THE PREDICTABILITY OF SPECULATIVE BUBBLES
AN EXAMINATION OF THE LOG-PERIODIC POWER LAW MODEL

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In this thesis we examine the ability of the log-periodic power law model to accurately predict the end of speculative bubbles on financial markets through modeling of asset price dynamics on a selection of historical bubbles. The methods we use are based on a nonlinear least squares estimation which yields predictions of when the bubble will change regime.

We find evidence which support the occurrence of LPPL-patterns leading up to the change in regime; asset prices during bubble periods seem to oscillate around a faster-than-exponential growth. In most cases the estimation yields accurate predictions, although we conclude that the predictions are quite dependent on at which point in time the prediction is conducted. We also find that the end of a speculative bubble seems to be influenced by both endogenous speculative growth and exogenous factors. For this reason we propose a new way of interpreting the predictions of the model, where the end dates should be interpreted as the start of a time period where the asset prices are especially sensitive to exogenous events. We propose that negative news during this time period results in a regime shift of the bubble. This study is the first to address both the possibilities and the limitations of the LPPL-model, and should therefore be considered as a contribution to the academia.

**Keywords:** Econophysics, mathematical finance, LPPL, log-periodic power law model, JLS-model, power law, speculative bubbles, bubble forecasting, modeling asset price dynamics, financial bubbles, bubbles and crashes.
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Marcus Gustavsson  
Daniel Levén

Linköping 2015
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<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in regime‡‡</td>
<td>The event in which the long-run growth rate of asset prices changes.</td>
</tr>
<tr>
<td>Crash</td>
<td>A substantial drop in asset prices as a result of either endogenous or exogenous influences.</td>
</tr>
<tr>
<td>Crisis</td>
<td>A crash is sometimes followed by a crisis which we define as a longer period of economic instability alongside decreased economic activity. The crisis usually affects the banking industry or society as a whole.</td>
</tr>
<tr>
<td>Critical point‡‡</td>
<td>The point in time at which the model anticipates the asset prices to change regime, signified by $t_c$ in the LPPL-equation. Note that $t_c$ is iteratively chosen in the fitting procedure, yielding a great number of critical points.</td>
</tr>
<tr>
<td>End date‡‡</td>
<td>The critical point expressed as a date.</td>
</tr>
<tr>
<td>Endogeneity</td>
<td>Market movements are classified as endogenous if they cannot be explained by exogenous factors and instead speculation is the driving force.</td>
</tr>
<tr>
<td>Exogeneity</td>
<td>Market movements are exogenous if they are the results of external shocks, often caused by political factors.</td>
</tr>
<tr>
<td>Ex-ante prediction</td>
<td>A prediction conducted before the actual occurrence of the event of interest.</td>
</tr>
</tbody>
</table>

‡‡ These concepts are related to each other and may by the casual reader be regarded as synonymous. Which word is used in each situation is mostly dependent on context, although there are minor differences in their meanings.
<table>
<thead>
<tr>
<th><strong>Ex-post prediction</strong></th>
<th>A prediction conducted after the actual occurrence of the event of interest.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Last observed date</strong></td>
<td>The last date of data that is used in the estimation process. The point in time where we imagine doing the prediction.</td>
</tr>
<tr>
<td><strong>Maturity of oscillations</strong></td>
<td>Oscillations reach maturity when they have reached the stage at which they are frequently occurring and their amplitude is close to zero.</td>
</tr>
<tr>
<td><strong>Regime shift‡‡</strong></td>
<td>Synonymous to <em>change in regime</em>.</td>
</tr>
<tr>
<td><strong>Speculative bubble</strong></td>
<td>A situation where asset prices are traded in high volumes at prices that are considerably deviated from intrinsic or fundamental values.</td>
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CHAPTER 1 · INTRODUCTION

1.1 Background
During the 20th and 21st centuries the increased occurrence and the unpredictability of speculative bubbles on financial markets and their accompanying crashes have confounded economists worldwide. These crashes have shaken the belief in the capitalist financial system and unraveled the lives of millions of people. A striking example is the dawning of the financial crisis of 2008, one of the worst crashes in living memory, where losses in potential GDP in the aftermath of the crash is predicted to amount to 7.6 trillion U.S. dollars in the United States alone (bettermarkets.com, 2012). In addition, the crisis left tens of millions of people unemployed worldwide, while millions of people fell into poverty (Stiglitz, 2010).

The most common definition of a speculative bubble is that an asset experiences trade in high volumes at prices that are considerably deviated from intrinsic or fundamental values (Smith et al., 1993). Speculative bubbles are not a new phenomenon, but can in fact be traced back many centuries. A famous example is the so called South Sea Bubble of the early 18th century. The South Sea Company was a British enterprise with a monopoly on trade with South America, but since Britain was at war with Spain there was no realistic prospect of value in the company. However, according to Kindleberger (1978/2011), the board of the company managed to get the interest of the British public through the spreading of well-placed rumors, leading to a speculative bubble which peaked in 1720. Sir Isaac Newton famously said of the speculation, “I can calculate the motions of the heavenly bodies, but not the madness of people”, before selling at a 100% profit. Later in the same year he was gripped by the frenzy and lost a large part of his fortune.

The South Sea Bubble is only one example in a long line of speculative bubbles that continue through modern times. The many crashes of the 20th and early 21st centuries have revived the debate on the causes and implications of speculative bubbles. Several renowned economists, among them many Nobel Prize winners, have proposed new ideas about bubbles. Nobel laureate Robert Shiller (1987), after the 1987 stock market crash, proposed that many
investors are driven by emotion rather than rationality and established himself as a proponent of the importance of behavioral factors in financial markets.

While Keynesian economists like Shiller (1987), Stiglitz (2010) and Krugman (1999) insist that the solution for avoiding financial instability lies in correctly regulating financial markets, the people in power, e.g. Alan Greenspan followed by Ben Bernanke, often hold the monetarist view that markets are self-regulatory.

Although the phenomenon of speculative bubbles on various markets has long been known by economists and policy makers, the unpredictability of crashes still remains. Many economists have come to accept bubbles, crashes and crises as inevitable consequences of the capitalist system. Marx (1867) was one of the first economists to claim that crises are inevitable. He was of the opinion that these crises will be increasingly severe until the contradictions between the mode of production and the development of productive forces reach the final point of failure. He thus linked the inevitability of crises to the ultimate failure of the capitalist society. Schumpeter (1942/2014), expanding on Marx’s theories, had a different view on the consequences of market crashes. He believed that crashes are a necessary component in an evolving economy. By introducing a concept later referred to as creative destruction, he proposed that new technology must inevitably render some old technology obsolete. This obsolescence means a decrease, or a crash, in value of the particular technology.

However, even if crashes and crises are inevitable this doesn’t necessarily mean that they are unpredictable and thereby impossible to dampen. Minsky (1974), another influential economist, argues that crashes on financial markets are inevitable, although the effects could be dampened through government and central bank actions, following in the tradition of Keynesian economics. In fact, there are several economists who argue that excessive speculation is the effect of regulatory mismatches. Besides the previously mentioned Shiller (1987), Stiglitz (2010) and Krugman (1999), institutional economists like Ostrom (1990), Acemoglu (2012) and North (1997) argue for the importance of institutional factors in
governing the behavior of market actors, and that exaggerated speculation in financial markets can be avoided through strategic regulation.

When it comes to the prediction of crashes on financial markets, one of the most recognized economists is Robert Shiller who correctly predicted both the crash of the dot-com bubble in 2000 and the crash of the housing bubble in 2007 (Shiller, 2000). Another acknowledged economist, Nouriel “Dr. Doom” Roubini, accurately predicted the crash of the housing market 2007 and its effects upon the economy. Others who warned of the impending crash were Joseph Stiglitz, George Soros and Stephen Roach, all Keynesian economists, sharing the view that markets are not entirely self-correcting (Stiglitz, 2010). Although these authors use different approaches to arrive at these conclusions, they all face the same fundamental drawback; by the usage of their methods they are unable to prove the situation mathematically and accurately predict the end of a bubble which is more difficult than just acknowledging that the market will eventually have to correct itself.

1.2 Problem

One of the most influential theories in economics is the efficient-market hypothesis (EMH), developed by Fama (1965). It is by many economists accepted as one of the core tenets of financial theory. However, it is also one of the most heavily debated theories. The EMH states that financial markets are informationally efficient, meaning that an asset's price will reflect all available information regarding the particular asset. This means that in an efficient market, prices at every point in time represent the best estimates of intrinsic value and that all crashes are the result of exogenous variables. This relation between price and intrinsic value suggests that the intrinsic value of an asset rises together with its price even during periods of price rallying.

The common definition of a speculative bubble, however, is as previously mentioned, a period when asset prices substantially deviate from intrinsic value. This definition is obviously very much in contradiction with the hypothesis of efficient markets, and the occurrence of bubbles may be regarded as one of many signs that financial markets might not be as efficient as what is often claimed. This inefficiency of financial markets gives rise
to the need for a model with the ability to identify speculative bubbles and predict their end in advance.

Sornette et al. (1996), at least to some degree, satisfy this need by presenting a quantification of the asset price dynamics leading up to a crash, inspired by earlier work on the prediction of earthquakes. The authors propose that financial time series during speculative bubbles exhibit the same properties and patterns as seismic activity leading up to a critical point signifying the end of a bubble or, in the case of seismic activity, the beginning of an earthquake. They claim that all speculative bubbles are the results of endogenous market dynamics, and therefore they contradict the efficient-market hypothesis. The authors propose that asset prices during a speculative bubble increase as a power law decorated with log-periodic oscillations. In everyday language this means that asset prices during a speculative bubble increase faster than exponentially, and fluctuate systematically around this faster-than-exponential increase. The log-periodic power law model, henceforth called the LPPL-model, suggests that the amplitude of the oscillations decrease in size as the bubble approaches its peak, and when the amplitude turns to zero, it signifies the end of the bubble. This critical point doesn’t necessarily have to be a crash, the model simply predicts the most probable point in time for a change in regime, i.e. a change in growth rate of asset prices. The patterns of the LPPL-model are illustrated graphically in Figure 1 below, where the model has been fitted to the S&P 500 Index prior to the 1987 crash, colloquially known as Black Monday.
The LPPL-model proposed by Sornette et al. (1996) is interesting since it has proven useful in predicting the end of speculative bubbles both ex-post and ex-ante, i.e. both after and before the occurrence of the actual change in regime. Further studies within this area are of interest since there are a lot of markets and bubbles to which the LPPL-model has not yet been fitted. Also, all previous studies have in common that they only present results that reinforce the theory while no one has highlighted both the potential and the limitations of the LPPL-model. In addition, no previous study has examined the robustness, i.e. the time dependence of the model, which is of interest from a scientific point of view. By addressing these issues we give ourselves the opportunity to further assess the predictability and the accuracy of predictions of the model, which should be considered as a contribution to the academia.

Despite that Sornette and his companions have written several papers where they conduct ex-ante predictions, in many cases with successful results, the authors are not willing to reveal their methods and the programming code they use. We are of the opinion that this
kind of research should be publicly held so everyone who is interested in the research has the possibility to replicate it.

1.3 Purpose of the Study

The purpose of this thesis is to explore and analyze the possibilities and limitations of the LPPL-model in predicting the end of speculative bubbles on financial markets. To this end we aim to answer the following questions

- Are the patterns suggested by the LPPL-model observable on time series during speculative bubbles?

- How well does the LPPL-model manage to predict the end dates of speculative bubbles?

- How are the results affected by the point in time at which the predictions are conducted, i.e. how robust are the results?

1.4 Delimitations

In theory, the LPPL-model should be applicable to all time series and all markets influenced by speculative behavior. Despite this we choose to limit this study to only analyze time series of stock and commodity markets. Bond and money markets are excluded since the purpose of this study is to analyze the LPPL-model and the inclusion of these markets would not contribute to the understanding of the model, but rather weaken the focus of this study. We also choose not to include house price bubbles in this study, simply for practical reasons. The code we use in the estimation is only suitable for time series comprised of daily data, while house prices usually are quoted on a monthly or quarterly basis. It would be possible to statistically construct a price index with daily observations, although this would be quite time-consuming. Since time is scarce we exclude analysis of house price bubbles and accompanying data processing. Because of the issue of the code’s inability to process non-daily data, and the lack of data availability regarding older bubbles, this study is limited to relatively modern bubbles.
Some earlier studies on the LPPL-model have also included so called anti-bubbles, i.e. periods of negative returns and reverse LPPL-patterns. We choose to exclude such time series from this thesis since we want to keep the focus of presentation. We also consider anti-bubbles to be of less interest since they are not generally as impactful on financial systems and on the lives of investors, as “regular” bubbles.

1.5 Procedure
This study is quantitative in its nature and based on a nonlinear least squares estimation. The LPPL-model is fitted to a few selected time series where asset prices have increased substantially over the time period. The nonlinear least squares estimation is conducted over a great number of iterations, where the start and end date of the analyzed time period is changed in between iterations. The sum of squared errors is minimized, returning the two fits with the lowest sum of squared errors for each iteration. This procedure yields thousands of results, where the bad fits are separated from the good fits with the help of carefully chosen constraints on the different parameters of the LPPL-equation. The end date of each good fit is what signifies the predicted critical point of the bubble, i.e. the point in time at which the model anticipates the asset prices to change regime. These end dates are the objects of analysis, where they comprise a confidence interval of critical points, indicating where it is most likely for the analyzed bubble to change regime.

1.6 Disposition
The subsequent chapters of this thesis are organized as follows. Chapter 2 outlines previous research on financial bubbles and crises. In chapter 3 we present in detail the LPPL-model and its underlying assumptions. The methods used in this thesis are presented and discussed in chapter 4, while chapter 5 discusses the credibility of the thesis. The reason why this credibility discussion does not appear earlier is that chapter 5 contains a comparison to previous research which is related to chapter 6 and dependent on chapter 4. In chapter 6 we present and analyze the results of this thesis, which are further discussed in chapter 7. Chapter 8 discusses the implications of these results and suggests topics for further research while chapter 9 concludes.
Several renowned economists have attempted to explain the development and different phases of financial crises, as well as the occurrences of crashes. Before going into specific studies within these areas it is important to explicate a couple of definitions. Subsequently, we define a crash as a substantial drop in asset prices, while a crisis is what sometimes follows and is a longer period of economic instability alongside decreased economic activity.

Kindleberger (1978/2011) presents a model for explaining the development of financial crises. His model, heavily influenced by Minsky’s (1974) financial instability hypothesis, emphasizes the relation between financial crises and the business cycle. Especially highlighted is the supply of credit which is said to increase during an economic boom and decrease during an economic slowdown. Kindleberger and Minsky assume that the events leading to a financial crisis start with some exogenous shock to the macroeconomic system, what they call a displacement, in turn leading to an altered economic outlook. During this phase of economic prosperity investors are more optimistic about the future and more willing to borrow, while at the same time lenders’ assessed risk is decreased. This leads to a situation with overvalued asset prices, sensitive to small exogenous shocks which can quickly reverse the economic outlook. When economic activity slows down, debtors, due to decreased cash flows, are unable to pay their large debts and are forced to liquidate their assets, with a decline in asset price level and demand as a result. Minsky was in turn influenced by several classical economists who focused on the instability of financial markets and the role of debt in the emergence of crises, in particular Fisher (1932) who explored this debt-deflation as a possible explanation for financial crises and who argued that it must continue until bankruptcies and bank losses have eliminated indebtedness.

Knutsen & Sjögren (2010) criticize the Kindleberger-Minsky model and expand upon it by emphasizing the importance of institutional factors in explaining the occurrence of financial bubbles and crises. The authors propose a seven-stage cyclical process to explain financial crises. The cycle starts with increasing demand for financial capital in the nonfinancial sector, leading to organizational adaptations within the financial sector. This in turn leads to
institutional deregulation and the coexistence of new and old rules, allowing for financial innovations. In the next stage there is a surplus of idle capital, with investors increasingly attracted to “false” innovations and peripheral economic sectors. This can either lead to debt gearing and financial fragility, in the cases where the supply of liquidity coincides with a strong demand in the real sector, or asset price inflation in certain sectors if there is a speculative boom. The final two stages are characterized by financial decline as asset prices deflate after which panic sets in and the crisis forces stricter financial laws and a contraction of the financial sector.

Reinhart & Rogoff (2009), with their crisis sequence, compile empirical findings of previous work on the unfolding of financial crises. The authors suggest that crises often are preceded by financial liberalization which acts as a triggering factor. What follows is a boom in lending and asset prices after which weaknesses in bank balance sheets appear. The central bank then begins to provide support to institutions by extending credit to them. Due to this, the central bank cannot control the currency, if this is an objective of the bank, why a currency crash occurs, which often leads to increased inflation. At this stage the banking crisis either peaks following the currency crash or keeps getting worse as the economy approaches sovereign default.

Rodrigue et al. (2009) have made an important contribution to the study of crash prediction by illustrating that asset prices follow a certain pattern during a speculative bubble and in its aftermath. These patterns are similar to those recognized by Sornette et al. (1996), with the largest difference that Rodrigue does not take into account the empirical short-run oscillations around the price increase during a bubble. Rodrigue identifies four stages of the bubble, which are illustrated graphically in Figure 2 below. In the stealth phase a few investors with better access to information than others realize that there is potential for fundamental appreciation of the asset price. In the awareness stage many institutional investors take notice and there is a large inflow of money, driving prices further up. Eventually the public finds out about the “potential” of the asset, often spurred on by media reports, bringing us to the mania phase. This is where most of the psychological factors affect the price, through herding and positive feedback, discussed in section 3.1.1. This
unsustainable growth leads to a so called bull trap where a smaller fall precedes a brief return to “normal” before the market crashes. The bull trap and the return to “normal” can be explained by investor mentality, where the small crash is followed by denial before the asset price plunges.

Figure 2. The different phases of a bubble
Source: Rodrigue et al. (2009).
Before going into the details of the LPPL-model it is of importance to distinguish between exogenous and endogenous market movements. This distinction is important since the patterns of the model, the faster-than-exponential growth and the oscillations, only characterize endogenous market movements, i.e. speculative bubbles, leading up to a change in regime. We follow the definitions used by Johansen and Sornette (2010) who define exogenous market movements as the results of external shocks, often caused by political factors. For example, the Russian stock market fell by more than 20% in December 2014 as the result of rapidly decreasing oil prices and international economic sanctions against the Russian government. Exogenous market movements are by their nature impossible to predict through econometric modeling. Market movements are, on the other hand, classified as endogenous if they cannot be explained by exogenous factors and instead speculation is the driving force. In the case where speculation is what drives asset prices upwards we observe a speculative bubble. It cannot be stressed enough that the LPPL-model only applies to speculative bubbles, and it cannot be used to foresee exogenously driven bubbles.

3.1 The Patterns of the Model

The LPPL-model is characterized by two main attributes, faster-than-exponential growth and oscillatory movements. These characteristics are vital for the understanding of the model and will be discussed more thoroughly in this section.

3.1.1 Faster-than-exponential Growth

In the field of economics, students learn that markets in general tend to adjust towards equilibrium. This adjustment is the result of negative feedback, which means that when prices go up demand decreases. In a market driven by the laws of supply and demand, negative feedback acts as a limiting factor, under normal circumstances resulting in an economic equilibrium. One of the underlying mechanisms of the LPPL-model, however, is the concept of positive feedback (Sornette et al., 2013). Positive feedback means that when prices go up, investors tend to buy because they are expecting further price increases (Shiller, 2000). This is what an observer of a market might see during a speculative bubble. When
positive feedback becomes dominant the result is a self-reinforcing loop, which drives the market out of equilibrium. This loop means that demand for an asset increases as its price rises, until it reaches its critical point, which is when a change in regime occurs. The mechanism of positive feedback is what explains the faster-than-exponential growth in asset prices during a speculative bubble.

According to Shiller (1984) and Nofsinger (1999) the underlying reason why positive feedback loops occur, is the psychological phenomenon referred to as herding behavior. The authors define herding as a group of investors trading in the same direction over a period of time, as the result of the traders responding to fads or common sentiments among their communities. Herding behavior is based on learning from others, meaning that investors disregard their own personal beliefs and instead choose to act like other actors on the market.

Johansen et al. (2000) as well as Geraskin & Fantazzini (2013) describe the presence of two types of traders in the financial markets, one group characterized by rational expectations and the other characterized by irrationality, called noise-traders. While the group of rational investors contributes to the negative feedback, it is the noise-trading group that is responsible for herding behavior, being driven by external influences and the sentiment of other traders in their social networks.

3.1.2 Oscillations
The oscillatory patterns of the LPPL-model are more difficult to explain in theory and many of the authors that have explored the model simply explain the occurrence of the patterns as being empirically proven. As pointed out by Sornette et al. (1996), however, the oscillations bear resemblance to the Elliott wave principle as posited by Elliott (1938/2012), which is sometimes used in technical analysis. According to Elliott, collective investor psychology, or crowd psychology, alternates between optimism and pessimism which creates observable patterns in price movements. One of his reasonings for this pattern occurring is that every action must be followed by a reaction. An important difference between the oscillations of the LPPL-model and the Elliott wave principle is that Elliott’s model assumes that all cycles contain five waves consisting of three impulsive waves and two corrective waves. The
theoretical framework of the LPPL-model does not specify any specific number of waves during a bubble cycle. Instead, Sornette et al. propose that the oscillations during a speculative bubble decrease in amplitude as the price movements approach the regime shift. The theory suggests that the regime shift should occur when the amplitude of the oscillations turn to zero.

3.2 The Equation

The patterns of the LPPL-model are quantified by the LPPL-equation. When this model was first presented by Sornette et al. (1996) the equation was defined as

\[
p(t) = A + B(t_c - t)^z + C(t_c - t)^z \cos(\omega \log(t_c - t) + \phi)
\]

(3.1)

where asset price, \( p \), is a function of time, \( t \), and \( t_c \) expresses the critical point, the most probable time of a change in regime, i.e. a change in growth rate. The power law component of the function is defined by

\[
B(t_c - t)^z
\]

(3.2)

which captures the faster-than-exponential growth of the time series and hence the positive feedback mechanism. The accelerating log-periodic oscillations of the model are captured by the component

\[
C(t_c - t)^z \cos(\omega \log(t_c - t) + \phi)
\]

(3.3)

The parameter \( z \) in the equation controls the strength of the feedback mechanism as well as the amplitude of the oscillations, as shown graphically in Figure 3. The uppermost graphs in the figure illustrate the growth rate when \( z \) is varied, and the lower graphs illustrate the frequency of the oscillations when \( z \), once again, is varied. The values of \( z \) in this figure are for illustrative purposes set at 0.2 and 0.7. \( z \) must lie between 0 and 1, else we are dealing with some other type of process and not a power law characterized by faster-than-
exponential growth. $\omega$ denotes the frequency of the oscillations and empirically takes on values between 4.8 and 7.9 according to Johansen et al. (2010). The effect of $\omega$ is shown graphically in Figure 4, where $\omega$ is varied between 6 and 15 for illustrative purposes, while other parameters are kept fixed. $A$ is a positive parameter which naturally takes on the asset price value of the LPPL-fit at the critical point. This follows from the fact that the power law component and the oscillatory part of the equation approach zero as $t$ approaches $t_c$. The parameter $B$ will in the case of a speculative bubble take on negative values since asset prices are increasing during a bubble, while $C$ must take on values between 1 and -1. $\varphi$ in the equation simply signifies the direction of the oscillations.

Figure 3. The effect of $z$ on the feedback mechanism and oscillations
Source: Own figure.
Equation (3.1) contains three linear parameters \((A, B, C)\) and four nonlinear parameters \((t_c, z, \omega, \phi)\) that are to be estimated. Since the objective of the LPPL-model is to predict the end of speculative bubbles we aim to find the best estimation of the critical time \(t_c\). Authors of previous studies most commonly start by subordinating the three linear parameters to the four nonlinear parameters. There is no easy way to describe how such a subordination is done, although the end result is that the linear parameters become dependent on the nonlinear parameters which decreases the number of parameters that need to be estimated simultaneously. The authors then use the method of nonlinear least squares to find the best estimation of \(t_c\) (Johansen et al., 2000). The objective of nonlinear least squares is, just like with ordinary least squares, to minimize a cost function, that is the sum of squared errors (SSE), which in this case looks as in Equation (3.4) below.

\[
SSE(t_c, m, \omega, \phi, A, B, C) = \sum_{i=1}^{N} [p(t) - A - B(t_c - t)^2 - C(t_c - t)^2 \cos(\omega \log(t_c - t) - \phi)]^2
\]  

Minimization of such nonlinear multivariate cost function is a very complex task due to the presence of multiple local minima. An illustrative example of the minimization problem of this cost function is presented in Figure 5 below, where two of the nonlinear parameters are kept fixed in (a), (b), (c) and (d) respectively, while the other two parameters are varying along their axes. The y-axis of the plots measures the cost function, SSE. For example Figure

**Figure 4. The effect of \(\omega\) on the oscillations**
Source: Own figure.
Let $m$ be 0.7 and $\omega$ be 7.5 while $t_c$ and $\phi$ are varying along their axes and SSE is measured on the vertical axis.

Due to the many local minima illustrated in Figure 5, optimization based on the original equation requires determination of the global minimum using some metaheuristic method such as taboo search or genetic algorithm. These estimation methods are very demanding since they require many iterations for detecting the global minimum. The optimization algorithm, in addition, faces a big risk of getting trapped at local minima, and there is no guarantee that the found minimum is the actual global minimum, which means that the predictions of $t_c$ might be severely biased.

For these reasons Filimonov & Sornette (2013) present a modification of the equation where they first expand the cosine term of the original equation and rewrite the equation as follows.
\[ p(t) = A + B(t_c - t)^2 + C(t_c - t)^2 \cos(\omega \log(t_c - t)) \cos \varphi + C(t_c - t)^2 \sin(\omega \log(t_c - t)) \sin \varphi \]

(3.5)

The authors thereafter rewrite Equation (3.5) as

\[ p(t) = A + B(t_c - t)^2 + C_1(t_c - t)^2 \cos(\omega \log(t_c - t)) + C_2(t_c - t)^2 \sin(\omega \log(t_c - t)) \]

(3.6)

where

\[ C_1 = C \cos \varphi, \]
\[ C_2 = C \sin \varphi \]

(3.7)

As can be seen from Equation (3.6) the function now contains three nonlinear parameters \((t_c, z, \omega)\) and four linear parameters \((A, B, C_1, C_2)\), meaning that \(\varphi\) is now contained in \(C_1\) and \(C_2\) which are subordinated to the other nonlinear parameters. By transforming the original equation in this manner the authors pose no new constraints on the parameters in the equation. The new cost function to be minimized is presented in Equation (3.8) below.

\[
SSE(t_c, m, \omega, A, B, C_1, C_2) = \\
= \sum_{i=1}^{N} \left[ p(t) - A - B(t_c - t)^2 - C_1(t_c - t)^2 \cos(\omega \log(t_c - t)) - C_2(t_c - t)^2 \sin(\omega \log(t_c - t)) \right]^2
\]

(3.8)

This transformation has two very important implications. Firstly, the transformation decreases the complexity of the fitting procedure since the optimization problem is converted from a four-dimensional space to a three-dimensional space. Secondly, and possibly of even greater importance, the cost function to be minimized now contains a single minimum instead of multiple minima, as long as the model is appropriate for the empirical data. The stability of the model is thereby significantly improved. Thanks to this transformation the need for complex search algorithms such as the taboo search is eliminated and more simple algorithms, e.g. the Gauss-Newton algorithm, can be used without any reduction of the
robustness of the model. An example of the minimization problem of the new cost function is illustrated graphically in Figure 6, where one of the now three linear parameters is kept fixed while the other two are varying along their axes, and SSE is measured on the vertical axis.

From the discussion above it is easy to determine that the newer edited equation proposed by Filimonov & Sornette (2013) yields a simpler, more stable and more effective estimation procedure compared to that of the original equation. For these reasons we in this thesis choose to base our methods and estimation process on Equation (3.6), with the related cost function as in Equation (3.8).

![Figure 6](source: Filimonov & Sornette (2013).)
4.1 Bubble Selection

Before fitting the LPPL-equation to time series of asset price appreciation it is of course important to first choose which time series, or bubbles, to analyze. The common practice in previous studies is to either choose bubbles based on historical context or by identifying bubbles with the drawdown methodology as described by Johansen & Sornette (2010). According to the authors a drawdown is defined as a persistent decrease in an asset’s closing price over several consecutive days. It is thus the cumulative loss from the last local maximum to the next minimum. The authors select which bubbles to fit the model to by first identifying the largest drawdowns on the market at hand, and thereafter fit the LPPL-equation to the period preceding this drop.

The main problem with identification by drawdowns is that it does not take into account the fact that not all speculative bubbles end in a crash. Recall that the model predicts regime shifts, not necessarily crashes. Therefore the drawdowns methodology excludes speculative bubbles that do not end in a crash. Simultaneously this methodology includes bubbles that are not of any real interest of the model. Exogenous bubbles may as well as endogenous bubbles end in a crash, which leads to the inclusion of both exogenous and endogenous bubbles with the use of this methodology. Fitting the LPPL-equation to time series of exogeneity would yield deceptive fits since the model is only applicable to speculative bubbles.

To circumvent these problems we choose to focus this study on a selection of bubbles where their historical contexts make them particularly interesting. This might for example be when it is well known that the underlying assets were objects of speculation during this period of time, or when differences in interest rates cause increased capital flows between economies leading to increased anticipation of rising asset prices. In addition, when choosing bubbles to analyze, we take into account the geographic and economic positioning of the markets where the bubbles existed. We strive to achieve geographic and economic diversity among
the analyzed bubbles since bubbles in different parts of the world and under different
economic circumstances might behave differently. By diversifying the selection of bubbles
we hope to strengthen the generalizability of this thesis.

Based on these selection criteria we select the following five bubbles to base our analysis
upon; the Japanese asset price bubble of the 1980s, one of the most spectacular and
speculative bubbles in living memory; the London stock price bubble of the 1990s, where
changes in interest rates possibly led to excessive speculation; the emerging markets bubble
of the 1990s, where investment trends contributed to excessive price growth in emerging
markets; the dot-com bubble of the late 1990s, where an entirely new market led to
disproportionate anticipations; and the wheat price bubble of the 2000s, where the possible
occurrence of speculation has been a topic of discussion.

4.2 Data

The data sets used throughout this thesis consist of daily closing prices of various well-
known stock price indices and commodity indices. The data sets consist of time series
ranging from the 1980s to the late 2000s. All data is retrieved from Thomson Reuters
Datastream, and has not been processed by us in any way. There has been a discussion in
the literature whether real prices or log-prices should be used when conducting LPPL-
estimations. Sornette et al. (2013) suggest that log-prices should be used in most cases
although they propose that in practice one should try the fitting procedure with both real
prices and log-prices and compare the results. By doing so we have found that our results
are not majorly affected by whether we use log-prices or real prices (see appendix), why we
choose to use real prices in the analysis. This is motivated by the fact that we do not wish to
process the data more than necessary. We also consider that the use of real prices is
preferable for graphical reasons since this makes the faster-than-exponential growth and
thereby the positive feedback mechanism easier to distinguish and understand.

When analyzing the different bubbles we use data from several indices; for the Japanese
asset price bubble we use data from the Nikkei 225 Index; for the London stock price bubble
we use data from the FTSE 100 Index; for the emerging markets bubble we use data from
the Hang Seng Index of Hong Kong; for the dot-com bubble we use data from the Nasdaq Composite Index; for the wheat price bubble we use American Soft Red Winter wheat futures prices.

We choose not to include data from undeveloped financial markets since these are less liquid compared to those in more developed regions and the reliability of the data might in many cases be questioned. By using these data we would take the risk of producing misleading or inaccurate results as a consequence of their unreliability.

4.3 Estimation

When fitting the LPPL-equation to a historical time series it is important to consider in what time interval the estimation process should be conducted, since it is often difficult to determine exactly when the speculative behavior did commence. In many previous studies the start date of the analyzed interval is quite arbitrarily chosen to be where the LPPL-signatures first seem to appear. The estimation is then conducted by fitting the LPPL-equation from the start date up until some time after the peak, that is the peak date plus a prediction interval. However, the data that is used only includes price data from the start date up until the peak or some time before the peak.

To eliminate the arbitrariness of choosing one specific start date we in this study use a rolling window of estimation and thereby let the estimation be iterated with both different start dates and different end dates. This is consistent with the recommendations of Sornette et al. (2013) to make the predictions more statistically robust. In each iteration the model is fitted from the start date to the end date by varying the equation parameters. The start date is rolling with increments of twenty days, while the end date is subsequently moved forward two days at a time. The data that is used when predicting the regime shift in each iteration ranges from the rolling start date, up until the last observed date, usually one month before the actual peak of the bubble. This is because we imagine that when doing the estimation we, in time, are located one month before the actual peak. Thus, the results of the estimation give us exactly the same results as we would have arrived at if we performed an ex-ante prediction one month before the actual peak occurred. We choose the period of one month since it is
close enough to the peak for the time series to potentially have developed clear LPPL-patterns, while there is still time left to take the results into consideration. Since we imagine ourselves at this point in time we are not interested in fits that predict a change in regime before this date, why the end date of each iteration is always set later than the last observed date. We make this assumption since we want to establish whether the model can accurately predict a critical time ex-ante, in which case we would not be interested in predicted critical points prior to the last observed date.

The method of rolling windows is illustrated in Figure 7, where graphs 1 to 6 should be looked upon in numerical sequence and show subsequent steps in the estimation process. The green line in each graph illustrates the start date of the estimation, while the red line signifies the end date of the estimation. The black line illustrates the last observed date before the peak of the bubble where we imagine doing the prediction. It thereby also shows the end of the data used in the estimation. As can be seen from graphs 1 to 3 the end date is incrementally moved forward, and in graphs 4 to 6 the start date is set at a later date, while the end date incrementally increases by each iteration. The LPPL-equation is fitted in between the green and the red lines in the graphs, while the data used only ranges from the green to the black lines. Thus we arrive at estimated critical points that are the same as if we would have done the LPPL-fit one month before the actual peak of the bubble. Note that the increments in Figure 7 are larger than those used in this study. This is simply so for illustrative purposes.
This method with a rolling window of estimation gives us more fits that exhibit acceptable properties compared to many previous studies, and make the fitting procedure less sensitive to input values. In practice the estimation process is a complex procedure which solves the minimization problem of Equation (3.8) using a Gauss-Newton algorithm, a method commonly used to solve nonlinear least squares problems. The SSE is thus minimized, returning the two fits with the lowest SSE for each iteration. Since the time series analyzed usually are a few years long and since we use the rolling window of estimation we end up with thousands of fits, both good and bad.

It has been shown in previous studies that the last observed date, the date chosen for ex-ante prediction, is of importance to the accuracy of the estimated critical time. In theory, the further from the peak the last observed date is set, the less accurate the prediction is. When testing the robustness of the model we test this theory by setting the last observed date two months and two weeks prior to the peak, respectively, and redo the estimation process. By
doing this we can establish whether there is any distinct difference in results based on the last observed date, and whether these results match the theoretical assumption.

4.4 Filtration
Since the estimation process described above leaves us with both good and bad fits it is important to filter out the bad fits from the good. We accomplish this by introducing a number of constraints presented below.

As previously mentioned Johansen et al. (2010) empirically find that the parameter for the frequency of oscillations, \( \omega \), lies between the values of 4.8 and 7.9. We choose to follow the guidelines proposed by Filimonov & Sornette (2013) in broadening the constraints on \( \omega \) to allow for values between 3 and 15. This is in order to avoid incorrectly rejecting fits that are of significance for predicting the critical point. By imposing this constraint we choose not to allow for fits where the oscillations are either too short or too long, since these fits are obvious mismatches for capturing the LPPL-signatures.

When it comes to \( z \), the parameter for feedback and amplitude of oscillations, we do not enforce any further constraints other than the accepted values between 0 and 1. Some previous studies have proposed stricter constraints on \( z \), but we find further restrictions not to contribute significantly to our results. The chosen restriction makes sure that we in our results only allow for fits that are characterized by a faster-than-exponential growth, with oscillations that are neither too small nor too big.

Since asset prices during a speculative bubble increases up until the critical point and \( A \) is the highest point of the fit, it follows from the equation that \( B \) must take on negative values. If \( B \) instead were to be positive, asset prices would decrease during the bubble. For this reason we constrain \( B \) to be less than 0.

In addition, we introduce constraints on the augmented Dickey-Fuller and Phillips-Perron values of the fits. We only accept fits that are non-stationary at a 1% significance level. Thus we filter out the stationary fits of the model, which obviously have no real explanatory power
in predicting the critical point, since asset prices during a speculative bubble surely are not mean reverting and therefore not stationary. This constraint filters out obvious misfits that the process otherwise in some cases presents as valid results.
Chapter 5 · Credibility

An important aspect to consider regarding the methods used in this thesis is their validity and reliability. Validity concerns whether the methods and results are well-founded, correspond to the real world and measure what they are intended to measure (Bryman & Bell, 2003). Reliability concerns the robustness or repeatability of the study; whether our methods will yield the same results over several repetitions and whether they are sensitive to small changes in input.

5.1 Validity

One issue concerning the validity of this thesis is that of the types of markets analyzed. The model should in theory be applicable to any market where speculative behavior might occur, but for reasons discussed in section 1.4 we delimit this study to stock and commodity markets. To increase the validity one could also include house prices, money markets, bond markets etc. However, we do not view the exclusion of these markets as a major problem since this should not detract in any major way from the conclusions we draw from the results of this thesis.

Another issue regarding the validity of this thesis lies in the selection of bubbles to be analyzed. Since this is done on the basis of historical context there is a risk that the selection is influenced by arbitrariness and bias. When selecting bubbles we have continuously had this issue in mind to reduce the influence from such forces.

The ideal way to select bubbles for analysis would emerge if there existed a reliable way of quantifying speculative behavior. If this were possible it would be easy to determine during which time periods asset prices were influenced by speculation. The quantification of speculation would give birth to an alternate method that could be used in theses like this one. The authors could then focus the study on bubbles that they know are speculative in their nature. Thanks to this the risk of biased selection would almost diminish. Until such validity increasing possibilities arise we regard selecting bubbles on their historical context as the most appropriate method.
5.2 Reliability
The data used as a basis for this study, as described in section 4.1, is retrieved from Thomson Reuters Datastream. All data comes from renowned indices with both high credibility and accessibility. The thesis is entirely based on raw data, which means that it has not in any way been processed, after being downloaded from Datastream. This means that replication of this thesis, based on the same data, should yield the same results and be possible to perform without any major difficulties. Possible problems may arise however, because of differences in coding techniques when developing the fitting procedure. There are many different ways to write the programming code and this can possibly yield differences in results. Difficulties may also stem from the choice of coding language used. To establish whether this issue is in fact a problem we in section 5.3 below present a comparison of the results produced by our methods compared to those of previous studies.

5.3 Comparison
Since the authors of previous studies in general are unwilling to reveal their estimation methods it is of importance to show that the methods and programming code used in this thesis yield similar results to those of previous studies. This is important mainly because it may illustrate that our methods are not completely at odds with the academic consensus and that our results can be regarded as a contribution to the academia, strengthening the reliability of our results. To accomplish this we, before presenting results on bubbles that have not yet been analyzed, first present our results for two historical bubbles that have been analyzed before.

5.3.1 The Speculative Bubble of the Late 1980s
The Black Monday crash of 1987 is one of the most well-known crashes in modern times. The downfall in asset prices was the result of one of the most apparent examples of a speculative bubble, and it has therefore been a popular target for LPPL-modeling. Johansen & Sornette (2010), when modeling the crash, arrive at an estimated critical point on September 2nd 1987, while Johansen et al. (2000) find that the regime shift should have occurred on the 27th. The results for this bubble yielded by the methods used in this thesis are presented below in Figure 8. The red line in the figure illustrates the median of the
predicted critical dates, while highlighted in grey is an 80% confidence interval of predictions. The bold black line prior to the confidence interval signifies the last observed date for the data used in the estimation, in this case set one month prior to the peak. The median gives us a critical point on September 3rd, while the 80% confidence interval ranges from August 4th to October 6th. Confidence intervals have been plotted in some of the previous studies. The reason for why this might be a good idea is because the date of the regime shift is a highly stochastic process and the prediction of one specific crash date in fact might be misleading. The actual regime shift occurred on August 25th, the peak date of the bubble. Black Monday, however, took place on October 19th and should not be mistaken for the regime shift. A summary of the predicted crash dates yielded by these studies is illustrated in Figure 9 where the arrows illustrate predicted critical dates and the red curly bracket shows the confidence interval yielded by our model. The conclusion of this comparison is that our methods and programming code yields similar results to those of previous studies on this particular bubble.

Figure 8. The LPPL-model fitted to S&P 500 Index during the speculative bubble of the late 1980s
Source: Own figure.
5.3.2 The Oil Price Bubble of the 2000s

During the energy crisis of the new millennium oil prices rose by over 400% from early 2003 to mid-2008. The reasons behind this extreme increase in price are said to be many; e.g. the decreasing value of the U.S. dollar, increased tension in the middle east, worries over peak oil, but also untenable oil price speculation (Conway, 2009). Sornette et al. (2009) perform both ex-ante and ex-post analysis of this bubble. In their ex-post analysis they find an 80% confidence interval of crash dates ranging from May 17th to July 14th. When we fit the LPPL-model to the same bubble with the same inputs we arrive at a confidence interval ranging from May 25th to August 2nd. The output of our analysis is illustrated in Figure 10 where the 80% confidence interval is highlighted in grey and the red line shows the median of the predicted crash dates. The bold black line represents the last observed date, in this case set at 41 days prior to the crash since this is the date when Sornette et al. imagine doing the prediction. Figure 11 shows the confidence intervals graphically, where the actual change in regime on July 3rd is shown by the black line. Once again we draw the conclusion that our methods and programming code yield similar results to those of previous studies.
5.4 Criticism of Sources

Consistently throughout this thesis we reference previous studies done in the area of LPPL-modeling. As previously mentioned, several of the authors of these studies have been unwilling to share the programming code used to achieve their results with few or no reasons given. This raises the question of whether they might be uncertain of the replicability of their studies or whether there is biased selection in their presented results. However, we have shown in section 5.3 that the code we use yields results that are comparable to previous
studies, alleviating suspicions of low replicability to some degree, although the problem of biased selection remains. Since authors of previous studies have only presented results that reaffirm the model it is quite probable that they are making active choices not to present results that might undermine the LPPL-model.

Most sources cited in this thesis are either published books or articles published in economics, physics or statistics journals. We regard these sources as reliable since the standard quality of articles published in these journals is usually high and in many cases frequently cited in other articles. In the cases when we reference working papers, the reason is that the paper has not been published although the author is respected in his or her field. We choose only to cite newspapers and internet sources when the aim is to substantiate our discussion.

As previously mentioned the data used in the fitting procedure is retrieved from Thomson Reuters Datastream, which is a reliable source of information, why the data should be regarded as dependable.

### 5.5 Ethics

Throughout this study we consistently consider the ethical guidelines for scientific research of the Swedish Research Council. Because of the quantitative nature of this thesis the ethical aspects to consider are limited. For example, since this study is not based on qualitative research we do not have to take into account the ethics regarding how to treat respondents. We are, however, aware of the importance of transparency in presenting our results and we actively try to avoid bias in our presentation.
In this chapter we present the results of this thesis in the form of graphs on the bubbles that have been chosen for analysis. In order to analyze and contextualize what is shown, we present the results for each individual bubble in a separate section. In each section we first present the main graph where data is being used up until one month before the actual regime shift, as described in section 4.3. Thereafter we present the same analysis instead based on data up until two weeks and two months before the regime shift, respectively. We do this in order to investigate the robustness and consistency of the LPPL-model. By doing so we can determine how sensitive the results are depending on when an ex-ante prediction is assumed to be conducted. In theory, since a fitting procedure conducted closer to the peak utilizes more information, prediction at a later date should be more accurate.

Highlighted in grey in each figure in this chapter is the 80% confidence interval of the critical points, $t_c$, that are the results of the fitting procedure described in chapter 4. This interval thus represents the most probable time for a change in regime. We exclude the peripheral results outside of the confidence interval since we do not want outliers to affect the estimated $t_c$. The confidence interval is plotted with the motivation that the date of the regime shift is a highly stochastic process and that the prediction of one specific crash date in fact might be misleading. The median date of the critical points is marked by a red vertical line and gives guidance to where it is most likely for the regime shift to occur. In each graph only a dozen of the resulting LPPL-fits are plotted, regardless of how many resulting fits are produced. We do this for the sake of visibility while the total number of fitted curves is given in the upper left corner of each figure. The bold black line in each figure illustrates the last observed date, which is where the ex-ante prediction is assumed to be conducted.

6.1 The Japanese Asset Price Bubble of the 1980s

One of the most spectacular financial bubbles during the 20th century was the Japanese asset price bubble of the 1980s, where stock prices on the Nikkei 225 Index during the period analyzed increased by around 200%. According to Kindleberger (1978/2011) the Japanese asset price bubble is one of the most classic speculative manias where overconfidence and
excessive speculation combined with a relaxed monetary policy led to rapid increases in asset prices.

During most parts of the 20\textsuperscript{th} century Japan had been a more or less closed economy to the rest of the world. In the first half of the 1980s things began to change as Japan experienced pressure primarily from America to open up its financial markets to foreign investors. The Japanese economy had grown substantially during the previous decades why American companies and investors wanted to gain access to clients, customers and trading opportunities in this market. The economic success of Japan led to an increased interest in investments in Japan, and in combination with the financial liberalization the result was a massive inflow of capital from foreign investors. The inflow of capital was spurred on by a relaxed monetary policy kept by the Bank of Japan. In addition, the authorities wanted to maintain a positive trade surplus why the Bank of Japan bought U.S. dollars to keep the yen from appreciating in the foreign exchange market. This in turn led to rapid growth in money supply and credit throughout Japan. Industrial firms could borrow as much money as they wanted and money seemed free of charge.

The increase in stock prices in Japan was clearly faster-than-exponential and therefore the price level eventually had to be corrected. The bubble imploded and stock prices plunged in early 1990 which led to massive failures of several banks and other financial institutions. The reasons behind this sudden downturn is often attributed to a tightened monetary policy and new credit regulations kept by the Bank of Japan, restricting Japanese banks to lend money. The implosion of the asset price bubble put Japan through more than a decade of sluggish economic growth, later referred to as the Lost Decade.

Since the Japanese asset price bubble is regarded as having been driven by excessive speculation the LPPL-model, in theory, should be able to accurately predict its ending. Both foreign and domestic investors chose to invest in Japanese stocks for reasons discussed above. However, all investors most likely had in common that they were expecting further price increases and thus contributed to the herding behavior on the market. When fitting the LPPL-equation to the time series preceding the peak we arrive at the results presented in
Figure 12. It can be seen from the figure that the stock prices during this time period follow the LPPL-characteristics. The prices seem to oscillate around a faster-than-exponential growth where the oscillations become smaller closer to the peak. These price movements act exactly as they are expected to during a speculative bubble, according to the LPPL-framework.

![NIKKEI225](image)

**Figure 12.** The LPPL-model fitted to the NIKKEI 225 Index during the Japanese asset price bubble of the 1980s
Source: Own figure.

It can also be seen from Figure 12 that the 80% confidence interval captures the actual peak date, or regime shift, of December 29th, while the median date is only six days later. This means that if an ex-ante prediction would have been performed on the Nikkei 225 Index one month before the actual peak it would have given us a good estimation of the upcoming date of the regime shift. These are promising results since they might validate the utility of the LPPL-model. However, before jumping to such conclusions it is important to consider the robustness of the model. It is clear that the model foresees the regime shift when conducted one month before the peak, although it can be of significance to consider if the model
predicts the regime shift with accuracy if the prediction is instead conducted either earlier or later. In Figure 13 below we present the results of the LPPL-model when fitted to the time series with the last observed date set two months and two weeks prior to the peak date, respectively. By doing so we strive to achieve an increased understanding of the robustness of the model.

Figure 13. The LPPL-model fitted to the NIKKEI 225 Index during the Japanese asset price bubble of the 1980s with different last observed dates
Source: Own figure.

The left graph of Figure 13 illustrates the results of an ex-ante prediction performed two months before the peak date. It is apparent that both the start and the end of the confidence interval are moved roughly one month to the left in the graph compared to the interval in Figure 12, although it still captures the actual peak date. The reason for why the interval is moved in this manner is unclear, but one possible explanation is that the amplitude of oscillations in this case already are quite small when the prediction is conducted. Since the theory suggests that the regime shift should occur when the amplitude of the oscillations turns to zero, the model in this case expects the regime shift to happen earlier. The confidence interval will continue to move to the left in the graph when moving the last observed date to the left, as long as the oscillations soon before the last observed date are close to zero. This means that the LPPL-framework would expect the regime shift to occur earlier. Why this speculative bubble lives for so long is difficult to tell. One possibility is that an exogenous
trigger might be needed when the oscillations have reached maturity to offset the positive feedback loop and end the bubble. Perhaps in this case there was no negative news big enough to start the downward spiral prior to the implementation of a tightened monetary policy and new central bank regulations. These possible explanations will be discussed more thoroughly in chapter 7.

The right graph of Figure 13 shows how the results of an ex-ante prediction would have looked when performed two weeks before the peak. The actual regime shift is once again captured by the confidence interval, while the interval is moved roughly two weeks to the right in the graph. This is probably for the same reason as the one above, namely that the oscillations in this case are so far gone when the predictions are conducted.

**6.2 The London Stock Price Bubble of the 1990s**

During the early 1990s the United States Federal Reserve pursued an expansionary monetary policy, where the federal funds rate was kept at a globally low level. This policy led to American and foreign investors seeking alternative investments with higher returns (Blakey, 2010). Hence there was a large inflow of capital to European and emerging financial markets. This capital inflow gave birth to a rapid increase in stock prices on these markets, where the growth rate, when looking back at the event, can be said to have been unsustainably high. One market that was affected by the inflow of capital is the London Stock Exchange in Great Britain where the FTSE 100 index increased from a low of roughly 2,000 to a peak of 3,520 in early 1994, before suddenly changing course. The reason behind this downturn in asset prices following the peak has been attributed to the Federal Reserve’s shift in monetary policy to a more contractionary policy with higher interest rates. The capital flows thereby changed direction and capital started flowing towards the United States once again.

When the LPPL-equation is fitted to the FTSE 100 Index during the time period preceding the downturn of 1994 the results are as illustrated in Figure 14 below. It can be seen from the figure that the asset prices during this time period follow the LPPL-characteristics. It is also evident that the LPPL-predictions in this case yield accurate results when conducted
one month before the peak date, since the actual change in regime on February 2\textsuperscript{nd} is captured by the confidence interval.

\textbf{FTSE100}

![Graph showing FTSE100](image)

\begin{itemize}
\item Last observed date: 1994-01-02
\item Number of curves: 135
\item Median: 1994-01-28
\item Quantile 10/90: 1994-01-06 --- 1994-03-04
\end{itemize}

\textbf{Figure 14. The LPPL-model fitted to the FTSE 100 Index during the London stock price bubble of the 1990s}

Source: Own figure.

It is apparent from the historical context of this bubble that both its initiation and end were influenced by an exogenous factor, the federal funds rate. Recall that if the bubble is purely exogenous by nature, it should not be explained by the LPPL-model. However, it is difficult to determine the importance of the federal funds rate when trying to explain the movement of the index. Perhaps this exogenous shock started a wave of positive feedback where speculation on further increases drove the market to new heights. However the case, it is apparent that the price development follows the LPPL-signatures leading up to the regime shift. The explanation could be that the market was in fact driven by speculative herding behavior.
Another related question that arises when considering Figure 14 regards how the model manages to accurately predict the critical date, given that the regime shift may have been triggered by an exogenous factor. There are several possible explanations for why this happens. Once again, the importance of the federal funds rate’s impact on the FTSE 100 Index may be questioned. It is possible that the critical date in fact is a result of endogenous unsustainable speculation as the theory suggests, and the federal funds rate is not actually triggering the downturn. It could also be that the asset prices are especially sensitive to exogenous shocks after the LPPL-patterns have reached maturity, increasing the chances of a regime shift during the interval of critical points. All that is needed for the asset prices to take a downturn is an exogenous factor to affect the prices downwards. Perhaps the federal funds rate is only one of many exogenous factors that in this case and at this point could start the downward spiral. It is also possible that the patterns leading up to the regime shift and the accuracy of the critical point may merely be coincidence. To achieve a more clear understanding on why the model yields accurate predictions in this case, we below in Figure 15 present how the results of an ex-ante prediction conducted two months and two weeks prior to the crash would have looked.

Figure 15. The LPPL-model fitted to the FTSE 100 Index during the London stock price bubble of the 1990s with different last observed dates
Source: Own figure.
When the last observed date is set two months prior to the peak date, as in the left graph of Figure 15, we observe interesting results. The interval of predicted end dates is shifted to the left in the graph. These results are reminiscent of those for the Nikkei 225 Index, with the exception that the confidence interval in this case does not capture the actual peak which occurs on February 2nd. As in the case of the Japanese asset price bubble the oscillations are already quite small when the predictions are conducted and this is probably why the confidence interval is simply moved sideways. These results indicate that the model might not be as robust as one would desire, which will be further discussed in chapter 7.

When conducting the prediction closer to the peak, as seen in the right graph of Figure 15, the interval of critical points is smaller than in previous predictions and also captures the actual peak date. This indicates that the prediction might be more accurate when done closer to the peak.

Another interesting observation when comparing the two graphs in Figure 15 is regarding the last oscillations prior to the peak. Since the data in the left graph is cut off earlier it assigns the last occurring price movements more weight which means that the estimation done two months before the peak finds more oscillations right before the last observed date compared to the estimations that are conducted at later dates. Note that the fits in the left graph are excluded from the results in the right graph since the additional information given to the model when postponing the prediction date disregards the left graph fits as bad fits. The estimation process of the right-hand graph finds less interest in the smaller last oscillations captured by the fits in the left graph, why the results look so different.

6.3 The Emerging Markets Bubble of the 1990s

In the middle of the 1990s institutional investors, mainly in Europe and North America, found particular interest in emerging markets around the world (Kindleberger, 1978/2011). These markets were characterized by a high economic growth rate and offered potentially high rates of return. Investments in emerging markets were highly sought after and were often seen as must haves in a well-diversified portfolio. By 1997 the large inflow of capital into several Asian economies led to dramatic rises in asset prices, possibly fueled by
speculative behavior. When the Thai government, due to lack of foreign currency, was forced to end the peg of the baht to the U.S. dollar the Thai financial market took a severe turn. The crisis in Thailand rapidly spread to other emerging markets in Asia, eventually even to Russia and South America. We have chosen to apply the model to the Hang Seng Index of Hong Kong since this was the largest stock exchange in the area at the time, and likely provides the most reliable data.

Figure 16 shows the result of the LPPL-model fitted to the Hang Seng Index, one of the markets heavily affected by the crisis, between 1995 and 1997. The time series shows clear LPPL-characteristics with a rather low frequency of oscillations. The confidence interval encapsulates the actual regime shift which occurred on August 7th. There is, however, a heavy skew to the left, indicated by the red median line, meaning that the regime shift is expected to occur during the earlier dates of the interval. We also observe an interesting pattern in that there seems to be two different main fitting structures, where some fits exhibit a steep incline and some have a more drawn out ascent, ending at later dates. It seems like the fits that end late in the interval do not take into consideration the small dip that occurs just prior to the last observed date, which leads to expected oscillations of a lower frequency and later end dates. The fits that end early in the interval, however, use the information contained in the small dip, why these fits expect more frequent oscillations and an earlier change in regime. This probably has to do with the design of the estimation method used in this study. Recall that the code iterates the fitting procedure a myriad times with different start and end dates and thereafter filters out the fits that do not fulfill the constraints used in this thesis. Apparently there are two kinds of fits that exhibit parameters with acceptable and reasonable parameter values in this case. These results might actually be regarded as affirmation of this method since the model finds different paths the asset prices might follow, and considers that the last dip in price before the last observed date might be nothing but an anomaly that should not affect the estimation results.
As in the case of the FTSE 100 Index, the Hang Seng Index is during the analyzed time period influenced by exogenous factors, mainly the Thai currency crisis. The results in Figure 16, however, indicate that there may have been speculative behavior on the market in which case the downturn, at least to some degree, was caused by endogeneity. As in the previous case though, the exogenous Thai currency crisis may have acted as a trigger, causing speculators to withdraw from the market.

The left graph of Figure 17 below shows the resulting fits when setting the last observed date two months ahead of the peak date. Interestingly this yields a slimmer interval of predicted end dates, with the median more centered than before. These results are somewhat unexpected since a prediction conducted closer to the peak utilizes more information and therefore, in theory, should yield more accurate predictions. The slimmer interval in this case is due to that the estimation process when conducted earlier does not find as big a proportion of fits with late critical dates as in Figure 16. This leads to that the fits with critical points
occurring late, end up outside of the 80% confidence interval, as illustrated by the purple line in the left graph of Figure 17.

In the graph on the right of Figure 17 the last observed date is set two weeks prior to the peak. This yields results that are similar to the prediction done one month prior to the peak, with the interval and the median shifted slightly to the right. These results do not conform to the theory that the prediction should be more accurate closer to the peak, since the prediction two weeks prior to the peak date do not yield more accurate results, compared to the prediction conducted one month before the peak. This is due to there being two different kinds of fits where the fits with later critical points are sufficiently many to be included within the confidence interval.

Figure 17. The LPPL-model fitted to the Hang Seng Index during the emerging markets bubble of the 1990s with different last observed dates
Source: Own figure.

6.4 The Dot-com Bubble of the Late 1990s

The dot-com bubble of the late 1990s is perhaps one of the most well-known examples of a highly speculative bubble. The establishment of a new market with the advent of internet based companies garnered great public interest following in the wake of successes for companies such as Google and Yahoo (Lowenstein, 2004). Speculators hoped to find the
next big successful company and were willing to pay higher and higher prices to not miss out, leading to herding behavior. During the bubble much of the speculative behavior was seen in speculators’ reactions to so called initial public offerings (IPOs), where many companies that had never made a profit but with grand business plans could raise unproportionate amounts of capital. Several new startups managed to garner interest from the public only by adding a ‘.com’ to the end of the company name. Since this bubble mostly affected companies in the technology sector it is most observable in indices mainly composed of high tech companies, and since the asset price increases were largely due to IPOs it is best observed in a broad index. The Nasdaq Composite Index matches these criteria and should represent the bubble well. From 1998 to its peak on March 10th 2000 the index increased more than fivefold to roughly 5,050, a record high point. The drastic downturn following the peak was not triggered by any particular major event, but was seemingly self-inflicted. Perhaps the market simply sobered up to realize that many of the Nasdaq companies were wildly overvalued. However, it is probable that this sudden shift in investor mentality was in fact inflicted by some kind of exogenous trigger.

The results of the LPPL-model fitted to the Nasdaq Composite Index are presented in Figure 18. Since the dot-com bubble is said to be one of the most evident examples of a speculative bubble, the price movements on the market are expected to closely follow the LPPL-patterns. It is apparent from Figure 18 that so is the case. The index clearly follows the LPPL-patterns leading up to the crash, while the oscillations are rather long in the beginning of the time series and decrease rapidly in amplitude closer to the peak. The actual peak date on the Nasdaq Composite Index occurred on March 10th 2000 and thus fits within the 80% confidence interval of predictions.
Figure 18. The LPPL-model fitted to the Nasdaq Composite Index during the dot-com bubble of the late 1990s
Source: Own figure.

It is apparent from Figure 18 that the LPPL-estimation conducted one month before the actual peak yields trustworthy results. The left graph of Figure 19 below illustrates the results of a prediction when the last observed date is instead set two months prior to the peak date. It can be seen from the graph that the interval of predictions is comprised of very few curves and misses the actual peak date by more than two months. This is not an accurate result, and it seems that the model only finds one long oscillation prior to the observation date and therefore predicts that the bubble will continue for several months. The explanation to why this happens is that the dot-com bubble is quite short and because of this there is a great deal of information needed for an accurate prediction contained in the price data of the last 2 months before the regime shift. For the model to accurately predict the regime shift it in this case needs more data closer to the peak. When conducted one month before the crash, as in Figure 18, the data includes a second oscillation why the estimation finds a regime shift closer to the observation date. Without this data the model is unable to anticipate how rapidly the oscillations actually are decreasing in size, which is crucial for the prediction.
When the last observed date is instead set two weeks before the actual peak date, as in the right graph of Figure 19, the results are again similar to those of Figure 18, although the interval is shifted slightly to the right. The confidence interval now contains more fits compared to both when the prediction is conducted one and two months before the peak. This is what one might expect when a bubble only ranges over a short period of time and the information contained in the last few months are of great significance for the prediction.

### 6.5 The Wheat Price Bubble of the 2000s

During the time period of 2006 to 2008 the price of wheat futures increased more than threefold to a high of 1,280 U.S. cents per bushel. Wheat was not the only commodity to experience high increases in price during the 2000s. Similar price developments were also seen on the markets for e.g. soybean and corn futures. Whether widespread speculative behavior was the reason behind this rapid price increase or not, has been a topic of discussion after the drastic downturn in prices during 2008. Some hedge fund managers, commodity end-users and policy makers argue that commodity index funds by buying commodity futures on such a wide scale are responsible for creating the bubble (Irwin & Sanders, 2010). In fact, between 2006 and 2008 index fund investments in commodities increased from 90
billion U.S. dollars to just under 200 billion U.S. dollars (Fawley & Juvenal, 2011). Some economists, on the other hand, have claimed that the commodity markets were driven by fundamental factors that pushed the prices higher (Irwin & Sanders, 2010). Most probable, however, is that the wheat prices during 2006 to 2008 were affected by several factors including rising demand for food, climate change, high oil prices, increased interest in alternative use of land through biofuels, but also speculative behavior (Robles et al., 2009).

As a result of the global financial crisis starting in 2007 and the associated credit crunch with more restrictive lending, overall speculative behavior saw a major drop as capital became harder to access (Mizen, 2008). The wheat futures prices reached its peak in March 12th 2008 which was followed by a sharp downturn that lasted far into 2009, likely as a result of the credit crunch, although other factors possibly could have been influential as well.

Figure 20 below presents the results of the LPPL-equation fitted to the time series preceding the downturn in the price of wheat futures with the last observed date set one month before the peak. Although the bubble in wheat futures is said to have begun in 2006 the model only finds fits starting in April 2007. For this reason we only present the shorter period between April 2007 and March 2008 in Figure 20. It is possible that the prices started to rise in 2006 but the actual herding behavior commenced in 2007. Note that the futures prices have two peaks, where we have chosen to define the second peak as the regime shift, since it is at this time the growth rate of the prices is dramatically changed. It can be seen from Figure 20 that the LPPL-model yields a quite inaccurate prediction where the actual peak date is not encapsulated by the 80% confidence interval and the median date is several weeks prior to the peak date. It can also be observed that the price of wheat futures, despite the inaccurate prediction, does follow the LPPL-characteristic patterns quite well leading up to the last observed date.
A question that arises when examining Figure 20 is regarding how it is possible for the model to miss the actual regime shift with such significance despite the fits of the curves being quite accurate. It seems that the oscillations are so far gone when the prediction is conducted that they indicate a regime shift close to the last observed date, while the movements later than this date are not possible to explain from the underlying theory of the LPPL-model.

To further examine this bubble we continue by presenting Figure 21 containing two estimations with different last observed dates. The left graph of the figure illustrates that an ex-ante prediction conducted two months before the regime shift yields results indicating that the change in regime should occur right after the last observed date. This prediction is, yet again, probably due to that the oscillations already have reached maturity and are quite small when the prediction is conducted. As in Figure 20 most of the fitted lines do follow the actual price data closely and should be of explanatory significance.
Figure 21. The LPPL-model fitted to wheat futures prices during the wheat price bubble of the 2000s with different last observed dates.
Source: Own figure.

It is apparent from the right graph in Figure 21 that the model yields more accurate critical points when the ex-ante prediction is assumed to be conducted two weeks before the actual change in regime. However, the 80% confidence interval just barely captures the actual peak date and yet again occurs just after the last observed date. The median date also occurs ten days earlier than the actual peak date, indicating that the results are not very accurate.

The three predictions on the wheat price bubble all have in common that they anticipate the critical point in time just after the last observed date. It is important at this point not to rush into any premature conclusions. These results might be a sign of the drawbacks of the LPPL-model, although they might as well be an indication that the prices of wheat futures to a large extent were driven by exogenous factors over this time period. As in the case of several bubbles in this chapter the oscillations in this case have come quite far when the predictions are conducted. Perhaps a negative exogenous trigger is needed to disrupt the herding behavior, why the price movements do not change regime as early as they should according to the model. These possibilities will be discussed more in general in chapter 7.
In the results presented in chapter 6 we have, when analyzing the different bubbles, encountered a number of interesting observations that are worth further exploration and discussion. Many of these observations are recurring, some of them expected and some unexpected based on the theoretical framework. In this chapter we aim to discuss these observations, answer the research questions of this thesis, and propose a new idea on how to interpret the results of the LPPL-model.

We find that the expected characteristics of the LPPL-model do in fact occur on all analyzed markets during bubble periods. The price movements of all bubbles leading up to the regime shift are characterized by faster-than-exponential growth, which is possible to conclude since the estimation process in all cases ends up with fits that exhibit allowed values of $z$, the strength of the feedback mechanism. All of the analyzed bubbles, in addition, show clear oscillatory patterns where the oscillations decrease in amplitude leading up to the regime shift. The smaller oscillations close to the regime shift are in some cases difficult for the model to fit perfectly, although it is clear that these oscillations in general are smaller than the earlier oscillations, and that deviations from the LPPL-patterns are uncommon. We conclude that the basic assumptions of the LPPL-model are legitimate, since the suggested patterns are possible to observe on markets during bubble periods. Recall that the patterns of the LPPL-model are similar to those proposed by Rodrigue, why the actual movements of the different bubbles also correspond to Rodrigue’s patterns leading up to the regime shift. In addition, most analyzed bubbles exhibit a bull trap followed by a return to “normal” following the regime shift, as proposed by Rodrigue.

One recurring observation of the results is the apparent influence of exogenous factors on both the initiation of a bubble and its regime shift. The initiation of the bubble being influenced by an exogenous factor is consistent with the Kindleberger-Minsky framework discussed in chapter 2, which suggests that a bubble is usually initiated by an economic displacement. This displacement might in turn lead to an increase in speculative behavior. According to both Reinhart & Rogoff and Knutsen & Sjögren, financial liberalization is most
commonly what initiates a bubble. This influence is clearly seen in the case of the Japanese asset price bubble. However we do not find any evidence to suggest that financial liberalization is the most common initiating factor, based on the bubbles we have analyzed. However, this causal relation might be difficult to observe since financial liberalization influences the macroeconomic situation which in turn may contribute to the potential influence of exogenous factors. For example, the London stock market would not have been influenced by U.S. interest rates if not for financial liberalization.

Several of the analyzed bubbles also exhibit regime shifts that seem to have been triggered by an exogenous event while still being captured by the LPPL-model’s predicted interval of end dates. In some cases we also observe that the entire interval of predicted end dates is shifted when moving the last observed date, calling into question the nature of the model’s predictive ability. These results are in practice problematic for the prediction of regime shifts since the actual peak date is unknown when conducting ex-ante prediction. We propose a couple of possible explanations for the exogenous influence on the regime shifts.

One possibility is that the exogenous events that seem to be plausible explanations for the regime shifts are accredited more influence than what is justified. Since it is difficult to measure speculative behavior, the influence of endogenous speculative growth might be mitigated in favor of more easily observable exogenous events. For example, in the case of the London stock market bubble in section 6.2, the regime shift seems to have been the effect of unsustainable speculative behavior based on the results of the LPPL-curves, but the prevailing explanation for the downturn is the more easily observed external change in the interest rate. The disregarding of speculation in favor of an exogenous trigger may explain the apparent influence of exogenous factors although it does not serve as a satisfactory explanation for why the confidence interval of predicted end dates is shifted when the last observed date is changed.

Another possible explanation is that the endogenous speculative growth and the exogenous trigger events complement each other. We propose the idea that the LPPL-patterns by themselves are not enough to explain the end of a speculative bubble, but there is a need for
an exogenous event to trigger the regime shift after the oscillations have reached maturity. We find this to be the most feasible explanation since it also explains the shift in the interval of predicted end dates. Consider, for example, the case of the Japanese asset price bubble in section 6.1. From the graphs it seems that the oscillatory patterns of the LPPL-curves become very small and likely reach maturity roughly halfway through 1989. By setting the last observed date two months prior to the peak the model then concludes that the time series is sensitive to a regime shift just following the last observed date. The fact that the asset prices continued to grow for two months more indicates, if our hypothesis holds, that there was no exogenous event large enough to trigger the regime shift during this period, and it took until the implementation of a tightened monetary policy and stricter central bank regulations for the herding behavior to be disrupted. At this point the asset prices have risen a great deal, why the impending downturn is more severe.

This hypothesis of ours is strengthened by the logical assumption that for speculators to stop buying and start selling, something more than endogenous factors is needed. During bubble periods asset prices continue to rise due to anticipation of further price increases. For the prices to take a downturn, or change regime, the investors have to change their expectations to anticipating decreasing asset prices. It is easy to assume that this does not happen by itself. An exogenous event is probably needed to disrupt the herding behavior and the investors’ anticipations. In some cases this disruption might crystallize as reversed speculative behavior where investors sell because they are anticipating decreasing asset prices, causing a crash on the market. If no exogenous event occurs the herding behavior and increasing asset prices will continue until negative news big enough reach the market. On the other hand, if such an exogenous event occurs before the speculative behavior has reached maturity, i.e. when the oscillations are still quite large in amplitude, the prices will not fall drastically since they are not sufficiently overvalued to begin with. We expect a dip in prices since they still are affected by exogenous shocks, although the asset prices quickly return to the trend and thereafter continue to rise in accordance with the LPPL-patterns.

This need for an exogenous influence after the endogeneity has reached maturity is consistent with the Kindleberger-Minsky framework, which proposes that overvalued asset
prices are more sensitive to small exogenous shocks. The theory of Kindleberger-Minsky does not address the price movements during a bubble, although the hypothesis proposed in this thesis is consistent with their contribution.

If our hypothesis holds, the model’s predictive ability is weakened, although with this newfound knowledge it still might contribute to an increased understanding of how bubbles develop and in anticipating when they might burst. However, following this hypothesis the results of a fit have to be interpreted in a more careful manner. When conducting an ex-ante prediction and interpreting the results, the predicted critical points should not be interpreted as best estimates of a change in regime, but rather as the start of a time period when the asset prices are more sensitive to negative external events. It can be noted from the results of this thesis, however, that the model in many cases accurately predicts the change in regime. We are of the belief that this happens since negative exogenous events during most time periods occur quite often. Because of this a regime shift is usually what closely follows when the oscillations have reached maturity. The asset prices do not change regime earlier since the oscillations at this point have not reached far enough, the assets are not overvalued enough. The high frequency of negative exogenous events is therefore what can explain the accuracy of the LPPL-model in most cases.

Before going into our results on the robustness of the LPPL-model it is first important to consider what robustness actually means in this case. Consider an ex-ante prediction conducted two months before the actual regime shift. For the model to yield similar results when conducting an ex-ante prediction one month later, it is necessary that the price movements during the past month have followed the predicted patterns of the estimation done two months prior to the peak. If this is not the case the new prediction will yield different results than those of the earlier prediction, since the later one uses data that the first one does not access. We can observe this on several of the bubbles analyzed where the results cannot be considered robust. A representative example is that of the dot-com bubble in section 6.4, where the predictions yield different results since the patterns in between the dates of predictions are in contradiction with what could be anticipated after conducting the earlier prediction. A contradictory example is that of the emerging markets bubble in section
6.3, where the patterns following the earlier prediction track what is anticipated. These specific results are clearly to be considered robust.

Even though the analysis of the emerging markets bubble yields robust results, the results in general cannot be regarded as robust since the rest of the bubbles of chapter 6 exhibit predictions that are dependent on when they are conducted. Why the predictions regarding the dot-com bubble are not robust is quite easy to tell. The reason is that this bubble spreads over a quite short time interval while the asset prices increase very rapidly. Due to this there is a great deal of information contained in the price data of the last few months, why the earlier prediction does not capture the real price movements and the oscillations leading up to the regime shift. The Japanese asset price bubble, the London stock market bubble, and the wheat price bubble all seem to suffer from the same condition, where the results of the different predictions cannot be regarded as robust. Instead of drawing the conclusion that the LPPL-model is lacking predictive ability due to this non-robustness, we stick to the hypothesis we presented earlier in this chapter. The reason why these predictions are so dependent on the time of prediction is that the oscillations are so far gone when the predictions are conducted. For this reason the predicted end dates end up just after the last observed date. We are of the belief that this, once again, has to do with that the model in fact predicts a point in time when asset prices start being easily influenced by negative exogenous factors. Since no negative news big enough occur right after the oscillations reach maturity the prices continue to rise until this happens, why the model continuously predicts closely following regime shifts. That the model does not yield robust results is therefore of no major significance, since this is totally logical if the estimation results are interpreted as predictions on when the asset prices start being more easily influenced by exogenous factors.

We have previously mentioned that in theory the predictions should become more accurate when the last observed date is closer to the actual peak date. When observing the results in chapter 6 it is clear that this is not consistently the case. Only in the instance of the London stock price bubble does the prediction seem to increase in accuracy when setting the last observed date closer to the peak. This relates to the third research question of this thesis regarding how the prediction is affected by the date at which the prediction is conducted. In
many of the analyzed bubbles we see that moving the last observed date yields inconsistent results, sometimes increasing in accuracy but sometimes shifting the interval of predicted end dates entirely. This should, however, be considered in combination with our proposed hypothesis that exogenous factors have a larger impact on the outcome of the bubble.

In contrast to the stage-models established by Kindleberger & Minsky, Knutsen & Sjögren and Reinhart & Rogoff which focus on the entire development of a crisis, the LPPL-model examined in this thesis mainly focuses on the evolution of the bubble itself. The stage-models emphasize the role of debt accumulation and its impact on bank crises. However, the conclusions we draw based on the LPPL-model neither contradict nor reinforce the assumptions of the stage-models.
Regardless of whether one considers the implications of the original LPPL-model or the hypothesis proposed in this thesis, the potential usability of the model bears with it a great advantage for individual speculators and investors. An investor or a group of investors with the sole possession of knowledge of the model might be able to use it to beat the market. This potential is even greater when considering that the patterns in theory should occur at every scale of time, meaning it could even predict price movements on intraday data as long as speculative behavior is at hand. We consider examination of the usability of the LPPL-model on intraday data as an interesting topic for further research.

In a wider perspective we imagine that the LPPL-model can be used to assist policy makers and public institutions to identify speculative bubbles on financial markets before they end. We visualize a future where the LPPL-model can be of help for policy makers to alleviate or prevent the consequences of excessive speculation. In practice this might mean that the government and other policy makers, with the use of this knowledge, through strategic regulation can counteract market dysfunctions and dampen the effects of crashes. Before such a future can exist the LPPL-model should, however, be further examined by the academia.

A question that remains, however, is whether widespread knowledge and acceptance of the LPPL-characteristics would cause the occurrence of the LPPL-patterns to cease entirely. If all market participants are aware of the prevailing market psychology and the overvaluation of asset prices, speculative bubbles might never occur in the first place, or at least end earlier than they otherwise would. In that regard, further exploration and widespread knowledge of the LPPL-patterns might contribute to a stabilization of financial markets.

The LPPL-model is still quite unknown in the academia and only a handful of scientists have yet come across the model. For this reason there are some aspects of the LPPL-model that we consider worthy of further examination. We would see further exploration of the relation between endogenous speculative growth and exogenous effects to determine whether our
proposed hypothesis has any truth to it. We see that this could be done by examining bubbles that do not change regime when the oscillations have reached maturity. It is of interest to investigate if exogenous shocks have or have not occurred following the predicted critical points. If it turns out that no exogenous shocks have occurred, our hypothesis is strengthened, and vice versa. This examination would contribute to a further understanding of how the predictions should be interpreted.

As described in section 1.4 we have chosen to focus this study on speculative bubbles on stock markets and a commodities futures market. To gain an increased understanding of the potential and the limitations of the LPPL-model we suggest that a topic of further research should be to apply the model to price data of different kinds of markets, e.g. bond markets, money markets and house markets. We also suggest that the model should be applied to more stock market bubbles, yet again to gain an increased understanding of the LPPL-model.
In this thesis we have investigated the ability of the LPPL-model to accurately predict the end of speculative bubbles on financial markets. To our knowledge, all previous studies have only presented results where the predictions turn out to be successful. Through this thesis we are first to highlight both the potential and the limitations of the LPPL-model. We have done so by applying the model to time series of five bubbles chosen based on their historical context and critically analyzed the results. Our results reinforce the underlying theory of the model that asset prices during bubble periods oscillate with decreasing amplitude around a faster-than-exponential growth. We find that the predictions of the LPPL-model in most cases are quite accurate, where the actual peak date is encapsulated by the confidence interval of critical points. The robustness of the model, however, can be questioned since the predictions seem to be dependent on when they are conducted.

One recurring observation of the results is that asset prices in several cases continue to rise with small oscillatory patterns even after the predicted regime shift. We are of the belief that this is because there is an interaction between exogenous and endogenous influences, where an exogenous factor acts as a trigger and is needed for the speculative behavior to be disrupted, thus changing the regime. With this motivation we propose that the results of the LPPL-model should be interpreted differently. Our hypothesis is that the interval of critical points in fact is not a prediction of the regime shift itself, but rather as a prediction of when the asset prices are more sensitive to the influence of exogenous factors. The interval thus gives an indication of after which point in time an exogenous factor is enough to disrupt the speculative behavior. With this hypothesis in mind, the reason why the LPPL-model yields accurate predictions in most cases is due to exogenous events happening quite frequently. Although when no such event occurs the asset prices continue to rise even though the oscillations have reached maturity.
REFERENCES


This appendix illustrates how the results of this thesis depend on whether real or log-prices are used in the fitting procedure. The left and right graph in each figure illustrate the results when, in the fitting procedure, using log-prices and real prices, respectively. The conclusion we draw is that the difference is negligible, why usage of either is acceptable.

**Figure A. 1. The Japanese asset price bubble**
Source: Own figure.

**Figure A. 2. The London stock price bubble**
Source: Own figure.
Figure A. 3. The emerging markets bubble
Source: Own figure.

Figure A. 4. The dot-com bubble
Source: Own figure

Figure A. 5. The wheat price bubble
Source: Own figure