Final thesis

Using Rigid Landmarks to Infer Inter-Temporal Spatial Relations in Spatio-Temporal Reasoning

by

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Abstract

Spatio-temporal reasoning is the area of automated reasoning about space and time and is important in the field of robotics. It is desirable for an autonomous robot to have the ability to reason about both time and space. $\text{ST}_0$ is a logic that allows for such reasoning by, among other things, defining a formalism used to describe the relationship between spatial regions and a calculus that allows for deducing further information regarding such spatial relations. An extension of $\text{ST}_0$ is $\text{ST}_1$ that can be used to describe the relationship between spatial entities across time-points (inter-temporal relations) while $\text{ST}_0$ is constrained to doing so within a single time-point. This allows for a better ability of expressing how spatial entities change over time. A major obstacle in using $\text{ST}_1$ in practice however, is the fact that any observations made regarding spatial relations between regions is constrained to the time-point in which the observation was made, so we are unable to observe inter-temporal relations. Further complicating things is the fact that deducing such inter-temporal relations is not possible without a frame of reference. This thesis examines one method of overcoming these problems by considering the concept of rigid regions which are assumed to always be unchanging and using them as the frame of reference, or as landmarks. The effectiveness of this method is studied by conducting experiments where a comparison is made between various landmark ratios with respect to the total number of regions under consideration. Results show that when a high degree of intra-temporal relations are fully or partially known, increasing the number of landmark regions will reduce the percentage of inter-temporal relations to be completely unknown. Despite this, very few inter-temporal relations can be fully determined even with a high ratio of landmark regions.
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1 Introduction

This chapter covers the motivation and background of this project. The goals and objectives of the project are discussed and finally, an outline of the report with an overview of the chapters is provided.

1.1 Motivation

The mobile autonomous robot research area is a growing field with several applications, one of which is the Unmanned Aerial Vehicles (UAVs). Applications for both military and civilian use are plenty and examples of these are reconnaissance, transport, search and rescue, law enforcement and usage in agriculture, such as crop spraying. Different variants of vehicles exist with varying levels of autonomy, some of which are merely remote-controlled while others are completely autonomous. Some also have a hybrid approach where the robot is capable of autonomous operations, but relies on human input to some degree.

As the level of autonomy of a robot increases, so do the requirements for the ability to reason about the world surrounding it. When the environment changes and the robot moves through it, initially correct plans may need to be reevaluated and perhaps revised to fit new circumstances. To do this, a robot must have the ability to reason about both the flow of time and the space surrounding it. This is called spatio-temporal reasoning. Another challenge is that of reasoning with limited data, as complete information of the environment rarely is available, and new data that may be collected incrementally needs to be reasoned with “on the fly”. Further complicating things is the fact that the data received from sensors often is noisy and difficult for high-level reasoning components to process. For handling this, Heintz [1] proposed and developed DyKnow, which is a knowledge processing middleware framework for “bridging the gap between sensing and reasoning in a physical agent”. DyKnow is capable of bridging this gap between low-level noisy data from sensors and the crisp, structured and qualitative data at a higher abstraction level by incrementally refining the information into a more accessible format. A key feature of DyKnow is its ability to process incrementally received information from streams of data and it can continually evaluate formulas in real-time as new information is made available [2].

DyKnow has been successfully implemented and used in practice as part of an architecture used in autonomous UAVs [3]. Extensions to DyKnow have been made, giving it the ability to perform spatio-temporal reasoning on formulas incorporating components similar to those in the spatio-temporal logic ST₀ [2, 4]. This enables evaluating logic formulas describing the relationship between spatial entities while also allowing for the use of temporal logic operators and thus provides spatio-temporal reasoning capabilities. By utilizing these spatio-temporal abilities it can therefore, for example, be used to monitor the execution of a plan according to set rules such as the UAV is never allowed to enter any restricted area, or if the UAV enters an urban area, it must exit that area immediately. A formula describing the first of those rules can be formalized as \( \forall ra \in \text{RestrictedAreas} \, \square DC(uav, ra) \). In addition to using common logic constructs, the formula uses the spatial relation \( DC \) (disconnected) and the temporal operator \( \square \) (always) to express that the UAV (represented by the spatial entity \( uav \)) should always be disconnected from every restricted area. The second
A rule may be expressed as \( \forall ua \in \text{UrbanAreas} \ PO(uav, ua) \rightarrow \Box DC(uav, ua) \). This formula is similar to the first, but introduces the new spatial relation \( PO \) (partially overlapping) and the temporal operator \( \Box \) (next) to state that if the UAV starts to overlap with an urban area, it will in the next time-point be disconnected from that area. If that is not the case, the formula evaluates to false and the execution monitor may take appropriate action.

A way of extending this functionality further would be to give it the ability to reason about spatio-temporal formulas from a more expressive language than \( ST_0 \), such as \( ST_1 \) [4]. The difference between the two is that \( ST_1 \) can describe the relationships of spatial regions from different time-points, while \( ST_0 \) only can describe the relationship between two relations within a single time-point. The difference is subtle, but opens up the possibility to describe how a spatial entity changes over time. As an example of this, consider the formula \( EQ(uav, \Box uav) \) containing the spatial relation \( EQ \) (equal) and also the next-operator acting on a spatial region term, rather than on a relation symbol like before. This means that while the formula only contains a single spatial entity \( (uav) \), the region occupied by that entity is described twice, once for the current time-point and once for the next time-point. The formula thus states that the UAVs position is unchanged (equal) between the current and the next time-point. Such a relation that describes the relationship between two regions from different time-points is called an inter-temporal relation and can not be described using only \( ST_0 \).

There is one inherent challenge to overcome when extending the functionality from \( ST_0 \) to \( ST_1 \) and it stems from the fact that it’s not possible to observe the relation between regions from different time-points. This is because every observation made is constrained to a single time-point, so any observation made can only capture the relation of regions within that time-point. Therefore, even if it fairly straight-forward to construct an expression containing inter-temporal relations, it’s not possible to determine if such an expression holds true or not without making certain assumptions. Because of this, it is not possible to actually evaluate formulas containing inter-temporal relations since the status of these are unknown. This thesis will examine a way of overcoming this limitation.

A possible way of bridging the gap between time-points is by introducing the concept of landmarks where a landmark is a region that can be used to reason across time-points. One such landmark is a region that is assumed to be rigid, meaning that it is assumed to never change, i.e. that it is equal to itself in all time-points. Such an assumption can act as a connection between time-points and allow for inferring additional knowledge about the relations of regions from different time-points. This thesis will explore to what extent such landmarks can be used to bridge the inter-temporal gap.

1.2 Goal

Landmark regions are used to infer information on inter-temporal spatial relations. The goal of the project is to examine the effectiveness and scalability of using landmark regions assumed to be rigid. The objective is to conduct empirical experiments to measure to what degree such landmarks can aid the inference of inter-temporal relations under different circumstances.
1.3 Delimitations

Work will not go towards actually integrating ST$_1$ functionality in DyKnow, but will work towards facilitating such functionality by investigating a method for allowing deduction of inter-temporal relations. The study limits itself to the rigid landmark regions. Due to the theoretical nature of the subject, the report will not discuss societal or ethical aspects related to the work since it is my assessment that no meaningful discussion regarding such topics can be had.

1.4 Contributions

Work on this thesis has gone towards investigating a method of bridging the inter-temporal gap in order to extend the spatio-temporal reasoning capability of DyKnow from ST$_0$ to ST$_1$. This work has also been in conjunction with the research and work towards a paper submission on the same subject. The tools for experiment measurements and scenario generation created for this thesis was also adopted for experiments in that paper.

1.5 Related work

Koymans [5] presented Metric Temporal Logic (MTL), a logic for temporal reasoning providing the binary temporal operators until and since extended with the unary operators eventually and always for both past and present tense. Allen [6] has presented a different temporal logic with his Interval Algebra that focused on the relationship between intervals with relations such as before, overlaps with and during. As spatial logic, RCC-8 is commonly used which describes eight distinct spatial relations [7]. RCC-8 has been combined with Allen’s Interval Algebra to create the spatio-temporal logics ARCC-8 [8] and STCC [9]. RCC-8 has also been used together with MTL as a foundation for the ST$^i$ family [4] of spatio-temporal logics, with ST$_0$ being the most basic one.

Heintz [1] incorporated temporal reasoning capabilities through MTL into DyKnow which was later extended by Lazarovski [10] to also handle spatial reasoning with RCC-8, resulting in spatio-temporal functionality similar to ST$_0$. To our knowledge, no previous studies has been done on the problem of deducing inter-temporal spatial relations. Heintz and de Leng [2] however, studied how regions can be separated into dynamic and static (rigid) ones in order to increase the effectiveness of the spatial reasoning by reducing the runtime of their reasoner.

1.6 Thesis outline

- Chapter 2 discusses earlier work that relates to this thesis by describing, DyKnow, a stream-based knowledge processing middleware.
- Chapter 3 provides a basic rundown of the spatio-temporal logic at the heart of this work. It begins with a description of the spatial logic RCC and its variant RCC-8 followed by the spatio-temporal logic ST$_0$ and its extension ST$_1$.
- Chapter 4 covers the experiments made to test the effectiveness of the rigid regions used as landmarks. It first describes the goal of the experiments.
and then presents how the experiments are setup and created and how the experiment scenarios are generated.

- Chapter 5 presents the results from the experiments conducted.
- Chapter 6 discusses the results from the experiments and also discusses the chosen method of conducting the experiments.
- Chapter 7 summarizes the work and the conclusions drawn from it and briefly discusses possible future work.
2 Background

This chapter briefly discusses the main concepts of the processing middleware framework DyKnow and the Robot Operating System on which it is built.

2.1 DyKnow

For an autonomous robot acting in the real world to be effective, a large number of different problems need to be solved, and solutions to these problems must be integrated and combined. Examples of such problems are chronicle recognition, task planning and execution monitoring. Modules handling such tasks often rely on formalisms that require crisp and clear information represented as abstract symbols. However, the available data often originate from sensors and other components that provide low-level quantitative data that is commonly noisy and hard to interpret. This creates a gap between the low-level numerical information available and the structured symbolic data required for high-level reasoning.

Heintz [1] expressed that bridging this gap is a complex task that often requires several steps of incremental refining and interpretation of information. Doing this in an ad hoc way is inappropriate and problematic, and to facilitate doing this in a structured manner he proposed the term knowledge processing middleware and defined it as a “systematic and principled software framework for bridging the gap between the information about the world available through sensing and the knowledge needed when reasoning about the world”. Heintz also presented DyKnow [1] a concrete implementation of a stream-based knowledge processing middleware. DyKnow was first implemented using CORBA (the Common Object Request Broker Architecture) as a method for internal communication but was later also implemented in ROS (Robot Operating System), instead relying on the ROS Message communication model [11]. This implementation of DyKnow has successfully been used as part of an architecture for autonomous unmanned aircraft systems by the AIICS division of Linköping University [2].

DyKnow views the world using two concepts: objects and features [1]. An object may be a representation of a real-world object, but may also be some kind of abstract entity. A feature is some sort of attribute, for example a property of an object, or a relation between objects. Every feature has some kind of value that may change over time, since the world is dynamic. It is ultimately these objects and features that are used for reasoning, as they represent the state of the world and environment the robot operates in.

Knowledge processing in DyKnow is performed by units called knowledge processes that depending on the type may either refine and reason around the received data, or make data available by gathering input from sensors or other external sources. DyKnow uses a layered approach where information is incrementally refined, so data may pass through several knowledge processes before it can be used for high-level reasoning. To pass information between knowledge processes, they are connected by streams of data. Doing computations based on the incrementally available information in a stream is a key concept called stream reasoning [1]. This chapter will be covering these topics in more detail by first describing the knowledge processes followed by the streams.
2.1.1 Streams

As described earlier, a stream can be viewed as a method of transferring data between knowledge processes. The streams used in DyKnow are called fluent streams and consist of a set of stream element where each stream element is a value coupled with two time-stamps: the available time and the valid time. A stream element may thus be denoted as a tuple \( (t_a, t_v, v) \) where \( t_a \) is the available time, \( t_v \) is the valid time and \( v \) is the value. The available time marks the time where the stream element was first sent to the receiver after being inserted into the stream, i.e. when the data was made available. The valid time denotes the point in time when the data in the value of the sample is representative of the attribute represented by the fluent stream.

An example of a traffic monitoring robot demonstrating these concepts is as follows: assume a source process receives an image \( i \) of a road with several cars on. The image was taken by an on-board camera at time \( t_1 \) and the image was sent to an image analysis process at time \( t_2 \), this particular stream element would thus be denoted \( (t_2, t_1, i) \). The image analyzer has the task of counting the number of cars \( n \) on the picture and after completing this task sends this data to two other processes, a logger and a task planner. The data is sent to the planner first, at time at time \( t_3 \) and slightly afterwards to the planner, at time \( t_4 \). These two stream elements are \( (t_3, t_1, n) \) and \( (t_4, t_1, n) \) respectively. Notice that the available times \( t_3 \) and \( t_4 \) are different from the two stream elements, while the valid time \( t_1 \) is the same for the two and also for the stream element with the received image. This is because the car count \( n \) and the image \( i \) both represent the state of the world at the same time. The available time has no relationship with the valid time. In the case that an approximation for a future value is provided, the available time may even be an earlier time-point than the valid time.

2.1.2 Knowledge processes

A knowledge process is a computational component in DyKnow. Formally, the general concept of a knowledge process is defined as “an active and sustained process whose inputs and outputs are in the form of streams” [1]. These processes usually have well-defined functions and responsibilities such as refining the received data and outputting a more structured interpretation of it for other processes to use, or making a decision based on the input. Two different kinds of knowledge processes exist in DyKnow: sources and computational units. A source is a process which takes data from external sources and makes them available for other processes to use. Examples of such external sources are sensors, cameras, databases or GPS systems. The other type of knowledge process in DyKnow is the computational unit. A computational unit uses one or more sources to provide input data and does some sort of computation on that data. A computational unit always receives its input from one or more streams of data that may originate either from a source or from other computational units.

2.1.3 Spatio-temporal logic in DyKnow

One of the main usage areas for DyKnow is that of autonomous unmanned aircraft systems (UAS) [2]. As a part of this, an automated planner (TALplanner [12]) is used to create high-level plans to complete the desired objectives. To
make sure that these objectives are met in an orderly fashion, and that the execution of these plans are done accordingly, a separate module, the execution monitor, is used. The execution of plans is monitored by specifying a set of restrictions or rules that the robot must adhere to in the form of logic formulas (see introduction chapter for examples). The evaluation of such formulas is done by the formula progressor, and if a formula is evaluated to false then some of the specified rules have been broken, which means that the execution is not going as planned and appropriate action should be taken.

The formulas constructed can be temporally quantified using Metric Temporal Logic (MTL) which provides the temporal operators $\bigcirc$ (next), $\Diamond$ (eventually) and $\Box$ (always). As a later extension, functionality of the spatial logic RCC-8 was added, giving the ability to describe the relationship between spatial entities in the formulas in the form of eight distinct relation symbols [10]. Knowledge about these spatial relations are maintained by an additional module, the spatial reasoner which also has the ability to use initial knowledge to deduce further spatial information in the form of new relations between other regions. To successfully evaluate formulas containing such RCC-8 symbols, the spatial relationship described by the symbol must either have been observed or deduced through other observations. If that is not the case, then the truth-value of the symbol is unknown. The result of combining the temporal logic MTL and the spatial logic RCC-8 closely resembles the spatio-temporal logic ST$_0$ [2]. An extension of ST$_0$ is ST$_1$, which allows for the expression of inter-temporal relations, meaning that one can make statements about the relationship between to spatial regions from different time-points. ST$_1$ functionality is as the moment not integrated into DyKnow. A major challenge when doing this is the fact that such inter-temporal relations may not be observed, and making deductions about them is only possible if certain assumptions are made. A closer description about these problems, as well as the logic behind ST$_1$ is provided in chapter 3.
3 Theory

3.1 Spatial and temporal logic

This chapter covers the theory behind the spatial logic language RCC and the spatio-temporal logic family ST\textsubscript{i}. The first part of the chapter discusses the kind of formulas that can be constructed using these logics. The second part shows how initial spatial knowledge in the form of RCC-8 region relations can be used to deduce further information in the form of additional relations. It also discusses the underlying problem of deducing inter-temporal relations, and how it may be mitigated using the concept of landmark regions. Finally, a particular software tool that may be used for RCC-8 reasoning, GQR is briefly discussed.

3.1.1 Region Connection Calculus (RCC)

The Region Connection Calculus (RCC) [7] is a spatial logic used for qualitative spatial reasoning in terms of regions in topological space. A region may be viewed as a set of spatial points and RCC defines a set of binary spatial relations that describes the relationship between two such regions. These spatial relations may in a logic sense be viewed as binary predicates. RCCs foundation is the relation $C(x,y)$ (reads as “x connects with y”) which holds true when regions $x$ and $y$ share one or more points in space. By extending this single relation, RCC defines a larger number of relations describing the relationships between spatial regions. Those relations are the following: DC (disconnected from), P (part of), PP (proper part of), EQ (equal to), O (overlapping with), PO (partially overlapping with), DR (discrete from), TPP (tangential proper part of), EC (externally connected with) and NTTP (non-tangential proper part of). The formal definition of these relations as defined by Randell et al. [7] is listed below.

- $DC(x,y) \equiv_{df} \neg C(x,y)$
- $P(x,y) \equiv_{df} \forall z [C(z,x) \Rightarrow C(z,y)]$
- $PP(x,y) \equiv_{df} P(x,y) \land \neg P(y,x)$
- $EQ(x,y) \equiv_{df} P(x,y) \land P(y,x)$
- $O(x,y) \equiv_{df} \exists z [P(z,x) \land P(z,y)]$
- $PO(x,y) \equiv_{df} O(x,y) \land \neg P(x,y) \land \neg P(y,x)$
- $DR(x,y) \equiv_{df} \neg O(x,y)$
- $TPP(x,y) \equiv_{df} PP(x,y) \land \exists z [EC(z,x) \land EC(z,y)]$
- $EC(x,y) \equiv_{df} C(x,y) \land \neg O(x,y)$
- $NTTP(x,y) \equiv_{df} PP(x,y) \land \exists z [EC(z,x) \land EC(z,y)]$

Note that all of the relations except for $P$, $PP$, $TPP$ and $NTPP$ are symmetrical so that if one relation holds between $x$ and $y$, the same relation holds between $y$ and $x$. The non-symmetrical relations have inverses so that for example, $TPP(x,y) \leftrightarrow TPP^{-1}(y,x)$. The inverse relations $P^{-1}$, $PP^{-1}$, $TPP^{-1}$ and $NTPP^{-1}$ may also be written as $Pi$, $PPi$, $TPPi$ and $NTPPi$. Apart
from the spatial relations, RCC also contains the functions \( \text{compl}(x) \), \( \text{sum}(x, y) \), \( \text{prod}(x, y) \) and \( \text{diff}(x, y) \). These functions operate similar to the set-theory operations \text{complement}, \text{union}, \text{intersection} and \text{set-complement} respectively. Notice that it is possible and sometimes necessary for two regions to be in more than one relation at one point in time. For example, \( P(x, y) \) must hold whenever \( EQ(x, y) \) holds, and whenever \( PP(x, y) \) holds, either \( TPP(x, y) \) or \( NTTP(x, y) \) must also hold.

Often times, a subset of the RCC-relations are used instead of the full set. RCC-8 is a formalization of such a subset. The name stems from the eight RCC relations DC, EC, PO, EQ, TPP, TPI, NTTP and NTPPi that are retained while the rest of the RCC relations are discarded. These eight relations are called the RCC-8 base relations and are jointly exhaustive and pairwise disjoint, meaning that any two regions must be in exactly one of the RCC-8 base relations. With this logic, it’s possible to express statements like \( NTTP(\text{Sweden}, \text{Europe}) \), \( TPP(\text{Stockholm}, \text{Sweden}) \), and \( EC(\text{Sweden}, \text{Norway}) \).

The simple meaning of these expressions is that Sweden is an internal part of Europe, and Stockholm is a part of Sweden, being connected to its border. By reasoning about these expressions, further knowledge can be extracted. Since Sweden is a non-tangential proper part of Europe, and Stockholm is a (tangential) part of Sweden, we can conclude that Stockholm must be a non-tangential proper part of Europe, i.e. the relation \( NTTP(\text{Stockholm}, \text{Europe}) \) must hold. The information we have is insufficient to determine the definite relationship between Norway and Stockholm, but some things can still be concluded. Since Sweden and Norway share no space except for the border, no part of Stockholm can be a part of Norway and vice-versa. From this we can conclude that none of the relationships TTP, NTTP, TPI, NTPI, PO, EQ can hold between Norway and Stockholm. Because RCC-8 is jointly exhaustive, one of the two remaining relations, namely DC and EC must hold, but we can’t know which one without gaining further information. A more formal description of how inference of RCC-8 relations work is provided in chapter 3.2.

### 3.1.2 ST\(_i\) family of spatio-temporal logic

The RCC-8 fragment of the region connection calculus is an efficient way of reasoning about spatial entities but since it only deals with space it provides in itself no way of reasoning about time. Therefore, if one wants to do spatio-temporal reasoning, RCC must be complemented with some temporal logic. A way of doing this is by adding the temporal component PTL (Propositional Temporal Logic) to RCC-8, resulting in the Spatio-temporal logic family ST\(_i\) [4]. The most basic variant, ST\(_0\), and its extension ST\(_1\) will be described here.

### 3.1.3 ST\(_0\)

The most basic variant of the ST\(_i\) family of spatio-temporal logic is ST\(_0\), which allows for the usage of temporal operators on the spatial relation predicates. These operators, together with their intended meaning on well-formed formulas \( \psi \) and \( \phi \) are listed below.

- \( \psi U \phi \) : \( \psi \) will hold until \( \phi \) holds (\( \psi \) until \( \phi \))
- \( \psi S \phi \) : \( \psi \) has held in all time points since \( \phi \) ceased holding (\( \psi \) since \( \phi \))
• $\square \phi$: $\phi$ will always hold in all future points in time (always $\phi$)

• $\Diamond \phi$: $\phi$ will hold in at least one future point in time (eventually $\phi$)

• $\bigcirc \phi$: $\phi$ will hold in the next point in time

Versions of the latter three operators operating in the past tense are also defined: $\square^{-}$ “has always”, $\Diamond^{-}$ “at some time in the past” and $\bigcirc^{-}$ “during the previous point in time”. To avoid ambiguity, the future tense operations can be denoted as $\square^{+}$, $\Diamond^{+}$ and $\bigcirc^{+}$ [4].

With these operators it is possible to construct basic statements regarding both space and time. Examples of such statements are $\Diamond \ EQ(EU, Europe)$ - “At one future point, the EU will encompass the whole of Europe” and $\square \ DC(Europe, NorthAmerica)$ - “Europe and North America will always be disconnected”.

Figure 1: Continuity network of RCC-8 [13]

With these operators it is also possible to express the progressional constraints in RCC-8 (see figure 1). For instance, if two regions are disconnected (DC) and are to be partially overlapping (PO) they must first become externally connected (EC) as a direct transition from DC to PO is impossible. This notion is called the conceptual neighborhood [13]. This can be described using the next ($\bigcirc$) operator to denote that for every relation in RCC-8, in the next state the relation can only either stay the same or progress to a relation adjacent to the previous.

- $DC(a,b) \Rightarrow \bigcirc (DC(a,b) \lor EC(a,b))$
- $EC(a,b) \Rightarrow \bigcirc (EC(a,b) \lor DC(a,b) \lor PO(a,b))$
- $PO(a,b) \Rightarrow \bigcirc (PO(a,b) \lor EC(a,b) \lor TPP(a,b) \lor EQ(a,b) \lor TPPI(a,b))$
- $TPP(a,b) \Rightarrow \bigcirc (TPP(a,b) \lor PO(a,b) \lor EQ(a,b) \lor NTPP(a,b))$
- $NTPP(a,b) \Rightarrow \bigcirc (NTPP(a,b) \lor TPP(a,b) \lor EQ(a,b))$
- $TPPI(a,b) \Rightarrow \bigcirc (TPPI(a,b) \lor PO(a,b) \lor EQ(a,b) \lor NTPPI(a,b))$
- $NTPPI(a,b) \Rightarrow \bigcirc (NTPPI(a,b) \lor TPPI(a,b) \lor EQ(a,b))$
EQ(a, b) ⇒ (EQ(a, b) ∨ PO(a, b) ∨ TPP(a, b) ∨ NTPP(a, b) ∨ TPPi(a, b) ∨ NTPPi(a, b))

One may also define time intervals during which the temporal operations are to be valid. Examples of such expressions are: □[t_1, t_2]PO(X_1, X_2), “regions X_1 and X_2 will be partially overlapping constantly between time-points t_1 and t_2”. This is not, however applicable to the next-operator since it always denotes a single time-point rather than an interval. However, several ○ operators can be applied to the same relation symbol to let it denote a time-point more than one step in the future. For example, ○○○PO(x, y) means that x and y are partially overlapping in the time-point that exists three steps into the future. This can also be written using the shorthand form ○^{+3}PO(x, y).

3.1.4 ST_1

A limitation of ST_0 is that one can not describe the relations of regions from different time points, since the temporal operators only are allowed to operate on predicate symbols such as the RCC-8 relation symbols. ST_1 adds this functionality by allowing the usage of ○ (next) on the region terms. Thus ○x denotes the area occupied by region x in the next state. Because of this, ST_1 allows for describing the change and progression of individual regions across time. For example, the simple statement □NTPP(x, ○x) means that the x in one state is always x in the internals of x in the next state, i.e. x will always be expanding in all directions. One may also state the opposite as □EQ(x, ○x) which means that the area occupied by the x never changes. If both region terms in a relation is preceded by a single ○, it can be simplified into a ST_0 expression as R(○x, ○y) is equivalent to ○R(x, y) for every relation R.

3.2 Qualitative spatio-temporal reasoning

The previous sections of this chapter presented the spatial logic RCC-8 and the spatio-temporal logics ST_0 and ST_1 and showed how they can be used to construct formulas containing, among other things, RCC-8 symbols. Actually evaluating these formulas requires knowledge about the relations contained in the formula. In the robotics field, such knowledge may be obtained by various sensors such as cameras, and GIS systems. This however is often not enough to achieve full knowledge about the relations between all regions in the formulas that are to be evaluated, which creates the need to use known relations to infer further knowledge to get a better chance of successfully evaluating the spatio-temporal formulas.

3.2.1 RCC-8

Since RCC is defined using first-order logic and set theory, common first order logic resolution using the relation definitions may be used to attempt new inferences. However doing so is computationally complex and unsuitable in practice. An alternative, more efficient method is to use the RCC-8 composition table which can be used to draw conclusions about the relationships between regions x and y if there is a region y so that we already have knowledge about the relationship between x and z as well as z and y. The full composition table can be found in Table 1. It can be used by selecting the row by the known relation.
$R_1(x,z)$ and the column by relation $R_2(z,y)$. The intersecting cell contains a set of possible relations between $x$ and $y$. Generally, we may view these RCC-8 scenarios as undirected graphs where the regions are edges and relations about which we have some knowledge as vertices. We have a potential of deducing information about the relation between two regions if there is a path in the graph between the corresponding edges. If there is more than two vertices in the path between two regions or edges we wish to infer relations between, we may infer intermediate information and thus shortening the path until there is a path of two vertices and the composition table may be used. Note however that there is no guarantee that knowledge can be deduced even if there is such a path. For example, $DC(x_1,x_2)$ and $DC(x_2,x_3)$ gives no information about the relationship between $x_1$ and $x_2$.

Table 1: Composition table of RCC-8 [10]

<table>
<thead>
<tr>
<th></th>
<th>DC</th>
<th>EC</th>
<th>PO</th>
<th>TPP</th>
<th>NTTP</th>
<th>TPPI</th>
<th>NTTPi</th>
<th>EQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC</td>
<td>DC EC PO, TPP NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PO</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPP</td>
<td>DC EC PO, TPP NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPPI</td>
<td>DC EC PO, TPP NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTTPi</td>
<td>DC EC PO, TPP NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EQ</td>
<td>DC EC PO, TPP NTTP</td>
<td>DC EC PO, TPP NTTP</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

For example, if we know $EC(x,z)$ and $TPPi(z,y)$, we can conclude using the composition table that $EC(x,y) \lor DC(x,y)$ holds. Because exactly one relation must hold, we can further conclude that either $EC$ or $DC$ holds between $x$ and $y$, however we may not determine which one without acquiring new information. We may say that the (observed) relation is between $x$ and $y$ is the disjunctive set $\{EC, DC\}$, we may not however say that it is the actual relation since only one of them may hold. Figure 2 shows how both of these relations may be valid for the given relations between $x$ and $y$, and $y$ and $z$. Suppose additionally that we know about a fourth region, $v$ and that $EC(x,v)$ and $TPP(v,y)$ holds. Using these two relations, the composition table tells us $EC(x,y) \lor PO(x,y) \lor TPP(x,y) \lor NTTP(x,y)$ or that the set of possible relations between $x$ and $y$ is $\{EC, PO, TPP, NTTP\}$. Since both disjunctions must be true, we can construct a new expression $(EC(x,y) \lor PO(x,y) \lor TPP(x,y) \lor NTTP(x,y)) \land (EC(x,y) \lor DC(x,y))$ that also must hold. This expression is a conjunction of the earlier obtained ones since both of them must be true. From this conjunction it’s easy to see that the only consistent interpretation is that $EC(x,y)$ must hold. Alternatively, one may take the intersection of the two sets which in this case is the singleton $\{EC\}$. This shows how even if no single path gives us complete information, we may if possible combine knowledge from different paths to gain more precise information.

Similarly, if the initial knowledge is incomplete so that we start out with
disjunctive information, we may pick one of the possible RCC-8 base relations for every pair of regions as a hypothesis and construct a disjunction of the resulting relations. This time, the result is a disjunction because only one of the hypotheses are correct and not all of them need to hold true. For example, assume that we know that \( NTPP(b, c) \) holds and that either \( DC(a, b) \) or \( EC(a, b) \) holds, but not which one. We may say that the (observed) relation between \( a \) and \( b \) is \( DC, EC \). To attempt to deduce the relation between \( a \) and \( c \) we may first pick a hypothetical RCC-8 base relation to hold between \( a \) and \( b \), such as \( DC \). \( DC(a, b) \) and \( NTPP(b, c) \) means that the set of possible relations between \( a \) and \( c \) is \{DC, EC, PO, TPP, NTPP\}. The other hypothesis, \( EC(a, b) \) together with the known \( NTPP(b, c) \) would mean that the possible relations between \( a \) and \( c \) are \{PO, TPP, NTPP\}. Since either of the hypotheses may be correct, we can only conclude that \( (DC(a, c) \lor EC(a, c) \lor PO(a, c) \lor TPP(a, c) \lor NTPP(a, c)) \lor (PO(a, c) \lor TPP(a, c) \lor NTPP(a, c)) = DC(a, c) \lor EC(a, c) \lor PO(a, c) \lor TPP(a, c) \lor NTPP(a, c) \) holds. Also, just as we earlier took the intersection of two sets when we dealt with a conjunction, we could now take the union of the sets since we have a disjunction. This shows that starting out with incomplete information often leads to incomplete inferences, even if some knowledge can be gained.

It is important to note however, that even partial knowledge, such as these conjunctions, may be sufficient to evaluate formulas containing RCC-8 symbols. Assume that we are presented with the simple formula \( \varphi = TPPi(a, c) \) and that we are to evaluate weather the formula holds true or not using what be previously concluded about the relationship between \( a \) and \( c \). We were unable to determine the exact relationship between \( a \) and \( c \), but we established that the set of possible relations is \{DC, EC, PO, TPP, NTPP\}. Since \( TPPi \) is not included in that set, we can conclude that \( TPPi(a, c) \) cannot hold and we can therefore conclude that \( \varphi \) evaluates to false. Thus, even imprecise knowledge may be used to successfully evaluate formulas. Although, in other situations, that may not be the case. Consider a second formula, \( \psi = PO(a, c) \). Since \( PO \) is included in the set of possible relations it may hold true, however since the set contains additional relations that also may hold true it is not at all guaranteed that \( PO(a, c) \) holds, therefore the truth value of \( PO(a, c) \) is unknown and \( \psi \) may thus not be evaluated with the current knowledge. In a more general sense, an RCC-8 relation symbol can be determined to be false if the relation described is not contained in the set of possible relations for the pair of regions in the
symbol. Similarly, a relation symbol can only be determined to be true if the relation is the only base-relation in the set of possible relations. Therefore, even if disjunctive information may be sufficient in some cases, having more precise information is always preferable, and reducing the size of the disjunctions is something to aim for when doing RCC-8 reasoning.

A different way to view the problem of deciding RCC-8 relations is to view the scenarios as Constraint Satisfaction Problems (CSPs). A general CSP consists of a set of variables, a domain or a set of possible values the variables may take on and a set of constraints that restrict which variable may take on what value. In the case of viewing RCC-8 problems as CSPs, the variables are the relations, i.e. the pairs of regions, the domain is the set of RCC-8 base relations and the constraints are the restrictions described in the RCC-8 composition table. Solving a CSP consists of assigning to a variable a value that is consistent to all restrictions. Here, that would be assigning a RCC-8 relation to every region relation. If this is not possible to do, the problem can be said to be unsatisfiable. In the case of RCC-8, every problem should be satisfiable since every pair of regions is in one of the RCC-8 base relations, but unsatisfiability may still occur as a result of for example faulty observations or if one makes up a scenario of arbitrary initial relations. The benefit of viewing RCC-8 problems as CSPs is that any CSP may be solved by a general constraint solver. Since all CSPs have the same basic components, a constraint solver does not need to be tailored to the actual underlying problem. Therefore, a constraint solver that was initially created to solve puzzles such as Sudoku can be used to solve an RCC-8 problem without needing to modify the search algorithm, assuming a general algorithm is used. Special algorithm tailored to a specific type of problem may be used for efficiency.

3.2.2 ST1

Since the spatial component in ST0 is pure RCC-8, the spatial reasoning can be done exactly the same way. With the extension of ST1, the same holds true, but there are some limiting complications that needs to be overcome. Remember that ST1 allows for slight modification of the RCC-8 relation symbols since the region terms can be preceded by the next operator (∘). Therefore, while ST0 allows for the expression of things like ∘PO(x₁, x₂) (in the next time step, regions x₁ and x₂ will be partially overlapping), ST1 can express things like PO(x₁, ∘x₂) (the current state of region x₁ and the next state of region x₂ is partially overlapping). The difference between the two is subtle, but the implications of this are large. This stems from the fact that we now are reasoning about region terms from different time points, something that may not be entirely trivial. The problem is that we are unable to observe inter-temporal relations because every observation made is constrained to a single time-point. Thus, even though the transition from ST0 to ST1 only involves a very slight difference in the formulas, actually evaluating formulas containing inter-temporal relations is not possible without inferring knowledge about those relations.

Before continuing, a good idea might be to define what “next” means in this context. As mentioned, ∘r denotes the state of region r in the next time point, but what the next time-point implicates is not obvious. Even at a theoretical level, this is a non-trivial question to ask. Since time is continuous, there
is no evident meaning of a “next” time-point. Therefore, one must partition
the time-points into discrete and ordered state samples that can be reasoned
with. Only then may we use the next and previous operators to traverse these
states. By moving from a continuous time-space to a set of discrete time-points
we must be aware that we may never have full knowledge about the world at
times, and this must be kept in mind. One may for example be tempted
to use the continuity network of the RCC-8 Relations (see figure 1 in section
3.1.3) to deduce possible future relations based on the current relation. For
example, if \( PO(x_1, x_2) \) holds at time \( t \), then the constraints from the continuity
network should tell us that either \( PO(x_1, x_2) \) or \( EC(x_1, x_2) \) must hold at \( t' \)
(the time-point after \( t \)). This may seem like a sound conclusion in theory but
is problematic in practice. Since we have to rely on observations as samples
from the world, we may observe \( PO(x_1, x_2) \) in one sample, and \( PO(x_1, x_2) \)
in the next. This does not violate the theory from the continuity network since
we only are able to make these observations with a limited frequency and we
must account for the possibility of things happening between the samples we
observe. We can, however draw the conclusion that \( EC(x_1, x_2) \) must have held
at some time between our two state samples, but we can never guarantee that
we will observe every state that occurs. The fact that we are unable to observe
or detect an event is not sufficient proof that it did not happen. Because of this,
the constraints from the continuity network can provide no help in determining
relations at previous or next time-points.

Remember from the RCC-8 reasoning discussion that we may only deduce
information about the relation between two regions if there is a “path” between
the two regions where we have knowledge about the relation for every adjacent
region in that path. If these is no such path between two regions, the relation
between these regions can not be deduced. This restriction is a large problem
when it comes to inter-temporal reasoning since the regions from every time-
point are separated from each other and since we can not observe inter-temporal
relations, we have no knowledge combining them. Thus, even with complete in-
formation within the time-points we would be entirely unable to deduce relations
between the two. To do so, we need to connect the different time-points with
some knowledge. A possible way of dealing with this would be to introduce the
concept of rigid regions. We consider a rigid regions to be a region that is
unchanging, i.e. always equal in all time-points such that \( □EQ(r, □r) \) holds for
all rigid regions \( r \). By having some of our regions being rigid, we can use these
as “anchoring points” or landmarks between the different time-points, and the
inter-temporal relations this provides may be used to infer new inter-temporal
relations between other regions.

We may consider a simple example scenario where we have two regions \( x \)
and \( r \) where \( r \) is rigid. Assume that we have made the observation \( PO(x, r) \)
and \( □DC(x, r) \), so we have some information about intra-temporal relations
from two time-points. Now assume that we are interested in knowing how \( x \) has
changed between the two time-points, in other words, we would like to know
the relationship between \( x \) and \( □x \). Without the concept of rigid regions we
would not be able to deduce this information since there is no “path” between
\( x \) and \( □x \). However, since \( r \) is rigid, we know that \( EQ(r, □r) \) which also
allows us to deduce that \( PO(x, □r) \) holds. This can in turn be used to deduce
that the possible relations between \( x \) and \( □x \) is \{\( DC, EC, PO, TPPi, NTPPi \)\}.
This shows that using the concept of rigid regions may help bridge the gap
between different time-points, and adding more rigid regions to the domain would potentially help deducing more exact knowledge and reducing the size of the disjunction.

### 3.2.3 Rigid regions

We have previously introduced the concept of rigid regions, and defined them as a region that is equal to itself in every time-point. However, it is still not evident what the rigid regions comprise in the real world. Determining if something in the real world is rigid or not is very much dependent on the perspective, both in a spatial and temporal manner. If one tries to observe if something is rigid or not, then given a short enough time-span, almost everything can be deemed to be rigid since there is insufficient time for change. Similarly, if the time-span considered is large enough, then there is a chance that everything changes sooner or later, meaning that nothing may be considered truly rigid.

The spatial perspective is perhaps even more important to consider and is related to the fact that space is perceived in a relative manner. A mobile robot operating aboard a cruise ship would probably benefit from perceiving features such as the rooms and walls of that ship as rigid. At the same time, for a UAV flying above it would make more sense to instead consider features of the environment such as forests, lakes, roads and buildings as rigid. However, if we strap rockets to that UAV and send it off for interstellar travel, considering features on the surface of earth as rigid and unchanging may be very inappropriate since the earth itself is moving. With these considerations in mind, we may draw the conclusion that we must manually define which regions are deemed to be rigid for every environment that we intend to operate in. This means that we may only assume that a region is rigid since in reality we probably would not be able to determine if it really is.

Another important point to consider is in what manner the rigid regions are to be used, and particularly how many rigid regions should be introduced to the domain. The more regions and relations we have knowledge about, the greater the chance that we are able to deduce new information about these regions. The drawback of introducing additional regions is that the time for computing the algebraic closure of the relations grows with the number of regions [2]. This means that if we have too few rigid regions we may not be able to profit considerably from them, and if we have too many the computations may take unnecessarily long.

An approach trying to minimize the amount of regions would be to not add additional unnecessary regions to the domain reasoned about. Instead, we may settle for the relevant regions already defined in the domain and separate them into rigid and dynamic regions. These rigid regions would represent features in the environment that are assumed to not change during operations, such as roads, buildings and different sections of the terrain like hills, pieces of woodland and lakes\(^1\). If these regions are already used for reasoning, making them rigid comes at no additional cost and may even give a performance increase since we don’t have to account for the possibility of the relations between rigid regions changing [2]. A possible drawback from this approach is that if the rigid regions are

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\(^1\)As discussed earlier, the nature of these rigid regions are very dependent on the application, and may vary.
too few and far between they may not provide a sufficient amount of information for it to be useful.

A more greedy approach would be to populate and cover the domain with an amount of rigid regions not representing actual objects in the environment, but rather acting as a tiling and providing a set frame of reference to facilitate reasoning about the dynamic regions within the environment. Because such an approach would provide complete coverage of at least a subset of the environment, it would supply more constant and reliable information for reasoning, but would likely result in a significant increase in regions of the domain. This is a trade-off that is important to consider when deciding how large each such rigid region should be. The smaller the regions are, the more exact information they would provide about the regions they interact with, but this also leads to a larger number of regions. If these regions are of uniform shape and placement in the domain, they may also be used to estimate quantitative information about the regions. An example of such an estimation would be the distance between to disconnected regions by computing the least number of adjacent (externally connected) rigid regions one would need to traverse to get from one region to the other.
4 Method

In the earlier chapters, the concept of landmarks were introduced and it was shown how it can allow for deducing inter-temporal relations from given relations constrained to a single time-point. Also, a concrete type of landmark utilizing the assumption of rigid regions has been discussed. To examine how effective this type of landmark is for reasoning across time-points, experiments will need to be conducted. Since reasoning over RCC-8-relations often lead to partial knowledge in the form of disjunctive relation sets, we are interested in measuring the size of of the resulting disjunction sets after reasoning. The size of a disjunctive relation set can be viewed as the quality of the relation since a large disjunction for a particular relation means that we have little knowledge about it. The primary objective of these experiments is therefore to measure the quality of the resulting inter-temporal relations after applying the landmark assumptions and using these together with some initially known relations to make additional inferences.

The scenarios used in the experiments will consist of a set of regions, \( \mathcal{R} \) and a set \( \mathcal{LM} \subset \mathcal{R} \), constituting the landmark set. The regions in the landmark set will represent regions assumed to be rigid, meaning that the restriction \( \forall r \in \mathcal{LM} \ EQ(r,r) \) will be enforced. It will also be assumed that we have full knowledge about all relations between two landmark regions. The set of all non-landmark regions will be denoted \( \mathcal{D} \). To reason over relations between two time-points, a set of initial relations between the regions in \( \mathcal{LM} \) and \( \mathcal{D} \) will be provided for each of the time-points. These initial relations, along with the landmark restrictions will allow for deducing relations between the regions in \( \mathcal{D} \) between the time-points. To investigate how the resulting quality of the inter-temporal relations changes depending on the initial circumstances, the ratio of landmark-to-non-landmark region will be varied, as well as the quality of the initially provided intra-temporal relations.

The first section of this chapter discusses GQR, a tool used in the experiments to deduce additional RCC-8 relations from a set of initially known ones. After that, a way of generating experiment scenarios is presented.

4.1 GQR

As stated before, a RCC-8 problem may be represented as a CSP which may be solved by any generic constraint solver. One such solver is GQR\(^2\) (Generic Qualitative Reasoner) by Gantner, Westphal and Wölfl [14]. GQR is implemented in C++ and designed to be generic in the sense that it can be used to solve problems from different calculi without the need to modify its implementation. To use it for a new calculi, the only thing needed is to create a specification for the calculi in text or XML-format. Support for RCC-8 is provided by the authors. Even though GQR is generic, it is still comparative in performance to specialized reasoners [14].

GQR uses the path-consistency algorithm to aid the solving of CSPs. This means removing values from the domain of a variable that are inconsistent with the restrictions. To fully solve a CSP, i.e to fully determine the value of every variable while keeping it consistent, GQR removes arbitrary values from a

\[^2\text{http://sfbr8.informatik.uni-freiburg.de/R4LogoSpace/Tools/gqr.html} \]
variables domain and checks whether the result is still satisfiable. If it is, the path-consistency algorithm is applied again to prune the search space and the process is repeated until completion. To fully solve a RCC-8 scenario would thus be to extract a consistent scenario where every relation is fully determined. Note however that fully solving an RCC-8 problem is usually not what we want to do when we do spatial reasoning. This is because the resulting chosen solution may be one of many consistent solutions and therefore may not be consistent with the real world which the input relations represent. We can however use GQR to remove inconsistent RCC-8 base relations from the set of possible relations by using it to only apply path-consistency to the CSP representing the RCC-8 scenario. The domain of a variable in a CSP is equivalent to the set of possible RCC-8 base relations that hold for the corresponding pair of regions. Therefore, removing variables from the domain is similar to removing uncertainty from the RCC-8 representation of the region relations. The more specific and accurate the initial knowledge is, the more restrictive the CSP gets and thus the resulting path-consistent CSP contain less uncertainty and will have more precise information.

4.2 Generating problem instances

The first step is to generate a set of fully determined relations which hold between the regions in the landmark set. A way of doing this would be to simply pick a random RCC-8 base relation for every pair of landmark regions, but that would almost certainly lead to an unsatisfiable scenario. A satisfiable scenario is required for reasoning, so that would be very inefficient. A more efficient way would be to randomize only a few relations in an initial restriction set, and use GQR to acquire a fully solved scenario that is consistent with those restrictions. This would result in a consistent and fully determined set of relations between all landmark regions.

The second thing that is required in these experiments are the relations between the regions in $D$ and $LM$, step two is therefore to generate these. To be able to examine the results under different circumstances, the quality of these relations will be varied and therefore not fully determined, like the landmark-to-landmark relations are. However, the challenges involved in are similar and the problem of generating a sufficient number of relations without creating unsatisfiability is still a real concern. The approach to deal with this will be similar as in the previous step. Initially, a few disjunctive relations between the regions in $D$ and $LM$ will be generated, and again GQR will be used to fully solve this scenario. The result of this is a complete set of fully determined consistent relations between all regions in $R$. This act of generating a full set of relations for $R$ out of the relations in $LM$ can be repeated a number of times to generate several scenarios for the same set of landmarks. Any two such scenarios can be combined to create a single scenario consisting of two time-points.

These different scenarios can be used to represent different time-points of the same region domain. The relations in these scenarios are, as stated before, fully determined and therefore of perfect quality. Because there is no uncertainty, these relations can be viewed as the true relations. Since we want to vary the quality, the third step of the scenario generation is therefore to modify these relations before reasoning over the scenarios. This modification will be an act of
reducing the quality, i.e. adding uncertainty to the scenarios. To keep the scenarios consistent, no base relation from the relation disjunctions can be removed and also, the assumption of complete knowledge about the landmark relations remain, so those relations remain untouched. The act of adding uncertainty to the previously fully determined relations is done by generating disjunctions of a number of RCC-8 base-relations for some of the relations, and adding these disjunctions to the true relation. The relations for which no disjunctions were generated are set to contain the entire set of the base relations, making the relation in essence unknown. In contrast to the true relations, the resulting disjunctive relations can be denoted as the observed relations as these are the relations which are provided to the reasoner.

A list of the parameters involved in scenario generation are provided in list 4.2. Since some characteristics of the scenarios are to be varied

\[ n \] Total number of regions in the scenario, equivalent to \(|R|\).

\[ m \] Total number of rigid regions in the scenario, equivalent to \(|LM|\).

\[ l \] Label size, or average number of base relations that are generated and added to the known true relation to create the observed relation in the third step of scenario generation.

\[ d \] The expected degree of known relations in the scenario. Represents the number of observed relations that are generated for every region. For \(d = 0\) no observed relations are generated, so the average disjunction size of the scenario (before any reasoning is done) is 8. On the other extreme, at \(d = 1\) every relation is partially determined and assigned a disjunction of size \(l\).

\[ r_s \] Number of single-timepoint scenarios to generate for every landmark set generated. The final experiment scenarios consisting of two timepoints are created by combining two such scenarios, therefore \(r_s^2\) experiment scenarios are created for every set of landmark relations.

A pseudo-code description of the generation of experiment scenarios can be found in algorithm 1. Using the values of \(n\), \(m\) and \(d\), \(r_s\) single time-point scenarios are created. For every pair of such scenario, a dual time-point scenario are created for reasoning in the experiment. The total number of dual time-point scenarios for every given set of the other parameters is thus \((r_s^2)\). In addition to using the parameters specified in list 4.2, the algorithm also makes use of \(d_{seed}\), a value that does not directly influence the quality of the generated observed relations. What it does, however, is determine the average degree for the initial restrictions that are to be fully solved to create the true relations. Having a too small \(d_{seed}\) would lead to few initial restrictions which could lead to a low variation in the true relations while a too large value would be too restrictive and cause a lot of initial relations to be unsatisfiable.

### 4.3 Experiment setup

To measure how well these landmarks work in different circumstances, the parameters needs to be set accordingly. The label size, \(l\) is set to 4 for all experiments. For measuring how well it scales with different number of regions in the
Algorithm 1 Algorithm for generating experiment scenarios

1: procedure GenerateScenarios(n, m, d, l, r)
2: scenarios ← MakeEmptySet()
3: for 1 to r do
4: repeat
5:     LM–Seed ← generateRestrictions(m, d, seed, l)
6:     LM ← solve(LM–Seed)
7: until LM ≠ null
8: timePoints ← MakeEmptySet()
9: for 1 to r do
10: repeat
11:     R–Restr ← generateRestrictions(n, d, seed, l)
12:     R–Seed ← merge(rRestrictions, LM)
13:     R–True ← solve(rSeed)
14: until R–True ≠ null
15:     R–Observed ← addUncertainty(R–True, d, l)
16:     timePoints.add(R–Observed)
17: end for
18: for all {t1, t2} | t1, t2 ∈ timePoints do
19:     scenarios.add({t1, t2})
20: end for
21: end for
22: return scenarios
23: end procedure

domain, the parameter n is set to range from 40 to 160 in increments of 20 (9 distinct values). Another crucial factor is the number of landmark regions in the scenario. To measure how the resulting quality of inter-temporal relations changes depending on the amount of landmark regions, the parameter m ranges from 0 \cdot n to 0.9 \cdot n in increments of 0.1 (8 distinct values). Since this value is proportional to n, it will be denoted the landmark ratio. Another aspect of interest is how the resulting quality changes with the amount of initially known intra-temporal relations. To account for this, the expected degree, d takes on values between 0.1 \cdot n and 1 \cdot n also in increments of 0.1 (9 distinct values). With the lowest value of d (0.1), the average disjunction size before reasoning will be close to 8 since 90% of the intra-temporal relations are completely unknown, while the rest have a disjunction size of l (4). On the other hand, the average disjunction size of the intra-temporal relations for the highest value of d (1) is equal to l since every relation has been assigned a disjunctive relation of size l. Remember that these disjunctions are before any reasoning is done, and will decrease after inferences are made during the experiments. Similarly to the landmark ratio, this value is denoted as the degree ratio and it determines the degree of intra-temporal relations for which there is some disjunctive knowledge before reasoning. With the lowest ratio will only about a tenth of the relations contain some knowledge while with the highest ratio (1) will every single relation be observed to some degree. To create a number of r is given the value of 5, meaning that \binom{5}{2} = 10 dual time-point scenarios are created for every landmark region. In addition to this, the whole process is repeated.
20 times, meaning that a total of 200 scenarios are created and run for every combination of $n$, $m$ and $d$. 

5 Results

Figure 3 shows the overall resulting average disjunction size of the inter-temporal relations. Although no experiment runs were done with a landmark ratio of 0, the values for this have been added to the graph to show the effects of having no landmark regions (no inter-temporal relations can be deduced). The results are shown first by total number of regions ($n$) and by the landmark ratio, and then by degree ratio and landmark ratio. The landmark-to-landmark relations are not taken into account since those are guaranteed to be fully determined. The first plot shows that the number of regions has a very small effect on the resulting disjunction size; the disjunction size seems to decrease with increasing region count, but the difference is minute. In contrast, the landmark ratio is shown to have a big effect on the resulting disjunction size, and unsurprisingly, a high ratio of landmark regions yields lower uncertainty in the form of smaller disjunctions. The second plot again shows the importance of the landmark ratio, but also that the degree ratio has a similar effect on the average disjunction sizes since low degree ratios lead to consistently high disjunction sizes.
Figure 3: Average resulting disjunction size of inter-temporal relations.
Figure 4 shows the percentage of inter-temporal relations to be fully unknown i.e. have a disjunction size of eight and thus represents cases where no knowledge at all could be gained for a particular relation. Here too has the values for landmark ratios of 0 been added and again, landmark-to-landmark relations are not included. The first plot again shows that the total number of regions seem to have little to no effect on the resulting inter-temporal knowledge. Overall is the shape of these plots very similar to those in figure 3.

Figure 5, in a similar manner as figure 4, shows the percentage of relations to be fully known, meaning that the disjunction is of size one. The first plot shows the results for inter-temporal relations while the second plot does the same for intra-temporal relations for comparison. This figure shows that in some cases can the inter-temporal relations be fully determined, but this is very rare and it does not happen nowhere as often as for the intra-temporal relations. Having
a higher landmark ratio seems to increase the percentage of fully determined inter-temporal relations, but even with the highest ratios is the percentage still extremely low (less than half a percent).

Since so many of the relations are completely unknown, it may be beneficial to again look at the average inter-temporal disjunction sizes but to filter out the relations that are unknown, i.e. have a disjunction size of 8. A plot of this can be found in figure 6. As earlier, the first plot shows it by region count and landmark ratio and the second graph by degree ratio and landmark ratio. As expected, the disjunction values in those graphs are consistently lower than in figure 3 since the relations with disjunction size 8 has all been removed.
Figure 6: Average resulting disjunction size of partially known inter-temporal relations.
Figure 7 shows the average runtime in milliseconds for computing the resulting relations (time for the generation of the scenario is not measured). The first graph shows that the time increases by a great deal by increasing the number of regions involved. This is expected since a higher number of regions is more computationally demanding. The same graph also seems to show that a mid-range landmark ratio is slower than for either higher or lower ratios; this is more visible for the higher region counts. The second graph shows how this phenomenon changes for different degree ratios. For the lowest degree ratios, the highest runtimes occur at the maximum landmark ratios. When the degree ratio increases, the higher runtimes seem to occur in the middle of the landmark ratio range, and with the maximum degree ratio, the landmark ratio 0.4 looks to have the highest runtimes. In addition to this, the runtime appears to be strictly increasing with increasing degree ratios for any given landmark ratio.

Figure 7: Average runtime for computing resulting relations.
6 Discussion

As seen in figure 3 and 4, for a given landmark ratio the total number of regions in the scenario seems to have little to no effect on the resulting quality of inter-temporal relations in terms of average disjunction sizes and percentage of unknown relations. This indicates that the effectiveness of these landmarks is largely independent of the region count, and also that it is the landmark ratio that is important, and not the actual absolute number of landmark regions. The same figures also showed that the degree ratio has a large impact on the inter-temporal quality as both the percentage of unknown relations and the average disjunction size decreases with increasing degree ratio. It is fairly natural that the degree ratio also has a large effect since it determines the amount of initial intra-temporal knowledge by providing knowledge about more relations and thus decreases the average disjunction size for the intra-temporal relations before any inferences are made. This initial knowledge is what allows for any deduction to be made, intra-temporal and inter-temporal alike, and the landmarks are what allows for the inter-temporal gap to be bridged. Thus, to decrease the inter-temporal relation disjunction size, it’s important to both have a high ratio of landmark regions and a high degree of intra-temporal information to start out with. If either of these are lacking, very few inter-temporal inferences can be made, as seen in the second graph in figure 4. Another thing of interest is that while the average disjunction size for partially known inter-temporal relations (figure 6) decreases with increased landmark ratio, the difference is more slight than for the overall average disjunction sizes of all inter-temporal relations (figure 3). This could indicate that increasing the landmark ratio does more to decrease the percentage of fully unknown inter-temporal relations than to decrease the disjunction sizes of the already partially known ones.

In figure 5 it can be seen that in some cases an inter-temporal relation be fully determined, i.e. reach a disjunction of size one. However, this happens for very few relations, especially compared to the intra-temporal relations that get fully determined much more often. This seems to indicate that even for the highest landmark ratios will the quality of the inter-temporal relations be much lower than for the intra-temporal ones. Interestingly, a higher region count seems to allow for a higher percentage of intra-temporal relations to be fully determined while the region count has barely affected the inter-temporal relation quality in any of the plots. Another interesting thing is that a low landmark ratio decreases the percentage of fully known for the intra-temporal relations, and thus not only does it affect the inter-temporal relations. This is because full knowledge is provided for the landmark-to-landmark relations, so having a higher ratio of landmark effectively increases the quality of the initial knowledge and allows for more precise inferences to be made.

As seen in the runtime measurements found in figure 7, it can be seen that the landmark ratio has a moderate effect on the runtime. A mid-range landmark ratio (around 0.4-0.5) seems to have the highest runtimes. An exception to this is for the lowest degree ratios where the runtimes instead seem to be strictly increasing with an increasing landmark ratio. The reason for the runtimes being lower for lower degree ratios is probably because with low degree ratios, many of the relations are completely unknown and thus, few inferences can be made. Since few inferences can be made, the reasoner terminates more quickly and can not do as much work as when more information is available at the higher degree.
ratios. The landmark ratio’s effect on runtime is harder to explain and we are not entirely sure what causes it. It could be that, for the lower landmark ratios, increasing it has a similar effect to the degree ratio where we introduce more knowledge and allow for further inferences to be made and thus increase the runtime. For the higher landmark ratios, a larger portion of the relations are between two landmark regions which always are fully known beforehand. This could cause the runtime to drop for the higher landmark ratios.
7 Conclusions

7.1 Summary

The experiments conducted show that using rigid regions as landmarks provides
the ability to deduce information about inter-temporal regions. Although it’s
very rare that the inter-temporal relations are fully determined, in many cases
partial knowledge can be obtained. The quality of the resulting inter-temporal
relations heavily depend on both the percentage of landmark regions (and not
the actual number in absolute terms) and the quality of the initial intra-temporal
information provided. The total number of regions involved seems to in itself
play a very small role in the resulting quality, so this method seems to scale well
with changing amounts of regions in the domain.

7.2 Future work

There are a few possible ways to go from here when it comes to further develop-
ing the spatio-temporal reasoning capabilities of DyKnow. Since this thesis only
has examined a method of attaining inter-temporal relation knowledge without
actually integrating ST$_1$ reasoning into DyKnow, it would be a natural progres-
sion to do so. Apart from adding the ability to perform inter-temporal reason-
ing to the spatial reasoner used this also involves, among other things, updating
the formula syntax for allowing the new type of next-operator and changing
the formula progressor to properly progress over and evaluate such formulas.
A delimitation of this thesis is the fact that only one method for bridging the
inter-temporal gap was examined, while there may be other methods that also
are worth investigating. Finally, there is the possibility of extending the spatio-
temporal reasoning to a more expressive language than ST$_1$, such as ST$_2$. ST$_2$
allows the application of the temporal operators $\square$ and $\lozenge$ on regions variables.
The informal meaning of these is that $\square x$ is the region of space that always is
occupied by $x$, similar to an intersection of all $x$’s and correspondingly, $\lozenge x$
is the region of space that at any point is occupied by $x$, like the union of all $x$’s.
References


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