Correction: Marginal AMP chain graphs (vol 55, pg 1185, 2014)

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In the original paper, we present a new family of models that is based on graphs that may have undirected, directed and bidirected edges. We name these new models marginal AMP chain graphs (MAMP CGs) because each of them is Markov equivalent to some AMP chain graph under marginalization of some of its nodes. Among other results, we describe global and pairwise Markov properties for MAMP CGs and prove their equivalence for compositional graphoids.

Unfortunately, the definition of descending route given in the original paper has to be modified so that Theorems 5 and 6 hold. Specifically, we have to redefine a descending route as a sequence of nodes $V_1, \ldots, V_n$ of a MAMP CG G st $V_i \rightarrow V_{i+1}$, $V_i - V_{i+1}$ or $V_i \leftrightarrow V_{i+1}$ is in G for all $1 \leq i < n$. The original definition only allowed edges of the form $V_i \rightarrow V_{i+1}$ or $V_i - V_{i+1}$. Therefore, the descendants of a node are now a superset of the descendants in the original paper. Recall that the descendants of a set of nodes X of G is the set $de_G(X) = \{V_n |$ there is a descending route from V_1 to V_n in G, $V_1 \in X$ and $V_n \notin X\}$. This implies that we have to redefine the pairwise separation base of a MAMP CG, since this builds on the concept descendant. Specifically, we have to define the pairwise separation base of a MAMP CG G as the separations

- $A \perp B | pa_G(A)$ for all $A, B \in V$ st $A \notin ad_G(B)$ and $B \notin de_G(A)$,
- $A \perp B | ne_G(A) \cup pa_G(A \cup ne_G(A))$ for all $A, B \in V$ st $A \notin ad_G(B)$, $A \in de_G(B)$, $B \in de_G(A)$ and $uc_G(A) = uc_G(B)$, and
- $A \perp B | pa_G(A)$ for all $A, B \in V$ st $A \notin ad_G(B)$, $A \in de_G(B)$, $B \in de_G(A)$ and $uc_G(A) \neq uc_G(B)$

where the notation not explained here can be found explained in the original paper.

Another consequence of the redefinition above is that we have to rewrite the proof of one of the main results in the original paper, namely that the global and pairwise Markov properties for MAMP CGs are equivalent for compositional graphoids. We sketch the proof below. A detailed proof can be found in the corrected version of the paper that is available at `http://arxiv.org/abs/1305.0751`

Theorem 5. *For any MAMP CG G, if $X \perp_{cl(G)} Y | Z$ then $X \perp_G Y | Z$.*

Proof. Since the independence model represented by G satisfies the compositional graphoid properties by Corollary 3 in the original paper, it suffices to prove that the pairwise separation base of G is a subset of the independence model represented by G. We sketch the proof for this next. Let $A, B \in V$ st $A \notin ad_G(B)$. Consider the following cases.

Case 1: Assume that $B \notin de_G(A)$. Then, every path between A and B in G falls within one of the following cases.

Case 1.1: $A = V_1 \leftarrow V_2 \ldots V_n = B$.

Case 1.2: $A = V_1 \multimap V_2 \ldots V_n = B$.

Case 1.3: $A = V_1 - V_2 - \ldots - V_m \leftarrow\!\!\circ\, V_{m+1} \ldots V_n = B$.

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Case 1.4: $A = V_1 - V_2 - \ldots - V_m \rightarrow V_{m+1} \ldots V_n = B$.

It is relatively easy to prove the path in each of the cases above either is not $pa_G(A)$-open or implies a contradiction.

Case 2: Assume that $A \in de_G(B)$, $B \in de_G(A)$ and $uc_G(A) = uc_G(B)$. Then, there is an undirected path ρ between A and B in G. Then, every path between A and B in G falls within one of the following cases.

Case 2.1: $A = V_1 \leftarrow V_2 \ldots V_n = B$.

Case 2.2: $A = V_1 \multimap\!\!\rightarrow V_2 \ldots V_n = B$.

Case 2.3: $A = V_1 - V_2 \leftarrow V_3 \ldots V_n = B$.

Case 2.4: $A = V_1 - V_2 \multimap\!\!\rightarrow V_3 \ldots V_n = B$.

Case 2.5: $A = V_1 - V_2 - V_3 \ldots V_n = B$ st $sp_G(V_2) = \varnothing$.

Case 2.6: $A = V_1 - V_2 - \ldots - V_n = B$ st $sp_G(V_i) \neq \varnothing$ for all $2 \leq i \leq n-1$.

Case 2.7: $A = V_1 - V_2 - \ldots - V_m - V_{m+1} - V_{m+2} \ldots V_n = B$ st $sp_G(V_i) \neq \varnothing$ for all $2 \leq i \leq m$ and $sp_G(V_{m+1}) = \varnothing$.

Case 2.8: $A = V_1 - V_2 - \ldots - V_m - V_{m+1} \leftarrow V_{m+2} \ldots V_n = B$ st $sp_G(V_i) \neq \varnothing$ for all $2 \leq i \leq m$.

Case 2.9: $A = V_1 - V_2 - \ldots - V_m - V_{m+1} \multimap\!\!\rightarrow V_{m+2} \ldots V_n = B$ st $sp_G(V_i) \neq \varnothing$ for all $2 \leq i \leq m$.

Again, it is relatively easy to prove the path in each of the cases above either is not $(ne_G(A) \cup pa_G(A \cup ne_G(A)))$-open or implies a contradiction.

Case 3: Assume that $A \in de_G(B)$, $B \in de_G(A)$ and $uc_G(A) \neq uc_G(B)$. Then, every path between A and B in G falls within one of the following cases.

Case 3.1: $A = V_1 \leftarrow V_2 \ldots V_n = B$.

Case 3.2: $A = V_1 \multimap\!\!\rightarrow V_2 \ldots V_n = B$.

Case 3.3: $A = V_1 - V_2 - \ldots - V_m \leftarrow\!\!\circ V_{m+1} \ldots V_n = B$.

Case 3.4: $A = V_1 - V_2 - \ldots - V_m \rightarrow V_{m+1} \ldots V_n = B$.

Again, it is relatively easy to prove the path in each of the cases above either is not $pa_G(A)$-open or implies a contradiction.

□

Theorem 6. *For any MAMP CG G, if $X \perp_G Y | Z$ then $X \perp_{cl(G)} Y | Z$.*

Proof. This proof is more technical than the previous one. It is a proof by induction where the trivial cases are proven in the three lemmas below. The proofs of these lemmas are also rather technical and they make extensive use of the properties of compositional graphoids, i.e. symmetry, decomposition, weak union, contraction, intersection and composition.

□

Lemma 5. *Let X and Y denote two nodes of a MAMP CG G st $X, Y \in K_m$, $X \perp_G Y | Z$ and $Z \cap (K_{m+1} \cup \ldots \cup K_n) = \varnothing$. Let H denote the subgraph of G induced by K_m. Let $W = Z \cap K_m$. Let W_1 denote a minimal (wrt set inclusion) subset of W st $X \perp_H W \setminus W_1 | W_1$. Then, $X \perp_{cl(G)} Y | Z \cup pa_G(X \cup W_1)$.*

Lemma 6. *Let X and Y denote two nodes of a MAMP CG G st $Y \in K_1 \cup \ldots \cup K_m$, $X \in K_m$ and $X \perp_G Y | Z$. Let H denote the subgraph of G induced by K_m. Let $W = Z \cap K_m$. Let W_1 denote a minimal (wrt set inclusion) subset of W st $X \perp_H W \setminus W_1 | W_1$. Then, $X \not\perp_G C | Z$ for all $C \in pa_G(X \cup W_1) \setminus Z$.*

Lemma 7. *Let X and Y denote two nodes of a MAMP CG G st $Y \in K_1 \cup \ldots \cup K_{m-1}$, $X \in K_m$, $X \perp_G Y | Z$ and $Z \cap (K_{m+1} \cup \ldots \cup K_n) = \varnothing$. Let H denote the subgraph of G induced by K_m. Let $W = Z \cap K_m$. Let W_1 denote a minimal (wrt set inclusion) subset of W st $X \perp_H W \setminus W_1 | W_1$. Then, $X \perp_{cl(G)} Y | Z \cup pa_G(X \cup W_1)$.*