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Design Information Entropy**

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Design Space Configuration for Minimizing Design Information Entropy

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Abstract: In this paper concepts for quantitative theory of design are developed. The design process is viewed as a process of increasing the information of the product/system using concept from information theory. The creation and evolution of design space and information entropy are central concepts that are studied in this paper. Furthermore, axiomatic design is discussed in relation to this, and it is shown that the two fundamental axioms are likely to be dependent.

The design information entropy is used as a state that reflects both complexity and refinement, and it is argued that it can be useful as some measure of design effort and design quality. It provides a tool to describe the relation between a design, and the design space from which it is derived as well as for quantitative description for various aspects of design, both structural information regarding architecture and connectivity, as well as for parameter values, both discrete and continuous. In this paper it is also shown to be useful as a measure of quality when configuring design spaces, such as in product platforms, and when setting up parameterization for design.

Keywords: *design entropy, information theory, design space*

1 Introduction

During the process of design, information is gradually increased as the design progress, and the uncertainty of the design is reduced. Every design decision makes it more defined and thus reduces the uncertainty of the design, as well as parameter refinement do. It could be argued that this is a central aspect of design.. This can be viewed as a learning process, as described in Ullman [1]. Therefore design theory should really be a theory of design information. Although there is a rich literature regarding design process, there has been little or no effort to describe the generation of information in quantitative terms.

A central concept in this study is the *design space*. In fact, design information have no meaning unless it is related to some kind of design space. The design space is here defined as the space within which the design can vary through parameter variations. The parameters are here any coding used to define the design, e.g. design parameter values, information about architecture, components, connectivity etc. A product platform also defines a design space through the different combinations of components that can be chosen. Corresponding to this is also a *functional space*, which is the space with the functional characteristics corresponding to designs within the design space.

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Typically, first a system level design space is first refined into a single product/system level specification. In the next step the design space is expanded by adding more details and releasing more parameters into the design space, that is then further refined.

1.1 Information theory and design

The classical information theory of communication was founded in 1948 by C.E. Shannon with his paper “A Mathematical Theory of Communication”, Shannon [2]. Subsequently it has been recognized that information is a key property of design, and for describing and analyzing the design process. The notion of information theory in design has been introduced by several authors. Notables are N.P. Suh [3], Kahn and Angeles [4] and Frey and Jahingir [5]. The two first are discussed later in the text. Frey and Jahingir deals with the transformation of information content in design parameters to information content in the functional characteristics, so is Bras and Mistree [6] where robustness is defined from maximizing the signal to noise ratio, which also deals with the relation between design parameters and functional characteristics. Information theory was also used to analyze design optimization in Krus and Ölvander (Andersson) [7] and [8]. Information theory has also been used to define complexity in software, and notably by Bansiya et al [9] to describe complexity in object oriented systems, which is very close to general design.

The notion of information is used in the second axiom of N.P. Suh’s Axiomatic design, which states that the information content in a design should be minimized, Suh [10].

In Krus [11] design information entropy was introduced as a measure of information needed to define a particular design relative to a design space. In this study this is extended to modular design and to parameterization.

1.2 Design information entropy

The definition of Information entropy for the discrete case is defined by Shannon [3]

$$H_d = \sum_{i=1}^n p_i \log_2 p_i \quad (1)$$

Here the system can be in n different states with probabilities p_i for each of them.

A more general definition than the information entropy for the discrete case is the differential information entropy for continuous signals, defined by Shannon [3] as:

$$H_c = - \int_{-\infty}^{\infty} p(x) \log_2(p(x)) dx \quad (2)$$

The differential entropy of the probability density function $p(x)$ can also be related to another distribution $m(x)$. The result is called the Kullback-Leibler divergence [12] from

the distribution $m(x)$. This is the relative entropy, and it is defined as (multi variable case):

$$H_{rel} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p(x_1 \dots x_n) \log_2 \left(\frac{p(x_1 \dots x_n)}{m(x_1 \dots x_n)} \right) dx_1 \cdots dx_n \quad (3)$$

A rectangular distribution of $m(x)$ in the bounded interval $x \in [x_{0,\min}, x_{0,\max}]$, with $x_R = x_{\max} - x_{\min}$, would mean that the distribution $m(x)$ of the *design space* is a space of equal possibilities, where no particular region can be considered more likely than another a priori. This can then be rewritten as:

$$H_x = H_{rel}(x) = \int_{x_{\min}}^{x_{\max}} p(x) \log_2(p(x)x_R) dx \quad (4)$$

The H_x is here used here to indicate relative information entropy related to a rectangular distribution, and it has the unit *bits*. It is here defined as the *design information entropy*. For the multidimensional case it can also be written in a more compact form as:

$$H_x = \int_D p(\mathbf{x}) \log_2(p(\mathbf{x})S) d\mathbf{x} \quad (5)$$

where D is the design space within where the design $\mathbf{x}$ is defined. S is the size of the design space and is defined as:

$$S = \int_D \mathbf{x} d\mathbf{x} \quad (6)$$

If the range of one variable is divided into equal parts Δx that have the same probability, the probability density distribution will be:

$$\begin{aligned} p(x) &= \frac{x_R}{\Delta x} : x \in [x_{\min}, x_{\max}] \\ p(x) &= 0 : x \notin [x_{\min}, x_{\max}] \end{aligned} \quad (7)$$

This yields the information content (in bits) for that variable as:

$$H_x = \int_{x_{\min}}^{x_{\max}} \frac{x_R}{\Delta x} \log_2 \left(\frac{x_R}{\Delta x} \right) dx = \log_2 \frac{x_R}{\Delta x} = \log_2 \frac{1}{\delta_x} \quad (8)$$

where Δx is the uncertainty of the variable, and x_R its design range. δ_x is introduced as the relative uncertainty in parameters. For the general multivariable case it can be expressed simply as:

$$H_x = \log_2 \frac{S}{s} \quad (9)$$

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Here S is the size of the design space and s the region of uncertainty for the final design.

2 Design space

2.1 Design space for modular design

Modular design is an effective approach for defining design spaces that can be used to efficiently produce products with different functional characteristics with a low design information content, and low design effort. In fact, the introduction of standardized components means that a much smaller design space is needed to create useful design, since large design spaces for the individual component are effectively removed.

Modular design is a popular approach to create product families. In modular design, common subsystem or subassemblies are defined, further reducing the need for a large design space. In order to assess how efficient a product platform/product family is, there is a range of different definitions for commonality indices. One overview is found in Thevenot et al. **[Error! Reference source not found.]**. Interestingly, efficiency index based on design information entropy can also be derived. The components that makes up a product platform, defines a design space. Each particular design/product then represents design entropy relative to the original design space. Small design information entropy is desirable, since that would imply a low design effort to create a particular variant from the product platform, which would constitute of a limited number of modules/components to achieve a wide variety of products with different functional characteristics. In addition to design information, it is here useful also to introduce information entropy to describe the diversity in the functional space. The simplest would be a simple function of the number of different product variants n_v (with different functional characteristics).

$$H_f = \log_2 n_v \quad (10)$$

Example: The modular light vehicle in Figure 1, from Hallberg et al (2008), can have two alternative front ends, one and two wheels respectively, and also two alternative rear ends, one with a single wheel and one with the two wheels and accommodating a flatbed for carrying goods. This results in four different possible vehicles: The four wheeler, the two wheeler and two three wheelers, with four different functional characteristics.

Figure 1. Modular light utility electric vehicle. (Photo: Peter Hallberg)

To define the design information entropy of each design, the design space has to be defined. A vehicle must have one front section, one centre section and one rear section. There are two front ends, one centre, and two rear ends. This means that a particular design has a design information entropy (with respect to that design space) of:

$$H_x = -\log_2 \frac{s}{S} = \log_2 n_s = \log_2 (2 \times 1 \times 2) = 2 \quad (11)$$

Here n_s is the number of unique design defined by the design space.

In reality however, only three of these variants, where considered practical, since the variant with two wheels in front and one in the rear, was impractical for various reasons. This means that the product platform can produce three different viable sets of functional characteristics. The information entropy of the *constrained design space* of viable designs can then be defined as:

$$H_c = \log_2 n_v = \log_2 3 = 1.58 \quad (12)$$

One useful measure for the quality of a modular design would be the relative size of the part of the design space that is outside the constrained design space. That can be formulated as wasted design information entropy as:

$$H_w = H_x - H_c = -\log_2 \frac{S_x / s}{S_c / s} = -\log_2 \frac{S_x}{S_c} \quad (13)$$

then H_w represents waste of design information entropy, and should thus be minimized.

It is regarded as waste, since it means that more information is needed to select a concept, without offering more viable concepts, than would be needed without the “waste”. In the case of the example, the waste would be 0.42 bits. I.e. it would have been better if that part of the design space was not included in the design space in the first place. If all four had been viable then the wasted entropy would be zero, since the whole design space would represent viable designs. The waste of design entropy corresponding is similar to the definition in Suh [10] i.e. what is there meant by minimizing

“information content”, although it is there used in the context of robust design, not product families. Note that this is also very relevant for parameterization of models. I.e. if a parameterized simulation model, or CAD model is used for optimization, the parameterization should be such that as large portion as possible of the design space is within the constraint design space of viable designs, in order to have an efficient optimization. It could also, in the author’s opinion, be argued that more “knowledge” is built into a product platform that has little waste of design space.

2.2 Design space for parameterization

To consider the quality is very appropriate when defining design parameters in a design. One existing well documented case is the parameterization of aircraft wing plan form. The wing plan form for a wing can be defined with three parameters (disregarding wing sweep). These are wing span, b , root cord, c_r , and tip cord, c_t as defined in Figure 2. These are here referred to as the primary parameter set. In general, however, it is common to use wing area, S , aspect ratio, AR , and tapering, λ , as parameters instead. See e.g. Torenbeek [14]. These are here referred to as the secondary parameter set. The relationships between these are:

$$\begin{aligned} S &= \frac{b}{2}(c_r + c_t) \\ AR &= \frac{b^2}{S} \\ c_t &= c_r \lambda \end{aligned} \tag{14}$$

Figure 2. Wing parameters.

To study the design space a data set of existing aircraft are shown in the table below.

Table 1. Parameter values for existing aircraft. (Data is from open literature such as Wikipedia and might be inexact, but are just used as an illustrative example)

	b	cr	ct	S	AR	λ
AN225	88.4	16.5122	3.96293	905	8.63487	0.24
A380	79.75	17.6594	3.53187	845	7.5267	0.2
A320	34.09	5.99394	1.19879	122.6	9.47902	0.2
Gulfstream IV	23.7	5.73191	1.71957	88.3	6.36116	0.3
U2	32	4.83333	0.966667	92.8	11.0345	0.2
F-16	9.96	4.47711	1.11928	27.87	3.55944	0.25
Mirage 2000	9.13	8.5537	0.427685	41	2.0331	0.05
Cessna 172	11	1.59214	1.35332	16.2	7.46914	0.85
Max value	88.4	17.6594	3.96293	905	11.0345	0.85
Min value	9.13	1.59214	0.427685	16.2	2.0331	0.05

Plotting the region spanned by the extreme values in the secondary parameter set, in the coordinates of the primary parameter set gives **Error! Reference source not found.**

Figure 3. Design space using the secondary parameter set.

In order to be able to deal with large variation of parameter values it is convenient to use the logarithm of the parameters instead. This can also be seen as reallocation of the probability density function of the design space.

Figure 4. Design space defined by the logarithms of the secondary parameter set, displayed in the logarithms of the primary parameter set. To the right the outer box represents limitations on primary parameters.

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This is also justified by the fact that it is more relevant to define parameter tolerance in relative terms of the nominal parameter value, due to equation (9).

In the left part of **Error! Reference source not found.** the design space defined by the logarithm of the secondary parameters is shown as the slanted block. The parameters of all the studied aircraft are fitted inside this box. To the right, the outer box represents the design space defined by the primary parameters. There is a small part of the design space of the secondary parameters that is outside. This represents an empty part of the design space, which can be regarded as a sort of “waste” of design space. On the other hand all the space inside the outer box that is not occupied by the secondary parameter design space is wasted if the primary parameters are used. Doing volume integration over the design space of the second parameter sets reveals that the ratio between its size S_2 and the size of the design space S_1 of the primary parameter set is

$$\frac{S_2}{S_1} = 0.413 \quad (15)$$

This means that the extra amount of information needed to specify a design using the primary parameter set compared to the secondary parameter set to within the same tolerances is:

$$H_{diff} = H_{x,1} - H_{x,2} = \log_2 \left(\frac{S_1 / s}{S_2 / s} \right) = \log_2 \left(\frac{S_1}{S_2} \right) = 1.24 \quad (16)$$

Hence this is a measure of the relative quality of the two design spaces. In this way it can be argued that the design information entropy is a way to assess the quality of a parameterization and consequently a design space. An interesting consequence is that this provides a mechanism to optimize the design space itself provided there is a sample sets of designs that can be used to span the design space. There is of course also other consideration regarding parameterization. One is to have a parameterization such as the functional characteristics are as uncoupled as possible with respect to the design parameters. However, this is likely to be coherent with a minimization of waste of design information entropy, since an uncoupled design would mean that the design space can be fit around the design space of the functional characteristics more tightly. Example: For a linear design the functional characteristics can be calculated as:

$$\mathbf{y} = \mathbf{A}\mathbf{x} \quad (17)$$

Where $\mathbf{y}$ is the functional characteristics an $\mathbf{x}$ is the design parameters. $\mathbf{A}$ is the design matrix as in Suh [10]. If there are constraints on $\mathbf{y}$ such that

$$\mathbf{y} \in [\mathbf{y}_{\min}, \mathbf{y}_{\max}] \quad (18)$$

The only way this can coincide with a function design space resulting from design parameters with the limitations

$$x \in [x_{\min}, x_{\max}] \quad (19)$$

is if the matrix $\mathbf{A}$ is diagonal and hence uncoupled. A consequence of this is that the independence axiom and the information axiom in Axiomatic design Suh [10], are not really independent from each other, but that an uncoupled design also tends to minimize the information needed to specify a design. The overriding axiom should be that the best design space formulation is the one that minimize waste of design space.

3 Discussion

The notion of design space is relevant both in design automation, where it is very concrete, and also in manual design, although it is perhaps less obvious there. In this paper design space has been defined by a rectangular probability distribution of the design space. In a manual design it is usually implicit, and can be seen as a more irregular distribution. For design to be effective, it is desirable to have a small design space, but that still contain sufficiently good designs. It could be argued that an experience designer knows early on how to limit the design space to quickly arrive at a design. In a good design space, in the form of a product platform, is that it is easy to assemble viable designs from a limited set of design elements that can be combined into new products. Design information entropy can also be applied to parameterization of a design. In a good parameterization, all parameter combinations should yield viable designs. In fact the fraction of viable parameter combinations can be seen as a quality of a parameterization, see Amadori et al. [Error! Reference source not found.]. Ideally a good design space should be well aligned with the functional design space of possible requirements. This is consistent with axiomatic design [3] that a design should be as uncoupled as possible

The concept of design information entropy also provides a sound base for defining creativity as the process of selecting areas for expanding the design space “to think outside the box” which here takes on a more literal meaning. Creativity can also be the process of navigation in highly dimensional multimodal design spaces for concept generation, which on the other hand is simplified by a limited design space, like an efficient product platform, with few unviable designs. One path of future work is to use the result to automate formation of the optimal design spaces in cases where known samples of designs exist, like in aircraft. Another path is to extend the theory into dealing also with unbounded design spaces.

4 Conclusions

In this paper it is demonstrated that introducing design information entropy as a state, can be used for quantitative description for various aspects of design, both regarding structural information regarding architecture, as well as for parameter variables, both discrete and continuous. The design information entropy is a measure of the precision with which a design is defined within a design space. Since the information entropy

increase with a large design space, it is desirable to have a small design space that still contains the good designs. In particular it has been shown that design information entropy can be used as a measure of quality of product platforms for modular design. It has also been shown that it can be used as a measure of the quality of parameterization in design. It is also clear that the design of the design space as such, is critical to promote creativity by making viable design alternatives clear to the designer.

It is also suggested by this kind of analysis that the independence axiom and the information axiom in Axiomatic design are not independent from each other.

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