promoting or "selling" the subject and the chosen work approach, and success in motivating will be minimized, delayed or at worst absent (Bentley, 2003; Franke, Kazemi, & Battey, 2007).

Revisiting motivational aspects, motivation to learn and stimulating interest by the teacher's personal approach, the teacher can encourage students' motivation to learn by modelling their own classroom discourse, communicating enthusiasm, personalizing, and stimulating interest, for the content as well as work method (Brophy, 1998; Kilpatrick et al., 2001). Waage (2010) undertook a study where the focus was on development of Norwegian upper secondary school students' motivation for learning when they experienced an inquiry based mathematics teaching approach. Using her own theoretical framework based on the study she concludes that:

The study shows that students' motivation for learning mathematics, although it is considered relatively stable, can be influenced by changes in the teaching approach.... (p.90)

Furthermore she suggests a variety in approach that:

A combination of working with mathematics problems and routine tasks from the textbook, and the fact that the students were given opportunities to find their own solutions and rules

for solving the problems, in collaboration with peer students and with guidance from the teacher, contributed to the sense of understanding and mastery (p.90-91).

Waage's findings are well in line with the implications of Hannula (2006a, 2006b) who expresses the limited space of the teacher-centred mathematics classroom, which tends to focus on the learning of routine procedural skills and individual work and therefore does not fulfil the students' needs of autonomy and relatedness. Waage and Pantziara claim that

More student-centered or reform classrooms where the emphasis is on meaning making and collaborative work would give the students more opportunities to fulfil their needs (Wæge & Pantziara, 2013, p. 5).

As already implied, the pursuit of *one teaching approach* that satisfies all individual's learning is comparative with the pursuit of the unachievable. What we successfully can pursue, and have partially identified, is more or less effective teaching approaches and learning strategies under certain circumstances. Hattie's meta analyses (Hattie, 2013) highlights some of the better known teaching strategies and their effects. Of relevance in our study are the teaching strategies *Problem-solving*, *Cooperative learning* and *Individualistic learning*. Problem-solving and Cooperative learning in the sense of being provided for in the SLV method, and Individualistic learning in the sense of being the primary method applied in the traditional approach. All three strategies stand out as effective approaches and are only exceeded in effect by *Teaching of strategies* (mainly used among students with learning difficulties) and *Reciprocal teaching* (mainly used in reading comprehension). Problem-solving teaching provides students with tasks involving defining the problem, selection of alternatives to solve the problem, solving the task and finally evaluating the outcome. Cooperative learning suggests learning by interacting with other students and peers and is more effective than individualistic methods. Furthermore cooperative learning has a prime effect on enhancing interest and problem-solving, provided it is set up with high levels of peer involvement. Finally, in comparison, cooperative learning exceeds individualistic learning in effect, which clearly indicates the power of peers and human interaction in teaching and learning (Hattie, 2013).

3.5 THE NATURE AND NECESSITY OF SYSTEMIZING VARIATION

In the national quality review “*Lusten att lära – med fokus på matematik*” (Skolverket, 2003) it is claimed that

A varied education, with elements of theoretical, practical and perceptual parts satisfies the conditions of human difference of nature and different learning styles, broadens learning opportunities and increases the desire to learn (p.30).

In the Swedish Mathematics Delegation report (2004) one can read the editors’ emphasize that variation in mathematics teaching has a value of its own, but only achieves a high level of quality if prepared well, exclusively relating to the mathematical contents and carried out by well-educated teachers. Furthermore, the authors highlight that research and experience show that this variation of method in teaching provides fruitful conceptions, which enables maintained interest in and enjoyment of mathematics in later school years and adult life.

The notion of the “milieu” (Brousseau et al., 1997) arises in the sense of systemizing and normalizing components and settings that offer, among other things, possibilities for student interaction and discussions of solution strategies. Familiarity in context and lesson design paves the way for adaptation and optimization of learning environment. Changing the context for each learning situation and teaching sequence is neither practical nor recommendable and would lead to unnecessary waste of time and anxiety (Ruthven, Laborde, Leach, & Tiberghien, 2009, p. 332). In the evaluation of the National Mathematics Initiative (Skolverket, 2011a), the authors reported that what was currently missing was didactic analysis of how different work approaches and exploratory settings can be part of the long-term planning of mathematics education. Many teachers had expressed their concern for the more varied and exploratory work approach being time-consuming for both teachers and students. Because the students were not familiar (normalized) with the exploratory settings, each exploratory lesson took too much time to initialize, instruct and complete, which further suggests systemizing a varied approach.

4. PREPARATIONS AND DESIGN

When preparing for the intervention, a number of logistical issues had to be solved. Decisions also had to be taken, to design the intervention to ensure the best possible research opportunities:

- Locating appropriate school(s), classes and students.
- Scheduling the classes.
- Scheduling the teachers to enable them to alternate between both classes.
- Parents and guardians being informed of the research study.
- Deciding on content and design of the booklets for the textbook elements.
- Deciding on the problem-solving and exploratory tasks for the SLV method.
- The questionnaire design and appropriate questioning.
- Design of mathematical skills tests.

Furthermore, practical elements such as copying and producing booklets from textbook components, purchase and arrangement of physical objects and materials had to be provided for.

4.1 THE SYSTEMATICALLY VARIED LESSON METHOD (SLV)

The lesson series design was inspired by personal teaching experience and the already mentioned educational ideologies and relationships to classroom practice (Brousseau et al., 1997; Ernest, 1991; Samuelsson, 2007) that meets, and in some sense ensures curriculum implications. Furthermore, the design of the study emerged in parallel with the recent (2009-2013) national initiatives to engineer mathematical educational settings that are more dynamic and provide for a variety of teaching and learning approaches. Motivating learning and promoting the necessity of a basic mathematical ability is a natural part of the teacher's everyday work. Bearing this in mind, the search for lesson designs that could create and maintain enjoyment of learning mathematics seems relevant. Hattie's (Hattie, 2012, 2013) overall results regarding teaching approaches point out *Problem-solving and Cooperative learning* methods as successful teaching approaches and these

Preparations and design

conclusions have also served as a basis for the design of the teaching method. The lessons series was developed with the intention of including as many elements of access to *possible knowledge development* (Engvall, 2013), foremost with goals of meeting possible development of *reasoning abilities*, *ability to communicate mathematics*, *interest in mathematics*, *efficacy* and finally *the context and relevance of mathematics*. Furthermore the structure of the 2nd and 3rd lesson in the series provides teacher access to assessment methods beyond the conventional approach of written tests. Targeting the above mentioned aims of developing abilities in line with the national curricula, the SLV method also enables adequate assessment opportunities. As demonstrated in the previous chapter, the concept of persistent textbook use is both an asset and a limitation. The textbook provides structure and stability in the sense of specifying what knowledge should be obtained, or at least which tasks seem relevant to obtain necessary knowledge. The consistence of measurability in number of tasks or number of pages seems to fit both teachers and most students, but this may well be due to indoctrination and acclimation through numerous years of schooling. As demonstrated by results and surveys, the monotonous textbook use does not meet the diverse nature of learning and motivating students in today's comprehensive schools by far. But, at the same time, the textbook dominance has not been established without reason. The pupils being introduced to the textbooks approach in early school years and adapting to this customized approach is so common that it is neither appropriate nor defensible to completely abandon textbooks as educational tools. This is the main reason for the SLV method maintaining a lesson containing textbook based (booklet) individual work. During the SLV method, the students only worked on selected questions in the booklets, because of the other activities in lessons two and three.

The overall structure is that of three different lessons per week, repeating this weekly approach over the time at my disposal, hoping that the time span is sufficient enough for the students to familiarize themselves with the design. As will be discussed later, the design and exploratory/collaborative and problem-solving character of the lessons, the second lesson especially, is more or less favourable depending on the mathematical content to be studied. Working in an explorative, hands-on, constructive and investigating way with the contents of, for example, three-dimensional geometry is probably easier to do than using the same approach with algebra.

4.1.1 THE DIFFERENT LESSONS OF THE SLV METHOD

Lesson 1: The teacher initiates the mathematical content of the week, explains and gives instructions. The teacher provides examples of solutions, specifies tasks and instructions prior to the students studying independently in their text booklets. The pupils work with a variation of textbook tasks, which are deliberately chosen to ensure learning of basic skills in each section, but also in view of the more explorative and problem-solving nature of the tasks in lesson 2. This serves the purpose of learning and maintaining basic numeracy, the ability to read and correctly interpret questions and practice choice of appropriate methods to solve standard textbook tasks. It also provides the opportunity to take the time to connect the teacher's instructions and explanations with examples of procedures from the textbooks.

Lesson 2: The students are given exploratory/problem-solving tasks on worksheets. These tasks are produced at the National Centre of Mathematics of Education (NCM) in Sweden (list of tasks appended) and each task is carefully chosen with regard to progress and content from the first lesson. The students work in pairs or small groups solving the worksheet tasks. The pupils were able to choose method solving approach and also of choose of how to present and explain results. Focus is on interpretation, practicing, judging and estimating appropriate methods to solve the tasks and also on practising to communicate mathematical terms and expressions as well as argue, defend, and motivate standpoints.

Lesson 3: The groups and pairs take turns presenting their solutions and findings from the exploratory tasks in lesson 2. These presentations continue with group and whole class discussions of results and methods, ending with teacher and/or students summarizing the discussion and agreeing on solutions or differences. Computers, smart boards and other ICT devices for presenting results that activate students and create settings for discussion and argumentation were available during the exploratory task lesson and the pupils were encouraged to use these aids to present results in this third lesson.

4.2 THE BOOKLETS

With the intention of maintaining both well recognizable individual settings and simultaneously presenting a different approach, we (the teachers and I) selected relevant parts from the textbook *Matte Direkt år 9* (Bonnier Utbildning AB) in subjects of *Algebra* (Algebra) and *Relationships and rate of change* (Samband och förändring) and produced booklets. The pupils were well accustomed to using the *Matte Direkt* textbook series having used it since 7th grade. All booklets were in full colour and had answer sections in the back of the booklet; the layout was thereby the same as the textbook they normally used. We assumed that the pupils would equate the booklets with the textbooks that they had previously worked in and thereby consider the booklets as a recognizable textbook artefact. This assumption, to some extent, proved inaccurate and will later be elaborated in the discussion chapter. The contents of booklet was chosen so that it merged well with the problem-solving/exploratory tasks in the *systematically varied lessons* section and at the same time guaranteeing access to fundamental content knowledge and abilities according to the national curriculum. The booklets supplemented the books the school had already purchased, which each pupil possessed and normally used in mathematics education. Both classes used identical booklets during the different teaching methods. The class applying the SLV method, however, did not have to solve all the questions, but only certain, carefully selected, tasks and problems.

4.3 THE PROBLEM-SOLVING AND EXPLORATORY TASKS

Activity sheets from the National Centre for Mathematics Education in Sweden (NCM; <http://ncm.gu.se>) formed the basis of the practical/problem-solving and collaborative lesson (two) in the SLV lesson series. These activity sheets were chosen from the Strävorna matrix according to algebraic and relationships and rate of change (samband och förändring) content. Strävorna (<http://ncm.gu.se/node/5302>) are intended to be used both as inspiration and support in teaching mathematics. They consist of different types of materials such as articles, activities, and ideas about forming the lesson content into learning goals. Certain articles describe reasoning about abilities and competencies and others teaching processes. The activities and problem-solving tasks are well defined, ready for classroom use and specified to

learning objectives. The design of the KappAbel mathematics competition (Wedegge et al., 2006) inspired our selection of appropriate activities in lesson two. Teacher explanation of worksheets/problems/manipulatives, finding examples in textbooks, collaboration in small groups or pairs and presentation of results were the main features under consideration.

Middleton and Spanias (1999), when approaching the motivational aspect of choice of tasks, state that what is essential is for students to feel successful in mathematics and teachers must structure tasks in such a way that they supply an appropriate level of difficulty and challenge, and that tasks should be chosen to allow a high degree of success in relation to effort seems critical. For this reason the problems and questions among the NCM activity sheets selection were chosen given the mathematical content but also chosen to allow success and participation by all students as well as providing the possibility to develop and complicate tasks if found suitable.

The consistent structure of the worksheets facilitated working with the activities for both the pupils and the teachers. Unfortunately, due to heavy initial workloads and other responsibilities, the teachers were, to some extent for some of the activities, poorly prepared, and had not spent the time necessary practicing and preparing for the specific tasks. Most of the worksheets are organized in the same manner and consist of explanations of the purpose of the activity, the aimed mathematical contents, what materials are necessary, what prior knowledge is appropriate and descriptions of how to go about organizing the actual work on the tasks. Some of the more hands-on worksheet problems required hardware and specific material and were more time-consuming in preparation and implementation while others, less hands-on tasks required only paper and pencil. On all worksheets and activities the pupils had to work together, collaborate and agree on certain aspects of the solving approach and how to present results.

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4.3.1 THE DIFFERENT EXPLORATORY AND PROBLEM-SOLVING TASKS

Section 1: Algebra section

- *Agenternas hemliga tal* (The agents' secrets number)
Learning objectives: Communication, forms of expressing algebra and mathematical reasoning.
http://ncm.gu.se/media/stravorna/4/b/4B5B_agenternas_hemliga_tal.pdf
- *Ekvationsleken* (The equation game)
Learning objectives: *Mathematical* reasoning, communicating and expressing forms of algebra.
http://ncm.gu.se/media/stravorna/4/b/4B5B_ekvationsleken.pdf
- *Algebraisk triangel* (Algebraic triangle)
Learning objectives: Practicing algebra skills and identifying patterns.
http://ncm.gu.se/media/stravorna/4/b/4B_algebraisktriangel.pdf
- *Vad muttrar du om?* (What are you nuts about?)
Learning objectives: Mathematical modelling, communicating and expressing forms of problem-solving.
http://ncm.gu.se/media/stravorna/9/b/9BF5B_mutterpasar.pdf

Section 2: Relationships and rate of change

- *Biografer* (Bio graphs)
Learning aims; Interpretations and how functions can be illustrated.
http://ncm.gu.se/media/stravorna/9/e/9E5E_biografer.pdf
- *Skostorlek* (Shoe size)
Learning objectives: Problem-solving, mathematical relationships and interpretations.
http://ncm.gu.se/media/stravorna/1/e/1E5E_skostorlek.pdf
- *Vasgrafer* (Vase graphs)
Learning objectives: Problem-solving, mathematical relationships, interpretations and rate of change.
http://ncm.gu.se/media/stravorna/5/e/5E_vasgrafer.pdf

- *Bungeedockan* (the Bungee doll)
Learning objectives: Mathematical modelling, communicating, expressing forms of problem-solving and functions.
http://ncm.gu.se/media/stravorna/4/e/459E_Bungeedockan.pdf

4.4 THE CONVENTIONAL METHOD (CM)

Lessons 1-3

Designed as the conventional, well-established common teaching method with textbook dominance, as described in Chapter 2 (Skolverket, 2003), the CM consisted of the teacher explaining, providing instructions and examples of solutions, prior to the students studying independently in their booklets for the rest of the lesson, as well as the following two lessons of the weekly lesson series. These lessons therefore mainly consist of individual work with specific exercises in the booklet. The student reads examples and explanations and then solves the questions and problems in their booklets. Collaboration is confined to the closest classmate and the settings in the classroom are predefined and not deliberately altered. These lesson settings are those described in “Standard settings in educational methods of mathematics”. The contents of the booklets were identical to those participating in the SLV class and all copied material was taken from the students’ regular textbook prior to the intervention study. During the CM section the students worked on all questions in the booklets.

5. METHODOLOGY

5.1 CHRONOLOGICAL DISPOSITION

The empirical intervention was conducted according to the following time schedule:

May 2012

- Inquiries into what schools were available.
- Identification of adequate schools with sufficient possibilities to provide conditions for the intervention.
- Approval from the school principal.
- Two teachers consent to participation with their two classes.

June 2012

- Consultation and planning with the two participating teachers.
- The school scheduler produced at workable schedule enabling the teachers to alternate between classes in the autumn semester.
- Content specific (algebra and functions) problem-solving, exploratory and collaborative tasks are chosen.

August 2012

- Parents, guardians, and students are informed of the study.
- Both school classes are simultaneously presented to the pre-test/post-test/questionnaire design and lesson approach.

September -December 2012

- Lesson design intervention.

December 2012

- Student and teacher interviews are conducted.

January 2013

- Analysis of questionnaire data and interview comments commences.

5.2 THE INTERVENTION

The teaching structure intervention consists of the “Systematically varied lesson series” in two comprehensive school classes. An eight week teaching approach intervention divided into two sections of four weeks each. The mathematical content (Swedish national curricula) of the autumn semester in 2012, at the participating school, was equations and variables (algebra) and relationships and rate of change (samband och förändring).

The participants are two 9th grade classes (Class A; N=21, Class B; N=22) and their two mathematics teachers. The classes differ significantly in experience of mathematics education prior to the study, and this is discussed and elaborated later in the discussion chapter. The same two teachers taught both classes, alternating between them every second lesson in both sections, which served the purpose of minimizing single teacher influence of students’ perceptions of the teaching methods.

One of the two participating classes was taught with the systematically varied lesson method (SLV), alternating lesson structure and work approach in the first section (algebra). The other class was taught simultaneously with a conventional method (CM) using only the textbook. After finishing the first section (algebra), the classes swapped teaching methods for the second section (relationships and rate of change). Comparisons between both classes’ experience of the different teaching methods were thereby possible. After consultation with the teachers, Class A was chosen for the SLV teaching approach first because of their insufficient mathematics education in the 8th grade, and consequently their somewhat less motivated attitude to mathematics. The intention was to provide a structure, independent of approach, early in the 9th grade, and in a best-case scenario, the students would experience mathematics education as more pleasant and stimulating than in previous years. This, choice of being first exposed to the SLV method, was to reclaim conditions for good learning settings for the rest of the final comprehensive school year.

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Figure 4 The Intervention disposition and mathematical core contents

5.3 RESTRICTIONS AND LIMITING FACTORS

During the spring of 2012 I was able to establish contacts with teachers participating in a municipal mathematical project and thereby could lay the foundation for planning and executing the intervention study. This municipal mathematics project provided an opportunity to apply the intervention with relevant framing as the project consisted of initiatives to develop methods to vary mathematics education. These contacts provided access to two ninth grade classes and the possibility of applying the intervention during the autumn semester of 2012. This opportunity was the only visible opening at the time and due to the licentiate study time span, two and a half years, necessary to grasp. However, making the best of this opportunity resulted in time frame limitations. Available time was limited and wedged in between the start of the autumn semester and the autumn holiday, which is adjacent to the first weekend in November. The students also had a trainee (PRAO, PRaktisk Arbetslivs-Orientering) during the intervention period, which disrupted the study somewhat. The schedule planner did what was possible to schedule a

sufficient number of lessons but the approach turned out too pressed for time and with too little time in lessons two and three of the SLV method. Both of these lessons would have benefited from longer lessons.

In the context of the practical and collaborative approach in the SLV method the testing of the mathematical skills development was also limited due to lesson time restrictions.

5.4 THE PERCEPTION OF LEARNING QUESTIONNAIRE

Before and after the first section of the intervention (Class B conventional and Class A systematically varied) and after the second section of the intervention, the students answered the specially designed perception of learning questionnaire. The post-questionnaire from the first section was used as a pre-questionnaire for the second section. The design of the questionnaire was inspired and developed from the Swedish 2012 PISA student questionnaire, sections C and D, which relate to learning mathematics and the experience of learning. The questionnaire was designed to enable the students to express how they experienced the different teaching methods and how the method affected interest, enjoyment, and learning possibilities. Furthermore, it intended to cover the students' relations to, and experience of, the textbook as the main source of learning and their appreciation of other learning settings within the framework of the two methods applied. The questionnaire consisted of 22 questions divided into three headings (Appendix A). Five of the initial questions were removed during data analysis as these questions were found to be of no relevance for the research questions. The questions are clustered and subtitled in the results chapter as follows (* originally a PISA question):

Interest and enjoyment

- *I look forward to mathematics lessons. **
- *I enjoy trying to solve mathematical tasks. **
- *I am interested in the things I learn in mathematics. **

Anxiety

- *I often worry that it will be difficult for me in mathematics classes.**
- *I often feel unsure when solving a mathematical problem.**
- *I worry about presenting results in front of others.*

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Relations to Learning by means of textbook

- *I learn mathematics best by studying the textbook.*
- *I enjoy learning mathematics the best when I am studying my textbook.*

Relations to Cooperative and dynamic learning settings

- *I learn best by studying with a variation of educational methods.*
- *I enjoy working with other students in mathematics.*
- *I enjoy working with practical problems and experimenting in mathematics.*
- *I would like to work more with problem-solving in mathematics.*

Perception of Appreciated learning settings

- *I am content with the lesson design.*
- *I learn well from our teaching method.*
- *I experience that I am learning.*

Perception of Unappreciated learning settings

- *I do not learn well with this teaching method.*
- *I think the mathematics lessons are boring.*

Questionnaire questions removed from analysis;

- *I enjoy reading about mathematics.* (Interest in and enjoyment of mathematics)*
- *Making an effort in mathematics is worth it because it will help me in the work that I want to do later on. * (Instrumental motivation)*
- *Learning mathematics is worthwhile for me because it will improve my career chances. *(Instrumental motivation)*
- *I do well in mathematics. * (Self-concept)*
- *My mathematics lessons are varied.*

5.5 MEASUREMENT OF MATHEMATICAL SKILLS

Pre and post tests for assessment of mathematical core content specific skills were performed in both teaching method sections; the first section with the core content of algebra (algebra) and the second section with the core content of relationships and rate of change (samband och förändring). The tests were produced from relevant parts of the textbook tests (*Matte Direkt, Bonniers förlag AB*) as well as a selection of questions from the diagnostic tests provided by the Swedish National Agency for Education (PRIM-gruppen, 2010). In addition to these tests each worksheet's specific tasks from the second lessons in the SLV method were taken into consideration when choosing test questions.

5.6 INTERVIEW DESIGN

Intending to create conditions to freely express perceptions and experiences of the different teaching methods, a semi-structured student (S)/teacher (T) interview was drafted with question-areas comprising:

- Individual experience of previous mathematics education (S).
- Personal perception of the classes previous mathematics education (S/T).
- Individual experience of the *Systematically Varied Lesson* method (S).
- Individual experience of the *Conventional Method* (S).
- Individual perception of how s/he learns best (S).

The semi-structured approach was further intended to enable space to elaborate on, not only what they themselves experienced, but also what they thought or heard other students experience or express.

The teacher interview was not intended to evaluate effects of the different teaching methods but to establish background information of the different classes' prior mathematics education, the teachers' experience of teaching the classes and to discuss the general teaching approach of the SLV and how it may further be developed.

5.7 DATA ANALYSIS

Data from the tests of mathematical skills as well as questionnaire answers were compiled in the SPSS data analysis software program. Results of mathematical skills development is reported as class means and show the standard deviation of both pre- and post-testing. Questionnaire answers have been analyzed in order to monitor individual changes of perceptions from the different teaching sequences. Changes of perceptions between the two significantly different sides of opinions *Strongly Agree/Agree* and *Strongly Disagree/Disagree* are summarized and indicated separately at the bottom of each table in the results section. The questionnaire answers of the students interviewed are also monitored separately and have individual index markings in tables as further clarified in the results chapter.

5.8 VALIDITY OCH RELIABILITY

A short-term intervention of this kind implies low expectations regarding impact on long-term motivational aspects. All the same, even minor changes of opinions are indicators of effect. It could be expected that knowledge development would occur regardless of the applied teaching method and this development was secured in content by the equivalent approaches in both methods. We could not assume an overwhelming change of opinion or favouring of either method due to the time span available for acclimation to the practice. However, we could expect influence regarding perception of method specific features and appreciation due to the classes' previous experiences.

It is questionable if a small-sized intervention of this kind, targeting motivational aspects, can be properly evaluated by means of questionnaires. The qualitative nature of the intervention implies a closer, more participating and on-site documentation of the classroom interaction. The questionnaire designed for the study was, however, crafted with questions of a specific nature regarding perceptions of the different teaching methods applied. These questions could be targets of criticism if results were to be used in a careless way and transformed into indications of generalizing methods. At the same, time taking findings from an intervention of this size and transforming them into generalizing methods would be way out of line of good research. The supplemented questions are, however, viable in this small-scaled intervention,

targeted at research specific aims and framed in the classroom settings and teaching approach, which we at least to some extent control. In a large-scale teaching-method intervention, being able to guarantee the applied method is the planned method decreases, and the number of variables that we do not have reasonable control over increases (i.e. facilities, resources, schedules, teacher relations).

Possible weakness of using questionnaires in a short-term, small-sample intervention with the objective of assessing changes in perception is somewhat compensated by the semi-structured interviews. The four interview participants were voluntary, which in turn implies motivated students, willing to dedicate time and share their experience of the teaching methods. Their opinions are explicit and in no way representative the whole class. All the same, they provide descriptions of prior settings and the teaching methods and can therefore contribute to analysis and verification of questionnaire findings. Both participating teachers alternated between classes every second lesson to minimize the effects of either teacher having influence on teaching method specific features.

5.9 ETHICAL CONSIDERATIONS

All students in both participating classes were informed of the planned teaching-method invention at the same time. Parents and guardians were informed of the planned research project in writing and they were informed of the option of declining their children participation. Written consent from students and parents/guardians was collected prior to interviews. All questionnaires were individually coded on all three questioning occasions and could only be traced by the researcher. Mathematical skills tests were likewise coded. Marking of the mathematical tests was carried out by the researcher and individual test scores was communicated to the teachers in order to provide additional assessment data. All interviews were digitally recorded and transcribed later. Research in the compulsory school setting is of a serious nature due to its unspoken compulsory element. Participation is considered voluntary, but at the same time, to decline participation is stepping outside of both companionship and class belonging. The seriousness implies experimentation with teaching approaches within certain frames of what I choose to call “not causing lost mathematical generations with reduced abilities and desire to learn”. What is meant by this is that when doing

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classroom research we have to ensure the students access to acquirement of basic abilities. This, in turn, enforces the precision of knowing the curriculum and providing for its content even when experimenting with design or teaching approach.

6. RESULTS

This chapter consists of three sections. In the first section results from the *Perceptions of Learning mathematics* questionnaires for the SLV and CM sections are presented. Sequentially subtitled, Class A precedes Class B in all tables, and results from the systematically varied lesson method (SLV) are presented first and followed by results from the conventional method (CM). The answers provided by the students interviewed, Alva (a), Bea (b), Cecilia (c) and Diana (d), are specified in each class's tables. Results are listed in tables as the number of students giving a certain answer before (pre) method application and after (post) method application. The number of students' changing opinions from the significant areas of Agreeing/Strongly agreeing to Disagreeing/Strongly disagreeing (Movements) is also listed in the tables. Each subtitled question cluster is disruptively summarised. Unless otherwise indicated Class A; N=21, and Class B; N=22. Any differences in number of respondents to each question prior/post are indicated and student identification number specified.

In the second section, test results from mathematical skills tests are presented. The third section concludes the chapter with a compilation of results from the student and teacher interviews including essential elements relevant for analysis of results in relation to the research questions.

Results SLV

6.1 RESULTS FROM THE *PERCEPTIONS OF LEARNING* QUESTIONNAIRES PRE AND POST THE *SYSTEMATICALLY VARIED LESSON* METHOD OF TEACHING

Interest and enjoyment

- *I look forward to the mathematics lessons.*
- *I enjoy trying to solve mathematical tasks.*
- *I am interested in what I learn in mathematics.*

Class A

- *I look forward to mathematics lessons*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	0	6ab	12	3
Post SLV	0	10ab	8	3
Agree/Disagree Movements		0→	←4	

- *I enjoy trying to solve mathematical tasks*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	2a	9b	10	0
Post SLV	0	11ab	8	2
Agree/Disagree Movements		3→	←3	

- *I am interested in what I learn in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	1b	10a	8	2
Post SLV	0	9ab	10	2
Agree/Disagree Movements		4→	←2	

Class B

- *I look forward to mathematics lessons*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	5c	11d	5	1
Post SLV	2	16cd	4	0
Agree/Disagree Movements		1→	←3	

- *I enjoy trying to solve mathematical tasks*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	6c	13	2d	1
Post SLV	3	16c	2d	1
Agree/Disagree Movements		1→	←1	

- *I am interested in what I learn in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	4	15cd	1	2
Post SLV	4	10cd	7	1
Agree/Disagree Movements		6→	←1	

Summary of results Interest and enjoyment (SLV)

Class A expresses minor changes of opinion about *interest and enjoyment* except for an increase of students (4) looking forward to their mathematics lessons post the SLV method. Both interviewed students maintain a positive attitude in terms of interest and enjoyment throughout the SLV method.

Class B has a majority of students who are *interested in and enjoy* mathematics. Class B also shows an increase of students (2) expressing opinions of looking forward to their mathematics lessons post the SLV method. Class B exhibits a decrease of five (5) students in content specific interest (relationships and rate of change) during the SLV method. Both interviewed students maintain a mostly positive attitude during the SLV method (with the exception of Diana disagreeing to enjoying trying to solve tasks and maintaining this opinion).

Results SLV

Anxiety of mathematics or for mathematical situations

- *I often worry that it will be difficult for me in mathematics classes.*
- *I often feel unsure when doing a mathematical problem.*
- *I worry about presenting results in front of others.*

Class A

- *I often worry that it will be difficult for me in mathematics classes*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	0	11	7	3ab
Post SLV	2	9	8a	2b
Agree/Disagree Movements		1→	←1	

- *I often feel unsure when doing a mathematical problem*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	2	7	11ab	1
Post SLV	0	10	10a	1b
Agree/Disagree Movements		2→	←3	

- *I worry about presenting my results in front of others*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	4	7	4	5ab
Post SLV	6	3	7	5ab
Agree/Disagree Movements		2→	←0	
N prior; 21				
N Post; 20(2)				

Class B

- *I often worry that it will be difficult for me in mathematics classes*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	0	2	9	11cd
Post SLV	0	2	11	9cd
Agree/Disagree Movements		1→	←1	

- *I often feel unsure when doing a mathematical problem*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	0	6d	9c	7
Post SLV	1	8	8 cd	5
Agree/Disagree Movements		2→	←5	

- *I worry about presenting my results in front of others*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	1	4d	10c	7
Post SLV	2	6 d	6	8c
Agree/Disagree Movements		0→	←3	

Summary of results Anxiety of mathematics or for mathematical situations (SLV)

Approximately half of Class A expresses agreement in worrying or feeling unsure in all questions related to anxiety (11, 9, and 11)ⁱ and this distribution remains the same throughout the SLV method. Class B, however, has a majority of students initially not feeling unsure (20, 16, 17)ⁱ, but has an increase of students (3) changing views, agreeing to feeling anxiety and being unsure in both doing mathematical tasks and presenting results. The students interviewed, Alva and Bea in Class A maintain the standpoint that they do not feel anxiety, but in Class B interviewed student Diana feels unsure doing tasks and presenting results while the interviewed student Cecilia maintains a feeling of being sure during the SLV method.

ⁱ Number of answers to each question

Results SLV

Relation to learning by means of the textbook

- *I learn mathematics best from studying the textbook.*
- *I enjoy learning mathematics best when studying my textbook.*

Class A

- *I learn mathematics best from studying the textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	1a	7	10b	3
Post SLV	2a	7	8b	4
Agree/Disagree Movements		2→	←3	

- *I enjoy learning mathematics best when studying my textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	1a	8	9b	3
Post SLV	3a	7	7b	3
Agree/Disagree Movements		1→	←2	
N prior; 21				
N Post; 20(2)				

Class B

- *I learn mathematics best by studies my textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	1d	3	11c	6
Post SLV	2d	5	6	9 c
Agree/Disagree Movements		2→	←4	
N prior; 21				
N Post; 22(8)				

- *I enjoy learning mathematics best when studying my textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	2d	4	10c	6
Post SLV	2d	4	7	9c
Difference	0	0	-3	+3
Agree/Disagree Movements		2→	←2	

Summary of results Relation to learning by means of the textbook (SLV)

A majority of students (13) in Class A disagree with the statements *best learning* and *enjoying* textbook studies. These opinions maintain more or less stable throughout the SLV method. There are no changes in view among the interviewed student's from Class A's prior or post the SLV method in terms of a agreeing and b disagreeing with *best learning* from textbooks. In Class B an overwhelming majority of the students disagree (17) with *best learning* from and *enjoying* studying by means of the textbook. This experience of *enjoyment* and appreciation remains while agreeing with *best learning* from textbooks increases by 3 students. The only interviewed student to change opinion post the SLV method is Cecilia in class B. Diana remains in strong agreement with best learning from and enjoying textbooks while Cecilia initially disagrees and ends up strongly disagreeing.

Results SLV

Relations to Cooperative and Dynamic Learning Settings

- *I learn best by studying with a variation of educational methods.*
- *I enjoy working with practical problems and experimenting in mathematics.*
- *I enjoy working with other students in mathematics.*
- *I would like to work more with problem-solving in mathematics.*

Class A

- *I learn best by studying with a variation of educational methods.*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	6b	12	3a	0
Post SLV	5b	14	2a	0
Agree/Disagree Movements		1→	←2	

- *I enjoy working with practical problems and experimenting in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	7	11b	3a	0
Post SLV	7	9b	5a	0
Agree/Disagree Movements		3→	←1	

- *I enjoy working with other students in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	9b	9	2a	1
Post SLV	4	11b	4a	1
Agree/Disagree Movements		2→	←0	
N prior; 21				
N Post; 20(2)				

- *I would like to work more with problem-solving in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	3	13ab	3	2
Post SLV	2	11ab	4	3
Agree/Disagree Movements		2→	←0	
N prior; 21				
N Post; 20(2)				

Class B

- *I learn best by studying with a variation of educational methods*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	7c	10	4d	0
Post SLV	4c	13	4d	0
Agree/Disagree Movements		2→	←2	

- *I enjoy working with practical problems and experimenting in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	7	11	2c	2d
Post SLV	8	11	2c	1d
Agree/Disagree Movements		1→	←2	
N prior; 21				
N Post; 22(8)				

- *I enjoy working with other students in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	5	12c	4d	0
Post SLV	7	10	4cd	1
Agree/Disagree Movements		0→	←1	
N prior; 21				
N Post; 22(8)				

Results SLV

- *I would like to work more with problem-solving in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	5	11c	5d	0
Post SLV	5	12c	3d	2
Agree/Disagree Movements		1→	←1	
N prior; 21				
N Post; 22(8)				

Summary of results Relations to Cooperative and Dynamic Learning Settings (SLV)

In both Class A and B an overwhelming majority of the students agree with all 4 statements regarding relations to cooperative and dynamic learning settings (18, 18, 18, 16/17, 18, 17, 16)ⁱ. For the most part these views do not change, with the exception of displacement between strongly agreeing and agreeing with views of enjoying working with other students prior and post the SLV method in Class A. The interview-students perceptions of relations to cooperative learning settings in Class A are opposites; Alva not agreeing and Bea agreeing in *enjoyment of working with others*.

In Class B Diana remains in disagreement side with positive relations to cooperative and dynamic learning settings in all four questions. Cecilia initially agrees with *enjoying of working with others* but changes view to disagreeing.

ⁱ Number of answers to each question

Perception of Appreciated Learning Settings

- *I am content with the lesson design.*
- *I learn well from our teaching method.*
- *I feel that I am learning.*

Class A

- *I am content with the lesson design*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	0	8b	13a	0
Post SLV	0	16b	4	1a
Agree/Disagree Movements		1→	←9	

- *I learn well from our teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	2	8b	8a	3
Post SLV	1b	10	8a	1
Agree/Disagree Movements		2→	←4	
N prior; 21				
N Post; 20(2)				

- *I feel that I am learning*

Questionnaire	Every lesson	Most lessons	Some lessons	Non/almost non lessons
Prior SLV	4b	6	9a	0
Post SLV	3 b	9	7 a	2
N prior; 19(4,17)				
N Post; 21				

Class B

- *I am content with the lesson design*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	2	10d	4c	4
Post SLV	3	9cd	10	0
Agree/Disagree Movements		5→	←4	
N prior; 20				
N Post; 22(8,15)				

Results SLV

- *I learn well from our teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	4	7c	7d	3
Post SLV	3c	9	9d	0
Agree/Disagree Movements		2→	←3	
N prior; 21				
N Post; 21				

- *I feel that I am learning*

Questionnaire	Every lesson	Most lessons	Some lessons	Non/almost non lessons
Prior SLV	3	8c	11d	0
Post SLV	5c	9d	8	0

Summary of results Perception of Appreciated Learning Settings (SLV)

There is a significant increase (8) in Class A's views of being *content* with the applied teaching approach, and an increase (2) in agreement with *learning well* from the SLV method. The students interviewed remain on different sides of agreement/disagreement; Alva disagrees with the statement *learning well* and being *content*, while Bea expresses the opposite, agreeing with the statements of *content* and *learning well*. Class B exhibit minor changes in views of methods promoting learning and enjoyment, but maintain a majority of students expressing agreement with the statements of *learning well* and being *content*. There is a displacement of opinions of *learning well* from the applied method from strongly disagreeing prior the SLV method, to disagreeing. The interviewed student Cecilia changes opinion from disagreeing with the statement *learning well* to agreeing with *learning well* from the SLV method, while the interviewed student Diana remains in agreement. Both Cecilia and Diana feel that there are more lessons where they learn.

Perception of Unappreciated Learning Settings

- *I do not learn well with this teaching method.*
- *I think the mathematics lessons are boring.*

Class A

- *I do not learn well with this teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	0	9	10a	1b
Post SLV	1	7a	13b	0
Agree/Disagree Movements		4→ ←3		
N prior; 20(20)				
N Post; 21				

- *I think the mathematics lessons are boring*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	5	12a	4b	0
Post SLV	3	9a	9b	0
Agree/Disagree Movements		6→ ←1		

Class B

- *I do not learn with this teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	6	3	7d	3
Post SLV	0	8d	10	4
Agree/Disagree Movements		5→ ←4		
N prior; 19(8,15,18(c))				
N Post; 22				

- *I think the mathematics lessons are boring*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior SLV	3	4d	10	4c
Post SLV	0	9	9d	4c
Agree/Disagree Movements		2→ ←4		
N prior; 21				
N Post; 22				

Results SLV

Summary of results Perception of Unappreciated Learning Settings (SLV)

Class A shows an increase (2) in disagreement with the statement *not learning well* from the teaching method, and an increase (5) in the number of students *not finding the mathematics lessons boring*. The interviewed student Alva changes opinion from disagreeing with the statement *not learning well* to agreeing with the statement *not learning well*, but remains in the agreement section of lessons being boring. The interviewed student Bea remains in the disagreement section of both *not learning well* and *mathematics lessons boring*. Class B expresses a displacement of students from strongly disagreeing with the statement *not learning well* to agreeing with the statement *not learning well* and an increasing number of students thinking the *lessons in mathematics are boring*. The interviewed student Diana changes opinion, from disagreeing with the statement *not learning well* to agreeing with the statement *not learning well*, but from agreeing with the statement *lessons being boring* to disagreeing with *lessons being boring*.

6.2 RESULTS FROM THE *PERCEPTIONS OF LEARNING* QUESTIONNAIRES PRE AND POST THE *CONVENTIONAL* *METHOD OF TEACHING*

Interest and enjoyment

- *I look forward to the mathematics lessons.*
- *I enjoy trying to solve mathematical tasks*
- *I am interested in what I learn in mathematics*

Class A

- *I look forward to the mathematics lessons*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	10ab	8	3
Post CM	1b	6a	11	3
Agree/Disagree Movements		4→	←1	

- *I enjoy trying to solve mathematical tasks*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	11ab	8	2
Post CM	2 ab	13	5	1
Agree/Disagree Movements		1→	←5	

- *I am interested in what I learn in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	9ab	10	2
Post CM	1b	11 a	7	2
Agree/Disagree Movements		1→	←4	

Results CM

Class B

- *I look forward to the mathematics lessons*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2c	10	9	1d
Post CM	5c	11d	5	1
Agree/Disagree Movements		3→	←7	

- *I enjoy trying to solve mathematical tasks*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	7	9c	6d	0
Post CM	6c	13	2d	1
Agree/Disagree Movements		1→	←4	

- *I am interested in what I learn in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	5c	13	4d	0
Post CM	4	15cd	1	2
Agree/Disagree Movements		3→	←4	

Summary of results Interest and enjoyment (CM)

Class shows an increase in *interest and enjoyment* questions (4, 3)ⁱ but a decrease (3) in the number of students *looking forward to lessons*. Both of the interviewed students Alva and Bea remain in the agreement section, but Bea moves from agreeing to strongly agreeing with all four questions (see interview statements). Class B show equivalent views of *interest and enjoyment*, however, there is an increase in the number of students looking forward to lessons (4). The interviewed student Diana makes a significant change from strongly disagreeing to agreeing with *looking forward to the mathematics lessons* (see interview statements) and even changes opinion from *disagreeing* with being *interested* to *agreeing* with being *interested*, while the interviewed student Cecilia only changes opinion within the agreement section.

ⁱ Increase in each question

Anxiety of mathematics or for mathematical situations

- *I often worry that it will be difficult for me in mathematics classes.*
- *I often feel unsure when doing a mathematics problem.*
- *I worry about presenting results in front of others.*

Class A

- *I often worry that it will be difficult for me in mathematics classes*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2	9	8a	2b
Post CM	2	5	12a	1b
Agree/Disagree Movements		3→	←0	
N prior; 21				
N Post; 20(2)				

- *I often feel unsure when doing a mathematical problem*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	10	10a	1b
Post CM	2a	8	9	1b
Agree/Disagree Movements		3→	←4	
N prior; 21				
N Post; 20(2)				

- *I worry about presenting my results in front of others*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	6	3	7	5ab
Post CM	4	5	5	6ab
Agree/Disagree Movements		1→	←2	
N prior; 21				
N Post; 20(2)				

Results CM

Class B

- *I often worry that it will be difficult for me in mathematics classes*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	3	9d	10c
Post CM	0	2	9	11cd
Agree/Disagree Movements		2→	←1	

- *I often feel unsure when doing a mathematical problem*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	1	6	8cd	7
Post CM	0	6d	9c	7
Agree/Disagree Movements		3→	←2	

- *I worry about presenting my results in front of others*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2d	7	6c	7
Post CM	1	4d	10c	7
Agree/Disagree Movements		4→	←0	

Summary of results Anxiety of mathematics or for mathematical situations (CM)

Initially, half of Class A's agrees with the statement *worrying that it will be difficult*, while the other half disagrees. This shifts to fewer students (4) *worrying* about it being *difficult* during CM. The interviewed students Alva and Bea remain in strongly disagreeing throughout CM. An overwhelming majority of Class B's students are not worried about it being difficult, nor do they feel unsure. Initial anxiety over *presenting results* in front of other students decreases (4). The interviewed student Cecilia remains in disagreement with anxiety while Diana switches over to agreeing with feeling anxiety both when tackling *mathematical problems* and *presenting results*.

Relation to learning by means of textbook

- *I learn mathematics best by studying the textbook*
- *I enjoy learning mathematics best when studying the textbook*

Class A

- *I learn mathematics best by studying the textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2a	7	8b	4
Post CM	1a	10	6b	3
Agree/Disagree Movements		0→ ←2		
N prior; 21				
N Post; 20(2)				

- *I enjoy learning mathematics best when studying the textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	3a	7	7b	3
Post CM	3a	10b	7	1
Agree/Disagree Movements		0→ ←2		
N prior; 20				
N Post; 21(2)				

Class B

- *I learn mathematics best by studying the textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	5d	5	12c
Post CM	1d	3	11c	6
Agree/Disagree Movements		2→ ←2		
N prior; 22				
N Post; 21(8)				

- *I enjoy learning mathematics best when studying the textbook*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	4d	8	10c
Post CM	2d	4	10c	6
Agree/Disagree Movements		1→ ←3		

Results CM

Summary of results Relation to learning by means of textbook (CM)

Initially, Class A has a slight surplus of students (9/12)ⁱ disagreeing with the statement *learning best by means of studying the textbook* and a 50/50 relationship when it comes to *enjoying mathematics by means of textbooks*. Answers to both questions switch over in favour of agreeing with the statement *best learning* (11/9)ⁱ and *enjoying by means of studying the textbook* (13/8)ⁱ during the CM Method. The interviewed student Alva remains in strong agreement while Bea switches from initially disagreeing to agreeing with the statement *best enjoying studying the textbook*.

Initially a majority of Class B strongly disagrees with the statement *learning best by means of studying the textbook* (17/5)ⁱ and on the whole, a majority remains on the disagreement side during the CM, even if there is a displacement from strongly disagreeing to disagreeing, and 2 students go from disagreeing to strongly agreeing. Both the interviewed students remain on opposite sides during the CM; Cecilia disagreeing with the statement *best learning from textbooks* and Diana agreeing with the statement *best learning from textbooks*.

Regarding *enjoyment of best learning from textbooks*, initially, half of Class A agrees and half disagrees, but this changes during CM with three (3) more students agreeing, one (1) of which is Bea. However, Class B initially has a majority (18/4)ⁱ of students disagreeing with the statement *best enjoying learning from textbooks*, but also here there is an increase of 2 students switching over to agreeing, though the majority though remains in disagreement.

ⁱ Number of students agreeing/disagreeing

Relations to Cooperative and Dynamic Learning Settings

- *I learn best by studying with a variation of educational methods.*
- *I enjoy working with practical problems and experimenting in mathematics.*
- *I enjoy working with other students in mathematics.*
- *I would like to work more with problem-solving in mathematics.*

Class A

- *I learn best by studying with a variation of educational methods.*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	5b	14	2a	0
Post CM	5b	13	2a	0
Agree/Disagree Movements		0→ ←0		
N prior; 21				
N Post; 20(2)				

- *I enjoy working with practical problems and experimenting in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	7	9b	5a	0
Post CM	6	11ab	3	0
Agree/Disagree Movements		2→ ←4		
N prior; 21				
N Post; 20(2)				

- *I enjoy working with other students in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	4	11b	4a	1
Post CM	3	15ab	1	2
Agree/Disagree Movements		1→ ←3		
N prior; 20(2)				
N Post; 21				

Results CM

- I would like to work more with problem-solving in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2	11ab	4	3
Post CM	2a	11b	6	2
Agree/Disagree Movements		1→	←1	
N prior; 20(2)				
N Post; 21				

Class B

- I learn best by studying with a variation of educational methods*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	8c	11d	3	0
Post CM	7c	10	4d	0
Agree/Disagree Movements		2→	←1	
N prior; 22				
N Post; 21(15)				

- I enjoy working with practical problems and experimenting in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	8	11	2cd	1
Post CM	7	11	2c	2d
Agree/Disagree Movements		1→	←0	

- I enjoy working with other students in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	6	11c	4	1d
Post CM	5	12c	4d	0
Agree/Disagree Movements		1→	←2	
N prior; 22				
N Post; 21(8)				

- *I would like to work more with problem-solving and tricky questions in mathematics*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	6	9c	7d	0
Post CM	5	11c	5d	0
Agree/Disagree Movements		1→	←4	
N prior; 22				
N Post; 21(8)				

Summary of results Relations to Cooperative and Dynamic Learning Settings (CM)

A majority of students in both Class A and B claim best learning and enjoying learning from using different methods and experimenting. In Class A there are minor changes, except for the increase (4) in enjoying working with other students during CM. The interviewed student Alva is one of the pupils going from disagreeing to agreeing with *enjoying working with others*, and also going from disagreeing to agreeing with experimenting and working with practical problems. The interviewed students Cecilia and Diana remain in their separate corners, Cecilia agreeing with the statements of enjoyment and learning and Diana disagreeing, with the exception of Cecilia disagreeing in *enjoyment of working with practical problems and experimenting*, and Diana switching to agreement of best learning using different methods.

Results CM

Perception of Appreciated Learning Settings

- *I am content with the lesson design.*
- *I learn well from our teaching method.*
- *I experience that I am learning*.*

Class A

- *I am content with the lesson design*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	0	16b	4	1a
Post CM	2b	12	5	1a
Agree/Disagree Movements		2→	←1	
N prior; 21				
N Post; 20(2)				

- *I learn well from our teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	1b	10	8a	1
Post CM	2b	15	4a	0
Agree/Disagree Movements		0→	←5	
N prior; 20(2)				
N Post; 21				

- *I experience that I am learning*

Questionnaire	Every lesson	Most lessons	Some lessons	Non/almost non lessons
Prior CM	3b	9	7a	2
Post CM	5b	8a	6	2

Class B

- *I am content with lesson design*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2	18cd	2	0
Post CM	4	7c	7d	3
Agree/Disagree Movements		8→	←0	
N prior; 22				
N Post; 21(8)				

- *I learn well our teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	7c	12d	3	0
Post CM	2	10d	4c	4
Agree/Disagree Movements		6→ ←1		
N prior; 22				
N Post; 20 (8, 15)				

- *I experience that I am learning*

Questionnaire	Every lesson	Most lessons	Some lessons	Non/almost non lessons
Prior CM	7	8c	6d	0
Post CM	3	8c	11d	0
N prior; 21(7)				
N Post; 22				

Summary of results Perception of Appreciated Learning Settings (CM)

Class A has a significant increase (6) in the number of students going from disagreeing to agreeing with the statement *learning well* from the applied teaching method CM. Class B, however, has a significant increase (5) in the number of students going from agreeing to disagreeing with the statement *learning well* from the applied teaching method CM. Furthermore, Class B shows a significant increase (8) in the number of students disagreeing with being content with the CM teaching approach. The interviewed student Cecilia goes from strongly agreeing to disagreeing with the statement *learning well* from the teaching method. Class A, however, remains content with the teaching approach and the interviewed students Alva and Bea remain in the opposite corners; Alva disagreeing with the statements in learning and being content, and Bea agreeing with learning and being content.

Results CM

Perception of Unappreciated Learning Settings

- *I do not learn well with this teaching method.*
- *I think the mathematics lessons are boring.*

Class A

- *I do not learn well with this teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	1	7a	13b	0
Post CM	0	3	15a	2b
Agree/Disagree		5→	←0	
Movements				
N prior; 21				
N Post; 20(2)				

- *I think the mathematics lessons are boring*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	3	9a	9b	0
Post CM	3	10a	6	1b
Agree/Disagree		1→	←3	
Movements				
N prior; 21				
N Post; 20(2)				

Class B

- *I do not learn well with this teaching method*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	1c	4	11d	6
Post CM	6	3	7d	3
Agree/Disagree		0→	←4	
Movements				
N prior; 22				
N Post; 19(8,15,18(c))				

- *I think the mathematics lessons are boring*

Questionnaire	Strongly Agree	Agree	Disagree	Strongly disagree
Prior CM	2	6	10d	4c
Post CM	3	4d	10	4c
Agree/Disagree Movements		2→	←2	
N prior; 22				
N Post; 21(8)				

Summary of results Perception of Unappreciated Learning Settings (CM)

Class A has a significant decrease (5) in the number of students agreeing with the statement not learning well from CM, and Class B shows the opposite with six (6) students shifting to agreeing with the statement not learning well from the CM method. The interviewed student Alva goes from agreeing to disagreeing with the statement *not learning*, while Bea and Diana remain in the disagreeing section throughout the CM method. There are no noteworthy changes in either class when it comes to finding the *lessons boring* or not.

Mathematical skills Results

6.3 TESTRESULTS OF MATHEMATICAL SKILLS PRIOR AND POST THE DIFFERENT TEACHING METHODS

Prior to studying the algebra and mathematical relationships and rate of change content during the intervention, Class B had, to some extent, studied the subject areas in late 8th grade mathematics. Class A, however, had not.

Both tests are included as appendices.

6.3.1 THE MATHEMATICAL SKILLS DEVELOPMENT IN SECTION 1, ALGEBRA

Means and standard deviation of tests of mathematical skills in the algebra section

Class (teaching method)	Pre-test	Std. deviation	Post-test	Std. deviation	Diff.
Class A (SLV)	9,05	5,43	12,81	6,47	+3,76
Class B (CM)	14,91	4,86	17,82	4,54	+2,91

Maximum 24 Points

6.3.2 THE MATHEMATICAL SKILLS DEVELOPMENT IN SECTION 2, MATHEMATICAL RELATIONSHIPS AND RATE OF CHANGE

Means and standard deviation of tests of mathematical skills in the mathematical relationships and rate of change section

Class (teaching method)	Pre-test	Std. deviation	Post-test	Std. deviation	Diff.
Class A (CM)	6,57	5,60	17,44	3,16	+10,87
Class B (SLV)	13,23	3,10	18,41	2,84	+5,19

Maximum 25 Points

6.4 COMPILATION OF INTERVIEWS COMMENTS

6.4.1 STUDENT INTERVIEW'S

Class A

Alva (girl)

The student has a positive attitude, is motivated, and usually displays mathematical skills above average.

She explains that the mathematics education in the 8th grade was boring, monotonous and textbook-dominated. Because the teacher had difficulties handling the class the conditions for an appropriate working atmosphere were less favourable. Sections started with the teacher explaining initial examples and then the lessons mostly consisted of individual work in the textbook. The teaching contained a few minor elements of collaborative work. Regarding her experiences of the intervention, she expresses some discontent with the third lesson in SLV where the students presented results and discussed solutions and methods. She felt that these lessons were something of a waste of time once one or two groups had presented. She mentions appreciating having two teachers, which provides different perspectives on the same content. She also mentions the dimension of fairer and better knowledge assessment due to two teachers being present. She found the explorative, problem-solving tasks in lesson two to be less challenging initially, but after some thought and work on the tasks she discovered that it was possible to find challenging aspects. She also thought these lessons provided a new stimulating way of working and mentioned that other students felt the same, even if some commented that “they thought time was wasted playing”, suggesting that these lessons were a waste of time and that this was not real maths. She mentions that the students making these comments would probably have learned more during these lessons if they had taken the tasks more seriously. Regarding the pairing of students for lesson two, she commented on the pairing as unfortunate because she was paired with someone she would not normally work with, and one who did not have the same appreciation of mathematics. This pairing resulted in her carrying a greater workload finding solutions and methods of solutions for the tasks. However, she was well aware of this being educational and developing for her peer. She clearly states best learning from individual work in the textbook and this is confirmed in the questionnaire answers regarding

Interview Comments

perceptions and enjoying textbook learning, but quite contradictory also disagrees with learning well from the CM method as well as the SLV.

Bea (girl)

The student has average mathematical skills and is moderately motivated.

She explains the mathematics education in grade 8 as being totally textbook-dominated and mainly working individually in the textbook. At best the students received a document including section content, goals, time disposition, tasks expected to be done, and test dates. She claims the SLV method suits her well and she enjoys the elements of collaborative, problem-solving, exploratory (manipulative materials) and presentations and argumentations. During the interview she never mentioned appreciating the CM method, but as questionnaire answers reveals, she expresses equivalent standpoints in both the SLV and CM methods regarding appreciation of textbook learning as well as other methods, even if she clearly prefers SLV according to the questionnaire answers in the *perception of appreciated learning settings* section. She also comments, that she believes both she and many others appreciate the systematic nature of regularity in lesson design. They felt comfortable knowing the design of the different lessons, the work approach, goals and expectations. She also mentions the tests of mathematical skills as a positive experience because of the pretesting giving some idea of the knowledge goals of the section and being able to recognize questions and examples from the booklet and connections with the exploratory tasks in lesson two. Her class had the CM after the SLV method and she comments on the monotony of the CM and that the whole class would have had better working conditions if they had kept at least some elements of explorative work in groups or pairs.

Class B

Cecilia (girl)

The student has average mathematical skills and is motivated.

She expresses content with the mathematics education in the 7th and 8th grade, and that the teaching approach, at least in the 8th grade, was varied with elements of collaborative work, exploratory and problem-solving tasks and use of ICT devices when appropriate. The textbook approach in CM did not work well for her and this dislike is also found in the questionnaire results

where she disagrees in with both the statement learning best and enjoying studying in the textbook. When asked what teaching method she would prefer she clearly states the SLV method but stresses appreciating of working in the textbook from time to time (which they did in the first lesson of the week during the SLV method). The presentations in the third weekly lesson during the SLV method became somewhat cut short as to the class did not have their ordinary teacher all the time. She comments that the discussions and argumentations might have been more productive if their ordinary teacher had been present.

Diana (girl)

The student has average mathematical skills and is motivated.

She expresses a feeling of best learning and enjoying the traditional approach as in the conventional method of teaching, preferring individual work in the textbook. This standpoint is verified in the questionnaire where the student agrees or strongly agrees with the statement best learning from textbook settings and disagreeing with best learning from other, more dynamic settings. The student becomes somewhat passive during the second lesson in SLV where the pairing and grouping was meant to invite to collaborative learning. She expresses feeling awkward having to work with other students with whom she would not work with under normal circumstances and this caused her to underperform. The compulsory element of presenting results and discussing solutions in lesson three did not appeal to her and this perception is reflected in increased anxiety levels because of the SLV method. She is aware that to some extent it is her own doing that the presenting results becomes an uncomfortable experience because of her, anonymous role in the collaborative work and therefore not feeling confident during presentations. Even though the student firmly states best appreciating the CM method she also stresses the necessity of the different teaching approaches to satisfy ways that stimulate a majority of students' way of learning. She also stresses the necessity of problem-solving and explorative settings to prepare for future real-life situations.

Interview Comments

6.4.2 TEACHER INTERVIEW

As mentioned, the teacher interview was not primarily intended to evaluate effects of the different teaching methods but rather to establish background information of the different classes' previous mathematics education, the teachers' experience of teaching the classes and to discuss the general teaching approach of the SLV method and how it may be further developed.

The teachers were interviewed together on the same occasion. Questions regarding previous mathematics education were foremost answered by the teacher with best inside experience of each class. One teacher had taught one of the classes in mathematics and science since the 7th grade, while the other teacher had taught the other class in the same subjects in the 7th but not in the 8th grade. Both teachers were well known and familiar to both classes.

The interview started with questions regarding the teachers' perceptions of the classes' experience of previous mathematics education. The one teacher expresses Class A's mathematics education in the 8th grade as "chaotic" and the other teacher as "almost chaotic". A varied teaching approach was attempted, and the teacher's intentions were good, but the student-teacher relation (neither of the teachers participating in the intervention) just did not work out. By early autumn all students, including those more motivated, had lost track of goals and aims. In consultation with school management another adult was relocated to the class during mathematics lessons, but this had little effect on improving their mathematics education even if it had some effect on the situation as a whole. The class was provided with a schedule and detailed plan of what to do in their textbooks. Some experimental, problem-solving elements were scheduled, but because of the deteriorating teacher-student relationship this never took place. The class was not homogeneous, pulling in different directions, and mostly refused to participate in any activity. Only a few students (girls) managed to maintain some kind of self-esteem, ability to concentrate during lessons, and make progress. One interviewed teacher comments that the situation in the classroom could not only be blamed on the teacher's ability to structure the lessons; the students did not see their own responsibility in learning.

The students do not see their responsibility in the failures of the mathematics education in the 8th grade and claim that the teacher alone is to blame for their lack of knowledge.

The experience of previous mathematics education seems quite the opposite in Class B. One of the participating teachers has taught the same class since the 7th grade.

Since the 7th grade we worked with practical elements and have worked with matlab on numerous occasions. We've always tried to vary our teaching approach, so they (the students) are used to varied settings, which meant that the first section of working traditionally caused some anxiety,

The teachers state that Class B is homogeneous and has not had conflicts with either teacher or lesson approach. The students are responsive and curious. They respect and help each other and try their best to contribute to a good learning environment. The teacher comment that it is likely that the class would appreciate their education regardless of teaching approach even if she thought they preferred a varied approach (which they are used to even if not systematic). She also notes that quite a number of students would probably have difficulties studying if they had only applied a conventional textbook approach.

Yes, but I think they think it's fun when the approach is varied and I have examples of students that actually respond very well and that would probably not have made any progress with the conventional approach.

Furthermore, the teachers explain their perception of what they think most students' previous education implies:

Over the years the students have become acquainted with a certain lesson design and then they have to learn a different approach to how to work and it's the same way with the exploratory work.

Both teachers also comment on the effects of the assessment methods that the SLV method provides and how invisible assessment of certain abilities has been for the students

I think that when we have a math lab I have the opportunity to listen and watch. I can assess abilities in a different way and it is a "fun activity", but they have to know how to

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communicate and actively participate because it's these abilities and communication skills that I assess. The students are surprised that it's these abilities that are assessed and are not used to this observation type of assessment...

Regarding the yield of the methods the teachers comment:

Interviewer:

I was thinking about how your classes feel now about mathematics compared to before the study? I was thinking about whether the systematically varied method has contributed to anything?

Teacher 1:

Absolutely, I think so. They may not have a positive view of solely working in the textbook, which we also agree is not preferable, but a lot has to do with their own motivational development, and if disregarding some aspects they may have a more positive view of mathematics and that it can be so much more than doing tasks in the textbook.

Teacher 2:

And that the math is so much more than just individual work, I think that's really valuable, and hopefully it's something that they will take with them. And I think of those who felt that they were not good at this individual textbook work, that they had not progressed as far or were less successful, and then they are confronted with doing something practical or collaborative, and you hear them going - oh I understand this- . I think it's absolutely amazing to just walk around and listening to their conversations. They can develop the problems.

And about the benefits of alternating teachers the one teacher comments:

I think we have benefited from working together, having the opportunity to see each other's approach and to help in assessing.

7. DISCUSSION

Before elaborating on questionnaire results and interview comments in the perspective of the intent of the research it seems appropriate to frame the classes' initial differences, which, to some extent, seem to affect the outcome of the teaching methods.

Initial differences are threefold. Firstly there is a significant difference between the classes' experience of previous education. Secondly there is a significant difference between the classes' motivation and desire to learn. This is probably because of previous experience of mathematics education. Thirdly there are significant differences in mathematical abilities, and these differences are certainly, at least in part, a consequence of insufficient opportunities to learn during the 8th grade for one of the classes. Based on questionnaire answers, test results, and interviews I will account for these differences.

Classes A and B have significant differences in initial motivational starting points. Questionnaire answers from the first survey (Class A before SLV and class B before CM) regarding *Interest and Enjoyment* show these differences. As previously claimed, the difference in motivational starting points is probably due to previous educational experiences. Class A has a history of unfavourable mathematics education in the 8th grade. Dissatisfaction with the teacher affected the mathematics lessons to such an extent that some students refused to participate and those that chose to participate mainly got by on their own. Individual motivation differs greatly in the class, and there are not even any small groups of "more motivated students". The few more motivated students work individually and preferably in textbooks as this does not include nor call for collaboration. Class B, however, has been a homogenous group throughout grades 7 to 9. They have had the same mathematics teacher and the teacher has been appreciated and respected for applying a variation of educational methods with good results.

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7.1 RESULTS DISCUSSION

To frame the conditions for the students to objectively relate to the teaching methods the classes' initial differences in previous mathematics education, in relation to the order of SLV and CM, has to be emphasized. Class A has experienced the SLV method prior to the CM and preceding the intervention study Class A was accustomed to a less structured approach in classroom settings and was confined to individual studies in textbooks in a rather lively classroom environment. Class B on the other hand has experienced a varied approach in mathematics education and not been dependent on the textbook. Class B left this varied approach, which they were used to, for the CM method in the first section of the intervention and was thereafter subjected to the SLV method. These substantial influencing factors are further discussed in the general discussion.

7.1.1 CONCLUSIONS

Both classes express low levels of *anxiety* and anxiety levels vary little regardless of the teaching method. Students in Class B are for the most part *interested* and *enjoy* mathematics while students in Class A are slightly less interested. The classes have significant differences in opinions of textbook learning settings. Class B has practiced a more varied education since the 7th grade whereas Class A has mostly depended on the structure of the textbook rather than professional teacher guidance. One can assume that the necessity of structure, independent of teaching method, has been profitable for Class A on the whole. Results show shifts from negative attitudes to positive attitudes during the SLV method and minimal movement in the other direction during CM, which suggests Class A maintain a more positive attitude to their mathematics education in general, regardless of method.

The consistent results from Class B regarding *appreciated learning settings* and *unappreciated learning settings* seem to demonstrate that the method of *systematic lesson variation* is favourable for Class B. Class A, however, seems to have developed a more positive attitude to their mathematics education regardless of method and without preferring either teaching method significantly. Conclusions of the teaching methods proving effective on mathematical skills and achievement are difficult to derive due to the classes' different starting positions in content knowledge. Class B had worked some with the mathematical contents of algebra and rate of change during the 8th grade, while Class A attempted studying the content but ended

up not fulfilling the attempt due to the situation in their mathematics education (see teacher interview results). The test results from the mathematical skills show significant differences in initial mathematical skills between both classes in each section (algebra and relationships and rate of change). These results confirm differences in previous education in the core content. There is a significant increase in mathematical skills in Class A after the conventional method section. This is probably due to the presence of what one could call an *appreciated structure*, independent of educational method, stabilizing the students' mathematics education. The structure and question character in the knowledge test was similar to tasks in the booklets, and the booklet also included beneficial explorative questions in the relationships and rate of change section. Combining these factors may well explain the significant increase in test scores for Class A during the *conventional method*.

Regardless of teaching method applied, all students in both classes improved mathematical knowledge in both algebra and functions. These are positive results and encourage further development of pre and post testing, even if reasons for the knowledge skills progress cannot be deduced from either the *systematically varied lesson* method or the *conventional method*. Revisiting the initial research aim and questions, identifying results that can bring some light on impact of the teaching approaches is necessary. In many ways, for rather obvious reasons, there are minor noteworthy changes in opinions in both classes. Changing an educational setting takes time and patience and bearing this in mind provides something of an explanation to the minor shifts in attitudes from this short-term study. There are, however noteworthy changes which, to some extent, tell us something about both changes of perceptions of motivation, desire to learn, and if either teaching method has been perceived as favourable.

7.1.2 RESULTS INDICATING THAT THE STUDENTS PERCEIVE THE SLV METHOD AS A FAVOURABLE TEACHING METHOD

Class A

- Increase (4) in *looking forward to mathematics lessons* (6 to 10).
- Increase (8) in *being content with the applied teaching approach* (8 to 16).
- Increase (5) in *disagreeing with mathematics lessons being boring* (4 to 9).

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Class B

- Increase (3) in *I feel that I am learning* (most/every lesson 11 to 14).
- Increase (2) in *looking forward to mathematics lessons* (16 to 18).

Furthermore, two of the interviewed students comment that the explorative second lesson provides a new stimulating way of working, and even mention that several other students felt the same. Targeting influence in the affective domain the emotional experience of *being content* with the teaching approach and progressive aspects of *looking forward to* in the sub domain attitude can be found in both classes.

7.1.3 ELABORATION ON IMPLICATIONS OF RESULTS THAT MAY CONTRADICT THE SLV METHOD AS A FAVOURABLE SETTING.

Casting a shadow on the appreciation of the SLV method are aspects of what actually are SLV specific features. The exploratory, problem-solving and collaborative aspects in the SLV method seem to have negative effects in Class A, at least regarding the number of students who distanced themselves from these components. Even though the majority of students in Class A maintain a “positive attitude” in *relations to other learning methods* (other than the textbook) there are noteworthy changes in opinions:

Interest and enjoyment

- Decrease (2) in *interested in the things I learn in mathematics* (algebra) (11 to 9).

Relations to other learning methods

- Decrease (2) in *enjoying working with practical problems and experimenting* (18 to 16).
- Decrease (2) in *enjoying working with other students* (18 to 15, N=20 post).
- Decrease (2) in *would like to work more with problem-solving* (16 to 13, N=20 post).

All changes can be considered minor but all the same in some sense affecting a number of students’ appreciation of a varied approach with collaborative and problem-solving elements.

Class B express similar opinions regarding SLV method specific features, even if these attitudes concern different areas of interest. *Anxiety* levels increase for a number of students even if the majority of students express low anxiety levels or no anxiety:

- Increase (3) in *feeling unsure when doing a mathematical problem* (6 to 9).
- Increase (3) in *worry about presenting results in front of others* (5 to 8).

As mentioned, the nature of presenting results in group and whole class settings during the third lesson in SLV is probably the reason for the anxiety increase. Individual expectations and lack of prior experience in presentation and argumentation technique can contribute to this anxiety. The Lgr 11 (Skolverket, 2011b) curriculum states similar skills to be assessed and without providing opportunities to practice, it becomes impossible to develop these skills. Getting acquainted with applying these techniques on a regular basis, with professional guidance and feedback are possible factors of development and success. Furthermore, Class B shows an increase (3) in *learning best studying my textbook* (4 to 7), where two of the students are identified as both changing opinion about *worry about presenting results in front of others* and *feeling unsure when doing a mathematical problem*. Like in most classrooms, some students thrive in the simplicity of individual work in the textbook without the necessity of collaborating with others. These opinions may well have been aroused by the nature of the SLV method and the enforced cooperation. As interview results show, the pairing of the students for work in the problem-solving, exploratory and collaborative second lesson may have led to the more capable and resourceful students taking a greater part in solving tasks and this in turn disturbing their otherwise individual approach. At the same time, students interviewed have commented on the advantages of working with peers and learning from one another.

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7.1.4 RESULTS INDICATING THAT THE STUDENTS PERCEIVE THE CM METHOD AS A FAVOURABLE TEACHING METHOD

The most significant indication of Class A perceiving the conventional method as favourable is the increase (6) in *I learn well from our teaching method* (11 to 17). These opinions are confirmed in answers to the opposite question *not learning well with this teaching method* where the number of students decreases by 5 (from 8 to 3). In addition to the indications of the CM method as *promoting learning and enjoyment* there are also increases in the overall area of *interest and enjoyment*;

- Increase (4) in *enjoying trying to solve tasks* (11 to 15).
- Increase (3) in *interested in things I learn in mathematics* (relationships) (9 to 12).

Class B had experienced a more varied teaching approach in the 7th and 8th grade and their attitudes towards the textbook-dominant settings can be observed in the number of students not expressing learning best from textbook studies (18) during the CM. All the same, there is an increase (2) in *enjoying learning best by studying the textbook* (4 to 6) during the CM approach. This may indicate dissatisfaction with previous methods and discovering a more preferable method once exposed to the opportunity. In any case, far-fetched conclusions of causality should not be drawn from the numerous outcomes. In addition to the textbook learning settings, Class B increases the number of students in *interest and enjoyment* expressing appreciation of the CM method of teaching;

- Increase (4) in *looking forward to mathematics lessons* (12 to 16).
- Increase (3) in *enjoying trying to solve tasks* (16 to 19).

7.1.5 ELABORATION ON IMPLICATIONS OF RESULTS THAT MAY CONTRADICT THE CM AS A FAVOURABLE SETTING.

There are, however, many implications of the questionnaire results that contradict CM as being favourable settings and an influencer of motivation and desire to learn.

The initial increase in *interest and enjoyment* from Class A's first section (SLV) reverses in *looking forward to mathematics lessons* for four (4) students switching to disagreeing with the statement *looking forward to mathematics*

lessons during the CM. Three of these four students are those who changed to agreeing with the statement *looking forward to mathematics lessons* during the SLV section. Furthermore, the significant increase (8) in being *content with the lesson design* in the SLV section decreases by two students during the CM and one of these students is among those changing to a favourable opinion during the SLV method. Only two students change opinions, which leave a majority of students (14 of 20) in Class A “staying on the content side after the SLV and CM”, which implies both methods as stabilizing elements and providing an appreciated structure in comparison to previous experience. Regardless of teaching approach, on the whole, the intervention has contributed to increase self-confidence, motivation and desire to learn, even if neither of the specific methods stands out as the cause.

Several students in Class B change their perception of learning well or not learning well from the CM teaching. The increase of 4 students *not learning well from this teaching method* (5 to 9, with the post survey containing only 19 student’s answers) and confirmation of opinions in the opposite questions, *I learn well from our teaching method*, decrease. There is also a significant decrease in *content with the lesson design* (20 to 11), which, if not implies SLV as favourable, then certainly express dissatisfaction with the CM.

- Decrease (4) in *I feel that I am learning* (most/every lessons, 15 to 11).

Regarding anxiety, the situation is the opposite from the SLV method approach. Even if there were a few students who initially expressed anxiety for presenting results during the SLV method, a decrease in anxiety (4) in *worry about presenting results in front of others* (9 to 4) can be observed. This could be expected and is probably explained by the nature of the CM not containing elements of presenting results in front of others.

7.1.6 THE MATHEMATICAL SKILLS TESTS AND ASSESSMENT

As stated, the initial differences in previous education in content areas have caused a number of problems regarding evaluation. Furthermore, the questions in the tests of mathematical skills in the relationships and rate of change section were very similar to those of the standard question arrangement in the booklets. This probably caused the significant increase in Class A’s results because they worked in the booklets with the conventional method in this section. The worksheet specific tasks were also not aligned with

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test questions and the mathematical test design did not provide any access to analysis of collaborative discussion and problem-solving abilities. In this sense, mathematical skills development from the actual collaborative, exploratory and problem-solving lesson design aspect in the SLV method (lesson 2) cannot be determined. Teacher interview comments stress the necessity of test design and both teachers agree that the tests were less favourable regarding providing evaluation of problem-solving and reasoning abilities. However, both classes exhibit knowledge development during both methods and the minor development in Class B in the *relationships* and *rate of change* section is best considered on the basis of the class having previous experience of the topics. The selection of exploratory tasks and the design of the SLV method provided for visible and listening assessment opportunities and are mentioned in the teachers' interview as an asset.

7.1.7 SUMMARY

The SLV method of teaching is compared with the common, national teaching approach in compulsory school mathematics, that of the textbook. This comparison, or at least the effects of the SLV approach vs. the textbook approach, is limited due to the classes' differences in experienced methods of mathematics education. Class A left a shattered and unstable learning environment in mathematics by the end of the 8th grade and spent four weeks in the beginning of the autumn semester working according to the SLV method and then proceeded with the Conventional Method. Class B left the familiar, stable, somewhat varied and dynamic mathematics education in the 8th grade, for the framed and monotonous Conventional Method. Next they were exposed to the SLV method, which in some ways resembles their previous mathematics education design from the 8th grade. The classes' perceptions of learning settings and implications of the one method being superior to the other are to be considered from these different perspectives.

Regarding aims of influence in affective domains there are none the less important findings from the intervention: During the SLV method approach Class A's students increase in *interest* and perception of *enjoyment*, while Class B's students increase in *interest* and *experience of learning*. A majority of the students express appreciation of the SLV specific elements such as *enjoying working with practical problems and experimenting* and *enjoying working with other students*, and there are minor changes in these perceptions. During the CM approach, Class B increase in perception of dissatisfaction with the teaching method while decreasing in *experience of learning*.

Isolating each class within each class framework of prior settings, Class A exhibits an overall positive progression in *interest* and *enjoyment* regardless of method, and this is well in line with the intention of providing for a more pleasurable and stimulating mathematics education than in earlier grades. Class B exhibits a clear dissatisfaction with the CM method.

7.2 GENERAL DISCUSSION

7.2.1 APPROACH TO TEACHING METHODS AND LESSON DESIGN

The overall picture of how teachers go about teaching mathematics and how students expect teaching in mathematics to be conducted is that of textbook-dominated content, even if elements of variation are included (Engvall, 2013; Samuelsson, 2007; Skolverket, 2003). The dominant position of the textbook and its use therefore implies that variation in practices is also confined to the variance and work approach defined in the same. The normalizing of textbook use and allowing the textbook to form the structure of the teaching and what is to be learnt has been going on for many years (Johansson, 2006). This textbook dominance was not established without reason or by chance, and the extensive use of the mathematical textbook is easily explained. The textbook provides a way to organize and get an overview of what has to be learned and, at best, why and when it is to be done. It provides a simple structure that makes work and planning easier for both the student and the teacher. In agreement with the textbook normalizing the structure of the lesson results from the application of both methods show minor changes in opinions about *relations to learning by means of the textbook* and changes that do occur tend to move towards textbook appreciation. The nature of *systematically varied lessons* does not ignore the structural needs nor belittle the benefits of maintaining textbook elements but provides opportunity for a systematic approach, varied context and an alternative way of working with mathematics as a school subject. The systematic approach is simply to provide normalization of variation in the sense of the students being accustomed to different work approaches in different lessons. The benefits, besides adjusting opportunities to learn and develop skills in alignment with relevant policy documents (Lgr 11), are those of stimulating motivational aspects such as desire to learn, interest and enjoyment, which, at least to some extent, is shown by the results from the intervention. The necessity of such educational engineering enterprises should appear evident in our enlightened world of evidence of

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deteriorating results in mathematics (Skolverket, 2008, 2012b, 2013). One can further assume that the mathematical ability goals in the national curriculum (Lgr 11) are specified with regards to what skills are expected to be necessary in our future society, meaning adult “working, living and social interaction” settings. Practicing these necessary abilities has to be included in classroom educational practices. As concluded by Boaler (2001): students learning a set of practices from a conventional teaching approach will confine their knowledge to that specific classroom practice.

If they only ever reproduce standard methods that they have been shown, then most students will only learn that particular practice of procedure repetition, which has limited use outside the mathematics classroom. (Boaler, 2001, p. 9)

But she also concludes that students who had engaged in different practices in the classroom were able to use mathematics in different “out of the classroom” situation, more similar to “the real world”.

At the same time we cannot neglect mathematical content knowledge and procedural skills, which are necessary foundations for developing mathematical abilities. Thus a variety of settings and teaching approaches including various practices may well be appropriate to meet the demands of post- compulsory education and adult life.

Whose responsibility is it to shape the educational settings to meet these demands? The question is rather rhetorical and the answer obvious. Initially, it is the responsibility of the overall school administration, to set time frames, schedule, provide facilities and sufficient resources. But it is primarily up to the individual mathematics teacher(s) to provide for settings that allow opportunities to learn. There are numerous descriptions of favourable teacher “qualities”, some of which are well worth mentioning in the context of this study.

Franke et al. (2007) note that changing mathematical classroom practice has proven to be a challenge.

Simply using manipulatives, putting students in cooperative groups or asking higher order questions does not lead to classrooms that support the development of mathematical understanding. (Franke et al., 2007, p. 229)

What is essential is how the above mentioned methods provide teacher and student involvement in these approaches, and involvement requires belief in the implementation of methods. Belief in changes in teaching approach having effect on learning outcome initially lies with the teacher. Teacher involvement in educational development relies on these initial beliefs and is probably maintained if encouraged by results showing development associated with the implementation of methods.

As mentioned in the student interviews, both participating teachers are appreciated and highly respected by their students. Hattie (2013) emphasizes the importance of a good teacher-student relationship for success in influencing student achievement. Without both teachers having good relationships with their students, the study could never have taken place. The exploratory, problem-solving second lesson and the discussion settings of the third lesson in SLV method presupposed the ability to “pilot” the sessions, maintaining mathematical content track, inviting reasoning and correction. These settings also provided opportunities for the teacher to assess knowledge and abilities beyond the methods of individual written tests. The teachers comment on the benefits of observing the students’ conversations and interaction during the two collaborative lessons of the SLV method. These observations provide assessment well in line with a number of abilities to be evaluated.

Maintaining guidance during the third lesson of SLV method (among other things in the study) is well in line with what signifies efficient teachers as defined by Pantziara & Philippou (2007):

Efficient teachers press students for understanding, they treat students’ misconceptions in mathematics and they use different visual aids in order to make mathematical learning more interesting and meaningful. Additionally, they give students opportunities to engage in mathematical conversations, incorporating students’ erroneous solutions into instruction and giving substantive feedback rather than scores on assignments. (Pantziara & Philippou, 2007, p. 58)

According to Yackel and Cobb (1996), one of the teacher’s roles in developing sociomathematical norms in the inquiry based classroom is that of encouraging the students to compare solutions and facilitate mathematical discussion. These comparisons were well provided for in the third lessons of the SLV

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series assuming the teachers were able to maintain mathematical discussions and thereby framing what are mathematically acceptable explanations.

The open atmosphere for discussing, tolerating and correcting is mentioned several times in interviews regarding presentations of solutions. Both teachers emphasize the third, presentation-lesson's positive aspects in terms of the "room for failure", corrections and discussions toward agreement on favourable problem-solving approaches and ways of solving tasks. However, the art of maintaining interest may well lie in the teachers' ability to steer presentations and results in such a way that they suggest new questions or challenges. As one interviewed student comments: she felt that these third lessons were something of a waste of time once one or two groups had presented. This is probably because of the confinement of the actual tasks from the second lesson, but at the same time it pinpoints the necessity of well-chosen tasks where several methods and solutions are possible, as well as the teacher's ability to "keep the presentations and discussions alive".

In terms of further advantages of the "open atmosphere" reasoning and discussing results in the third lesson in the SLV method, Bauersfeld (1992) points out the following regarding "open atmosphere questioning"

The regular use of such questions in the mathematics classroom can provide opportunities for many students who never get a chance to make explicit their ways of constructing, analyzing and reasoning.

(Bauersfeld, 1992, p. 468)

Modelling this questioning praxis is well in line with the national guidelines and educational board policy documents on abilities required in today's society (Skolverket, 2011b), and was therefore used by the teachers in the presentation lessons. Even though the students interviewed have mentioned disadvantages of presenting results in class, the less appreciated elements were those of lack of development opportunities due to limited questions and problems with too few variations of solutions. In addition to this area of adaption, the lesson approach with its reasoning and discussions was appreciated according to three of the four students interviewed.

Löwing's (2004, pp. 263-264) discussion regarding working methods, students differing in progress, the choice of pairing or grouping and the lack of routine work implies what influences the possibilities to develop successful alternative teaching approaches. One of the interviewed students mentions the

important aspect of putting work groups and pairs together accordingly. The student mentions feeling awkward having to work with students that she otherwise would not work with, and this causing her to underperform. This implies the importance and careful choices the teacher has to do. Pairing and grouping of equals or unequals in abilities and knowledge? Both kinds of pairing have different advantages and may best be seen as adequate in different situations with different goals. Regarding important points about learning intentions, teaching approach and planning, Clarke, Timperley and Hattie (2003) noted that students work at different intellectual levels and with different goals. It is therefore necessary to adapt learning activities and their intentions to all students. Adjusting the pairs and groups constellations does not oppose the SLV method because the work *method* and approach are the same independent of human components. The intended outcome, however, has to be taken into account, and this creates opportunities for adjusting levels of knowledge development according to the tasks and not to the method.

Löwing (2004) furthermore comments on the exploratory settings and the lessons she observed as including activities rather than opportunities to learn. The teachers did not possess a mathematical language to connect the abstract content knowledge to the concrete, physical experiences in the mathematical exploratory settings. These findings are consistent with findings of Rystedt & Trygg (2010) regarding exploratory mathematics with teachers losing track to the actual mathematical content goal and the students perceiving it as “fun mathematics” as opposed to “real mathematics” and happenings rather than regularities. Even though the SLV method proceeded for 4 weeks, which in some sense creates regularity, one student I interviewed commented that other students “thought we wasted time playing”, suggesting that the exploratory, problem-solving lessons were time wasters and not real mathematics. In terms of mathematical skills tests results we can conclude positive knowledge development despite elements of what students call “fun maths”. All the same, these comments imply connections to Rystedt & Trygg’s conclusions, but also imply the inertia in changing teaching approaches. The necessity of maintaining mathematical content and making it visible to the students is important even in the absence of a textbook and working in other settings.

There needs to be some un-learning of prior study skills before new learning can occur. (Hattie, 2013, p. 191)

Discussion

7.2.2 MOTIVATIONAL ASPECTS

Revisiting the research questions and aims of the study, the implementation of the SLV method was performed with the intention of finding favourable teaching settings, i.e. appreciated, stimulating, and enjoyable experiences of mathematics education. The purpose of finding these elements was to see if influence in the affective sub-domain *emotions* (McLeod, 1992) triggers motivational aspects where interest and enjoyment are foundations for intrinsic motivation. Results of perception regarding these affective issues, such as being content with the SLV teaching approach, looking forward to lessons and not considering the lessons boring during the SLV method indicate some affective impact which in turn may affect beliefs and motivation to learn. Motivational attitudes are stable over time (Middleton & Spanias, 1999) and rather difficult to change. Critical elements seem to be the student's first encounter with an academic activity and whether arousal (stimulation such as challenge, curiosity and imagination) and control requirements (dimension of freedom and severity) are met. If so, the student may choose to include the activity among his or her interests. Once included among his or her interests, the student will engage in the mathematics tasks. This boils down to the necessary, sensitive and careful decisions that have to be taken by the teacher regarding choices of activities that arouse stimulation. While stimulating motivation to learn by choice of activities, one has to take into consideration and remember the mathematical intentions. Stimulating the enjoyment of the lesson and lesson approach does not directly stimulate their motivation to learn what the lesson is designed to teach (Kilpatrick et al., 2001), but should always provide for an opening to change motivational attitudes. Results showing maintained enjoyment, increase positive expectations and being content with the lesson design during the SLV section for Class A are in line with Middleton & Spanias (1999) arousal requirements for changing motivational attitudes.

7.2.3 NORMS AND SETTINGS

Norms and adaptations to classroom and teaching settings are established over a long period of time. Adaptation to how we go about teaching mathematics and therefore also expectations of how mathematics education is to be done are established in the early school years. If accustomed to certain teaching approaches then these approaches are expected and changes must be carefully motivated. This normalization in approach implies a revision of teaching methods and reflecting on if, how, and when to attempt changes in lesson approach. Government initiatives towards a more varied approach in teaching and maintaining regularity in variation are evidently not decided without reason, and these changes, at least to some extent, will in the future affect norms, attitudes and settings in mathematics education. What is expected and normative today may well not be in a near future. As mentioned, the textbook-dominance (Johansson, 2006) may well decline in favour of other methods, and students expect regularity in variation of lesson design, tasks, and teaching methods. This will of course not come about without reason and only assuming that these methods are successful and appropriate.

Revisiting the notion of learning milieus from Chapter 2 and framing of didactical contracts (Brousseau et al., 1997), norms changing in the mathematics classroom also implies the changing, adaptation and acceptance of the changes in the didactical contract. There has to be a balance and harmony in the learning *milieu* regarding how tasks are organized, how the textbook is structured and used, and how student-student and student-teacher communication proceeds. Changing methods of teaching approach affects this balance and new contracts have to be identified and recognized by all participants if interaction is to proceed without problems (Skovsmose, 2001).

Discussion

7.3 METHOD DISCUSSION

Hiebert & Grouws (2007) accurately point out the vulnerability and weaknesses of comparative studies trying to establish whether “method A is better than method B”. They point out that when one teaching method is claimed to be better than another, it is the quality of the weaker method, or at least how it is described, that defines significance. However, when pinpointing the necessity of describing the different methods in detail, the comparative aspect is that of teaching and learning and not that of teaching and foremost affecting motivational aspects as in this study. All the same, issues of fully describing (and formulating) the different methods and to the extent possible in the social interaction, isolate or control variables, are essential for comparison. Furthermore, they have a perhaps better question in these contexts: *under what conditions is method A better than method B?* This question puts aspects of content specific comparison (algebra, relationships and rate of change) in a new perspective regarding the specific teaching method (SLV and CM). One comparison is that of method specific and the other is that of content specific. Furthermore the question of *under what conditions* is well justified based on results from the study regarding the continuous progression of positive attitudes in Class A regardless of teaching method.

The small-scale setting and the few participants at my disposal justify more adequate qualitative method of evaluation. Even in a larger-scale setting the quantitative approach using questionnaires would have to be fine-tuned to better answer questions regarding perceptions of learning, motivation, and desire to learn mathematics.

Furthermore, it is questionable if the mathematical skills tests serve other purposes than that of providing the teachers means of assessment. Admittedly they serve as measurements of progression in mathematical content knowledge but due to time limitations they were never developed and adjusted to show method specific skills development. The tests were identical in both classes in both teaching methods. Under other circumstances, the testing of skills in the SLV approach could have been adjusted to the collaborative, problem-solving and exploratory setting, extracting other abilities than those visual in the CM.

In *Adding it up* (Kilpatrick et al., 2001) the authors proclaim of examining methods of instruction from the perspective of interaction between

student, content, and teacher, and furthermore how to examine effectiveness through engagement and enactment in solving tasks. The quality of instruction depends on a number of things all, within the whelm of the teacher's responsibility:

The quality of instruction depends, for example, on whether teachers select cognitively demanding tasks, plan the lesson by elaborating the mathematics that the students are to learn through those tasks, and allocate sufficient time for the students to engage in and spend time on the tasks. (Kilpatrick et al, p.8-9)

Considering this statement, we have succeeded in providing quality aspects of elaborating the mathematics, but those of allocating sufficient time on tasks are underestimated regarding necessary time required. Furthermore, some of the tasks chosen are less cognitively demanding. These are all areas of development.

7.4 FURTHER RESEARCH

Regarding the engineering aspects of experimenting in designing mathematics teaching and learning, Artigue (Artigue, 1994) mentions the constraints on a didactical level in the engineering process, that of confronting the methods with reality and engineering after the first experimentation. Bearing this in mind, once again, evaluating, processing “what works”, and developing an approach can hopefully continue and develop from this study. The extent of the national curriculum in mathematics provides possibilities to experiment in approach and to do this content specific. Students seem to consolidate motivational attitudes towards mathematics in junior high school. Motivations are internalized into students' self-concepts, thus affecting how they see themselves with regard to mathematical activities (Middleton & Spanias, 1999), and with regard to these conclusions in addition to the motivational vulnerability, it might be appropriate to experiment, evaluate, and develop favourable teaching approaches before the students enter secondary grades where motivational attitudes are sensitive. The teachers interviewed, who have many years of experience, also mention the 6th and 7th grades as motivationally sensitive, and that favourable varied approaches in teaching mathematics should be identified at the latest in these grades.

Discussion

Wishful thinking would provide for a long-term SLV method application, for one school year at least. Appropriate time span for the systematically varied lesson series needs to be explored regarding different mathematical core content. The instruments for evaluation have to be fine-tuned, exploratory och collaborative tasks carefully selected and the tests of mathematical skills have to be adjusted to better access the problem-solving and collaborative aspects of the lesson design. Does the method and variation of method prove more favourable for different mathematical contents? Sequencing the number of different lessons in the series to allow appropriate time on task and time to present and discuss results are essential issues.

Further research in the development of the SLV method, applying and exploring its effects, could contribute to pursuing sustainable teaching methods in mathematics education.

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APPENDIX A: QUESTIONNAIRE

Undervisningsstudien i matematik ht. 2012

I detta häfte finner du frågor om:

- Att lära sig matematik

Läs varje fråga noggrant och svara så exakt du kan. I den här enkäten ska du svara genom att sätta ett kryss i en ruta.

Om du sätter ett kryss i fel ruta ska du stryka över ditt fel eller sudda ut det som är felaktigt och sätta kryss i rätt ruta. Om du gör ett fel när du skriver ett svar, stryk bara över och skriv det rätta svaret bredvid.

I den här enkäten finns inga rätta eller felaktiga svar. Svara det som är rätt för dig.

Du kan be om hjälp om det är något som du inte förstår, eller om du inte är säker på hur du ska besvara en fråga.

Enkät id; _____

ATT LÄRA SIG MATEMATIK

Tänk på din inställning till matematik: hur mycket håller du med om följande påståenden?

		<i>Håller absolut med</i>	<i>Håller med</i>	<i>Håller inte med</i>	<i>Håller absolut inte med</i>
a)	Jag tycker om att läsa om matematik.	☐ ₁	☐ ₂	☐ ₃	☐ ₄
b)	Det är värt att satsa på matematik för det kommer att hjälpa mig i det arbete jag vill ägna mig åt senare.	☐ ₁	☐ ₂	☐ ₃	☐ ₄
c)	Jag ser fram emot matematiklektionerna.	☐ ₁	☐ ₂	☐ ₃	☐ ₄
d)	Jag tycker det är roligt att försöka lösa matematikuppgifter.	☐ ₁	☐ ₂	☐ ₃	☐ ₄
e)	Det är värdefullt för mig att lära mig matematik, eftersom det förbättrar mina framtidsutsikter.	☐ ₁	☐ ₂	☐ ₃	☐ ₄
f)	Jag är intresserad av det jag lär mig i matematik.	☐ ₁	☐ ₂	☐ ₃	☐ ₄

Appendix A

Tänk på hur det är att lära sig matematik: hur mycket håller du med om följande påståenden?

	<i>Håller absolut med</i>	<i>Håller med</i>	<i>Håller inte med</i>	<i>Håller absolut inte med</i>
a) Jag oroar mig ofta över att det kommer att bli svårt för mig på matematiklektionerna	☐ 1	☐ 2	☐ 3	☐ 4
b) Jag klarar mig bra i matematik.	☐ 1	☐ 2	☐ 3	☐ 4
c) Jag lär mig bäst då jag får jobba på olika sätt i matematik.	☐ 1	☐ 2	☐ 3	☐ 4
d) Jag tycker att matematiklektionerna är tråkiga.	☐ 1	☐ 2	☐ 3	☐ 4
e) Jag lär mig dåligt genom vårt arbetssätt i matematik.	☐ 1	☐ 2	☐ 3	☐ 4
f) Jag lär mig matematik bäst genom att arbeta i matematikboken.	☐ 1	☐ 2	☐ 3	☐ 4
g) Jag trivs bra med upplägget på matematiklektionerna.	☐ 1	☐ 2	☐ 3	☐ 4
h) Jag känner mig oftast osäker när jag löser matematikuppgifter.	☐ 1	☐ 2	☐ 3	☐ 4

i) Jag tycker om att arbeta praktiskt/laborativt på matematiklektionerna.	☐ 1	☐ 2	☐ 3	☐ 4
j) Jag oroar mig över att redovisa mina svar inför andra.	☐ 1	☐ 2	☐ 3	☐ 4

Tänk på dina matematiklektioner: hur mycket håller du med om följande påståenden?

- | | <i>Håller
absolut
med</i> | <i>Håller
med</i> | <i>Håller
inte med</i> | <i>Håller
absolut
inte
med</i> |
|--|-----------------------------------|----------------------------|----------------------------|--|
| a) Jag trivs bäst då jag arbetar i matematikboken. | ☐ 1 | ☐ 2 | ☐ 3 | ☐ 4 |
| b) Jag skulle vilja arbeta mer med kluringar och problemlösning. | ☐ 1 | ☐ 2 | ☐ 3 | ☐ 4 |
| c) Mina matematiklektioner är varierande. | ☐ 1 | ☐ 2 | ☐ 3 | ☐ 4 |
| d) Jag lär mig bra med det undervisningssätt vi har. | ☐ 1 | ☐ 2 | ☐ 3 | ☐ 4 |
| e) Jag trivs med att jobba med andra. | ☐ 1 | ☐ 2 | ☐ 3 | ☐ 4 |
| f) Jag känner att jag lär mig. | ☐ 1 | ☐ 2 | ☐ 3 | ☐ 4 |

APPENDIX B: KNOWLEDGE TEST PRE/POST ALGEBRA

Algebra

Diagnos id; _____

Nu följer ett antal frågor som har med det matematik du kommer att läsa om den närmaste månaden. Detta är endast en diagnos för att få en uppfattning om hur mycket du kan om algebra innan undervisningen startar. Det är viktigt att du försöker svara på så många frågor som möjligt. Gör inga ”gissningar” utan försök att resonera dig fram till ett svar och svara därefter även om du är osäker. Fråga gärna om något är oklart.

LYCKA TILL

1. a) Ringa in vilken av ekvationerna som har lösningen $x = 2$

A: $7 + x = 5$

B: $x - 3 = 5$

C: $3 - x = 5$

D: $\frac{10x}{6} = 5$

E: $3 + x = 5$

- b) Skriv en annan ekvation som har lösningen $x = 2$.

Svar

2. Med hjälp av tändstickor kan man lägga femhörningar i rad.

Antal femhörningar	Bild	Antal stickor
1		5
2		9
3		13

a) Hur många tändstickor behövs om man ska lägga en rad med 4 femhörningar?

b) Hur många tändstickor behövs om man ska lägga en rad med 9 femhörningar?

Appendix B

- c) En tändsticksask innehåller 50 tändstickor. Tänk dig att du ska lägga femhörningar i rad med dessa stickor. Till hur många femhörningar räcker tändstickorna i asken? **Visa hur du löser uppgift c).**
3. a) Anna är 5 år äldre än sin syster Maria. Skriv ett uttryck för Annas ålder om Maria är m år.
- b) Anna har en bror som heter Carl. I år är Anna dubbelt så gammal som Carl. Skriv ett uttryck för Carls ålder om Anna är a år.
- c) Skriv en mening om två personers åldrar och skriv också ett uttryck till.
4. Hur skriver man
- a) hälften av x
- b) 5 mer än x
- c) dubbelt så mycket som x
5. Du vet att $a + b = 6$. Vad blir då
- a) $a + b + 2$
- b) $2a + 2b$
6. 8C ska ha klassfest. De köper in dukar, ljus och servetter för 230 kr. De räknar med att mat och dryck kostar 42 kr per elev.
- Skriv med ord och/eller formel vad festen kostar (y kr) om det kommer x elever på festen.

Svar

--

7. Denna tabell beskriver ett samband mellan x och y :

x	1	2	3	4
y	6	7	8	9

Sambandet kan beskrivas med ord och formel, t.ex.

- Ord; Om man adderar x och 5 får man y .
- Formel; $x + 5 = y$

I a) och b) finns tabeller som beskriver olika samband. Beskriv dessa samband med ord och/eller formel. Skriv gärna på fler än ett sätt.

a)

x	1	2	3	4
y	2	4	6	8

Svar

b)

x	1	2	3	4
y	1	4	9	16

Svar

8. 7 äpplen kostar x kr. Hur mycket kostar ett äpple i genomsnitt?

Ringa in vilket uttryck som stämmer med detta?

a) $7x$ b) $7 - x$ c) $x - 7$

d) $\frac{x}{7}$ e) $\frac{7}{x}$ f) $x + 7$

Appendix B

9. a) Beskriv med ord sambandet mellan längd och bredd i rektangel A.

Rektangel A

- b) Teckna ett uttryck för rektangel A:s omkrets och förenkla det så långt som möjligt.
- c) En annan rektangel, rektangel B, har 3 gånger så lång längd som längden i rektangel A. Skriv ett uttryck för längden i rektangel B.

10. Förenkla uttrycken så mycket som möjligt:

- a) $x + 5 + x$
- b) $5(3x + 7) - 23$
- c) $3a - 5 + a + 3$
- d) $3a - (5 + a)$

APPENDIX C: KNOWLEDGE TESTS PRE/POST SAMBAND OCH FÖRÄNDRING.

Samband & Förändring.

Diagnos id; _____

Nu följer ett antal frågor som har med den matematiken du kommer att läsa om den närmaste månaden. Detta är endast en diagnos för att få en uppfattning om hur mycket du kan om samband & förändring innan undervisningen startar. Det är viktigt att du försöker svara på så många frågor som möjligt. Gör inga "gissningar" utan försök att resonera dig fram till ett svar och svara därefter även om du är osäker. Fråga gärna om något är oklart.

LYCKA TILL

1. Petra springer ett 400-meterslopp. Hon öppnar snabbt men efter 200 m blir hon trött och minskar farten. När hon närmar sig målet spurtar hon.

Rita in i figuren hur farten varierar under loppet.

Appendix C

2. Figuren visar ett koordinatsystem. Punkten A (4, 1) är utsatt.

a) Sätt ut punkterna B (6, 3) och C (0, 3). Sammanbind de tre punkterna genom att dra streck.

b) Vilken figur bildas?

3.

Graferna visar samband mellan "Avstånd från hemmet" och "Tid" för en person som cyklar.

a) Vilka av graferna är omöjliga? Förklara varför de är det.

b) Tänk dig att de andra graferna beskriver en cykeltur.

Valj två av graferna och beskriv cykelturen för var och en av dem.

4.

Anna tar reda på vikt och ålder för fem av klasskamraternas katter. Hon redovisar det i figuren nedan.

- Vilken katt väger mest?
- Vilken katt är yngst?
- Jämför Max och Sluggos vikter. Vad kan du dra för slutsats?
- Jämför Gustafs och Lisas åldrar. Vad kan du dra för slutsats?

Appendix C

5.

Man tömmer en tank genom en kran i botten.
Grafen visar hur vattenvolymen förändras med tiden.

- När började man tömma tanken?
- Hur många liter vatten var kvar i tanken kl 8.40?
- Vad händer kl. 9.00?
- Hur mycket är klockan när hälften av vattnet har runnit ut?
- När rinner vattnet fortast – i början eller slutet? Hur kan du se det?
- Hur mycket vatten rann det ut under den sista kvarten?

6. Vilken linje (färg) hör ihop med vilken funktion?

$$Y=x+2$$

Svar

$$Y=x-2$$

Svar

$$Y=2x$$

Svar

$$Y=-x+2$$

Svar

7.

Studera följande talföljder. Vilket tal ska stå på raden?

a) 1 000 100 10 1 _____

b) 4 9 16 25 _____

c) 13 11 8 4 _____

d) Välj en av talföljderna. Beskriv hur den är uppbyggd på ett eller flera sätt: med bild, ord och/eller formel.

APPENDIX D: EXPLORATORY TASKS

Worksheets from the National Centre of Mathematics Education (NCM) used in lesson 2 of the different sections i the *systematic lesson variation method*.

Agenternas hemliga tal

strävorna

4B 5B

Agenternas hemliga tal

KOMMUNIKATION — UTTRYCKSFORMER — RESONEMANG — ALGEBRA

Avsikt och matematikinnehåll
 Denna aktivitet kan fungera som en introduktion till ekvationer. Eleverna ska få en känsla för att det hemliga talet x är ett obekant men bestämt tal och att ekvationslösnig går ut på att "avslöja" sådana obekanta tal. Eleverna får samtidigt analysera tals egenskaper samt uttrycka dem med symboler och med ett korrekt matematiskt språk.

Förkunskaper
 Känna till grundläggande egenskaper hos tal, som till exempel udda/ jämn och delbarhet.

Material
 Papper, penna och miniräknare (för att kontrollera beräkningar).

Beskrivning

- Dela in eleverna i grupper om 2–5 elever. En av grupperna ska vara agenter. De andra ska vara deckargrupper.
- Agenterna ska tillsammans välja ut ett hemligt tal x , exempelvis mellan 1 och 50, skriva talet på en lapp och lämna den till läraren.
- Deckarna i varje grupp ska diskutera ihop sig om en bra ja- eller nej-fråga. Därefter ställer en grupp i taget sin fråga till agentgruppen som bara får svara ja eller nej. Deckarna ska inte bara gissa på olika tal utan låta ledtrådarna hjälpa dem att komma fram till ett bestämt tal som förhoppningsvis också är det rätta.
- Slutsatsen av varje svar skrivs på tavlan, matematiskt formulerat, antingen av läraren eller av den deckargrupp som ställde frågan. På tavlan kan det till exempel se ut så här:
 $x < 40$
 $x > 25$
 x är ett heltal
- Efter varje svar diskuterar grupperna: Vilka tal kan det vara? Vilka tal kan det inte vara? Vilken fråga kan göra att vi snabbare hittar rätt tal?
- Sedan är det dags för en ny frågerunda och på tavlan kan det kompletteras med:
 x är ett udda tal, x är inte delbart med 5, x är delbart med 3, x är delbart med 9
 och så fortsätter det tills någon deckargrupp hittar det hemliga talet.
- Läraren bekräftar genom att visa den skrivna lappen och skriver som sista rad på tavlan: $x = \{\text{talet}\}$, i detta fall $x = 27$.
- Efter uppföljning får den deckargrupp som avslöjade talet bli den nya agentgruppen.

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strävorna

4B

5B

Ekvationsleken

RESONEMANG – KOMMUNIKATION OCH UTTRYCKSFORMER – ALGEBRA

Avsikt och matematikinnehåll
Denna aktivitet används för att introducera *lösning* av ekvationer.

Förkunskaper
Eleverna bör känna till vad "det obekanta talet x " innebär. Likhetsstecknets betydelse: eleverna måste veta och acceptera att det inte betyder "blir" utan "är lika mycket som". Gör tex 4B5B *Agenternas hemliga tal* eller någon annan liknande aktivitet före denna.

Material
Papper, penna och miniräknare för att kontrollera beräkningarna.

Beskrivning

- Dela in eleverna i grupper om 2–6 elever. Det är bra om det är olika antal elever i grupperna.
- Varje grupp bestämmer sig för ett gemensamt tal som var och en i gruppen skriver på en lapp. Var och en av eleverna representerar sedan detta valda, hemliga tal.
- Gruppen ska också komma överens om en likhet innehållande det valda, hemliga talet. Detta skrivs också på lappen som sedan göms.
- Välj en grupp (kanske den med minst antal elever) som får börja.
- Gruppen skriver likheten på tavlan och ställer upp sig så att likheten stämmer. Det kan till exempel se ut så här:

Grupp 1

$+ 8 = 17$

- Övriga grupper försöker att komma fram till vilket tal som står på tavelgruppens lappar. När någon grupp kommit fram till rätt tal visar eleverna vid tavlan sina lappar som bekräftelse. Gruppen som löste problemet berättar hur de resonerade för att komma fram till svaret.
- Efter uppföljning (se nedan) visar en annan grupp sin likhet på tavlan.

Uppföljning

Grupp 2

$- 10 = 20 - 8$

Grupp 3

$+ 54 = 84 -$

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Algebraisk triangel

strävorna

4B

Algebraisk triangel

FÄRDIGHETSTRÄNING — ALGEBRA — MÖNSTER

Avsikt och matematikinnehåll
Färdighetsträning av summering av bokstavsuttryck samt övning i logiskt tänkande.

Förkunskaper
Grundläggande kunskaper om att hantera bokstavsuttryck.

Material
Spelplan och sex kapsylar. I respektive kapsyl ska följande stå skrivet: a , $a + d$, $a + 2d$, $a + 3d$, $a + 4d$, $a + 5d$.

Beskrivning
Se elevbladet. Kapsylerna ska placeras i ringarna så att de tre sidornas summor blir lika.

Introduktion
Beskriv reglerna och förklara hur summan ska vara lika på triangelns sidor.

Uppföljning
Vilken är den största möjliga summan? Varför? Vilken är den minsta möjliga summan? Varför? Finns det andra summor? I uttrycken ovan är det bara ett a i varje. Kan du skriva uttryck med olika antal a n så att summan ändå blir den samma längs varje sida?

Variation
Variera de variabler som används. En enklare variant av denna aktivitet finns i Strävorna-aktiviteten *Magisk kvadrat*.

Utveckling
Låt eleverna göra nya bokstavsuttryck. Spara varje kapsyluppsättning i en egen zipplåse. Gör nya spelplaner, som t ex blommor och stjärnor.

Att läsa
Bergsten, C., Häggström, J. & Lindberg, L. (1997). *Algebra för alla*. (Nämnamn: TEM A). NCM, Göteborgs universitet.

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strävorna

9BF
5B

Vad muttrar du om?

MODELLER — KOMMUNICERA — UTTRYCKSFORMER — ALGEBRA — PROBLEMLÖSNING

Avsikt och matematikinnehåll

Eleverna får identifiera variabler samt sätta upp och lösa ekvationssystem i två snarlika problem. I ena aktiviteten ska enstaka muttrar viktbestämmas trots att de är dolda i påsar. I den andra är det bultar, muttrar och brickor som ska viktbestämmas. Först beskrivs *Mutterpåsar* och på följande sidor *Bultar och muttrar*.

Mutterpåsar

Förkunskaper

Areaberäkning och viss vana vid ekvationslösning.

Material

Våg som visar tiondels gram, linjaler samt påsar med muttrar. Varje grupp ska ha två olika stora påsar med olika antal muttrar av samma storlek. Dessa påsar tillverkas i förväg. Klipp i kraftigt tyg ut en bit med måtten $20\text{ cm} \times 15\text{ cm}$ samt en med måtten $24\text{ cm} \times 15\text{ cm}$. Vik dessa på mitten och sy ihop med sick-sack så att det blir två rektangulära påsar med måtten $10\text{ cm} \times 15\text{ cm}$ respektive $12\text{ cm} \times 15\text{ cm}$. Innan du syr igen den sista sidan stoppar du in exempelvis fem muttrar i den mindre påsen och två muttrar i den större påsen.

Beskrivning

Låt eleverna arbeta i grupper om 2–3 personer. Uppgiften är att bestämma vikten på en mutter. Varje grupp ska ha två olika stora påsar med olika antal muttrar i. De får känna på påsen och bestämma hur många muttrar det är i varje påse samt mäta hur stor påsen är. Genom att väga de båda påsarna kan de ställa upp två ekvationer där exempelvis x är antalet gram en mutter väger och y är hur mycket tyget väger per areaenhet.

Introduktion

Placera eleverna i grupper och ge dem mutterpåsar. Den enda information de får är att de ska bestämma vikten på en mutter utan att öppna påsen.

Uppföljning

Följ upp i helklass de olika metoder och variabelsättningar grupperna har använt sig av.

Variation

Med fler eller färre ledtrådar kan aktiviteten fungera på olika stadier. Det blir tex en enklare uppgift om muttrarna ligger i plastpåsar där vikten på påsen blir försumbar. Öka antalet variabler genom att lägga i skruvar eller brickor tillsammans med muttrarna i påsarna.

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Biografer

strävorna

9E
5E

Biografer

BIOLOGI — FUNKTIONER — REPRESENTATIONER — TOLKNING

Avsikt och matematikinnehåll

Eleverna ska gruppvis tolka grafer och koppla samman dem med tillväxt av olika organismer. Ämnesövergripande arbete sker genom den nära kopplingen till biologi.

Förkunskaper

Eleverna måste vara bekanta med några olika funktioner.

Material

Varje grupp får en uppsättning av grafer på elevsidan. Plocka fram eller be eleverna ta med sina biologiböcker. Internetsökning efter fakta kan också bli aktuellt.

Beskrivning

Tolkningen av graferna är en utmaning som inte klaras av på ett ögonblick. Gruppens uppgift är att koppla olika organismers tillväxt till graferna på elevsidan. Graferna är inritade i ett koordinatsystem med ograderade axlar. Eleverna ska sedan föreslå en skala på axlarna för att graferna ska bli troliga. De olika graferna behöver inte ha samma storhet på axlarna.

Ledtrådar på vägen kan vara att be dem diskutera var på kurvorna det sker en långsam tillväxt respektive snabb tillväxt. Låt eleverna ta en vanlig organism och rita upp tillväxten och se om den kan kopplas till någon av de givna graferna. Om x-axeln är en tidsaxel, vad kan y-axeln visa? Höjd, längd, massa eller population?

Introduktion

Se till att eleverna är införstådda med att de kan behöva hjälp av annan litteratur och sökning på internet.

Uppföljning

Möjliga tillväxter för respektive kurva skulle kunna vara att den svarta representerar ett typiskt däggdjurs tillväxt. Den röda skulle kunna beskriva ett krädddjurs tillväxt (där skalet begränsar tillväxten men då kräftan ömsar skal kan växa igen). Den gröna skulle kunna beskriva någon plantas tillväxt (där plantan frodas på våren men ligger i träda under höst och vinter) och den blå tillväxten av en bakteriepopulation. Eleverna kan komma att finna andra möjligheter som är minst lika riktiga som de här angivna förslagen.

Utveckling

Låt eleverna diskutera om de kan komma på några andra tillväxtkurvor för någon organism. En annan viktig fråga värd att diskutera är vilka funktioner som skulle kunna passa till de olika graferna?

Ursprung

Åktiviteten är inspirerad av ett problem från NRICH, se nrich.maths.org.

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strävorna

1E
5E

Skostorlek

FUNKTIONER – TOLKNING – PROBLEMLÖSNING – SAMBAND

Avsikt och matematikinnehåll
Att låta eleverna undersöka och tolka en enkel graf.

Förkunskaper
Grundläggande kunskaper om funktioner.

Material
Måtticka för barns skostorlekar finns att köpa eller få i skobutiker. Måtticka finns också att kopiera, se elevbladet.

Beskrivning
Arbetsgång beskrivs på elevbladet.

Introduktion
Eleverna kan jobba enskilt och får var sin måtticka. Arbetsgången är tydlig och lätt beskriven på elevbladet.

Uppföljning
Jämför i klassen om ni fått samma funktion och vad k - och m -värdet betyder i detta sammanhang. Det kan vara intressant att föra en diskussion om skillnaden mellan verklighet och matematisk modell. I modellen kan vi till exempel använda negativa tal, men i den verklighet som denna uppgift har är inte det tänkbart.

Variation
Hitta andra funktioner i reklam för telefonabonnemang, elräkningar eller dylikt.

Erfarenheter
På teoretiska gymnasieprogram (N och Te) räcker det vanligtvis att endast ge eleverna måttickan och be dem ta fram förhållandet mellan skostorlek och fotlängd, medan på grundskolan eller andra gymnasieprogram behövs lite mer diskussion i klassrummet innan eleverna jobbar enskilt.

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Vasgrafer

strävorna

5E

Vasgrafer

UTTRYCKSFORMER — SAMBAND — FÖRÄNDRING

Avsikt och matematikinnehåll

Elever får arbeta med en aktivitet där matematikens olika uttrycksformer blir synliga. Först ska de arbeta med en verklig situation. De ska studera hur vattennivån i vaser och flaskor med olika form förändras allt eftersom de fylls med vatten. Värdena skrivs in i en värdetabell och eleverna ska rita motsvarande graf. Grafen kan sedan användas på andra sätt, tex tolkas som en beskrivning av en annan händelse.

Förkunskaper

Att kunna göra en värdetabell och använda ett koordinatsystem.

Material

Vaser och flaskor, hushållsmåttstaser, trattar, linjaler, eventuellt vattenfärg och blompinnar samt tillgång till vatten och avlopp.

Beskrivning

Använd vaser och flaskor av skilda former och storlekar. De ska fyllas med vatten och minst 10 delfyllningar behövs för att eleverna ska få tillräckligt många värden att arbeta med. I genomsnittliga vaser är det enklast att mäta utvändigt hur vattennivån stiger. Med lite färg i vattnet blir det enklare att se hur högt vattnet har stigit. Gör eleverna uppmärksamma på att de behöver ta hänsyn till tjockleken på vasens botten. Är vasen inte genomsiktig mäts vattennivån invändigt, tex med hjälp av en torr blompinne.

Innan eleverna börjar fylla på vatten och mäta, ska de rita hur de tror att grafen kommer att se ut. De ska sedan under tiden de fyller på vatten och mäter göra en värdetabell där volym och vattenytans höjd skrivs in. Därefter ska de rita en graf till varje vas och flaska som de fyller.

Introduktion

Inled arbetet med att ställa fram och tillsammans uppskatta volymen på varje vas och flaska. Ordna dem efter volym, från minst till störst. Diskutera även vilka olika rymdmått vi brukar använda och vilket mått som är mest lämpligt till några enskilda vaser eller flaskor. Diskutera också lämpliga graderingar på axlarna.

Uppföljning

Diskutera graferna när de är färdigritade. Vad kan en graf berätta? Finns det olika typer av grafer och vad är i så fall gemensamt eller vad är det som skiljer? Vad är fördelen med grafer jämfört med andra uttrycksformer? När används grafer? Hur stämde elevernas gissning med den färdiga grafen? Vilka slutsatser kan vi dra då vi jämför vas eller flaska med grafen?

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45
9E

Bungeedockan

RESONERA – MODELLERING – MÄTNING – PROBLEMLÖSNING – STATISTIK – FUNKTIONER

Avsikt och matematikinnehåll
Bungeedockan handlar om att anpassa längden på en lina av gummisnoddar så att den inte blir för kort, då blir hoppet lite tråkigt, och inte för långt, det är inte bra för hälsan. Denna aktivitet kan innehålla mätning, statistisk behandling av mätdata i tabell, linjära funktioner och koordinatsystem, anpassning av rät linje till mätpunkter och medelvärde. Det går att anpassa aktiviteten till alla åldrar.

Förkunskaper
Längdmätning, medelvärde och koordinatsystem.

Material
Gummisnoddar (alla av samma sort), måttband, maskeringstejp, barbiédocka eller liknande.

Beskrivning
Se elevbladet.

Introduktion
Diskutera bungeehopp. Ställ frågor som: Hur viktigt är det att linan har rätt längd? Varför är det viktigt att inte ljugas om sin vikt? Vad är viktigt när man ska hoppa säkert, dvs inte för lång lina och roligt, dvs inte för kort lina? På nätet finns många olika filmklipp att visa för klassen innan ni gör aktiviteten. Visa eleverna hur man gör en kedja av gummiband och hur man fäster en docka i änden av kedjan.

Uppföljning
Klassen kan få testa hur väl deras resultat stämmer vid ett riktigt högt hopp. Om det finns möjlighet för ett hopp från tex en balkong, så kan eleverna få gissa hur lång linan ska vara för det "bästa" hoppet. Sätt gärna en vattenbalja under uthoppet.

Variation
Använd dockor eller andra saker med olika vikt. Undersök vad som händer. Behövs det fler eller färre gummisnoddar för samma hopp som dockan gjorde? Formulera en hypotes och testa den.

Erfarenheter
Använd aktiviteten tidigt på grundskolan. Följande filmklipp visar en liknande aktivitet från en norsk tredjeklass, www.skoleipraksis.no/matematikk-1-4/filmar/det-store-strikkhoppet/. Det går även utmärkt att genomföra aktiviteten i en gymnasieklasse som jobbar med linjära funktioner.

Ursprung
Aktiviteter som liknar denna finns att läsa om på flera håll. Denna är inspirerad av en lektion hämtad från NCTM, se illuminations.nctm.org/LessonDetail.aspx?ID=L646.

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APPENDIX E: LETTER TO PARENTS AND GUARDIANS

Om matematikundervisningen höstterminen 2012.

Hej!

Jag heter Laurence Russell och är forskarstuderande vid Linköpingsuniversitetet/Mittuniversitetet. Jag har matematikundervisning som mitt forskningsområde och har haft förmånen av att samarbeta med xxxxx vid xxxxxskolan i xxxxxx kommuns matematiksatsning. Xxxx är en av kommunens matematikpiloter och är i högsta grad delaktig i utveckling av matematikundervisningen i skolan.

Vi avser pröva ett annorlunda upplägg av matematikundervisningen under höstterminen 2012. Klass 9;x och 9;x kommer, omväxlande under en 8 veckors period att undervisas med alternativa upplägg av matematikundervisningen. Syftet att hitta en mer lustfylld, variationsrik och stimulerande matematikundervisning. Kvalitén i undervisningen kommer inte att påverkas.

Elevernas förförståelse för avsnittet, inställning till samt självkänsla för matematikämnet kommer att mätas före och efter undervisningsperioden. Enskilda elevprestationer kommer inte att kunna härledas till respektive elev och elever kommer inte att kunna identifieras. Fotodokumentation kommer inte att ske.

Resultat av studien beräknas vara publicerad senast hösten 2014.

Ni är välkomna med frågor om studien till

laurence.russell@miun.se

0611-86279

APPENDIX F: LETTER FOR INTERVIEW CONSENT

Mittuniversitetet

MID SWEDEN UNIVERSITY

Till förälder/vårdnadshavare av elever i 9;x och 9;x på xxxxxxskolan

Med anledning av din son/dotters medverkan i vår undervisningsstudie i matematik under hösten 2012 vill jag gärna genomföra en intervju med honom/henne. Intervjun syftar till att samla vidare underlag om vad respektive elev tyckte om den prövade undervisningsmetoden. Intervjun kommer att ljudinspelas. Det inspelade materialet sammanställs enbart av mig personligen och enskilda elevkommentarer och synpunkter kommer inte att kunna spåras till respektive elev.

Er dotter/son har först tillfrågats och efter deras medgivande har ni fått den här lappen i er hand.

Om ni samtycker till ert barns deltagande var vänlig signera talongen nederst och återsänd signerad talong med era ungdomar till skolan för överlämnande till respektive mentor.

Tack för er medverkan. Era ungdomars medverkan är ovärderlig för analysen av den undervisning som genomförts och utvärderas.

Välkomna med eventuella frågor om studien till

Laurence Russell

Mittuniversitetet

0611-86279

Jag samtycker till att mitt barn intervjuas i undervisningsstudien i matematik läsåret 2012-2013

Datum

Namnteckning

Namnförtydligande (textas)

Elevens namn (textas)

Elevens klass

1. Margareta Enghag (2004): MINIPROJECTS AND CONTEXT RICH PROBLEMS – Case studies with qualitative analysis of motivation, learner ownership and competence in small group work in physics. (licentiate thesis) Linköping University
2. Carl-Johan Rundgren (2006): Meaning-Making in Molecular Life Science Education – upper secondary school students' interpretation of visualizations of proteins. (licentiate thesis) Linköping University
3. Michal Drechsler (2005): Textbooks', teachers', and students' understanding of models used to explain acid-base reactions. ISSN: 1403-8099, ISBN: 91-85335-40-1. (licentiate thesis) Karlstad University
4. Margareta Enghag (2007): Two dimensions of Student Ownership of Learning during Small-Group Work with Miniprojects and context rich Problems in Physics. ISSN: 1651-4238, ISBN: 91-85485-31-4. (Doctoral Dissertation) Mälardalen University
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