On Electrohydraulic Pressure Control for Power Steering Applications

Active Steering for Road Vehicles

Alessandro Dell'Amico

Division of Fluid and Mechatronic Systems
Department of Management and Engineering
Linköping University, SE-581 83 Linköping, Sweden

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Per la mia famiglia, Barbara e Leonardo

” Dreams are what happen when you’re
asleep. Achieving is what you do when
you’re at work.

Abstract

This thesis deals with the Electrohydraulic Power Steering system for road vehicles, using electronic pressure control valves. With an ever increasing demand for safer vehicles and fewer traffic accidents, steering-related active safety functions are becoming more common in modern vehicles. These are functions that aim at assisting the driver in more or less critical situations and could be anything from applying a guiding steering torque to taking over the steering completely. Future road vehicles will also evolve towards autonomous vehicles, with several safety, environmental and financial benefits. A key component in realising such solutions is active steering. This refers to the possibility to control the steering angle or steering torque with an electronic signal.

The power steering system was initially developed to ease the driver's workload by assisting in turning the wheels. This is traditionally done through an open-centre hydraulic system and heavy trucks must still rely on fluid power, due to the heavy work forces. The system provides a robust solution but suffers from poor energy efficiency and lacks the possibility of active control. Since the purpose of the original system is to control the assistive pressure, one way would be to use proportional pressure control valves. Since these are electronically controlled, active steering is possible and with closed-centre, energy efficiency can be significantly improved on.

In this work, such a system is analysed in detail with the purpose of investigating the possible use of the system for Boost curve control, the most common control strategy in power steering systems, and position control for autonomous driving. Commercially available valves are investigated since they provide an attractive solution. A model-based approach is adopted, where simulation of the system is an important tool. Detailed models at both a component level and a system level are developed and form the basis of the analysis and control design. Another important tool is hardware-in-the-loop simulation. A test rig of the system, that also includes the original system, is developed. The rig supports the modelling and validation process, as it also makes it possible to verify control concepts on a real system.

This work has shown how proportional pressure control valves can be used for Boost curve control and position control and what implications this has

on a system level. As it turns out, the valves add a great deal of phase shift and with the high gain from the Boost curve, this creates a control challenge. The problem can be handled by tuning the Boost gain, pressure response and damping and has been effectively shown through simulation and experiments. For position control, there is greater freedom to design the controller to fit the system. The pressure response can be made fast enough for this case and the phase shift is much less critical.

Keywords: Electrohydraulic power steering, pressure control, closed-centre steering, Boost curve control, autonomous driving, hardware-in-the-loop simulation, state feedback control, nonlinear control

Populärvetenskaplig sammanfattning

I denna avhandling studeras elektrohydraulisk servostyrning för fordon, där elektroniska tryckstyrande ventiler används. Då kravet på säkrare fordon och färre olyckor ständigt ökar blir aktiva säkerhetsfunktioner som använder sig av servostyrningen allt vanligare. Dessa funktioner syftar till att assistera föraren i mer eller mindre farliga situationer. Detta kan göras genom att ett styrande moment guidar föraren i rätt riktning, till att styrningen tas över helt. Framtida fordon kommer även att utvecklas mot autonoma fordon, vilket ger flera säkerhetsrelaterade-, miljörelaterade- och ekonomiska fördelar. Nyckeln ligger i att realisera aktiv styrning. Med detta menas att hjulvinkeln, eller det styrande momentet, går att kontrollera via en elektronisk signal.

Servostyrning i fordon utvecklades från början med syfte att underlätta förarens arbete, genom att assistera när föraren vrider på ratten. Traditionellt har detta gjorts genom ett hydrauliskt, s.k. öppet-centrum, system och lastbilar måste än idag förlita sig på denna teknik då lasterna är väldigt stora. Denna lösning är robust men har dålig verkningsgrad och kan inte styras aktivt. Då syftet med original systemet är att styra tryck kan detta istället göras med proportionella tryckstyrande ventiler. Genom att de är elektroniskt styrda och har stängt centrum kan aktiv styrning realiseras, samtidigt som bränslebesparingar är möjliga.

I detta arbete analyseras detta system i detalj. Undersökningarna tittar på reglering via Boost kurva, vilket är det mest förekommande sättet i dagens system, samt positionsreglering för autonom körning. Möjligheten att använda kommersiellt tillgängliga ventiler studeras då detta skulle vara en attraktiv lösning. Arbetet med att undersöka systemet är modellbaserat där simulering är ett viktigt verktyg. Flera matematiska modeller har utvecklats på både komponentnivå och systemnivå och ligger till grund för reglerdesign och de analyser som gjorts. Ett annat viktigt verktyg är ”hardware-in-the-loop” simulering, som används som en del i valideringsprocessen och verifieringsprocessen av koncept.

Detta arbete visar på hur man kan använda tryckstyrande ventiler som ser-

vostyrning och hur dessa påverkar systemet. Ventilernas beteende skiljer sig från ursprungssystemet och ger en del utmaningar när Boost kurvan används som reglerstrategi. Dessa kan hanteras genom rätt inställning av prestanda, dämpning och förstärkning i reglerloopen. Detta har visats genom simulering och experiment. För positionsreglering finns större frihet att utveckla regulatorn för att uppnå dom krav som finns och ventilens prestanda är inte lika kritisk.

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Notation

α_f	Front slip angle	[rad]
α_r	Rear slip angle	[rad]
β	Bulkmodulus	[Pa]
δ	Jet stream angle	[rad]
δ_a	Desired closed loop damping	[-]
δ_m	Mechanical damping	[-]
λ	Flux linkage	[Wb-turns]
μ	Permeability of the material	[H/m]
ω_a	Desired closed loop response	[rad/s]
ω_c	Resonance of control chamber	[rad/s]
ω_h	Hydraulic resonance	[rad/s]
ω_m	Mechanical resonance	[rad/s]
ω_s	System volume resonance	[rad/s]
ω_v	Valve break frequency	[rad/s]
Ω_z	Yaw rate	[rad/s]
Φ	Damping function	[Pa]
ϕ	Magnetic flux	[Wb]
ϕ_p	Damping function derivative	[sPa/m ³]
ϕ_q	Damping function derivative	[-]
θ_{sw}	Steering wheel angle	[rad]
A_1	Opening area of open-centre valve	[m ²]

A_2	Opening area of open-centre valve	[m ²]
A_c	Pressure-sensing area	[m ²]
A_p	Piston area	[m ²]
A_s	Valve opening to pump	[m ²]
A_t	Valve opening to tank	[m ²]
A_{cc}	Large pressure-sensing area	[m ²]
A_{cr}	Area of damping orifice	[m ²]
A_{gap}	Gap area	[m ²]
A_{in}	Meter-in area	[m ²]
A_{out}	Meter-out area	[m ²]
B	Magnetic field	[T]
B	Viscous friction coefficient	[Ns/m]
b_v	Viscous friction coefficient	[Ns/m]
b_{rw}	Damping coefficient of the rack	[Ns/m]
b_{sw}	Damping coefficient of steering wheel and column	[Ns/m]
C_q	Discharge coefficient	[-]
C_{rw}	Equivalent tyre stiffness	[N/m]
C_{α_f}	Front tyre cornering stiffness	[N/rad]
C_{α_r}	Rear tyre cornering stiffness	[N/rad]
f_0	Spring pretension	[N]
f_1	Resistive current function coefficient	[A/W _b ³]
f_2	Resistive current function coefficient	[A/W _b ²]
f_3	Resistive current function coefficient	[A/W _b]
F_f	Flow force	[N]
F_s	Solenoid force	[N]
F_{fin}	Flow force, meter-in	[N]
F_{fout}	Flow force, meter-out	[N]
F_{frw}	Friction at the rack	[N]
F_{fstat}	Static friction level	[N]

F_{load}	Load force	[N]
F_{yf}	Front lateral force	[N]
F_{yr}	Rear lateral force	[N]
H	Magnetic field intensity	[A/m]
i	current	[A]
i_d	Dissipative current	[A]
i_r	Resistive current	[A]
I_{encl}	Enclosed current by magnetic path	[A]
J_{rw}	Total road wheel inertia	[kgm ²]
J_{sw}	Inertia of steering wheel and column	[kgm ²]
K	Spring stiffness	[N/m]
K_1	Pressure-flow coefficient of damping orifice	[m ³ /sPa]
K_1	Pressure-flow coefficient of damping orifice	[m ³ /sPa]
K_b	Boost gain	[Pa/Nm]
K_c	Flow-pressure coefficient	[m ³ /sPa]
K_e	Equivalent spring stiffness	[N/m]
K_h	Hydraulic stiffness	[N/m]
K_i	Integral gain	[V/sPa]
K_p	Proportional gain	[V/Pa]
K_q	Flow gain	[m ² /s]
K_T	Spring stiffness of torsion bar and column	[Nm/rad]
K_v	Static loop gain	[m ² Pa/N]
L	Inductance	[H]
l_f	Front axle distance	[m]
l_r	Rear axle distance	[m]
m	Mass	[kg]
m_v	Mass of spool	[kg]
$M_{f_{sw}}$	Friction at the column	[Nm]
M_{rw}	Total mass of rack	[kg]

N	Number of turns	[-]
p	Pressure	[Pa]
p_c	Pressure of damping chamber	[Pa]
p_L	Load pressure	[Pa]
p_s	System pressure	[Pa]
p_{Boost}	Boost pressure	[Pa]
p_{d1}	Dissipative current function coefficient	[A/V ³]
p_{d2}	Dissipative current function coefficient	[A/V ²]
p_{d3}	Dissipative current function coefficient	[A/V]
p_{d4}	Dissipative current function coefficient	[A]
p_{fs1}	Force function coefficient	[N/V ⁵]
p_{fs2}	Force function coefficient	[N/V ⁴]
p_{fs3}	Force function coefficient	[N/V ³]
p_{fs4}	Force function coefficient	[N/V ²]
p_{ref}	Reference pressure	[Pa]
q	Volumetric flow	[m ³ /s]
q_c	Flow through damping orifice	[m ³ /s]
q_l	Load flow	[m ³ /s]
q_s	Pump flow	[m ³ /s]
q_s	System flow	[m ³ /s]
q_s	System flow	[m ³ /s]
q_v	Valve flow	[m ³ /s]
q_{in}	Meter-in flow	[m ³ /s]
q_{out}	Meter-out flow	[m ³ /s]
R	Coil resistance	[Ω]
R_e	Reynolds number	[-]
R_T	Steering system gear ratio	[rad/m]
R_v	Static characteristic of the valve	[Ns/m ⁵]
t	Time	[s]

T_d	Driver's torque	[Nm]
T_{tb}	Torsion bar torque	[Nm]
U	Control signal	[V]
V	Voltage	[V]
V_1	Cylinder volume	[m ³]
V_2	Cylinder volume	[m ³]
V_c	Damping volume	[m ³]
V_L	Inductive voltage	[V]
V_l	Load volume	[m ³]
V_s	System volume	[m ³]
V_s	Total voltage	[V]
v_v	Spool velocity	[m/s]
v_x	Longitudinal velocity	[m/s]
v_y	Lateral velocity	[m/s]
w	Area gradient	[m]
x_v	Spool position	[m]
x_{rw}	Rack position	[m]

Papers

This thesis is presented as a monograph. Parts of the work are based on the following publications:

Journal articles

- Dell’Amico, Alessandro and Petter Krus (2015a). “Modeling, simulation and experimental investigation of an electrohydraulic closed-centre power steering system”. In: *IEEE/ASME Transactions on Mechatronics* 20.5, pp. 2452–2462.
- Dell’Amico, Alessandro and Petter Krus (2015b). “Modelling and experimental validation of a nonlinear proportional solenoid pressure control valve”. In: *accepted for publication in the International Journal of Fluid Power*.

Conference articles

- Dell’Amico, Alessandro and Petter Krus (2013). “A Test rig for Hydraulic Power Steering Concept Evaluation using Hardware-in-the-loop Simulation”. In: *The 8th International Conference on Fluid Power Transmission and Control (ICFP13)*. Hangzhou, China.
- Dell’Amico, Alessandro and Petter Krus (2014). “Closed-centre Hydraulic power steering by direct pressure control”. In: *the 9th JFPS International Symposium on Fluid Power*. Matsue, Japan.
- Dell’Amico, Alessandro, Jochen Pohl, and Petter Krus (2010). “Modeling and Simulation for Requirement Generation of Heavy Vehicles Steering Gears”. In: *Fluid Power and Motion Control (FPMC 2010)*. Bath, UK.
- Dell’Amico, Alessandro, Jochen Pohl, and Petter Krus (2012). “Conceptual evaluation of closed-centre steering gears in road vehicles”. In: *The 7th FPNI PhD Symposium on Fluid Power*.

Licentiate thesis

- Dell’Amico, Alessandro (2013). *Pressure Control in Hydraulic Power Steering Systems*.

Additional articles

Following articles do not directly relate to the work in this thesis but constitute an important part of the process.

- Dell’Amico, Alessandro, Marcus Carlsson, et al. (2013). “Investigation of a digital actuation system on an excavator arm”. In: *13th Scandinavian International Conference on Fluid Power*. Linköping, Sweden.
- Dell’Amico, Alessandro, Liselott Ericson, et al. (2015). “Modelling and experimental verification of a secondary controlled six-wheel pendulum arm forwarder”. In: *the 13th European Conference of ISTVS*. Rome, Italy.
- Dell’Amico, Alessandro, Magnus Sethson, and Jan-Ove Palmberg (2009). “Modeling, Simulation and Experimental Verification of a Solenoid Pressure Control Valve”. In: *The 11th Scandinavian International Conference on Fluid Power (SICFP09)*. Linköping, Sweden.
- Ericson, Liselott, Alessandro Dell’Amico, and Petter Krus (2015). “Modelling of a secondary controlled six-wheel pendulum arm forwarder”. In: *the 14th Scandinavian International Conference on Fluid Power*. Tampere, Finland.

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1

Introduction

With an ever increasing demand for safer vehicles and fewer traffic accidents, there is a clear trend within the vehicle industry where steering-related active safety functions are becoming more common. Active safety systems provide assistance to the driver in more or less critical situations. The most common ones on the market today are the Anti-lock Brake System, the Electronic Stability Program and Traction Control systems. Including the steering system in increasing safety has several benefits (J. Ackermann, Bunte, and D. Odenthal (1999)) and the system can intervene to avoid a collision with an oncoming vehicle or keep the car on the road. Typical functions are Emergency Lane Assist (Eidehall et al. (2007)), collision avoidance systems (Schorn et al. (2006)), roll-over prevention and vehicle stabilisation (Koehn and Eckrich (2004)). The future will also see road vehicles evolve into autonomous vehicles, with several safety, environmental and financial benefits. The key to realising steering related active safety or autonomous driving is active steering, referring to the possibility to control the steer angle or torque with an electronic signal.

The power steering system is the driver's mean to change the direction of the vehicle. Power steering was developed to ease the driver's workload in steering by adding an assistive force to the driver's applied force. Traditionally, this has been done with a hydraulic system, Hydraulic Power Assisted Steering (HPAS), consisting mainly of a fixed pump, an open-centre valve and a cylinder (Marcus Rösth (2007)). The valve is actuated by the driver's applied torque on the steering wheel. The actuation of the valve results in a controlled pressure in either side of the cylinder, generating the required force to assist the driver. The characterisation of the assistive pressure to the input of the driver is called the Boost curve. This is a simple and robust solution. The drawback is the inherent compromise between steering effort and road feel, the low energy efficiency and the lack of the possibility for active steering.

The solution to the energy problem in the passenger car industry was the introduction of the Electric Power Assisted Steering (EPAS) system. This has become a very common solution, though HPAS systems are still found in the

premium car and sport car segments. In addition to the energy savings, the electric motor also allows the assistive torque to be actively controlled and thereby opens up for implementation of active safety functions. Due to the high axle load on heavy vehicles, EPAS systems are not possible with today's limitations in the electric system of the vehicle. A hydraulic power steering system is therefore necessary in this segment. The most common solution to active steering for trucks is a combination of an electric motor in parallel to the hydraulic system. There are, however, several implications. The electric motor introduces inertia, friction and backlash. The open-centre system remains with its poor energy efficiency. A large electric motor can do the majority of the work but increases cost, could be difficult to pack and the large inertia needs to be handled. A smaller electric motor could be packed more easily but the open-centre hydraulic system needs to do much of the work. The electric motor can only work through the Boost curve of the hydraulic system, which could be a limitation.

Since the purpose of the hydraulic system is to control the assistive pressure, a possible solution would be to use proportional pressure control valves. These are actuated by a solenoid, thereby providing an electronic interface and they have closed-centre, which opens up for a reduction in energy consumption. In this work, this system is referred to as Electrohydraulic Power Assisted Steering (EHPAS) and should not be confused with the electric motor driven pump system, which is also often referred to as EHPAS (Forbes, Baird, and Weisgerber (1987)). An electrohydraulic solution takes advantage of the high power density from fluid power systems, which gives a very compact solution. The valves could completely replace the original system, giving only one Boost curve, or work as an add-on system. The latter solution could be beneficial with respect to safety issues.

Since the electrohydraulic pressure control valve with closed-centre behaves in a completely different manner to the hydromechanical open-centre valve, there is a need to understand how this affects the steering system performance and how it can be controlled. The purpose is to keep the Boost curve control strategy in order to maintain, as far as possible, the behaviour of the system. It is also interesting to investigate the possible use of a commercially available valve, since this would reduce development cost.

1.1 Aims and research questions

The main aim of this thesis is to investigate the possibility to use proportional solenoid pressure control valves in power steering applications for realisation of active steering. The valve behaves fundamentally differently to the original valve and an understanding of the impact on the steering system performance is therefore required. The intended application is Boost curve control and position control. The latter is aimed at autonomous driving of vehicles, while the former is the most common control strategy of today's power steering systems. The

process includes a deep understanding and analysis of the original system, which works as a reference point. A deep understanding of the control valves is also necessary, which leads to detail modelling and control design. Finally, the closed-centre system is analysed for performance and stability to give an idea of the usefulness of the valves in this application.

The aim of the thesis can be formulated into the following research questions, which relate to the different chapters and sections in the thesis.

- 1) How can the valve be modelled and what are the important phenomena that need to be considered in order to capture the essential behaviour of the valve?
- 2) How can the pressure control valve be controlled to increase and tune the performance and what control strategies are possible?
- 3) What are the main challenges with Boost curve control when using electronically controlled closed-centre valves and how can they be handled?
- 4) Under what circumstances does the pressure control valve represent a feasible solution to Boost curve control of the power steering system and how do they relate to the original system?
- 5) In what way can the pressure control valves be used for autonomous driving?
- 6) Why should a truck steering system be considered as a potential candidate for the solution with pressure control valves?

Question 1) is treated in chapter 5, while the remaining questions are treated in chapter 6.

1.2 Method

The working approach is model-based, resulting in a framework where the original and new systems can be analysed in order to draw conclusions. The main idea of the work flow can be illustrated by figure 1.1. Initial experiments are conducted on the original system in order to derive model parameters and validation data. The original system is modelled in order to provide a reference system and is validated against measurements. This results in an analysis in the time domain and the frequency domain. Both are valuable tools. Parts of the model are then reused for the modelling of the electrohydraulic system. Derived models are used for control design, which is a requirement of the system architecture. The system is simulated and the models are also used for analysis of the system, from which conclusions can be drawn regarding its behaviour. Final experiments are conducted to verify conclusions from simulation and analysis. The process is an iterative process where several steps could be undertaken several times.

Figure 1.1 Work flow illustrating the methodological approach.

1.3 Delimitations

The work in this thesis is bounded to study the steering system alone and the impact of the closed-centre system on power steering system performance. The approach is rather a comparison between the closed-centre system and the original system and the impact of vehicle dynamics is therefore excluded. Since the real reference system is related to the test rig, no vehicle is therefore present. A load is, however, necessary and an arbitrary load model, such as a vehicle model or a tyre hysteresis model, can be used. The same load model is used for both systems in order to equate the results from both systems.

Although the technology studied in this work is most interesting and perhaps suitable for heavy vehicles, the application has been a steering system for a passenger car. As will be discussed, there is no fundamental difference between how the steering systems work, especially with regard to the hydraulic system. The main difference lies in the pressure level, the flow rate level and the mechanical linkage. The reason why a passenger car steering system has been

studied as the application is related only to the available hardware during the project. The test rig, which is explained in this work, is from a passenger car. In order to facilitate the comparison of simulation results to measurements, the test rig constitutes the application in this work. However, the results are applicable to any steering system with the same working principle, whether it is for a passenger car or a truck.

Energy aspects of the steering system are not considered. It is well known that the traditional hydraulic power steering system is not primarily designed to be energy efficient. It is part of the fundamental technology and several researchers have investigated this, as well as alternative layouts of the system in order to improve energy efficiency. Whether or not it is an issue depends on the application, but in the end it must be answered by the vehicle manufacturer. It is therefore assumed in this work that the chosen technology offers significant improvements in fuel consumption but will be treated as a very positive side effect. The primary target thus remains realisation of active steering.

Safety aspects are not considered in this work. Of course, the new system brings additional safety-critical aspects compared to the original system. For instance, the system relies on active control of the assistive pressure, based on feedback control loops. At any point the system must not apply unexpected values and must handle erroneous sensor data and power loss. This is left for future work and discussion. However, the derived simulation and hardware-in-the-loop simulation environments provide a good platform for such an investigation.

The most common control strategy in power steering systems is Boost curve control. It is an inherent solution of the original system's hardware layout. Even though other control concepts are possible, the approach in this work is to keep this strategy in order to minimise the modification of the original system.

1.4 Contributions

The contributions in this work are the following:

- A model-based framework that analyses the closed-centre power steering system in both the time domain and the frequency domain.
- A model and analysis of a commercial solenoid pressure control valve is derived, with the level of detail sufficient to describe its particular behaviour. The model includes the behaviour of the electromagnetic characteristic of the solenoid, the detailed behaviour of the pressure-flow characteristic and varying flow induced forces.
- A control structure of the pressure control valve that increases performance and handles disturbances.

- An analysis of the control challenges for Boost curve controlled power steering with electrohydraulic valves and possible solutions. Since the work is focused on Boost curve control, the control strategy is already defined and constitutes a limitation.
- Position control of the steering rack with pressure control valves for realisation of autonomous driving.
- An analysis of the difference in performance and control conditions between a passenger car steering system and a truck steering system.
- A hardware-in-the-loop test rig for simulation and verification of the electrohydraulic closed-centre power steering system.

1.5 Outline

Chapter 2 gives a description of the original steering system and its purpose and functionality, both on a system level and a component level. This is to give an understanding of the application. This is followed by mathematical modelling and analysis of the original system in chapter 3. The analysis is important since it gives an understanding of the system's behaviour. The models are used for comparison with the closed-centre system and parts of the model are reused when modelling the closed-centre system. Chapter 4 gives an overview of existing solutions in realising active steering and discusses the selected solution in this work. Chapter 5 covers the detailed modelling and analysis of the commercial pressure control valve and contains a general discussion about pressure control. In chapter 6 the analysis of the pressure control valve with the application is outlined. This covers control design, Boost curve control analysis and position control. The test rig that was developed and used in this work is described in section 7. A brief discussion of the work can be found in chapter 8 while detailed discussions on specific topics are given in the relevant chapters. Finally, the conclusions, related to the research questions, are outlined in chapter 9.

2

Description of the steering system

The steering system of a vehicle serves two main purposes: to provide a means to control the direction of the vehicle and to provide a feedback information channel to the driver about the road conditions. Driving a vehicle can be seen as a closed loop system, which is well described by Marcus Röstl (2007). The intention of the driver is to follow a predefined route. This is done by controlling the steering wheel angle. Feedback channels are a visual channel, the driver sees where the vehicle is headed, the lateral acceleration from the vehicle and the steering wheel torque from the steering system. There are several requirements the steering system should fulfill, as is explained by Pfeffer (2006). These are related to the response of the vehicle to steering inputs, returnability of the steering wheel, torque build-up at on-centre, handling limits should be perceptible and during parking a low steering wheel torque and small steering angles are desired. With high axle loads, large tyres and front wheel drive, an assistance system is required to meet the demands for low steering wheel torque and comfort. Traditionally, this is done with hydraulic power, but in recent years the most common technology is electric power steering for passenger cars. For heavy vehicles and trucks, the axle loads are significantly higher and with today's electric system limitations, hydraulic power is still in use.

This chapter gives an overview of the power steering system, both the mechanical system and the hydraulic system. A comparison between the passenger car and heavy vehicles is presented. Energy aspects are discussed, as well as steering feel. This chapter gives the background to the system modelled in chapter 3.

Figure 2.1 *Illustrations of steering system arrangements.*

2.1 Design and functionality

There are basically two types of steering system arrangement: the rack and pinion and the worm gear steering box. Both arrangements are illustrated in figure 2.1. Even though both types can be found in both passenger cars and heavy vehicles, it is most common for the rack and pinion to be used in passenger cars and the worm gear steering box in heavy vehicles. The main benefits with the rack and pinion is the reduced complexity: it is much lighter, easier to accommodate in front wheel drive vehicles and has a better efficiency, which gives the driver a more accurate feeling of the tyre-ground interaction, see e.g. Gillespie (1992) and Genta and Morello (2009). The worm gear steering box allows for a greater gear ratio, which is needed with the heavy loads in trucks. The basic working principle of the two arrangements is the same. The difference lies in the mechanical linkage, which is more complicated for the worm gear steering box, with more moving parts that generate friction.

Figure 2.2 shows a photograph that compares the steering gears of two arrangements. Note the difference in size of the assistance piston. The rack and pinion system converts a rotational motion into a translational motion of the rack. The steering wheel is attached to the column, which connects to the pinion via a torsion bar. The torsion bar is the main control element of the assistance system. The rack is supported by a sliding block, called a yoke. The yoke is pushed by a preloaded spring in order to reduce backlash between the pinion and the rack. It also adds friction, which is controlled by the preload.

The most common worm gear is the recirculating ball gear, see e.g. Stone and Ball (2004). A nut is meshed onto a threaded worm. Inside the nut there are helical channels where balls rotate and move. The channels with balls are closed, which makes the balls recirculate. The worm rotates with the rotation

Figure 2.2 Photograph comparing the rack and pinion system and the recirculating ball screw steering box.

of the steering column, which makes the nut move up and down. The nut is meshed with a sector that rotates with the movement of the nut. The pitman arm is in turn attached to the sector at one end and to the mechanical linkage at the other. The purpose of the channels with balls is to reduce friction and backlash.

2.2 Assistance

In order to ease the driver's workload during steering manoeuvres the steering system is equipped with an assistance system. The assistance system is in parallel to the mechanical system. This ensures the haptic feedback to the driver. There is a compromise, however, since with the assistance system there is also a loss of information to the driver. The assistance system is characterised by the assistance curve, also known as the Boost curve. Two examples are shown in figure 2.3, where figure 2.3a could represent a passenger car and figure 2.3b a heavy vehicle. The Boost curve shows how much assistance is applied depending on the torque applied by the driver. The shape of the Boost curve is a consequence of the requirements of the steering system, that is, low effort to steer during parking and good feel during high speed driving. Two regions are marked in figure 2.3a: on-centre driving and parking. On-centre driving refers to for example to highway driving with small steering inputs. Here the need for good feedback to the driver is high. The steering torque is

Figure 2.3 Typical assistance curves of a passenger car to (left) and a heavy vehicle (right). Regions for on-centre driving and parking are also marked in the figure.

low, as well as the assistance. It is during parking that the loads are highest. The need for information feedback to the driver is less critical and the steering should be comfortable. Therefore, the assistance is high in this scenario. From here onwards, the Boost curve will be shown as the absolute value, which is common.

The traditional assistance system is a hydraulic system. In this case, the Boost curve represents the pressure applied by the hydraulic system. In case of an electric power steering system, the Boost curve represents the torque applied by the electric motor.

2.3 Hydraulic system

The hydraulic system can be treated equal for both the rack and pinion system and the recirculating ball gear system. Both systems are illustrated in figures 2.4 and 2.5, respectively. The main components of the system are a constant flow pump, rotational valve and cylinder. The pump is usually a fixed displacement vane pump driven by the combustion engine. It has an integrated pressure relief valve and a flow control valve. The rotational valve is controlled by the torsion bar. The spool is attached to the upper end of the torsion bar. The lower part of the torsion bar is attached to the pinion, where the valve body is also attached. The valve is of open-centre type with several control edges. At centre position, all control edges are equally opened and all ports are connected to tank. When the driver applies a torque on the steering wheel, the torsion bar is twisted. This causes an angular displacement between the spool and the valve body. This in turn will close the control edges on one side and open it on the other side. On the closing side, the pressure will increase

Figure 2.4 Schematic of the valve assembly of the rack and pinion steering system.

Figure 2.5 Schematic of the valve assembly of the recirculating ball screw steering system.

and generate the assistance force needed. Thus, the purpose of the hydraulic system is to control pressure. The torsion bar is linear, which means that the control of the valve is proportional to the torque applied by the driver.

2.4 Steering wheel torque

The steering wheel torque is the fastest information channel to the driver about the road conditions. The torque translated through the steering system comes from the tyre-road contact, steering geometry and suspension effects and friction in the steering system itself. In order to accurately describe the steering wheel torque related to vehicle characteristic and handling, a precise model of the vehicle and tyres is required. This is studied by Pfeffer (2006), where an enhanced bicycle model is suggested. The forces from the tyre-ground contact act in the x-, y- and z-planes and generate a torque around the kingpin axis. These are analysed by Gillespie (1992). The sum of the left and right wheel makes part of the steering wheel torque. The contribution from the vertical force is mainly a result of the lateral inclination of the kingpin axis and produces a centering torque while the front of the vehicle is lifted. The lateral force exists to counteract the centrifugal force of the vehicle during cornering. It produces a torque through the caster angle. The tractive forces produce a torque through the scrub radius in the opposite direction and is therefore only present when there is a force imbalance. The aligning torque is a result of the lateral force acting at a distance behind the centre of the tyre-ground contact, called pneumatic trail. The torque opposes the steering action.

2.5 Energy aspects and alternative systems

One of the major drawbacks with the hydraulic power steering system is its poor energy efficiency. Due to the open-centre of the valve and a fixed pump that always delivers flow, some part of the oil will always go to tank. The exception would be extreme cases where the valve is fully displaced and the open-centre path is closed, but this is hardly the case. The flow through the open-centre path creates a pressure drop even when no steering occurs and causes a power loss. The level of assistive pressure from the Boost curve is flow-dependent, which is shown in chapter 3. Since the pump is driven from the combustion engine, the pump must therefore be designed to provide enough oil even at idling speed. It is also here where the highest assistive pressure is needed. As soon as the engine speed increases, so also does the pump speed, resulting in an increase in oil flow since the pump is fixed. The surplus oil is fed back to the tank by the internal flow control valve of the pump. This property of the system is one of the major reasons for the poor energy consumption, see Marcus Rösth (2007). Additional losses also come from the pressure drop over the load ports.

In the passenger car industry, the Electric Power Assisted Steering (EPAS) system has become the most common system. The main advantage initially was low energy consumption. Here, the hydraulic system is replaced with an electric motor that provides the level of assistance necessary either through the steering column or directly through the rack, see Badawy, Zuraski, et al.

(1999). The low energy consumption comes from the torque being applied only when necessary and this dramatically reduces energy consumption compared to the HPAS system, see Wellenzohn (2008). Challenges with the EPAS system have been the steering feel, which is compromised due to the high inertia of the electric motor, friction, backlash, and torque ripples. Safety aspects are also a concern. A pure electric power steering system, however, is not a possibility for heavy vehicles due to the greater axle loads and a hydraulic system is still necessary. The exception would be hybrid vehicles, where a high voltage battery allows the use of an EPAS system, as was studied by Morton, Pickert, and Armstrong (2014).

Extensive research can be found that is aimed at improving the energy consumption of hydraulic power steering systems. A common approach is to drive the pump from an electric motor instead of from the combustion engine, which is discussed by Pfeffer, D. N. Johnston, et al. (2005), Forbes, Baird, and Weisgerber (1987), Suzuki et al. (1995), Inaguma, Suzuki, and Haga (1996), Gessat (2007) and Yu and Szabela (2007). The main purpose is to actively control the oil flow depending on the driving scenario and which can be turned down to a very low level during high speed driving. Other advantages of the system include improved packaging and the possibility to vary the level of assistance for different scenarios and in that way to some extent tune the steering feel. The solution can also be used with a hydromechanical closed-centre rotational valve, as presented by Suzuki et al. (1995) and Inaguma, Suzuki, and Haga (1996). With a small overlap of the valve the pump flow can be turned off when no steering action is applied. The characteristic of the steering system is also affected, providing a region of manual steer around zero position. An electronically controlled 4/3-valve with closed-centre is used by Bootz, Brander, and Stoffel (2003), in order to reduce energy consumption. The system uses a constant pressure, provided from fixed pump driven from an electric motor or the combustion engine via a clutch. Two accumulators are used to vary the system pressure depending on driving scenario. With this system, energy consumption was significantly reduced compared to the open-centre system.

Several solutions to control the output flow of a combustion engine driven pump for open-centre systems are also presented by Bootz (2006). For a fixed pump, a by-pass valve and an electro-hydraulic proportional valve can be introduced to control the oil flow. Instead of a fixed pump, a hydromechanical or an electrohydraulic variable pump can be used. If an accumulator is used in the supply system, initial flow demands can be taken care of by the accumulator and the pump can be run at very low speed. The use of an accumulator has the highest potential to reduce the energy consumption. The tests by Bootz (2006) showed that only 28% of the power, compared to the original system, was needed for the given driving cycle. In the hybrid power steering system for heavy vehicles, presented by Wiesel et al. (2009), an electric motor is combined with the open-centre power steering system. The use of energy can be optimised for any specific driving scenario. On the hydraulic supply side, different

solutions were investigated: a by-pass pump, a variable pump and a pump with clutch. The variable pump showed the highest potential with a reduction of up to 80% for the given driving cycle.

2.6 Steering feel and steering system influence on vehicle handling

A good steering feel is important when driving to allow the driver to accurately place the vehicle, Adams (1983). Steering feel can be defined as "vehicle reaction and steering wheel torque feedback within the driving task of directional control", Pfeffer (2006). As mentioned by Adams (1983), there is a different need of feel depending on driving situation. During low speed manoeuvre the driving can be undertaken with less feel, while at high speed cornering a higher feel is needed to give the driver information about the vehicle's response.

Several researchers have investigated steering system properties on on-centre characteristics. There are several measures that are possible to quantify that have an influence on the on-centre characteristics. These are derived by, for example, Norman (1984). Several vehicles were tested with sinusoidal input and parameters were defined from cross plots, such as steering wheel angle vs. lateral acceleration, which gives steering sensitivity, steering hysteresis and yaw rate lag time; steering wheel torque vs. lateral acceleration, which gives the returnability, indication of friction, steering torque gradient that indicates road feel and steering effort; steering wheel torque vs. steering wheel angle, which gives stiffness. Other quantitative measures are defined analytically by Baxter (1988). The stiffness is defined as the gradient of the rack force over steering angle and increases with the Boost gain. The steering gear feel is defined as the gradient of the steering wheel torque over rack force and decreases with the Boost gain. There is thus a contradiction between the two. An example is for on-centre driving where the Boost gain is low to give a good feel, but also generates a low stiffness.

An early paper by Segel (1964) investigated the influence steering force (or steering torque) versus lateral acceleration, that is torque gradient, has on driver rating of vehicle controllability. It was shown that there is a strong relation between steering torque gradient and driver ratings, e.g. too light a torque gradient results in difficulty in precise positioning of the vehicle as too high a gradient was experienced as the response was slowed down. The importance of tuning the torque gradient for a good steering feel is also stressed by Harrer, Pfeffer, and N. D. Johnston (2006). A typical relation between the steering wheel torque and the lateral acceleration is shown in figure 2.6. Due to the Boost curve the gradient decreases with lateral acceleration. At low lateral acceleration the gradient is steep to provide a good on-centre feel. An analytical expression for the steering wheel torque characteristic versus lateral acceleration at steady state was derived by Pfeffer and Harrer (2008),

by defining the steering assistance ratio. A linear ratio is desirable, which gives two parameters to tune. The Boost curve can then be defined from the assistance ratio.

Figure 2.6 *Steering wheel torque as a function of lateral acceleration.*

Several researchers have also investigated the relationship between the driver's subjective rating regarding the controllability of the vehicle and vehicle characteristic. Jaksch (1979) found that there is a strong relationship between subjective ratings and yaw velocity response time, steering wheel angle gradient and steering wheel torque gradient. Rothhämel (2010) investigated the correlation between steering feel and vehicle handling properties for heavy vehicles. The steering wheel returnability is correlated to the torque gradient while torque magnitude and friction levels are correlated to the driver's opinion of the heaviness of driving and stability.

It can be concluded that the steering wheel torque is very important for steering feel. In this work it is therefore assumed to be sufficient to study how the steering wheel torque is affected by the closed-centre system. Since no references are available, the original system will fill this role. Vehicle performance is also a contributor to the steering feel. By using the same arbitrary model for both systems when comparing, the effects of the vehicle can be excluded. It is important to have a model that predicts the steering wheel torque.

3

Modelling of the hydraulic power steering system

The modelling of the original hydraulic power steering system serves several purposes in this work. Since the alternative electro-hydraulic concept will work only to replace the existing hydraulic system, a model of the original system will therefore be useful for evaluation and development of the electro-hydraulic system. A simulation environment facilitates much of the development work, where control strategies and concepts can be evaluated without the need for hardware. Since the original system also works as a frame of reference, a good understanding of the system is essential. A model will help in this, where the effect of different parameters can be understood better. A frequency analysis of the system can further explain certain behaviour.

This chapter describes the modelling procedure for the hydraulic power steering system. To better compare results, the system studied is the test rig explained in detail in chapter 7, where the steering system has been isolated from the rest of the vehicle. A nonlinear model of the system is first derived and validated with measurements from the test rig. The model is then linearised and analysed in the frequency domain, where stability can be examined.

3.1 Nonlinear model

For modelling purposes, the hydraulic power steering system can schematically be described as shown in figure 3.1. The system is only studied from a steering wheel angle θ_{sw} to a rack displacement x_{rw} . It is convenient to divide the model into several submodels, which is illustrated in figure 3.2. A driver model serves the purpose of applying the right amount of torque at steering wheel in order

to follow given steering wheel angle driving cycle. This means that different steering systems can be evaluated for the same cycle. The main idea is to be able to compare different steering systems and the difference in behaviour is seen in the driver's torque. The development of a driver model is not within the scope of this work and a PID controller, implemented as a lead-lag filter, is chosen and manually tuned. This will not be explained further in this work. The driver submodel communicates with the mechanical submodel by sending the torque. The mechanical submodel will in turn calculate a steering wheel angle based on this input. The mechanical submodel also communicates with the hydraulic submodel and the load model. The mechanical submodel sends the torque at the torsion bar to the hydraulic submodel, which in turn calculates the pressure difference in the assistance cylinder. This is sent back to the mechanical submodel. The load model could be any model that generates the right amount of load felt by the steering rack during different driving scenarios. Detailed models of the load are not within the scope of this work and simple, but sufficient, models are used. For highway driving, the classic bicycle model is used. The highest load case represents the vehicle at stand-still. For this scenario, the load, or tyre, is simply regarded as a spring. The load takes the rack position from the mechanical submodel as input and calculates the force, which is sent back. The mechanical, hydraulic and load submodels are described in detail below. The equations are defined in continuous form, but all implementation has been done with the Transmission Line Method and Bilinear transform, see e.g. Auslander (1968), since it ensures a robust numerical solver and the models can be re-used for implementation in a real-time computer.

Figure 3.1 *Schematic of the hydraulic power steering system.*

Figure 3.2 The structure of the simulation model of the hydraulic power steering system. The interaction between the submodels is also illustrated.

3.1.1 Mechanical submodel

The mechanical subsystem can be modelled with different levels of complexity depending on the purpose and a great deal of research has been done on the subject. Preferably, the model should not be unnecessarily complex since it will increase computational time and be more difficult to analyse. Starting from a 2 DoF system, models up to a 5 DoF system have been considered where every inertia of the system is taken into account. Pfeffer (2006), see also Pfeffer, Harrer, et al. (2006), derives a 5 DoF model with the purpose of studying the steering wheel torque. However, a 2 DoF model is also derived for the same purpose and also shows that the two models perform in a similar fashion. A 2 DoF model is also used to study the power consumption of an HPAS system by Pfeffer, D. N. Johnston, et al. (2005). Baxter (1988) also uses a simple model and ignores friction to derive expressions for steering gear feel and stiffness. Post and Law (1996) measure the friction and stiffnesses of a rack and pinion steering gear and a recirculating ball screw steering gear and model the systems with both a high degree of freedom model and a 2 DoF model implemented with a vehicle model. The importance of including nonlinear effects, such as friction, stiffness and boost curve, in order to predict the behaviour of the system, was shown. Neureder (2002) studied steering wheel nibble, i.e. vibrations in the steering wheel, and derived a simple model of the steering system. Marcus Rösth (2007) derived both a 2 DoF model of a passenger car steering system and a 3 DoF model of a truck steering system, both models with pressure- and speed-dependent friction. Linear models were derived to study chattering and a nonlinear model to study catch-up. The same approach to modelling the mechanical subsystem is also used in EPAS systems, as shown by Badawy, Zuraski, et al. (1999). A column-assisted EPAS was modelled and a higher and a lower degree model were compared to show similar results. Ueda et al. (2002) investigated how changes in friction in different parts of the steering

system affect the system characteristics. A 4 DoF model was used for this. In the early paper by Segel (1966), the author derives a simplified model in order to predict the behaviour of the vehicle. The steering system is modelled with 2 DoF with Coulomb friction together with a 3 DoF linear vehicle model. In the work done by Zaremba and Davis, 1995, the choice was a 2 DoF model for the mechanical structure, with the purpose of analysing stability in the frequency domain of an EPAS system. Since the system studied is the test rig without wheel assembly and vehicle and since the operating range where the steering system is studied in this work is similar to that in, for example, Pfeffer (2006), it should be sufficient to use a 2 DoF model for the purpose of describing the behaviour of the system and to predict the steering wheel torque under different conditions.

Friction is an important aspect to properly describe the system and a proper friction model is therefore needed. There is friction in the column and in the rack. The rack friction is strongly pressure-dependent, but friction also comes from the yoke, mesh and steering valve sealings. One possible way would be to implement a Coulomb friction. As long ago as 1966, Segel (1966) stressed the importance of including friction, and found that without a certain amount of Coulomb friction in the steering system the vehicle would be unstable, in studies of a pure mechanical steering system. However, investigations by Neureder (2002) suggest that the friction in the steering system behaves differently than a Coulomb friction. It was found that cars exhibit nibble even at low-level forces, which contradicts the behaviour of Coulomb friction. Ueda et al. (2002) also investigated friction and suggested a spring/friction model to best describe its behaviour. This means that the friction force behaves like a spring up to a certain level where the force remains constant. Pfeffer (2006) used the same approach and further developed the model. This is also adopted in this work.

The equations of motion describing the steering system are described by equation 3.1 and equation 3.2. The steering wheel and column constitute the upper inertia, J_{sw} , and the rack with mass constitutes a translational mass, M_{rw} . Steering wheel torque is denoted T_{sw} , stiffness K_t , steering gear ratio R_t , viscous friction coefficient b , load force F_{load} , cylinder area A_p , column friction $M_{f_{sw}}$ and rack friction $F_{f_{rw}}(p_L)$. The steering wheel is denoted index sw and the rack index rw . The load pressure is denoted p_L and is defined as the difference between the right and left cylinder chambers, $p_L = p_1 - p_2$. The rack friction is dependent on the pressure in the assistance cylinder.

$$J_{sw}\ddot{\theta}_{sw} = T_d - K_T(\theta_{sw} - x_{rw}R_T) - b_{sw}\dot{\theta}_{sw} - M_{f_{sw}} \quad (3.1)$$

$$M_{rw}\ddot{x}_{rw} = p_L A_p + K_T(\theta_{sw} - x_{rw}R_T)R_t - b_{rw}\dot{x}_{rw} - F_{load} - F_{f_{rw}}(p_L) \quad (3.2)$$

The friction model is illustrated in figure 3.3. It is modelled as an exponential spring with limited force, F_{lim} , and hysteresis, and is well documented by Pfeffer (2006). The column friction uses this model without modifications. The rack friction uses the same model as well but with an extra term for the pressure dependency, which is assumed to be linear. It follows the relation in

equation 3.3 where cp_{Ff} is a constant that describes the pressure dependency.

$$F_{f_{rw}}(p_L) = F_{f_{stat}} + cp_{Ff} \frac{p_1 + p_2}{2} \quad (3.3)$$

Equivalent spring stiffness of the rack

Figure 3.3 Representation of the friction model according to Pfeiffer, 2006.

3.1.2 Hydraulic submodel

The open-centre valve consists of several control edges but is preferably modelled as a lumped Wheatstone bridge, as shown in figure 3.4. Opposite orifices are assumed to be equal. This has proven to be useful by M. Rösth and J-O. Palmberg (2007). A more general model and analysis of an open-centre valve can be found in Merritt (1967), which is also applicable here. For the modelling, the tank pressure is assumed to be zero and the pump is assumed to deliver a constant flow q_p . The model consists of the equations describing the flow through the four orifices, q_1 to q_4 , equations 3.4 to 3.7, and the continuity equation to describe the pressure change in the system volume V_s and the two cylinder volumes V_1 and V_2 , equations 3.8 to 3.10. Variables are defined in figure 3.4, where A_p is the piston area, C_q is the discharge coefficient, ρ is the oil density and β is the bulkmodulus. The volume are allowed to change their size with piston stroke, i.e., $dV = A_p x_{rw}$.

Flow equations

Figure 3.4 The open-centre valve represented as a Wheatstone bridge.

$$q_1 = C_q A_1 \sqrt{\frac{2}{\rho} (p_s - p_1)} \quad (3.4)$$

$$q_2 = C_q A_2 \sqrt{\frac{2}{\rho} (p_s - p_2)} \quad (3.5)$$

$$q_3 = C_q A_1 \sqrt{\frac{2}{\rho} p_2} \quad (3.6)$$

$$q_4 = C_q A_2 \sqrt{\frac{2}{\rho} p_1} \quad (3.7)$$

Continuity equations

$$q_p - q_s = \frac{V_s}{\beta} \dot{p}_s \quad (3.8)$$

$$q_1 - q_4 = A_p \dot{x}_{rw} + \frac{V_1}{\beta} \dot{p}_1 \quad (3.9)$$

$$q_2 - q_3 = A_p \dot{x}_{rw} + \frac{V_2}{\beta} \dot{p}_2 \quad (3.10)$$

Equations 3.4 to 3.10 represent how the system is implemented in the simulation environment. For further analysis of the system, a few assumptions can be

made and some variables defined to simplify the model. If it is assumed that the orifices in diagonally opposite arms are symmetric and matched and that the flow is also equal, as described by equations 3.11 and 3.12, it follows that the system pressure p_s can be expressed as shown in equation 3.13.

$$q_1 = q_3 \quad (3.11)$$

$$q_2 = q_4 \quad (3.12)$$

$$p_s = p_1 + p_2 \quad (3.13)$$

The load pressure p_L is defined by equation 3.14.

$$p_L = p_1 - p_2 \quad (3.14)$$

Combining equations 3.13 and 3.14, the cylinder volume pressures p_1 and p_2 can be defined as shown in equation 3.15 and 3.16.

$$p_1 = \frac{p_s + p_L}{2} \quad (3.15)$$

$$p_2 = \frac{p_s - p_L}{2} \quad (3.16)$$

The load flow q_l is defined by equation 3.17 or equation 3.18. The system flow q_s is defined by equation 3.19 or equation 3.20.

$$q_l = q_1 - q_4 \quad (3.17)$$

$$q_l = q_3 - q_2 \quad (3.18)$$

$$q_s = q_1 + q_2 \quad (3.19)$$

$$q_s = q_3 + q_4 \quad (3.20)$$

At steady-state, the load flow corresponds to the piston velocity. Combining the definitions of the cylinder volume pressures, the system flow and load flow, the hydraulic system can be described by equations 3.21 to equation 3.24, where the system is expressed in the variables system pressure p_s , load pressure p_L , system flow q_s and load flow q_l .

$$q_l = C_q A_1 \sqrt{\frac{p_s - p_L}{\rho}} - C_q A_2 \sqrt{\frac{p_s + p_L}{\rho}} \quad (3.21)$$

$$q_s = C_q A_1 \sqrt{\frac{p_s - p_L}{\rho}} + C_q A_2 \sqrt{\frac{p_s + p_L}{\rho}} \quad (3.22)$$

$$q_p - q_s = \frac{V_s}{\beta} \dot{p}_s \quad (3.23)$$

$$q_l = A_p \dot{x}_{rw} + \frac{V_0}{\beta} \dot{p}_L \quad (3.24)$$

$$V_0 = \frac{V_1 V_2}{V_1 + V_2}$$

At steady-state, the load pressure and system pressure can be derived from equations 3.21 and 3.22 and expressed by equations 3.25 and 3.26,

$$p_L(T_{tb}, q) = \frac{\rho q_p^2}{8C_q^2} \left(\left(\frac{1-q}{A(T_{tb})} \right)^2 - \left(\frac{1+q}{A(-T_{tb})} \right)^2 \right) \quad (3.25)$$

$$p_s(T_{tb}, q) = \frac{\rho q_p^2}{8C_q^2} \left(\left(\frac{1-q}{A(T_{tb})} \right)^2 + \left(\frac{1+q}{A(-T_{tb})} \right)^2 \right) \quad (3.26)$$

where $q = \frac{q_L}{q_p}$. The opening areas depend on the displacement of the valve, that is, the twisting of the torsion bar. The opening areas can therefore be expressed as functions of the torsion bar torque, as shown in equations 3.27 and 3.28. The load pressure in equation 3.25 is therefore also a function of the torsion bar and represents the Boost curve in analytical form. As can be seen, it depends on the opening areas, i.e. the geometry of the valve and the pump flow. It also varies with piston speed.

$$A_1 = A(T_{tb}) \quad (3.27)$$

$$A_2 = A(-T_{tb}) \quad (3.28)$$

The opening areas can be derived in several ways. One way is to measure the static Boost curve together with the pump pressure for a given pump flow. The opening areas can then be calculated according to equations 3.29 and 3.30. The Boost curve for the test rig is measured and the opening areas calculated in section 3.2.

$$A_1(T_{tb}) = \frac{q_p}{2C_q} \sqrt{\frac{\rho}{p_s(T_{tb}) - p_L(T_{tb})}} \quad (3.29)$$

$$A_2(T_{tb}) = \frac{q_p}{2C_q} \sqrt{\frac{\rho}{p_s(T_{tb}) + p_L(T_{tb})}} \quad (3.30)$$

3.1.3 Load model

The steering rack is subjected to forces due to the interaction between the tyre and ground. An exact model of the behaviour of the load is not within the scope of this work and arbitrary, but reasonable, load models have been used. Different forces and behaviour of the load arise during different driving scenarios. For example, the highest loads arise when the vehicle is at stand still. The tyre then behaves like a spring until it begins to slip and the force becomes more or less constant. The simplest way is to only study small steering wheel angle inputs and see the tyre as a pure spring. This has been done when analysing the closed-centre system for stability, which is described in chapter 6. For most study cases, however, the bicycle model has been used as load, which can be found in the book by Wong (2001). Small steering wheel and slip angles are assumed. The model only considers lateral acceleration and any forces that

could arise from longitudinal accerelation are ignored. The longitudinal velocity v_x is therefore constant. The lateral acceleration $\dot{v}_y$ and the yaw acceleration $\dot{\Omega}_z$ about the z-axis of the vehicle are described by equations 3.31 and 3.32, respectively, where m is the mass of the vehicle and J is the inertia about the z-axis. The equations are defined in the vehicle's fixed coordinate system, with parameters defined by figure 3.5.

Figure 3.5 Bicycle model used as load model.

$$m\dot{v}_y = F_{yf} + F_{yr} - mv_x\Omega \quad (3.31)$$

$$J\dot{\Omega}_z = F_{yf}l_f - F_{yr}l_r \quad (3.32)$$

The front and rear lateral forces, F_{yf} and F_{yr} , depend on the tyre model. For moderate lateral accelerations it is sufficient to consider a linear behaviour of the tyre and the lateral forces are given by equations 3.33 and 3.34, where C_{α_f} and C_{α_r} are the tyre cornering stiffnesses and α is the slip angle at the front and rear. The bicycle model lumps the inner and outer wheel and the cornering stiffness is therefore considered for both wheels.

$$F_{yf} = C_{\alpha_f}\alpha_f \quad (3.33)$$

$$F_{yr} = C_{\alpha_r}\alpha_r \quad (3.34)$$

The slip angles are defined by equation 3.35 and 3.36. The road wheel angle is here denoted θ_{rw} and is input to the model. Since the steering system is isolated about the rack, the rack position is tranformed to a corresponding road wheel angle.

$$\alpha_f = \theta_{rw} - \frac{l_f\Omega_z + v_y}{v_x} \quad (3.35)$$

$$\alpha_r = \frac{l_r\Omega_z - v_y}{v_x} \quad (3.36)$$

With the parameters according to table 3.1, the yaw response shown in figure 3.6 can be produced with a step input in road wheel angle of 0.01 rad at 70 kph.

The force on the steering rack is given by the front lateral force, which generates a torque through the pneumatic trail and caster angle, as explained

Figure 3.6 Yaw response of the vehicle used as load model to a step input in road wheel angle of 0.01 rad at 70 kph.

Parameter	Value	Unit
m	$1.66 \cdot 10^3$	kg
J	$4.88 \cdot 10^3$	kgm ²
C_{α_f}	$2.06 \cdot 10^5$	N/rad
C_{α_r}	$10.65 \cdot 10^5$	N/rad
l_f	0.7025	m
l_r	2.0125	m

Table 3.1 Vehicle parameter values

in chapter 2, and is transformed to a force on the rack through the suspension geometry. Here it works only through the total pneumatic trail, p_t . Whichever load model, it is dependent on the rack position. The load model could therefore be treated as part of the control loop or as a disturbance. Hereafter, when the load is referred to the vehicle model it is the vehicle model presented.

3.2 Measurements and parameter identification

Several parameters of the steering system model can be identified by measurements of the test rig. The test rig provides a very convenient way to study specific parts of the system, which could be difficult when installed in the vehicle. More on the test rig can be found in chapter 7. Parameters identified are the Boost curve, opening areas of the valve, rack friction, column friction, column and torsion bar stiffness, and steering wheel and column inertia.

Boost curve

The Boost curve is measured by clamping the rack. A steering wheel torque is applied slowly in both directions. Pump and cylinder pressures are recorded together with the torsion bar torque. The measured boost curve is shown in figure 3.7.

Figure 3.7 The measured Boost curve of the test rig shows differential cylinder pressure vs. torsion bar torque.

From the boost curve the opening areas of the valve can be calculated using equations 3.30 and 3.29. Both areas are shown in figure 3.8.

Rack friction

According to Pfeffer (2006), it is the pressure-dependent friction that is the most dominant friction in the rack. The friction is measured by disconnecting the column from the input shaft of the valve. The load cylinder is run as a position controller with a sinusoidal input with an amplitude of 0.004 m and a frequency of 0.5 rad/s. Both the force required to drive the rack and the rack position are registered. The servo valves are controlled to maintain a constant pressure in both chambers. Each measurement is made for 5, 10, 20, 40, 60 and 80 bar in each chamber. The results of the measurements can be seen in figure 3.9 and the increased amplitude of the hysteresis curve with increased pressure is clearly visible. A linear relation between the friction and the sum of the cylinder pressures is assumed according to equation 3.3. The static friction

Figure 3.8 The calculated opening areas of the open-centre valve.

level $F_{f_{stat}}$ is set to 357 N and the pressure coefficient cp_{Ff} to $2.94 \cdot 10^{-5}$ N/Pa.

Column friction

The column friction is measured by again using the load cylinder as a position servo with a sinusoidal input with an amplitude of 0.004 m and a frequency of 0.5 rad/s. The steering column is attached to the valve and the steering wheel is free. The torsion bar torque then gives an indication of the friction in the upper inertia. The chamber pressures are held at 10 bar. It turned out that at zero pressure the result is very oscillative, probably due to the friction in the rack. Figure 3.10 shows the torsion bar torque vs. the steering wheel angle. The result is still quite oscillative but the amplitude of the hysteresis curve is still clearly around 0.3 Nm.

Column and torsion bar stiffness

The column stiffness is determined by measuring the torsion bar torque and steering wheel angle with the rack clamped. This gives the total stiffness of the upper inertia, i.e. the torsion bar and steering wheel column. The result is shown in figure 3.11. The inclination of the curve gives the stiffness and is set to 1.06 Nm/deg.

Figure 3.9 Rack force vs. rack position for different chamber pressures.

Figure 3.10 Torsion bar torque vs. the steering wheel angle with a freely rotating steering wheel. The mean friction level at ± 0.3 Nm is also indicated.

Steering wheel and column inertia

The inertia of the upper system, i.e. the steering wheel and column, is measured by clamping the rack and applying a torque at the steering wheel. When

Figure 3.11 The figure shows the torsion bar torque vs. the steering wheel angle with the rack clamped.

the torque is suddenly released, the steering wheel will excite free oscillations according to equation 3.37

$$f = \sqrt{\frac{K_T}{J_{sw}}} \quad (3.37)$$

The result is shown in figure 3.12. The frequency is 5.15 Hz. Since the stiffness K_T is known, the inertia can be calculated to 0.058 kgm^2 . The damping coefficient is tuned to get a satisfactory result from simulations of the same test case compared to the measurements. A damping of 0.06 was achieved in this way.

Figure 3.12 *The free oscillations of the steering wheel.*

3.3 Validation of nonlinear model

The model of the hydraulic power steering system is validated by comparing the outputs of the model to measurements for the same input cycle. The input is the steering wheel angle, which is measured and fed as a reference value to the driver model. The outputs are the torsion bar torque, the rack position, the pump pressure and the assistance pressure in the cylinder. The steering wheel angle was manually applied with increased velocity. The vehicle model was used and set to run at 70 kph. The steering wheel angle reached an amplitude of $\pm 30^\circ$ and a velocity of $400^\circ/\text{s}$. The pump pressure reached ~ 40 bar. The oil temperature was maintained between 40 and 45°C .

Figure 3.13 shows a comparison between the simulated and measured steering wheel angle, which shows the performance of the driver controller. The driver model is able to follow the reference with very good result. Figure 3.14 shows a comparison between the simulated and measured torsion bar torque, which is the most important indication that the model is accurate enough to be used in the evaluation process of the electrohydraulic power steering system. Figure 3.15 shows a comparison between the simulated and the measured rack position. Figure 3.16 shows a comparison between the simulated and the measured pump pressure. Figure 3.17 shows a comparison between the simulated and the measured pressure differences, i.e. the assistance pressure. All figures indicate that the model performs sufficiently well for the given driving cycle.

Figure 3.13 Comparison between simulated (dashed) and measured (solid) steering wheel angle.

Figure 3.14 Comparison between simulated (dashed) and measured (solid) torsion bar torque.

Figure 3.15 Comparison between simulated (dashed) and measured (solid) rack position.

Figure 3.16 Comparison between simulated (dashed) and measured (solid) pump pressure.

Figure 3.17 Comparison between simulated (dashed) and measured (solid) pressure differences between left and right chamber of the assistance cylinder.

3.4 Linear modelling and analysis

The complete hydraulic power steering system is linearised and analysed in the frequency domain. Such an analysis is very useful. It helps to better understand the system, stability can be examined and any particular behaviour will be visible. The analysis is divided into first study the hydraulic system and then the complete system.

3.4.1 Hydraulic system analysis

The hydraulic system described by equations 3.21 to 3.24 is analysed. The first step is to linearise the flow equations, which gives the results in equations 3.38 and 3.39. The Δ -symbol indicates a change in the state.

$$\Delta q_l = \frac{\partial q_l}{\partial T_{tb}} \Delta T_{tb} + \frac{\partial q_l}{\partial p_s} \Delta p_s + \frac{\partial q_l}{\partial p_l} \Delta p_l \quad (3.38)$$

$$\Delta q_s = \frac{\partial q_s}{\partial T_{tb}} \Delta T_{tb} + \frac{\partial q_s}{\partial p_s} \Delta p_s + \frac{\partial q_s}{\partial p_l} \Delta p_l \quad (3.39)$$

The partial derivatives are defined by equations 3.40 to 3.45, where index 0 indicates the working point for which the equation is linearised.

$$\frac{\partial q_s}{\partial T_{tb}} = C_q w_1 \sqrt{\frac{p_{s0} - p_{l0}}{\rho}} + C_q w_2 \sqrt{\frac{p_{s0} + p_{l0}}{\rho}} = K_{q2} \quad (3.40)$$

$$\frac{\partial q_s}{\partial p_s} = \frac{C_q A_1}{2\sqrt{\frac{p_{s0} - p_{l0}}{\rho}} \rho} + \frac{C_q A_2}{2\sqrt{\frac{p_{s0} + p_{l0}}{\rho}} \rho} = K_{c1} \quad (3.41)$$

$$\frac{\partial q_s}{\partial p_l} = -\frac{C_q A_1}{2\sqrt{\frac{p_{s0} - p_{l0}}{\rho}} \rho} + \frac{C_q A_2}{2\sqrt{\frac{p_{s0} + p_{l0}}{\rho}} \rho} = K_{c2} \quad (3.42)$$

$$\frac{\partial q_l}{\partial T_{tb}} = C_q w_1 \sqrt{\frac{p_{s0} - p_{l0}}{\rho}} - C_q w_2 \sqrt{\frac{p_{s0} + p_{l0}}{\rho}} = K_{q1} \quad (3.43)$$

$$\frac{\partial q_l}{\partial p_s} = \frac{C_q A_1}{2\sqrt{\frac{p_{s0} - p_{l0}}{\rho}} \rho} - \frac{C_q A_2}{2\sqrt{\frac{p_{s0} + p_{l0}}{\rho}} \rho} = -K_{c2} \quad (3.44)$$

$$\frac{\partial q_l}{\partial p_l} = -\frac{C_q A_1}{2\sqrt{\frac{p_{s0} - p_{l0}}{\rho}} \rho} - \frac{C_q A_2}{2\sqrt{\frac{p_{s0} + p_{l0}}{\rho}} \rho} = -K_{c1} \quad (3.45)$$

w is the area gradient and is defined according to equation 3.46.

$$w = \frac{\partial A(T_{tb})}{\partial T_{tb}} \quad (3.46)$$

The linearised system equations are transformed into the Laplace domain

and are defined by equations 3.47 to 3.50, where s is the Laplace operator.

$$\Delta q_l = K_{q1}\Delta T_{tb} - K_{c2}\Delta p_s - K_{c1}\Delta p_l \quad (3.47)$$

$$\Delta q_s = K_{q2}\Delta T_{tb} + K_{c1}\Delta p_s + K_{c2}\Delta p_l \quad (3.48)$$

$$\Delta q_p - \Delta q_s = \frac{V_s}{\beta} \Delta p_s s \quad (3.49)$$

$$\Delta q_l = A_p \Delta x_{rw} s + \frac{V_0}{\beta} \Delta p_l s \quad (3.50)$$

From the linearised system equations the transfer function of load pressure p_L can be derived from an input in torsion bar torque. The transfer function is defined in equation 3.51.

$$\begin{aligned} \Delta p_L &= \frac{K_{c1}}{K_{c1}^2 - K_{c2}^2} \frac{1 + \frac{V_s}{K_{c1}\beta} s}{\frac{s^2}{\omega_0^2} + \frac{2\delta_0}{\omega_0} s + 1} \\ &\quad \left(\frac{1 + \frac{\frac{V_s}{K_{c1}\beta} s}{\left(K_{c1} + \frac{K_{q2}K_{c2}}{K_{q1}}\right)\beta}}{1 + \frac{V_s}{K_{c1}\beta} s} K_{q1} \left(1 + \frac{K_{q2}K_{c2}}{K_{q1}K_{c1}} \right) \Delta T_{tb} - A_p \Delta x_{rw} s \right) \\ &\approx \frac{G_{K_c}(s)}{K_c} (G_{K_q}(s) K_q \Delta T_{tb} - A_p \Delta x_{rw} s) \end{aligned} \quad (3.51)$$

where

$$\begin{aligned} \omega_0 &= \sqrt{\frac{K_{c1}^2 - K_{c2}^2}{V_0 V_s}} \beta \\ \delta_0 &= \frac{1}{2} \frac{K_{c1}}{\sqrt{K_{c1}^2 - K_{c2}^2}} \frac{V_0 + V_s}{\sqrt{V_0 V_s}} \\ K_c &= \frac{K_{c1}^2 - K_{c2}^2}{K_{c1}} \\ K_q &= K_{q1} \left(1 + \frac{K_{q2}K_{c2}}{K_{q1}K_{c1}} \right) \end{aligned}$$

At steady-state, the transfer function describes the static Boost curve, which in linear form is defined by equation 3.52.

$$\Delta p_L = \frac{K_q}{K_c} \Delta T_{tb} - \frac{A_p}{K_c} \Delta x_{rw} s = K_{boost} \Delta T_{tb} - \frac{A_p}{K_c} \Delta x_{rw} s \quad (3.52)$$

The frequency of the transfer function is plotted in figure 3.18. The frequency response depends on the working point and the figure shows the response at zero load flow for torque levels 0 Nm, 1 Nm, 2 Nm, 3 Nm and 3.5 Nm. The response is very similar to a first order system with a maximum phase shift of 90°. What is interesting to note is how the response changes with higher

torque, starting at around 150 Hz at 0 Nm and becoming as slow as approx. 5 Hz at 3.5 Nm. This will have an impact on the closed loop system of the complete steering system, as will be seen in the analysis in the next chapter, and can also be useful information when designing new control loops.

Figure 3.18 Frequency response from a torsion bar torque input to a load pressure at zero load flow for different torque levels. The upper plot shows the amplitude curve and the lower plot the phase curve.

3.4.2 HPAS system analysis

The hydraulic system makes up part of the complete steering system. By excluding static and pressure-dependent friction, the mechanical subsystem is already linear. By transforming equations 3.1 and 3.2 into the Laplace domain and with the hydraulic subsystem transfer function, the complete system in linear form is described by equations 3.53 to 3.56.

$$J_{sw}\Delta\theta_{sw}s^2 = \Delta T_d - K_T(\Delta\theta_{sw} - R_T\Delta x_{rw}) - b_{rw}\Delta\theta_{sw}s \quad (3.53)$$

$$M_{rw}\Delta x_{rw}s^2 = \Delta p_L A_p + K_T(\Delta\theta_{sw} - R_T\Delta x_{rw}) R_T - b_{rw}\Delta x_{rw}s \quad (3.54)$$

$$\Delta p_L = \frac{G_{K_c}(s)}{K_c} (G_{K_q}(s) K_q \Delta T_{tb} - A_p \Delta x_{rw}s) \quad (3.55)$$

$$\Delta T_{tb} = K_t(\Delta\theta_{sw} - R_t\Delta x_{rw}) \quad (3.56)$$

From the linearised system equations a block diagram can be derived, shown in figure 3.19.

Figure 3.19 Block diagram of the hydraulic power steering system.

The block diagram clearly shows that the steering system can be treated as a position control loop. The driver applies a torque which through the inertia of the steering wheel and column generates an angle at the steering wheel. There is a torque from the rack through the steering gear. The steering wheel angle serves as input, or reference, to the control loop. The output is the rack position. The corresponding rack position is compared to the steering wheel angle and the error constitutes the twisting of the torsion bar. The twisting of the torsion bar is the input to the valve and activates it. The valve controls the pressure that pushes on the piston of the assistance cylinder and together with the manual torque applied by the driver through the steering gear accelerates the rack. Hence, there are two control loops: the outer position control loop and the inner pressure control loop. The pressure control loop for the hydraulic power assisted steering system is an open control loop without any feedback. From the block diagram an expression for the closed loop transfer function from a steering wheel angle to a rack position can be derived according to equation 3.57. The valve controls the gain and increases the damping. The spring stiffness is also increased.

$$\frac{R_t \Delta x_{rw}}{\Delta \theta_{sw}} = \frac{K_T R_T^2 + G_{K_c} G_{K_q} A_p K_T K_q R_T / K_c}{M_{rw} s^2 + \left(b_{rw} + \frac{A_p^2}{K_c} G_{K_c} \right) s + K_T R_T^2 + \frac{G_{K_c} G_{K_q} A_p K_q K_T R_T}{K_c}} \quad (3.57)$$

In order to study stability the loop gain transfer function $A_g(s)$ can also be derived from the block diagram according to equation 3.58.

$$A_g(s) = \frac{R_t K_t K_q G_{K_q} G_{K_c} A_p G_{rw}}{K_c + A_p^2 G_{K_c} G_{rw} s} \quad (3.58)$$

$$G_{rw} = \frac{1}{M_{rw} s^2 + b_{rw} s + K_T R_T^2}$$

The frequency response of the loop gain at zero load flow is shown in figure 3.20 and the closed loop frequency response in figure 3.21.

Figure 3.20 Frequency response of the loop gain $A_g(s)$. The upper plot shows the amplitude curve and the lower plot the phase curve.

Figure 3.21 Frequency response of the closed loop, from a steering wheel angle input to a corresponding rack displacement. The upper plot shows the amplitude curve and the lower plot the phase curve.

3.5 Discussion

The simulation model of the hydraulic power steering system has been shown to be accurate within the working region of interest. With the help of the test rig, the different components of the system could be evaluated independently of each other. This has been valuable for the simulation model.

From the analysis it is clear seen that the purpose of the hydraulic system is to control and generate a certain amount of pressure in order to assist the displacement of the steering rack. As explained previously, this is done by changing the control edges of the valve and the characteristic of the hydraulic system is therefore set by the geometry of the valve. The complete steering system is a positioning follower. This is easily seen from the block diagram in figure 3.19, where the reference input is the steering wheel angle applied by the driver and the output signal is the rack displacement. The steering system therefore constitutes a closed control loop that must be designed for stability. The gain of this control loop is set by the hydraulic system, which in practice is the Boost curve. This is a critical point of the steering system. While the torque level increases, so does the boost pressure to assist the driver. While the boost pressure increases the gain of the control loop decreases, together with the stability margins. This is most critical during exiting a corner when the steering rack is pushed back by the external forces on the rack. The boost

pressure varies with load flow and is at its highest during this scenario. When the rack is pushed back, it will compress the chamber of the assistance cylinder with higher pressure and momentarily increase the pressure, and thereby the gain. If the stability becomes too small oscillations can occur in the system, causing small vibrations called chattering. This is analysed in detail by M. Rösth and J-O. Palmberg (2007). The positioning control can also be seen as a proportional controller with the gain set by the Boost curve.

The hydraulic system also contributes a dynamic, affecting the outer control loop. The bode plot of the pressure dynamic in figure 3.18 illustrates this. What is interesting is that for higher torque level, i.e. for a higher boost gain, the response of the pressure dramatically decreases. This has the effect of "pushing down" the amplitude curve of the loop gain as the torque increases, giving back some of the stability margin lost with the higher gain. This is best explained by studying figure 3.22, which shows the same loop gain frequency response as in figure 3.20 but only for the highest torque level. The dashed curve represents the loop gain with a fixed response, the same as for the lowest torque level where the response is at its fastest. The solid curve is the normal curve. It can be seen that the amplitude margin is higher with a slower response. However, this comes at the cost of a lower phase margin. There is a compromise between the two. How this information can be utilised is studied in detail in chapter 6, section 6.2.

Figure 3.22 Frequency response of the loop gain at highest torque level and zero load flow. The dashed curve is for a response with fixed dynamic. The solid curve is with a variable dynamic.

In addition to controlling the gain of the positioning control loop, the valve

also contributes a great deal of damping. The damping can be seen as part of the Boost curve, i.e. there is a gain and a damping coefficient, or be seen as a part of the plant, which is the case here. The damping is important in terms of steering feel but also affects the control from a stability point of view. Like the gain the damping is not controllable but is set by the valve's geometry.

Next chapter will look into how the system can be modified in order to realise active steering.

4

Active steering

Active steering is referred to as the possibility to control the output torque required to turn the wheels or manipulate the steering angle introduced by the driver with an electronic signal. The need for active steering comes from an increased attention to safer vehicles. Steering-related active safety is becoming more common and several solutions can already be found on the market. Another aspect is the possibility to increase the comfort and experience of the driver through the steering system and enhance the steering feel. A third aspect is autonomous driving. There are several safety, economic and environmental benefits that can be realised with autonomous driving and it is only possible if active steering is in place, see e.g. Tai, Hingwe, and Tomizuka (2004). This chapter gives a brief overview of different solutions to increase safety and existing solutions to how active steering is realised on the hardware side.

4.1 Active safety and increased comfort

There is active safety and passive safety. Typical passive safety systems are the seat belt and airbag, while active safety systems that rely on some kind of control and usually requires sensors and actuators. Typical active safety functions are the Anti-lock Brake System, Electronic Stability Program, ESP, and Traction Control. These systems use the brakes and engine torque control to increase safety. An additional way is to incorporate the steering system as well.

Several studies on yaw disturbance attenuation and roll-over avoidance using active steering have been made by J. Ackermann, Bunte, and D. Odenthal (1999), J. Ackermann and Bunte (1997), Jürgen Ackermann (1998), Jürgen Ackermann and Dirk Odenthal (1999). Yaw attenuation refers to when the yaw rate is no longer observable from the lateral acceleration of the front axle. The driver applies a desired yaw rate and the system measures the actual yaw rate and rejects any disturbances. A disturbance could come from a cross-

wind or μ -split, i.e. different grip on different tyres. The authors mention that active steering is more efficient than using the braking system for the same purpose but requires an additional actuator. These studies make use of an angle overlay system, which is described in the next section. Experiments were also conducted to show the benefit of the system. A roll-over prevention system for vehicles with an elevated centre of gravity was also developed, where the authors claim that active steering is more efficient than active damping. A PD-controller of roll rate and roll acceleration has been developed, where the controller gains are scheduled with velocity and height of centre of gravity.

The active steering system developed by BMW and ZF, presented by Koehn and Eckrich (2004), is also described in the next section. However, several safety and comfort functions can be applied with the system, such as variable steering ratio. Here the steering angle is increased compared to the driver's angle at low speed, e.g. during parking, in order to reduce the steering action. At high speed, the driver's steering angle is instead decreased to help stabilize the vehicle. The actuator can also be used to stabilize the vehicle during oversteering, like ESP. Using the steering system is more efficient since a faster reaction of the vehicle is achieved compared to using the braking system.

The research project by Eidehall et al. (2007) resulted in a method to evaluate active safety functions as well as a new function, Emergency Lane Assist, ELA. The problem mentioned with lane keeping assist functions is to separate intended lane changes and dangerous situations. The ELA function therefore only intervenes when there is a dangerous lane change and prevents the lane change by adding a torque to the steering wheel angle, i.e. a torque overlay system as described in the next section. Information to the system comes from a sensor and vision system that provides the distance to the lane markers, the angle of the host vehicle in relation to the lane, the road curvature and the relative position in relation to observed vehicles.

A study of an emergency manoeuvre of an articulated vehicle was conducted by McCann and Nguyen (2004). An angle overlay system is assumed to be available to the truck, referred to as Active Front Steering. The benefit of such a system is studied for an evasive manoeuvre where both braking and steering are applied. The AFS is used to stabilize the truck and trailer and avoid jack-knifing. Different metrics were developed to quantify the performance of the AFS interventions. The results show the trade-off between stabilizing the vehicle and the driver's perception of vehicle control and that there exists an optimal range. McCann and Le (2008) also investigated the use of gain scheduled assist torque in trucks with an Electrohydraulic Power Steering system, i.e. the hydraulic power steering system with electric motor driven pump. The assist torque is varied by controlling the pump speed and is linearly proportional to the rate of change of the articulation angle. The results show that the gain scheduling opposes the driver's tendency to overshoot the steering angle and helps stabilize the vehicle without interfering with the driver's perception of the vehicle's handling characteristics, since the lateral position change and

wheel steering angle are similar to the case without gain scheduling.

A collision avoidance system is developed and evaluated by Schorn et al. (2006). A passenger car is equipped with active front steering. Obstacles are detected using a vision system and a desired trajectory is calculated to avoid a collision. Two methods for trajectory control were evaluated, both with successful steering of the vehicle. A steering assistance system is developed by Enache et al. (2009) with the intention to assist the driver in steering the vehicle to the centre of the lane during situations where the driver lacks attention. An Electric Power Steering System is used, where the steering torque is controllable. A switching strategy is used to switch to the assistance steering during the driver's inattentions.

Active steering can also be combined with active braking for yaw stabilisation, as studied by Tjønnås and Johansen (2010). Five actuators are considered: braking at each wheel and steering angle correction. Several modules at different levels handle the vehicle motion control objective, braking control, and steering angle corrections. The design is such that the actual torque about the yaw axis tends to the desired calculated value by satisfying actuator constraints and minimal control effort. An active pedestrian safety system developed by Keller et al. (2011) also uses the brakes and steering to avoid collisions. The system combines sensing, situations analysis, decision making and vehicle control. The decision-making prioritises collision avoidance by first warning the driver. If the driver does not do what is necessary, full braking will be applied. If collision is not to be avoided, evasive steering will take place. This is done by applying a torque at the steering wheel with an additional actuator. During evasive steering, the motion controller considers the nonlinear dynamics of the vehicle during such a scenario.

With active steering it is also possible to tune the steering feel. One way of doing this is described by Birk (2010). The purpose of the described system is to suppress disturbances, such as side-wind or uneven road, and provide a good steering feel based on what the driver's intentions are. The benefit is also that the steering feel remains the same regardless of driving situation, vehicle class, vehicle velocity, front axle load, tyre choice, etc.

4.2 Realising active steering

As mentioned earlier, active steering is realised either by alternating the steer angle applied by the driver or in some way alternating the torque applied by the driver, see illustration in figure 4.1. The two approaches do not exclude each other. In fact, a combination can provide for full steer-by-wire functionality, which is explained by S. Müller, A. Kugi, and W. Kemmetmüller (2005). Steer-by-wire systems decouple the steering wheel and the road wheels by removing the mechanical connection and the actuation of the road wheels can be performed using electrical or hydraulic power. This kind of system is not considered in this work. A superposition of the steering angle is mainly used

Figure 4.1 Schematic principle of active steering. For superpositioning of the steering angle, the DC motor adds or subtracts an angle to the steering wheel angle θ_{sw} through the gear box, i.e. $\theta_{pinion} = \theta_{sw} + \theta_{DC}$. For torque overlay, the driver's torque T_d can be altered by the DC motor, i.e. $T_{pinion} = T_d + T_{DC}$.

to alter the steering ratio or for stabilisation of the vehicle, as described above. In order to change the level of assist and the steering feel in that sense, a superposition of the torque is necessary.

4.2.1 Superposition of steering angle

Together with BMW AG, ZF Lenksysteme GmbH has developed a superposition steering system which was first installed on the BMW 5 series. The system, as described by Koehn and Eckrich (2004), uses a planetary gear inserted between the steering wheel and steering gear. The planetary gear has external tothing and can be rotated by an additional actuator, in this case a brushless DC motor, through a worm gear. When the DC motor is deactivated, the steering system works like a conventional system. When the DC motor is activated, the steering angle from the driver and the steering angle from the DC motor are superimposed through the planetary gear. This system could be installed together with any power amplification system, such as hydraulic or electric. If a hydraulic power steering system was used, the superposition can generate a higher rack velocity compared to a conventional system and the maximum required flow is therefore higher. The system is illustrated in figure 4.2.

The solution by *Audi Dynamic Steering* (2011) is based on the same principle but instead uses a so-called harmonic drive with an electric motor to change the gear ratio between the steering wheel and the road wheels. The harmonic drive is very stiff, free of play, and very efficient. *Audi Dynamic Steering* (2011)

Figure 4.2 Schematic principle of active steering by BMW/ZF. The DC motors adds to or subtracts from the steering wheel angle through a planetary gearbox.

have integrated their gear in the steering column rather than at the steering gear, which was the case for Koehn and Eckrich (2004). A similar solution to that of *Audi Dynamic Steering* (2011) is also adopted by *Lexus Variable Gear Ratio Steering, VGRS* (n.d.) for their Variable Gear Ratio Steering, VGRS.

Similar solutions to that of Koehn and Eckrich (2004) can be found, e.g. by Bohm (2007) or by Rothhämel (2011), where the latter is adopted to heavy vehicles. The main difference between heavy vehicles and passenger cars is that the former can hardly be steered, or for some configurations not at all, without an assistance system. The solution today is to use an extra pump that will provide with hydraulic power if the primary unit fails. This is expensive and the solution by Rothhämel (2011) is to use superposition of the steering angle. The mechanical configuration can be designed with a higher gear ratio in order to facilitate manual steering in case of hydraulic loss. During normal driving, the superposition angle system corrects the gear ratio to the right one.

4.2.2 Superposition of steering torque

In the passenger car industry the most common approach to power assisted steering is a pure electrical system, the Electric Power Assisted Steering, EPAS, which is described by for instance Badawy, Zuraski, et al. (1999). An electric motor is attached either to the column, the steering gear, or the rack and assists the driver by adding a torque to the driver's torque. Since the electric motor is controlled from an ECU, active steering is already realised. For heavier applications, such as commercial vehicles, the load experienced by the steering system is too high and an EPAS system would stress the electrical system of the vehicle too much. A study by Roos (2005) shows this and a pure electrical

system is not interesting until a 600 V supply system is a reality, which can be found in for example hybrid buses. Morton, Pickert, and Armstrong (2014) studied such a system where the self-aligning torque is recovered through the steering system

This work assumes that a hydraulic system is required to deliver the amount of assistive force required to turn the wheels. There are different principles for how to change the torque experienced by the driver. One is to focus solely on the relation between the driver's torque and the level of assist. Another would be to actively regulate the pressure difference across the assistance cylinder piston.

From the model of the open-centre valve in section 3.1.2, it was seen that the assistance pressure depends on the flow from the pump and the opening areas of the valve. If one or the other can be altered by an electric signal, the pressure could be actively controlled. The Electrohydraulic Power Assisted Steering system, EHPAS, which is presented by for example Forbes, Baird, and Weisgerber (1987), Gessat (2007) and Badawy, Fehlings, et al. (2004), was primarily designed to overcome the poor energy efficiency of the HPAS system. Instead of driving the pump from the combustion engine directly, an electric motor is used to drive the pump. In this way the flow can be regulated and during no steering action the flow can be shut off or heavily reduced. Compared to the conventional system, fuel savings of up to 0.2 L per 100 km have been reported. With this possibility the level of assist can also be changed during different scenarios. If a fixed pump driven from the combustion engine is used instead, the flow to the valve can be controlled by using an electronically controlled valve prior to the open-centre valve that by-passes some of the oil to tank. Such a system is described by Duffy (1988).

The variation in pump flow is restricted to only affecting the level of assist without affecting the internal difference between the two cylinder chambers. Autonomous driving is not possible with such a system. One way to control the pressure difference across the piston is to actuate the open-centre valve with an electronic actuator. An example of this is the Active Pinion presented by M. Rösth, J. Pohl, and J-O. Palmberg (2003), illustrated in figure 4.3. With this concept an offset in valve displacement can be created that generates an offset in pressure difference across the assistance cylinder; a torque can thus be added to or subtracted from the driver's torque. The displacement of the valve is realised by a DC motor attached to an excenter, generating a linear motion. This linear motion is transferred to a rotational motion by an inclined groove in the valve body. The working envelope of the system is restricted to the offset angle of the valve body that can be created. A benefit with such a system is the low force required to actuate the valve and a relatively small DC motor can be used.

Several other similar solutions exists that aim to create an offset displacement in the valve body. One is presented by Leutner (2005), where a stepper motor is placed coaxially with the valve. The stator is connected to the pin-

Figure 4.3 Schematic of the Active Pinion concept developed by M. Rösth, J. Pohl, and J-O. Palmberg (2003). The DC motor generates a displacement of the valve, θ_{AP} , which shifts the boost curve.

ion and the rotor to the valve body. Another variant is the iHSA, intelligent Hydraulic Steering Assist, developed by tedrive steering, see *iHSA, intelligent Hydraulic Steering Assist* (n.d.). This solution is available for both rack and pinion systems and recirculating ball screw and displaces the valve with an electric motor and a planetary gear set.

The steering torque can also be modified by adding or subtracting a torque directly with an electromagnetic actuator. The Magnasteer presented by Pawlak, Graber, and Eckhardt (1994) uses an electromagnetic actuator consisting of a homopolar rotary actuator with a multipole ring permanent magnet that acts directly on the torsion bar.

For heavy vehicles an electromechanical actuator can be utilised together with the hydraulic system. The torque from the driver and from the electric motor are added at the steering column, before the steering gear. Since the hydraulic system is still in place, the electric motor does not have to provide the full assistance torque. A solution by ZF Lenksysteme is presented by Wiesel et al. (2009) and is referred to as hybrid steering, see the illustration in figure 4.4. The electric motor can add or subtract torque on the driver's torque. There is also a possible fuel saving by additionally controlling the pump-delivered flow. When low torque levels are required, the assistance from the electric motor alone is sufficient to turn the wheels and the pump flow delivered to the valve can be reduced or even shut off. When the load on the steering system is higher,

Figure 4.4 Schematic of the Hybrid steering, see Wiesel et al. (2009). The electric motor works in parallel to the hydraulic system and can add or subtract a torque to the driver's torque.

for example during parking manoeuvres, the pump flow can be increased and the hydraulic system will also provide the level of assistance torque required to turn the wheels. TRW also has a similar solution, the ColumnDrive, presented by Williams (2009). The electric motor on the steering column works to generate a synthetic torque feedback experienced by the driver while the hydraulic system generates the necessary amount of assistance torque. In this way, the steering feel can be improved on, such as a variable stiffness depending on the driving scenario, no hysteresis and better returnability. The ColumnDrive was also investigated together with a closed-centre hydraulic system by Yu and Szabela (2007). The valve is a rotational valve like the conventional system but in neutral position the flow paths to tank are closed. The valve is designed with a small overlap to reduce leakage but also to ensure a stable system. With the closed-centre valve the system can be supplied by an electric motor driven pump connected to an accumulator and the energy consumption can be reduced. The overlap of the valve has a negative effect on the steering feel generating what is called a "bump" steer. This is improved on by tuning the steering feel with the ColumnDrive. Volvo Trucks also uses a similar system to generate a superimposed steering torque through the hydraulic system, see *Volvo Dynamic Steering* (2013). Here the electric motor is attached directly on the hydraulic steering gear and the system is used to improve both the steering feel and the handling of the vehicle.

A completely different approach to varying the level of assist is to replace the conventional hydraulic system with a system that uses electronically controlled valves. Any pressure can then be set in the assistance cylinder based on any input of choice. If the valve also has a closed centre the supply system can be of a flow-regulated pump of some kind connected to an accumulator. One possible solution is studied by S. Müller, A. Kugi, and W. Kemmetmüller (2005), where a directional 4/3-valve is used. The energy saving potential is high with such a system where the closed-centre system only used 18% of that of the open-centre system for the given driving cycle. Since a directional valve is intended to control the flow, the system requires an active closed loop control of the pressure. With regard to the steering feel, there is a trade-off between a low leakage and a good feel. To control the pressure fast enough, an offset voltage to the valve is required but this increases the leakage flow and reduces the energy efficiency.

Another possibility is to use valves intended to control the pressure instead, such as pressure-reducing or pressure-relief valves, or a combination of both, that independently control the pressure in each cylinder chamber. Several similar solutions can be found. Wenzel, Stobbs, and Ward (1996) proposed a system with two 3-way pressure reducing valves, one for each cylinder chamber. A working pressure is controlled by a pressure control valve depending on the driving scenario. The 3-way valves then reduces the pressure to each chamber to generate the right amount of assist. Another approach presented by Pluschke and Stehli (2001) is to use similar 3-way valves to control the pressure in the assistance cylinder but in combination with a supply system consisting of pump with accumulator. The same system is also studied by Wolfgang Kemmetmüller, Steffen Müller, and Andreas Kugi (2007). A nonlinear controller is developed to control the pressure in the assistance cylinder with the desired performance and impedance matching is used to generate the right amount of assistance. From a failsafe perspective such valve will open to tank in case of loss of electricity. The vehicle will therefore always be steerable through the mechanical connection. Since the pressure is independently controlled in each chamber, the same pressure could also be set in both chambers, creating a hydraulic lock, for safety reasons. A variant of the same system is proposed by Mesiti and Avenatti (1995). One electronically controlled pressure control valve is used together with a 4/3-directional valve. The pressure control valve controls the required pressure to generate the assistance torque while the directional valve, that is mechanically connected to the steering column, connects the pressure control valve to one or the other chamber of the assistance cylinder depending on the direction of the steering movement.

Figure 4.5 Schematic principle of closed-centre power steering with pressure control valves.

4.3 Selected technology for active steering

The choice of technology in this work is the use of independently controlled pressure control valves as depicted in figure 4.5. There are several reasons why such a system is beneficial:

- The purpose of the hydraulic system in the power steering application is to control the pressure. It is therefore logical to use valves that are designed to control the pressure and the system can keep its fundamental structure.
- The valves are of closed-centre type. It has already been explained that a closed-centre system has great potential in reducing energy consumption. However, it requires a different structure of the supply system.
- Hydraulic systems are known for a high power density compared to electric systems. The valves are compact and provide a space-efficient solution.
- The actuation power of the valve is low. If a constant torque is to be applied by the steering system, e.g. during steady-state cornering, little energy is used to achieve this compared to an electric motor.
- Hydraulic power is today still necessary in order to overcome the high axle loads present in heavy vehicles. Since the hydraulic system is already in place, by using pressure control valves there is no need for additional technology.

- Pressure control valves already exist on the market for several different applications. If it were possible to use an off-the-shelf valve, the solution could also be cost efficient.

The next chapter will give a deeper analysis of the pressure control valve.

5

Pressure control

Based on the analysis of the HPAS system in chapter 3 the main purpose of the hydraulic system is to provide a pressure to assist the driver when turning the wheels. The open-centre valve controls the pressure through its metering edges. The valve is actuated by the driver when the steering wheel is turned, which results in a twisting of the torsion bar and a distortion of the metering edges of the valve. Pressure can be controlled in different ways by letting either a pump with variable displacement or a fixed pump with variable speed control the pressure or by using a valve. Generally, a valve is both faster and cheaper compared to a pump. One way is to use a kind of directional valve, e.g. a proportional valve or a servo valve. These mainly control flow by displacing the spool and to control the pressure they need to be actively controlled. Another way is to let the valve sense the pressure to control it directly. This can be solved hydro-mechanically and such a valve is called a pressure control valve and is considered in this work. Commonly of closed-centre type, it removes any additional flow paths to tank and the required flow can be greatly reduced. A valve of such a type that is also actuated by an electro-magnetic mechanism, a solenoid, can also be controlled from a computer.

A pressure control valve can be either a pressure relief valve or a pressure reducing valve. The relief valve is one of the most common components in every hydraulic systems, where it secures the maximum pressure. It can also be used to set a certain pressure level in a system. The valve is typically a seat valve with the schematics shown in figure 5.1. The controlled pressure acts on the seat whereas at the other end a spring is exerting a force, keeping the seat in balance. If the pressure is larger than the corresponding spring force, the valve will open to tank and relieve the pressure. The pretension of the spring corresponds to the opening pressure, or reference pressure, of the valve. In order to control the pressure, the pressure must be higher than the reference value and a flow will pass the seat.

A pressure-reducing valve reduces the system pressure from a higher to a lower value and is designed to maintain a certain pressure regardless of varia-

Figure 5.1 *Schematic of a pressure relief valve.*

tions in the supply pressure as long as the supply pressure is higher than the intended system pressure. This valve is usually of spool type, shown schematically in figure 5.2. Here again a spring exerts a force on the valve, keeping the valve in a normally opened position. For both types of valve, the static characteristics are constant in the ideal case, i.e. the pressure is the same for a given reference value regardless of the flow through the valve. In reality the pressure can change significantly with the flow. This has to do with both the mechanical spring and the flow forces acting on the seat or the spool.

Figure 5.2 *Schematic of a pressure reducing valve.*

A pressure valve can be of either a single-stage or a two-stage type. In the former case, the spool is controlled directly by the spring force. In the latter case, the pilot flow is needed to create a pilot pressure that acts on the main stage. The two-stage valve has much better static characteristics and can handle much higher flow rates than the single-stage valve. However, it is more complicated in its design. A pressure valve that is actuated by a solenoid is said to be proportional. For the single-stage valve the solenoid acts directly on the main spool. A current through the solenoid generates a force that is proportional to the current. The force corresponds to the reference pressure of the valve, hence the name proportional pressure control valve. A proportional solenoid can only provide moderate forces and becomes a limiting factor in the design of the valve. In the two-stage case, the solenoid acts on the pilot-stage

instead.

In the context of this work, a pressure control valve is a valve with both relieving and reducing capabilities. It can also be referred to as a reducing valve with relieving capability. Since the single-stage valve is simpler in its design, and therefore cheaper, it is the type of valve considered in the following analysis.

This chapter provides the modelling and an in-depth analysis of a single-stage pressure control valve. The chapter is divided into two parts. The first part gives a general analysis of the hydro-mechanical part of the valve, both from a static and a dynamic viewpoint. As it turns out, the hydro-mechanical subsystem constitutes a closed-loop control system. Both the nonlinear model and a linear model are presented. The second part of the chapter applies the theory on a commercially available valve. This valve is a proportional electronically controlled valve. The modelling and analysis must therefore also consider the solenoid. The theory is also extended in order to apply to the valve under consideration.

5.1 General modelling and analysis

Figure 5.3 Schematic of a pressure control valve.

The pressure control valve, illustrated in the simplified schematic in figure 5.3, works both as a reducing and a relieving valve. Since a single-stage valve is considered, the valve is actuated by a solenoid force acting directly on the spool. A spring force with a certain pretension works in the opposite direction. The load pressure p_l is also sensed by the valve against a defined area A_c which acts in the same direction as the spring. The solenoid force corresponds to the reference pressure. If the load flow q_l is zero, the valve will be in equilibrium when the corresponding force from the load pressure is equal to the solenoid force. If the load pressure is lower than the corresponding solenoid force, the valve will open up to the pump to fill the load volume with oil. If the pressure is higher than the force, the valve will open up to tank to relieve the pressure. The valve is then said to be self-regulating. This will also be clear from the linear

analysis presented below. If a load flow is present in either direction, the valve will find an equilibrium in an offset position to the closed one. The spring force will be different in this case due to the compression of the spring and since the valve is also subjected to flow-dependent flow forces the load pressure will have to change as well in order to maintain equilibrium. This change in pressure with flow is defined by the static characteristic of the valve. In many cases a small orifice is placed between the load volume and the small volume in which the load pressure is sensed, in this case called a damping chamber. This is to increase damping, which is further explained below. The modelling is divided into two parts. First, the nonlinear equations describing the valve's behaviour outlined. From these the static characteristics of the valve is defined. The equations are then linearised for an analysis in the frequency domain, which reveals some important behaviour of the valve.

The modelling of pressure control valves has been treated by several researchers. Merritt (1967) provides modelling and an analysis of single-stage pressure relief and reducing valves, with a focus on the dynamic response. A similar analysis of a pressure relief valve in both the frequency and time domain was made by Jan-Ove Palmberg (1983). A pilot-controlled solenoid pressure relief valve is modelled and analysed using the bondgraph technique by Dasgupta and Watton (2005). With a simplified model of the solenoid, the paper presents a good model of the valve. A direct-acting solenoid valve for clutch control is modelled and analysed by Walker, Zhu, and Zhang (2014). A nonlinear detailed model, where the solenoid characteristic is mapped, is presented. The model agrees well with both step inputs and steady state. A general linearised analysis of a pressure relief valve is presented in Jan-Ove Palmberg, Andersson, and Malmros (1983). It is shown how most valves can be characterised by a dominating break frequency and compliance and the system being connected to the valve by the hydraulic capacitance and the flow-pressure coefficient. This was also verified by experiments. A design process for a proportional pressure relief valve is proposed by Erhard, Weber, and Schoppel (2013). An inverse simulation technique is used to derive a design for both the hydraulic subsystem and the solenoid characteristics. The valve is simulated using both CFD for the hydraulic subsystem and the Finite Element Method for the solenoid.

5.1.1 Nonlinear model of the hydro-mechanical subsystem

Equation of motion

The movement of the spool is dictated by the equation of motion in equation 5.1. The forces that act on the spool are the solenoid force F_s , the pressure in the damping chamber p_c , which is the same as the static pressure in the static case, the spring force, the viscous friction, the pretension f_0 and the flow force F_f . The PWM controlled solenoid usually has superimposed dither frequency, which results in micromovements of the spool. This eliminates the Coulomb

friction to such an extent that it can be excluded in the model.

$$m_v \ddot{x}_v = F_s - p_c A_c - f_0 - K x_v - b_v v_v - F_f(x_v, p_s, p_l) \quad (5.1)$$

The reference pressure is set by the solenoid force and has the relation shown in equation 5.2.

$$F_s = p_{ref} A_c + f_0 \quad (5.2)$$

Flow equations

The valve has two main flow paths: from pump to load volume, q_{vs} , and from load volume to tank, q_{vt} . A small part of the main flow will go to the damping chamber. The flows are described by equations 5.3 and 5.4.

$$q_{vs} = C_q A_s(x_v) \sqrt{\frac{2}{\rho}(p_s - p_l)} \quad (5.3)$$

$$q_{vt} = C_q A_t(-x_v) \sqrt{\frac{2}{\rho} p_l} \quad (5.4)$$

Load volume

The pressure change in the load volume depends on the net flow into the volume and is described by the continuity equation in equation 5.5. q_l is the load flow.

$$q_{vs} - q_{vt} - q_c = q_l + \frac{V_l}{\beta} \dot{p}_l \quad (5.5)$$

Control chamber

The pressure in the control chamber depends on both the flow through the damping restrictor, equation 5.6, and the continuity equation for the volume, equation 5.7.

$$q_c = C_q A_{cr} \sqrt{\frac{2}{\rho}(p_l - p_c)} \quad (5.6)$$

$$q_c = -A_c v_v + \frac{V_c}{\beta} \dot{p}_c \quad (5.7)$$

Flow forces

The valve is subjected to flow forces that tends to close the valve. These are flow-dependent and exist for both main flows through the valve. The total flow force is expressed in equation 5.8.

$$F_f(x_v, p_s, p_l) = |2C_q A_s(x_v)(p_s - p_l) \cos(\delta)| - |2C_q A_t(-x_v) p_l \cos(\delta)| \quad (5.8)$$

5.1.2 Static characteristic

The static characteristic of the valve defines how the pressure will change for a given flow and, as said before, is an effect of the spring compression and

flow forces acting on the spool. It can also be interpreted as how accurately the valve is able to control the pressure. However, as will be seen from the dynamic analysis later on, there is a compromise between control accuracy and the stability of the valve. The valve is first analysed when it is open to pump and later when it is open to tank. The equations describing the static behaviour of the valve are equations 5.9 and 5.10. In order to be able to solve for the pressure, the opening area is considered to be linear, i.e. $A = wx_v$, where w is the area gradient. In reality the opening area is most likely a nonlinear function of the spool displacement and should therefore be linearised in order to solve the equations.

$$q_v = C_q wx_v \sqrt{\frac{2}{\rho}(p_s - p_l)} \quad (5.9)$$

$$p_{ref} A_c - p_l A_c - K x_v - K_f (p_s - p_l) x_v = 0 \quad (5.10)$$

The static equations can be solved for the spool displacement, equation 5.11, and the flow, equation 5.12.

$$x_v = \frac{A_c(p_{ref} - p_l)}{K + K_f(p_s - p_l)} \quad (5.11)$$

$$q_v = \frac{K_0 A_c(p_{ref} - p_l)}{K_e} \sqrt{p_s - p_l} \quad (5.12)$$

where

$$K_0 = C_q w \sqrt{\frac{2}{\rho}}$$

$$K_f = 2C_q w \cos(\delta)$$

$$K_e = K + K_f(p_s - p_l)$$

The term K_e can be regarded as an equivalent spring constant. For a given pressure drop a larger spool displacement will result in a larger flow and thus a larger flow force. The static characteristic can be solved by differentiating equation 5.12.

$$\begin{aligned} \frac{\partial q_v}{\partial p_l} &= \frac{K_0 A_c}{K_e} \sqrt{p_s - p_l} \left(-1 - \frac{p_{ref} - p_l}{2(p_s - p_l)} + \frac{K_f(p_{ref} - p_l)}{K_e} \right) \\ &\approx -\frac{K_0 A_c \sqrt{p_s - p_l}}{K_e} = -\frac{A_c K_q}{K_e} = -\frac{1}{R_v} \end{aligned} \quad (5.13)$$

$$\Rightarrow R_v = -\frac{K_e}{A_c K_q} \quad (5.14)$$

where $K_q = C_q w \sqrt{\frac{2}{\rho}(p_s - p_l)}$. The static characteristic is denoted R_v , equation 5.14, and corresponds to the incline of the pressure-flow curve. It is very

dependent on the operating point, such as spool displacement and pressure. In the case of a positive spool displacement, the static characteristic is negative meaning that a larger flow will reduce the load pressure, like a reducing valve.

The static characteristic of a negative spool displacement can be derived in the same way. The static equations for this case are expressed in 5.15 and 5.16.

$$q_v = C_q w x_v \sqrt{\frac{2}{\rho} p_l} \quad (5.15)$$

$$p_{ref} A_c - p_l A_c + K x_v + K_f p_l x_v = 0 \quad (5.16)$$

These can again be solved for the spool position, equation 5.17, and the flow, equation 5.18.

$$x_v = -\frac{A_c(p_{ref} - p_l)}{K + K_f p_l} \quad (5.17)$$

$$q_v = -\frac{K_0 A_c(p_{ref} - p_l)}{K_e} \sqrt{p_l} \quad (5.18)$$

where $K_e = K + K_f p_l$. The static characteristic is then derived by differentiating equation 5.18 and the result is expression 5.19.

$$R_v = \frac{K_e}{A_c K_q} \quad (5.19)$$

where $K_q = C_q w \sqrt{\frac{2}{\rho} p_l}$. In this case, for a negative spool displacement, the static characteristic is positive and a larger flow will increase the pressure, like a relief valve.

5.1.3 Linear model and analysis

A linear analysis in the frequency domain is very useful since the valve can be studied with regard to stability. Many properties are revealed and a better understanding of the valve is possible. The first step is to linearise the system equations, which is done around a specific operating point. Only the pressure reducing case is considered for this analysis. The pressure relief case is identical. The linearised system equations are presented in equations 5.20 to 5.29. The Δ -symbol indicates a change in the state variable. Index 0 indicates the state variable's value at a given operating point.

Equation of motion

$$\begin{aligned} m_v \Delta \ddot{x}_v &= \Delta p_{ref} A_c - \Delta p_c A_c - K \Delta x_v - b_v \Delta \dot{x}_v \\ &\quad - K_{fx} \Delta x_v - K_{fp} (\Delta p_s - \Delta p_l) \end{aligned} \quad (5.20)$$

The derivatives are defined according to equations 5.21 and 5.22.

$$K_{fx} = \frac{\partial F_f(x_{v0}, p_{s0}, p_{l0})}{\partial x_v} (p_{s0} - p_{l0}) = 2C_q w \cos(\delta) (p_{s0} - p_{l0}) \quad (5.21)$$

$$K_{fp} = \frac{\partial F_f(x_{v0}, p_{s0}, p_{l0})}{\partial p_l} x_{v0} = 2C_q w \cos(\delta) x_{v0} \quad (5.22)$$

The derivative K_{fp} is usually small in the context and will be ignored from here onwards.

Flow equation

$$\Delta q_v = K_q \Delta x_v + K_c (\Delta p_s - \Delta p_l) - \Delta q_c \quad (5.23)$$

The derivatives are defined according to equations 5.24 and 5.25.

$$K_q = \frac{\partial q_v(x_{v0}, p_{s0}, p_{l0})}{\partial x_v} = C_q w \sqrt{\frac{2}{\rho} (p_{s0} - p_{l0})} \quad (5.24)$$

$$K_c = \frac{\partial q_v(x_{v0}, p_{s0}, p_{l0})}{\partial p_s} = -\frac{\partial q_v(x_{v0}, p_{s0}, p_{l0})}{\partial p_l} = \frac{C_q w x_{v0} \sqrt{\frac{2}{\rho}}}{2\sqrt{p_{s0} - p_{l0}}} \quad (5.25)$$

Load volume

The linearised continuity equation of the load volume is expressed in equation 5.26.

$$\Delta q_v - \Delta q_l = \frac{V_l}{\beta} \Delta \dot{p}_l \quad (5.26)$$

Control chamber

The linearised flow and continuity equations for the control chamber are defined in equations 5.27 and 5.28.

$$\Delta q_c = K_1 (\Delta p_l - \Delta p_c) \quad (5.27)$$

$$\Delta q_c = -A_c \Delta v_v + \frac{V_c}{\beta} \Delta \dot{p}_c \quad (5.28)$$

The derivative is defined according to equation 5.29.

$$K_1 = \frac{\partial q_c}{\partial p_l} = -\frac{\partial q_c}{\partial p_c} = \frac{C_q A_{cr} \sqrt{\frac{2}{\rho}}}{2\sqrt{p_{l0} - p_{c0}}} \quad (5.29)$$

Laplace transformation

In order to perform an analysis in the frequency domain the linearised system equations are transformed into the Laplace domain. These equations are

defined in equations 5.30 to 5.34, where s is the Laplace operator.

$$\begin{aligned} m_v \Delta x_v s^2 &= \Delta p_{ref} A_c - \Delta p_c A_c - K \Delta x_v - B_v \Delta x_v s \\ &\quad - K_{fx} \Delta x_v \\ K_e &= K + K_{fx} \\ \Rightarrow m_v \Delta x_v s^2 &= \Delta p_{ref} A_c - \Delta p_c A_c - K_e \Delta x_v - B_v \Delta x_v s \end{aligned} \quad (5.30)$$

$$\Delta q_v = K_q \Delta x_v + K_c (\Delta p_s - \Delta p_l) - \Delta q_c \quad (5.31)$$

$$\Delta q_v = \Delta q_l + \frac{V_l}{\beta} \Delta p_l s \quad (5.32)$$

$$\Delta q_c = -A_c \Delta x_v s + \frac{V_c}{\beta} \Delta p_c s \quad (5.33)$$

$$\Delta q_c = K_1 (\Delta p_l - \Delta p_c) \quad (5.34)$$

Equations 5.30 to 5.34 can be rearranged into equations 5.35 to 5.37 and used to form a block diagram

$$\Delta x_v = \frac{A_c}{K_q} \frac{\Delta p_{ref} - \Delta p_c}{m_v s^2 + B_v s + K_e} \quad (5.35)$$

$$\Delta p_c = \frac{\frac{A_c}{K_1} \Delta x_v s + \Delta p_l}{1 + \frac{V_c}{K_1 \beta} s} \quad (5.36)$$

$$\begin{aligned} (K_c \Delta p_s - \Delta q_l) \left(1 + \frac{V_c}{K_1 \beta} s \right) + K_q \left(1 + \left(\frac{V_c}{K_1 \beta} + \frac{A_c}{K_q} \right) s \right) \Delta x_v \\ = K_c \Delta p_l \left(1 + \frac{V_l}{K_c \beta} s \right) \left(1 + \frac{V_c}{K_1 \beta} s \right) \end{aligned} \quad (5.37)$$

The block diagram describing the valve's dynamic behaviour is shown in figure 5.4, where the following definitions are used.

$$\begin{aligned} \omega_c &= \frac{K_1 \beta}{V_c} \\ \omega_m &= \sqrt{\frac{K_e}{m_v}} \\ \delta_m &= \frac{B_v}{2} \sqrt{\frac{1}{K_e m_v}} \\ \omega_s &= \frac{K_c \beta}{V_l} \end{aligned}$$

The behaviour of the valve depends very much on the damping restrictor. If no restrictor is used, the term K_1 would be infinite and the valve would be dominated by a lowly damped mechanical resonance. Such a system would be difficult to control and the valve would have to be designed for each specific

Figure 5.4 A general block diagram representation of the hydro-mechanical subsystem of the pressure control valve.

application. This was concluded by both Merritt (1967) and Jan-Ove Palmberg (1983). To analyse the effect of the damping restrictor the inner control loop around the mechanical resonance can be studied, as shown in equation 5.38.

$$\frac{\Delta x_v}{\Delta p_1} = \frac{\frac{A_c}{K_e} \left(1 + \frac{s}{\omega_c}\right)}{\left(1 + \frac{s}{\omega_c}\right) \left(\frac{s^2}{\omega_m^2} + \frac{2\delta_m}{\omega_m} s + 1\right) + \frac{s}{\omega_v}} \quad (5.38)$$

where $\omega_v = \frac{K_1 K_e}{A_c^2}$. There are basically two approaches depending on the size of the damping restrictor. If the restrictor fulfils the criteria in equation 5.39, the result is that of a valve without a damping restrictor but with an increased mechanical damping. The resulting block diagram representation of this system is shown in figure 5.5. The resonance ω_c is very large and has been ignored. There is also a pumping effect from the spool with resonance frequency $\frac{K_q}{A_c}$, which is usually very high and is therefore also ignored as well in the following analyses.

$$K_1 \geq \frac{A_c^2}{2\sqrt{K_e m_v}} \quad (5.39)$$

In this system, stability must be checked for each application since it depends

Figure 5.5 A block diagram representation of a valve with a large damping restrictor.

on the system parameters. The other solution is to have a damping restrictor

that fulfils the criteria in equation 5.40.

$$K_1 \leq 2A_c \sqrt{\frac{V_c}{\beta m_v}} \quad (5.40)$$

The result of this is that the expression in equation 5.38 can be factored into the expression in equation 5.41.

$$\frac{\Delta x_v}{\Delta p_1} = \frac{\frac{A_c}{K_e} \left(1 + \frac{s}{\omega_c}\right)}{\left(1 + \frac{s}{\omega_v}\right) \left(\frac{s^2}{\omega_h^2} + \frac{2\delta_h}{\omega_h} s + 1\right)} \quad (5.41)$$

where $\omega_h = \sqrt{\frac{K_h}{m_v}}$, $K_h = \frac{A_c^2 \beta}{V_c}$, $\delta_h = \frac{\omega_c}{2\omega_h}$. With the restrictor small enough, the oil inside the damping chamber will be entrapped and the result is that the mechanical resonance has been replaced by a hydraulic resonance. This resonance is much higher and will not dominate the system any more. It can therefore be ignored in the analysis, together with ω_c . The system will instead be dominated by a low frequency lag, ω_v , and the lag from the system volume. The resulting block diagram representation is shown in figure 5.6. The stability must be designed with respect to the phase margin, rather than the amplitude margin, as is the case with a large damping restrictor. Nevertheless, the design is very insensitive to variations in system parameters.

Figure 5.6 A block diagram representation of a valve with a large damping restrictor.

What can be seen from both cases is that the static loop gain K_v is defined by equation 5.42 and is also defined by the static characteristic R_v of the valve. This means that a desirable low static characteristic will yield a high loop gain and a valve that is difficult to control unless the damping is high enough.

$$K_v = \frac{A_c K_q}{K_e K_c} = \frac{1}{R_v K_c} \quad (5.42)$$

In the subsequent section, the above theory will be applied to a commercial valve. A model is derived and the valve is analysed using the techniques above.

5.2 Modelling and analysis of a commercial proportional solenoid pressure control valve

In this section a commercial valve is analysed as a potential candidate for the electro-hydraulic power steering system and the work relates to research question 1). Using a commercially available valve is an attractive solution since development costs can be reduced. The valve is the Hydac PDR08-11 proportional solenoid pressure control valve, which is primarily designed as a pilot stage for a spool valve. A simplified schematics of the valve is shown in figure 5.7. The valve is a single-stage valve. It has three ports: one to pump,

Figure 5.7 *Simplified schematics of the Hydac pressure control valve.*

one to tank and one to load system. The valve is actuated by a solenoid that pushes to open the valve to the pump directly on the spool. In the opposite direction a preloaded spring pushes on the spool. The spring is very weak in the context and serves only to keep the spool in place when no pressure or solenoid force is present. The load pressure also acts on the spool on a predefined surface in the opposite direction to the solenoid force. The solenoid is driven by a 200 Hz PWM-modulated voltage with an amplitude of 12 V. The low frequency ensures a dither effect in the signal, which will give micromovements of the spool and reduce stick-slip. The force from the solenoid is a function of the current through the coil. The reference pressure is therefore set by the solenoid current. Since the coil's resistance can vary with temperature the current will also vary for a given voltage. The common solution is to use some kind of

current controller to keep it at a desired value. This, however, is not used here. As described in the section above, this type of valve is self-regulating.

A small restrictor is placed between the load volume and a much smaller volume above the spool. This solution serves to increase the damping, as explained in the previous section. A further analysis of the restrictor is presented below, which reveals its true impact on the valve. The opening areas of the valve consist at the pump side of 12 circular holes around the valve body, each with a diameter of 1.5 mm. On the tank side, there are also 12 holes in the same manner. However, a single small hole of 0.8 mm diameter is placed before the larger holes with an offset of 0.3 mm. This generates a much smoother area gradient on the tank side compared to the pump side.

With the centre position of the spool at 0 mm, the spool's stroke is from -1 mm to 0.85 mm. The core's stroke is approximately from -1.6 mm to 0.85 mm.

The modelling of the valve serves to explain both the valve's particular behaviour and help the control design process. Measurements of the valve, shown in section 5.2.2, show non-typical behaviour, where the pressure-reducing curves have negative static characteristics. As it turns out, the design of the valve makes it less suitable for the intended application. However, the valve shows some intriguing behaviour and there are several lessons to be learnt during the modelling procedure. The behaviour of the valve suggests that it is important to consider in detail the flow-pressure relationship and flow forces of the valve, as well as the solenoid's characteristics when modelling the valve.

Merritt (1967) gives a good overview of different types of fluid flow. The flow is usually turbulent and the discharge coefficient can be assumed to be a constant value. The flow is then proportional to the square root of the pressure drop. In cases where the flow is more laminar, the discharge coefficient is instead modelled as a function of the square root of the Reynolds number. In a strict laminar case, this yields that the flow is directly proportional to the pressure drop. Wu, Burton, and Schoenau (2002) propose an exponential model of the discharge coefficient as a function of the Reynolds number. Since the Reynolds number itself is dependent on the flow, this model requires an iterative process or a pre-defined look-up table to solve. Ellman and Piché (1996) propose a two-region flow model for numerical reasons. This is also done by Åman, Handroos, and Eskola (2008), who use a cubic spline curve for the laminar and transition regions. This method has proven to be computationally efficient compared to the traditional orifice flow model. Borutzky, Barnard, and Thoma (2002) propose a single formula for the discharge coefficient for better numerical efficiency. This results in a linear and a quadratic term in the pressure flow relation.

The flow forces of the open-centre directional valve were measured and analysed by Amirante, Vescovo, and Lippolis (2006). Depending on the position of the spool, the flow forces initially act to open the valve, whereas they change direction to close the valve after a certain opening. Stationary flow forces were also analysed on compensated spool valves by Borghi, Milani, and Paoluzzi

(2000). Computational Fluid Dynamics (CFD) was used to analyse the jet flow angle and the flow forces in general for different designs.

A detailed model of a solenoid is presented by N. Vaughan and J. Gamble (1996). By measuring the current and voltage of the solenoid, the magnetization curves can be derived, which also include hysteresis effects. A PWM-driven solenoid for ABS valves is modelled by Branciforte et al. (2011). The fast dynamics are modelled using a fractional order system, while the hysteresis is covered by using neural networks.

5.2.1 Measurements and modelling of the solenoid

In this section the measurements and modelling procedure of the solenoid is presented. The test rig is first explained, followed by the mathematical description. A validation of the model is presented last. Magnetic properties are difficult to measure and no material properties of the coil are available. The measurements and modelling procedures are therefore based on the work done by N. Vaughan and J. Gamble (1996), which requires the voltage, current and force of the solenoid to be measured.

Test rig for solenoid and measurements

A schematic of this test rig is shown in figure 5.8a and a photo in figure 5.8b. A low-speed electro-mechanical actuator with a load cell attached to its end

(a) A schematic of the measurement set-up of the solenoid.

(b) A photo of the measurement set-up of the solenoid.

Figure 5.8 Solenoid test rig.

was used to control the position of the magnet core. The solenoid was clamped such that the core was resting against the load cell in the horizontal position. The solenoid was supplied with 12 V. Two types of test were conducted. The first tests measured the static relationship between current and force. Since the core was only resting against the load cell and no spring was used, no PWM-modulation was used. Instead, the supply voltage was slowly increased while current and force were registered. This was done for 25 positions over the entire stroke. The other tests applied step input signals, also at 25 different positions over the entire stroke. The voltage, current and force were measured.

Before each test the solenoid was heated by applying maximum current to avoid fluctuations in the coil resistance. All tests were sampled at a rate of 0.1 ms.

The results from the first tests are shown in figures 5.9 and 5.10. As can be seen in figure 5.9, the force is quite nonlinear for small currents and there is a variation with coil stroke which tends to be less influential within a certain region. This is confirmed by figure 5.10 which shows the solenoid force against stroke at different current levels. The static behaviour is strongly nonlinear but have a region where the behaviour tends to be more linear. This is where the valve has its working region. In these measurements, zero position corresponds to the coil's end position, furthest away from the spool. The fluctuations in current have been considered in the figure 5.10. The results from the second

Figure 5.9 *Static measurements of solenoid force against current.*

type of test, are shown in figures 5.11, 5.12 and 5.13 for coil positions -1.6mm, 0.01mm and 0.81mm. The current shows the expected first order behaviour and the force follows this.

Figure 5.10 Static characteristic of the solenoid represented as force against coil position at different current levels.

Figure 5.11 Dynamic response of solenoid voltage for coil positions -1.6mm, 0.01mm and 0.81mm.

Figure 5.12 *Dynamic response of solenoid current for coil positions -1.6mm, 0.01mm and 0.81mm.*

Figure 5.13 *Dynamic response of solenoid force for coil positions -1.6mm, 0.01mm and 0.81mm.*

Mathematical description

Figure 5.14 Representation of a solenoid.

A solenoid is basically a coil wired around an armature and in its simplest form it can be seen as a resistor in series with an inductor as shown in figure 5.14. By applying a current through the coil a magnetic field is produced where the magnetic field intensity H is described by Ampere's law as in equation 5.43, see e.g. Chapman (1998)).

$$\oint H dl = I_{encl} \quad (5.43)$$

where I_{encl} is the current enclosed by the magnetic path. If the magnetic path of the solenoid is seen as a closed core with an air gap and the coil wrapped around one leg, as illustrated in figure 5.14, the magnetic field intensity is described by equation 5.44.

$$H = \frac{Ni}{l} \quad (5.44)$$

How large the magnetic flux ϕ becomes depends on the material in the core, and in this case also the air gap. The relation between the magnetic field's intensity and flux density is given by equation 5.45.

$$B = \mu H \quad (5.45)$$

where μ is the permeability of the material. The total magnetic flux is given by $\phi = BA$. The permeability, however, is not constant for different fields. The easiest way to describe the relation between field intensity and field density is with a magnetization curve. A typical curve is shown in figure 5.15. The magnetization curve shows two typical phenomena: hysteresis, i.e. losses in the core, and saturation in the magnetic flux. Faraday's law, equation 5.46, states that if a magnetic flux passes through a turn of a coil, a voltage will be induced that is proportional to the rate of change of the flux in time.

$$V = -\frac{d\phi}{dt} \quad (5.46)$$

Figure 5.15 A typical magnetization curve represented as flux linkage against current.

The total flux passing through all turns is given by $\lambda = \sum_{i=1}^N \phi_i$. This yields that Faraday's law can be expressed as in equation 5.47.

$$V_L = \frac{d\lambda}{dt} \quad (5.47)$$

where V_L is the induced voltage over the solenoid. If it is assumed that the same flux passes through all windings, the flux linkage becomes $\lambda = N\phi$. This in turn gives an expression for the flux density as $B = \frac{\lambda}{NA}$. Since both B and H are difficult to measure, the magnetization curve can instead be expressed with flux linkage λ and current i , which are easy to measure. The current is directly measurable. Since the inductive voltage is the difference between the total voltage and the resistive voltage according to equation 5.48, the flux linkage λ can be derived by integrating the inductive voltage as in equation 5.49.

$$V_L = V_s - iR \quad (5.48)$$

$$\lambda = \int (V_s - V_R)dt = \int (V_s - iR)dt \quad (5.49)$$

The inductive voltage of the solenoid, calculated from the measured total voltage and current, is shown in figure 5.16a. The coil resistance is calculated from the voltage and current at steady-state condition. The corresponding magnetization curves are then calculated from the inductive voltage with the result shown in figure 5.16b. The slope of the curves varies for different coil positions.

The idea presented by N. Vaughan and J. Gamble (1996) is to model the solenoid as a resistor in series with a nonlinear inductor, which also considers the hysteresis seen from the magnetization curves. The total current through the solenoid consists of an energy-restoring part and an energy-dissipating part, as described in equation 5.50. The restoring function depends on both the flux

Figure 5.16 Inductive voltage and magnetization curves.

linkage and the core position, while the dissipating function depends only on the inductive voltage.

$$i = i_r + i_d = f(\lambda, x) + d(V_L) \quad (5.50)$$

The restoring function is found by calculating the mean current from the magnetization curves at each core position, as illustrated in figure 5.15, and fitting a polynomial function. The best fit for this solenoid is described by equations 5.51 to 5.54. Figure 5.17 shows the comparison of the fitted function coefficients to measured ones against the core position.

$$i_r = f_1 \lambda^3 + f_2 \lambda^2 + f_3 \lambda \quad (5.51)$$

$$f_1 = p_{f_{11}} x^5 + p_{f_{12}} x^4 + p_{f_{13}} x^3 + p_{f_{14}} x^2 + p_{f_{15}} x + p_{f_{16}} \quad (5.52)$$

$$f_2 = p_{f_{21}} x^5 + p_{f_{22}} x^4 + p_{f_{23}} x^3 + p_{f_{24}} x^2 + p_{f_{25}} x + p_{f_{26}} \quad (5.53)$$

$$f_3 = p_{f_{31}} x^2 + p_{f_{32}} x + p_{f_{33}} \quad (5.54)$$

The dissipating function is also fitted with a polynomial from the calculated dissipating current, with a best fit as in equation 5.55. The result is shown in figure 5.18. The dissipating current also shows a variation with coil position but it is assumed to be of less importance and is therefore ignored. The validation will confirm this assumption later.

$$i_d = p_{d_1} V_L^3 + p_{d_2} V_L^2 + p_{d_3} V_L + p_{d_4} \quad (5.55)$$

The existence of an air gap within the solenoid is the cause of a magnetic potential difference across the air gap that produces an attractive force between

(a) f_1

(b) f_2

(c) f_3

Figure 5.17 Coefficients for the restoring current function. Dashed curves are the fit.

Figure 5.18 Fitted dissipating current (solid) compared to calculated ones for coil positions -1.6mm, 0.01mm and 0.81mm.

the armature and the opposite stator. This force acts directly on the spool. N. Vaughan and J. Gamble (1996) proposed here again a nonlinear model of the force as a function of the flux linkage, since the flux linkage is easily obtained. However, since the force against current was convenient to measure, the force is modelled as a function of current and position. Again a polynomial model is fitted, described in equations 5.56 to 5.60. Due to the strongly nonlinear behaviour of the solenoid, the fit is limited to the valve's working region where the behaviour is more linear. The results from the fit are shown in figure 5.19.

$$F_s(i, x) = p_{f_{s1}} i^5 + p_{f_{s2}} i^4 + p_{f_{s3}} i^3 + p_{f_{s4}} i^2 \quad (5.56)$$

$$p_{f_{s1}} = p_{f_{s11}} x^3 + p_{f_{s12}} x^2 + p_{f_{s13}} x + p_{f_{s14}} \quad (5.57)$$

$$p_{f_{s2}} = p_{f_{s21}} x^3 + p_{f_{s22}} x^2 + p_{f_{s23}} x + p_{f_{s24}} \quad (5.58)$$

$$p_{f_{s3}} = p_{f_{s31}} x^3 + p_{f_{s32}} x^2 + p_{f_{s33}} x + p_{f_{s34}} \quad (5.59)$$

$$p_{f_{s4}} = p_{f_{s41}} x^3 + p_{f_{s42}} x^2 + p_{f_{s43}} x + p_{f_{s44}} \quad (5.60)$$

A resulting block diagram representation of the solenoid model is shown in figure 5.20. A time lag is added to the dissipating current to prevent it from growing too fast.

Figure 5.19 Coefficients for the force function. Dashed curves are the fit.

Figure 5.20 A block diagram representation of the solenoid model.

Validation

The validation of the solenoid model is done by applying step voltages of 12 V and 6 V at the fixed coil positions -0.6 mm, 0.095 mm and 0.81 mm, which is within the working range of the valve. The voltage reaches steady-state before a step to 0 V is applied. The steady-state characteristic is therefore

also validated from the same measurements. The coil resistance is calculated at steady-state current and used in the simulation model. Figure 5.21 shows the solenoid current for a 12 V step input. Figure 5.22 shows the solenoid force for a 12 V step input. Figure 5.23 shows the calculated inductive voltage for a 12 V step input. Figure 5.24 shows the calculated magnetization curves for a 12 V step input. Figure 5.25 shows the solenoid force for a 6 V step input. Figure 5.26 shows the solenoid force for a 6 V step input. Figure 5.27 shows the calculated inductive voltage for a 6 V step input. Figure 5.28 shows the magnetization curves for a 6 V step input.

(a) $x = -0.6$ mm

(a) $x = -0.6$ mm

(b) $x = 0.095$ mm

(b) $x = 0.095$ mm

(c) $x = 0.81$ mm

(c) $x = 0.81$ mm

Figure 5.21 Validation of the solenoid current for a 12 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

Figure 5.22 Validation of the force for a 12 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

(a) $x = -0.6$ mm

(a) $x = -0.6$ mm

(b) $x = 0.095$ mm

(b) $x = 0.095$ mm

(c) $x = 0.81$ mm

(c) $x = 0.81$ mm

Figure 5.23 Validation of the inductive voltage over the solenoid for a 12 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

Figure 5.24 Validation of the magnetization curves for a 12 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

(a) $x = -0.6 \text{ mm}$

(a) $x = -0.6 \text{ mm}$

(b) $x = 0.095 \text{ mm}$

(b) $x = 0.095 \text{ mm}$

(c) $x = 0.81 \text{ mm}$

(c) $x = 0.81 \text{ mm}$

Figure 5.25 Validation of the solenoid current for a 6 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

Figure 5.26 Validation of the force current for a 6 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

(a) $x = -0.6$ mm

(a) $x = -0.6$ mm

(b) $x = 0.095$ mm

(b) $x = 0.095$ mm

(c) $x = 0.81$ mm

(c) $x = 0.81$ mm

Figure 5.27 Validation of the inductive voltage over the solenoid for a 6 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

Figure 5.28 Validation of the magnetization curves for a 6 V step input at positions -0.6 mm, 0.095 mm and 0.81 mm. Dashed line is simulation results.

5.2.2 Measurements and modelling of the valve

This section presents the measurements, modelling of the hydro-mechanical subsystem and validation of the complete valve. The test rig for measuring the static and dynamic characteristics of the complete valve is explained first, followed by a mathematical description of the hydro-mechanical subsystem. The complete valve model is then validated. The analysis of the complete system is performed according to section 5.1.

Test rig for valve and measurements

(a) A schematic of the measurement set-up of the valve.

(b) A photo of the measurement set-up of the valve.

Figure 5.29 Solenoid pressure control valve test rig.

The test rig for measurements of the complete valve is shown in Fig. 5.29. The valve is connected to a constant pressure source at 200 bar. A small accumulator of 1 L is installed to minimise variations in the supply pressure. The load consists of a fixed volume of 0.1 L. A flow-meter is installed on the supply line, as well as a temperature sensor. The temperature varied between 37° C and 42° C for all tests. Two variable orifices are installed, one between the load volume and tank and the other between the pump and load volume. These are used to apply a positive or negative load flow. Load, supply and tank pressures are measured with transducers. The magnet core housing is fitted with a small screw at its end. The screw is used to bleed the core housing, since it is filled with oil. When the screw is removed, the hole gives access to the magnet core. A position transducer was installed to measure the core position. As long as the core is in contact with the spool, the core position corresponds to the spool position. The position transducer is spring-loaded to ensure it is always in contact with the core, at the same time as the spring force is kept low enough to have no impact on the valve.

Two types of test were conducted in order to capture the valve's static and dynamic characteristics. To measure the static characteristic of the valve, a constant duty cycle was applied. Either of the two variable orifices was slowly opened to increase the load flow while pressure, flow and spool position were

registered. This was done from 30% - 100% duty cycle with steps of 10%, both for positive and negative load flow, i.e. the valve is working either as a reducing valve or as a relief valve. Figure 5.30a shows the pressure flow curves and figure 5.30b shows the spool position against flow. At a certain negative flow the magnet and core separate and the core is pushed to its end position.

When measuring the dynamic characteristic, the load flow was set at zero. Steps were applied in the input signal for different levels and amplitudes while pressure, current and spool position were registered. If too sudden an input signal was applied when relieving the pressure, the magnet core and spool could separate and cause great undershots in the pressure. This was clearly seen from the measurements. Since this is an unwanted behaviour and is outside the scope of this paper, a rate limiter was implemented for a decreasing input signal. Different rates were evaluated. Figure 5.31 shows the response for a step input from 40% to 90% duty cycle and a negative rate of 5000% 1/s, which results in a large undershot in pressure. The rate limiter was tuned so that the core and spool stayed in contact. A negative rate of 2000% 1/s turned out to be sufficient. Figure 5.32 shows the response for a step input from 46% to 100% duty cycle.

Figure 5.30 Static measurements of the solenoid pressure control valve.

Mathematical description

This section describes the mathematical model of the hydro-mechanical sub-system. When studying the static characteristic of the valve in figure 5.30, a few things can be concluded that will affect the model of the valve. First, it can be seen that the spool works with an offset from the centre position towards the tank side, i.e. a negative stroke. This is due to the difference in area gradient and pressure drop over the meter-in and meter-out orifices. To cover the leakage from the pump side, the valve needs to open somewhat to tank.

Figure 5.31 Dynamic response of valve for a step input from 40% - 90% with negative rate of 5000% 1/s.

Figure 5.32 Dynamic response of valve for a step input from 46% - 100% with negative rate of 2000% 1/s.

It is therefore important to consider this when modelling the valve. It is done by regarding both the gap leakage and area opening on the pump side as two orifices, each with different characteristics. The gap leakage on the tank side is less important due to the smoother area gradient.

The second thing concluded is that generally the valve is working with low flows. It is far from certain that the flow reaches fully turbulent flow and the common assumption that the discharge coefficient is constant might be far from the truth. This is checked by calculating the discharge coefficient from measurements. The discharge coefficient is not purely a physical property and any assumption and modelling mismatch will affect its behaviour.

Third, the static characteristic of the valve is very specific when working as a pressure-reducing valve. As can be seen in figure 5.30a, the pressure begins

to increase for larger flows, which is not the expected behaviour of a pressure-reducing valve. The typical behaviour of a solenoid is an increased force with stroke, i.e. for a smaller air gap, which is also seen in Figure 5.9. The increase in force, however, is too small to fully explain the increase in pressure. The explanation lies in the flow forces, as will be described below.

The model considers both the meter-in and the meter-out port of the valve and is outlined in equations 5.61 to 5.82.

Spool motion

$$m\dot{v} = F_s(i, x) - A_c p + A_{cc} p_c - Kx - Bv - F_{fin}(x, p_s, p) + F_{fout}(x, p) - f_0 \quad (5.61)$$

The spool is subjected to flow forces F_f in both directions, a spring force with preload f_0 and viscous friction with coefficient B . The total mass is denoted m , A_c is the pressure sensing area, K is the spring stiffness and p is the pressure. Static friction and stick-slip phenomena are assumed to be negligible due to the dither frequency in the input signal. The spool's stroke is limited between -0.55 mm and 0.85 mm, which according to measurements is the working range of interest.

Continuity equation

The pressure change depends on both the meter-in and meter-out flow as well as a load flow as described by equation 5.62. The load flow could be a cylinder piston moving.

$$q_{in} - q_{out} - q_c - q_l = \frac{V}{\beta} \dot{p} \quad (5.62)$$

Orifice flow

The meter-in, q_{in} , and meter-out, q_{out} , flows are given by equation 5.63 and 5.64, where C_q is the discharge coefficient and ρ is the oil density.

$$q_{in} = C_q(x, p_s, p) A_{in}(x) \sqrt{\frac{2}{\rho} (p_s - p)} \quad (5.63)$$

$$q_{out} = C_q(x, p) A_{out}(x) \sqrt{\frac{2}{\rho} p} \quad (5.64)$$

On the pump side, the valve is modelled with two orifices, one for the gap leakage and the other when the spool passes the holes. Equation 5.63, which assumes turbulent flow, is used in both cases. The opening area is modelled according to equation 5.66 for the meter-in orifice and equation 5.67 for the meter-out orifice. The meter-in area considers the 12 circular holes and a plane leakage gap at each hole. On the tank side, the opening area is modelled in the same manner but also considering the small initial hole. The gap leakage is modelled with the opening area in equation 5.65, which is a plane gap. The reason to also include the gap in the hole opening areas is to get a smooth

transition from the gap to the holes. When implementing the opening areas, the range of the spool position x is considered with respect to the trigonometric functions.

$$A_{gap} = 2rh_{gap} \quad (5.65)$$

$$A_{in}(x) = 12 \left(\frac{1}{2} r^2 \left(2 \arccos \left(\frac{r - x \cdot 10^{-3}}{r} \right) - \sin \left(2 \arccos \left(\frac{r - x \cdot 10^{-3}}{r} \right) \right) \right) + 2rh_{gap} \right) \quad (5.66)$$

$$A_{out}(x) = \max \left(\pi r_s^2, \left(\frac{1}{2} r_s^2 \left(2 \arccos \left(\frac{r_s - |x| \cdot 10^{-3}}{r_s} \right) - \sin \left(2 \arccos \left(\frac{r_s - |x| \cdot 10^{-3}}{r_s} \right) \right) \right) \right) \right) + 12 \left(\frac{1}{2} r^2 \left(2 \arccos \left(\frac{r - (|x| - 0.3) \cdot 10^{-3}}{r} \right) - \sin \left(2 \arccos \left(\frac{r - (|x| - 0.3) \cdot 10^{-3}}{r} \right) \right) \right) \right) + 12 \cdot 2rh_{gap} \quad (5.67)$$

Since the flow can be anything from laminar to turbulent, the discharge coefficient C_q is modelled to take this into account. The discharge coefficient can be calculated from equation 5.63 or equation 5.64 and using the data from figure 5.30. The discharge coefficient is shown in figure 5.33a for the pump side and in figure 5.33b for the tank side. The discharge coefficient is split into

Figure 5.33 Measured (solid) and modelled (dashed) discharge coefficient for the pump side a) and tank side b) at corresponding 30%, 60% and 90% duty cycle.

two parts. For gap leakage, the flow is assumed proportional to the pressure drop. According to the literature, Merritt (1967), the discharge coefficient is

proportional to the square root of the Reynolds number R_e , which is calculated according to equation 5.68.

$$\begin{aligned} R_e &= \frac{q d_h}{A \nu} & d_h &= \frac{4A}{O} \\ \Rightarrow R_e &= \frac{4q}{O \nu} \end{aligned} \quad (5.68)$$

The discharge coefficient for the gap leakage is then modelled as in equation 5.69.

$$C_{q_{gap}} = g_g(x) \sqrt{R_e} \quad (5.69)$$

The function $g_g(x)$ is a geometrical term that will both compensate for any errors in the area opening function and take into account the effective cross area of the fluid stream. Studying the discharge coefficient, it can be seen that the value tends to approach a constant value when the spool crosses the holes at 0 mm. The flow goes from laminar to a mix of laminar and turbulent flow as the spool opens more. The discharge coefficient is no longer proportional to the square root of the Reynolds number and a more complicated relation would require an iterative process to solve. This is not practical when implementing the model in simulation software. In this case, the discharge coefficient is instead modelled as in equation 5.70, where it only depends on the spool position and pressure.

$$C_{q_{hole}}(x, p) = g(x) p^{\gamma(x)} \quad (5.70)$$

The function $g(x)$ is again a geometrical term, while $\gamma(x)$ handles the transition from laminar to turbulent flow and varies between 0 and 0.5. An arctan function turned out to be suitable and the three functions $\gamma(x)$, $g_g(x)$ and $g(x)$ are described in equation 5.71 to 5.73.

$$g_g(x) = 0.02477e^{1.314x} + 0.0642e^{24.41x} \quad (5.71)$$

$$\begin{aligned} g(x) &= 0.3973 \sin(3.493x - 0.6221) + \\ & 3.789 \sin(6.942x + 1.088) + \\ & 3.626 \sin(7.047x + 4.173) + \\ & 0.0009295 \sin(22.45x + 2.653) \end{aligned} \quad (5.72)$$

$$\gamma(x) = ((-\arctan(5(x - 0.3))) + \pi/2)0.3/\pi \quad (5.73)$$

The gap leakage on the tank side is assumed to be very small due to the smoother area gradient and no separate measurements of it were recorded. It is therefore lumped together with the main flow and the pressure-flow characteristic is modelled with the same method as on the tank side, resulting in the functions $\gamma_r(x)$ and $g_r(x)$, shown in equations 5.74 and 5.75.

$$g_r(x) = 0.1267e^{-\frac{|x| - 0.5556}{0.1016}^2} + 10880e^{-\frac{|x| - 3.269}{0.7823}^2} \quad (5.74)$$

$$\gamma_r(x) = ((-\arctan(10(|x| - 0.4))) + \pi/2)0.3/\pi \quad (5.75)$$

The resulting functions are also plotted in figures 5.33a and 5.33b. To obtain a smooth, continuous transition from the gap to the holes at the pump side the so called logistic function, equations 5.76 and 5.77, was used.

$$C_q = C_{q_{gap}} \cdot (1 - L(x)) + C_{q_{hole}} \cdot L(x) \quad (5.76)$$

$$L(x) = \frac{1}{1 - e^{-1000x}} \quad (5.77)$$

Damping orifice and chamber

The flow into the damping chamber, equation 5.78, depends on the size of the damping orifice and the leakage flow over the spool land. This flow is assumed to be laminar and is modelled with the constant K_d , which is tuned to give a good match with measurements. The pressure change due to this flow is modelled with the continuity equation as in equation 5.79.

$$q_c = K_d(p - p_c) \quad (5.78)$$

$$q_c + A_{cc}v = \frac{V_c}{\beta} \dot{p}_c \quad (5.79)$$

Flow forces

Since the mechanical spring is very weak, the static characteristic of the valve is mainly defined by the flow forces. The flow force is defined in equation 5.80, where only stationary flow forces are considered. Δp is the pressure drop over the orifice and is either the difference between pump and load pressure or the difference between load and tank pressure.

$$F_f = 2C_q(x, \Delta p)A(x)\Delta p \cos(\delta) \quad (5.80)$$

Since the discharge coefficient is known, the flow angle can be calculated from the spool equilibrium in equation 5.61. The resulting models are defined in equations 5.81 and 5.82. When implementing the models, the working range of the spool is considered. For convenience, the cosine of the flow angle is modelled rather than the flow angle itself.

Pump side

$$\begin{aligned} \cos(\delta) = \max & \left(0.7, 6.151 \cdot 10^{13} e^{-\frac{x+3.129}{0.5478}^2} \right. \\ & \left. + 0.1908 e^{-\frac{x-0.154}{0.106}^2} \right) \end{aligned} \quad (5.81)$$

Tank side

$$\begin{aligned} \cos(\delta) = \max & \left(1, 932 e^{-\frac{|x|+1.741}{0.7617}^2} \right. \\ & \left. + 0.3796 e^{-\frac{|x|-0.5226}{0.2083}^2} \right) \end{aligned} \quad (5.82)$$

Validation

The validation is performed by comparing measurements to simulation results. This is done for both static and dynamic measurements. Figure 5.34 shows the comparison of the measured and simulated pressure against flow and spool position against flow in the static case. Figures 5.35 to 5.37 compare the current, pressure and spool position of the measured and simulated results. The different cases are respectively 22%-62% duty cycle, 46%-100% duty cycle and 65%-85% duty cycle. Parameter data for the simulation model is found in the appendix.

Figure 5.34 Validation of the valve for static measurements. Measurements are solid lines and simulation results are dashed lines.

Figure 5.35 Validation of the dynamic response of the valve for a step input from 22% - 62% with a negative rate of 2000% 1/s. Measurements are solid lines and simulation results are dashed lines.

Simplified model

As explained in the general case in section 5.1, the purpose of the damping restrictor, is to increase the damping. Depending on the size of the restrictor

Figure 5.36 Validation of the dynamic response of the valve for a step input from 46% - 100% with a negative rate of 2000% 1/s. Measurements are solid lines and simulation results are dashed lines.

Figure 5.37 Validation of the dynamic response of the valve for a step input from 65% - 85% with a negative rate of 2000% 1/s. Measurements are solid lines and simulation results are dashed lines.

the results is either a very high hydraulic frequency with a low first order lag or an increased mechanical damping. With the parameters in table A.2 in the appendix, the valve fulfils criterion 5.39 and the restrictor only increases on the mechanical damping. This means that the nonlinear model can be simplified by removing the damping orifice and chamber and thus one state. What changes in the model is the equation of motion, equation 5.83, where the pressure acting on the spool is now the same as the load pressure and a new viscous friction, and equation 5.84 where the damping flow no longer takes place.

$$m\dot{v} = F_s(i, x) - A_c p - Kx - B_{simple}v - F_{fin}(x, p_s, p) + F_{fout}(x, p) - f_0 \quad (5.83)$$

$$q_{in} - q_{out} - q_l = \frac{V}{\beta}\dot{p} \quad (5.84)$$

The new total mechanical damping, which is represented by the viscous friction coefficient, is tuned to obtain an equal behaviour between the full model and the simplified model. The simplified model is advantageous when it comes to simulation. The very small damping chamber of the full model generates

a very stiff differential equation system which requires a small time step during simulation. The simplified model can be run with a larger time step and simulation is faster.

Linear modelling and analysis

A linear model of the system is very useful in both understanding the system and for control design, where linear control theory can be applied. The linear model is derived from the simplified nonlinear model. Since the characteristic of the valve is mainly defined by the flow forces, the solenoid is simplified to consist of a resistor and a linear inductor L . By applying the same method described in section 5.1, the linearised and Laplace transformed system is defined by equation 5.85 to equation 5.87, where s is the Laplace operator.

$$\Delta i s = \frac{1}{L} (\Delta V_s - R \Delta i) \quad (5.85)$$

$$\Delta x s = \frac{1}{m} (K_{fi} \Delta i - A_c \Delta p - K_e \Delta x - B \Delta x s) \quad (5.86)$$

$$\Delta p s = \frac{V}{\beta} (K_q \Delta x - K_c \Delta p + K_{c_{ps}} \Delta p_s + A_c \Delta x s - \Delta q_l) \quad (5.87)$$

The derivatives are defined according to equations 5.88 to 5.95, where index 0 indicates the working point for which the derivative is calculated.

$$K_{fi} = \frac{\partial F_s(i_0, x_0)}{\partial i} \quad (5.88)$$

$$K_{fx} = \frac{\partial F_s(i_0, x_0)}{\partial x} \quad (5.89)$$

$$K_{ff_{in}} = \frac{\partial F_{fin}(x_0, p_{s0}, p_0)}{\partial x} \quad (5.90)$$

$$K_{ff_{out}} = \frac{\partial F_{fout}(x_0, p_0)}{\partial x} \quad (5.91)$$

$$K_e = K - K_{fx} + K_{ff_{in}} - K_{ff_{out}} \quad (5.92)$$

$$K_q = \frac{\partial q_{in}(x_0, p_{s0}, p_0)}{\partial x} - \frac{\partial q_{out}(x_0, p_0)}{\partial x} \quad (5.93)$$

$$K_c = -\frac{\partial q_{in}(x_0, p_{s0}, p_0)}{\partial p} + \frac{\partial q_{out}(x_0, p_0)}{\partial p} \quad (5.94)$$

$$K_{c_{ps}} = \frac{\partial q_v(x_0, p_{s0}, p_0)}{\partial p_s} \quad (5.95)$$

The linearised equations are represented by the block diagram in figure 5.38. From the block diagram the loop gain transfer function $A_g(s)$ can be defined according to equation 5.96, which is used to study the stability margins of the valve.

$$A_g(s) = \frac{A_c(K_q + A_c s)}{(ms^2 + Bs + K_e)(K_c + \frac{V}{\beta} s)} \quad (5.96)$$

Figure 5.38 A block diagram representation of the valve.

The loop gain is evaluated for two working points. A working point should correspond to a point where the system is in steady-state. Any point on the pressure-flow chart in figure 5.34a and the corresponding pressure, current and spool position can be chosen. It is of interest to study what happens when the pressure starts to increase with a higher flow in pressure-reducing mode. The two points are listed in table 5.1 and are two points along the curve at 70% duty cycle. The first working point is where the pressure is declining with the flow. The second point is where the pressure is increasing with the flow, yielding negative characteristics. The loop gain frequency response is shown in figure 5.39. As can be seen, the system is stable in both cases. In the second case, however, the phase margin begins at -180° since the linearised spring constant is negative. Being quite low, the spring constant only affects the system at very low frequencies, allowing the system to maintain a positive, but low, phase margin at the crossover frequency. The resonance from the resulting pumping motion of the spool is very high and can be ignored.

	i	x_v	p
point 1	1.5 [A]	-0.0326 [mm]	120.4 [bar]
point 2	1.49 [A]	0.2482 [mm]	109.6 [bar]

Table 5.1 The working points for loop gain frequency analysis.

For controller design in chapter 6, it is of interest to study the closed loop transfer function of the valve, defined in equation 5.97.

$$\begin{aligned}
 G_o(s) &= \frac{K_q + A_c s}{(m s^2 + B s + K_e)(K_c + \frac{V}{\beta} s)} \\
 G_c(s) &= \frac{K_f i}{R + L s} \frac{G_o(s)}{1 + A_c G_o(s)}
 \end{aligned} \tag{5.97}$$

Six points covering the working region of interest are studied and defined in table 5.2. Figure 5.40 shows the resulting frequency plots. As seen, both the gain and resonance frequency varies with working point. The frequency resonance is in general quite low.

Figure 5.39 Loop gain frequency response of the valve. Dots indicate the amplitude and phase margins.

	i	x_v	p
point 1	0.57 [A]	-0.46 [mm]	27.5 [bar]
point 2	0.6 [A]	-0.2 [mm]	22.3 [bar]
point 3	0.6 [A]	0.01 [mm]	10.2 [bar]
point 4	1.27 [A]	-0.4 [mm]	108 [bar]
point 5	1.29 [A]	-0.1 [mm]	98.4 [bar]
point 6	1.29 [A]	0.046 [mm]	88.8 [bar]

Table 5.2 The working points for closed loop frequency analysis.

5.2.3 Discussion

The valve is constructed in a way that limits its performance. As can be seen from static characteristic curves in figure 5.30, in pressure-relieving mode the flow capacity is far from the capacity in pressure-reducing mode. The reason to this is probably that the magnet core connects with the spool in the damping chamber where the pressure is high. When the pressure is higher than the corresponding solenoid force, the magnet core is separated from the spool and pushed to its end position. This is also seen from the measurements. The solenoid loses its ability to control the pressure and what happens to the spool is not known since the spool position was not measured. This restriction makes the valve unsuitable for the intended application, where the flow requirement is equal in both directions.

Figure 5.40 Loop gain frequency response of the valve.

The static characteristic of the valve is mostly defined by the flow forces. The variation in force from the solenoid is comparatively small for the small stroke of the spool. The reason for a change in direction of the pressure in reducing mode is explained by studying the flow forces. Figure 5.41 shows the calculated flow forces at 70% duty cycle compared to the pressure. The normal behaviour is an increasing flow force with a higher flow rate, which initially occurs. However, the jet stream angle is continuously reduced with an increased area opening. At a certain opening, the jet stream angle reduces more than the flow is increased resulting in a reduced flow force. Equilibrium states that the pressure must therefore increase.

Since the static characteristic is defined by the flow forces, it also very likely to vary depending on circumstances. For instance, the smallest disturbance might change the jet stream angle and as a result a different behaviour is attained. The very flat characteristic yields low stability margins, as shown in figure 5.39. This could lead to difficulties when controlling the valve with an additional control loop. An alternative way to accomplish a good static characteristic and still maintain good stability margins and a more predictable behaviour would be to increase the size of the valve. However, a large solenoid force would be required and a large solenoid would not be practical and would be very slow. If the intention is to control the valve with an additional control loop, the static characteristic of the valve would be of less importance. The valve could instead be designed to use a smaller and faster solenoid.

The discharge coefficient compensates for the fact that the actual cross-sectional area of the stream is different to the area opening of the valve. The

Figure 5.41 Measured pressure (solid) and calculated flow force (dashed) against the flow at 70% duty cycle.

model of the discharge coefficient will also contain the uncertainties regarding the modelling of the area opening. There are uncertainties regarding the possible underlap or overlap of the spool and the gap between the spool and the spool body. The flow path of the leakage is also unknown but assumed to follow a certain direction and this will also affect the shape of the discharge coefficient. Nevertheless, the discharge coefficient clearly shows how the flow transfers from laminar in the gap region towards turbulent flow. The shape of the curve is also similar to the literature, Merritt (1967).

The effect of the solenoid on the loop gain response has been ignored in the analysis. There are two effects that probably have the most impact. One is the induced electromotive force, emf, that will change the current as the magnet core moves. A change in current is also a change in force. A change in force with the movement results in a damping. The other effect comes from the magnet core being surrounded by oil, which will increase the viscous friction as it moves in the magnet housing and as a result the damping is increased. The total viscous friction of the simplified model, which is tuned to obtain a good match with the actual valve, contains all effects that contribute to the damping.

6

Electrohydraulic closed-centre steering by pressure control

In this chapter the electrohydraulic closed-centre power steering system is analysed for the intended applications. These are Boost curve control and position control. Boost curve control is the inherent solution of the open-centre power steering system and is also widely used for e.g. electric power steering systems. It is therefore the intention to keep this strategy for the electrohydraulic power steering system in this work in order to, as far as possible, maintain the fundamental behaviour of the original system. Position control refers to the rack motion and is related to autonomous driving.

A description of the system is presented in section 6.1. A generalised analysis of the closed-centre steering system, with emphasis on stability for Boost curve control, is performed in section 6.2. This is done with high-response servo valves. The analysis shows the implications of a Boost curve controlled electrohydraulic power steering system with electronic closed-centre valves. A possible way of how this can be addressed is also presented. The ideas from that analysis are then extended to include the system with pressure control valves in chapter 6.4. The results from chapter 5 and chapter 3 are combined to derive a complete model of the closed-centre steering system. This is done by replacing the model of the open-centre hydraulic system in the steering system model in chapter 3, with the model of the pressure control valve in chapter 5. An external controller for controlling the performance of the valves is derived in section 6.3 and how the system can be utilised for position control is presented in section 6.5.

6.1 System description

There are several ways in which the system can be realised depending on the requirements of and the intentions with the system. This is not analysed in detail in this work but only discussed briefly. One way to realise the system is to replace the open-centre system with the pressure control valves from chapter 5. Schematically, the system could look as shown figure 6.1. The supply system needs to be modified to supply a constant pressure rather than a constant flow. This can be done in several ways but preferably by charging an accumulator to the required pressure by controlling the flow output of the pump. An electric motor driven pump or a pump with a shunt-valve can be used. In the remainder it is assumed that the pressure can be delivered.

There is no connection between the torsion bar and the valves as in the original system. The torque applied by the driver needs to be measured instead. This requires an additional sensor to be installed on the steering column. The existing torsion bar could also be used by attaching strain gauges. This was done to the test rig in chapter 7. Additional pressure sensors are also required, one for each chamber of the assistance cylinder.

Figure 6.1 Schematic principle of closed-centre power steering with pressure control valves.

It could also be possible to realise the system as an add-on to the existing power steering system. Under normal circumstances the open-centre valve is deactivated by shutting off the supply and tank lines and activating the closed-centre system. In a case of a failure the chambers of the assistance cylinder could be connected to the open-centre valve instead, providing a solution with fallback.

6.2 Stability analysis of the electrohydraulic CC-system by using servo valves

Figure 6.2 Illustration of the steering system with servo valves.

This section addresses the stability properties of the electrohydraulic power steering system with closed-centre valves. The purpose is to keep the analysis generalised and the results are applicable to any configuration of such a system, like the system with pressure control valves presented at the beginning of this chapter. In the system studied in this section, the hydraulic system of the original system is replaced by fast-acting servo valves from Moog. Four 4/3-servo valves are used. They are connected so that they individually control the meter-in and meter-out flow of each chamber of the assistance cylinder. The system can schematically be illustrated as in figure 6.2. Since the servo valves have a very high bandwidth, the system becomes more flexible which facilitates the study.

In chapter 3 the conventional power steering system was analysed with respect to stability. It was seen how the gain of the position control loop is defined by the Boost curve and how this affected stability. The pressure loop was also analysed, where it was seen how the response changed with working point, from being fast at low load pressure to become slow at high load pressure. The pressure response also affects the steering system stability. This section presents how the knowledge about the original system can be utilised in the control of the closed-centre system and how this affects stability and performance. The closed-centre system is analysed both in the frequency domain and the time domain and compared to the original system. Both a nonlinear and a

linear model of the system are derived for this purpose. The test rig, which is explained in chapter 7, was equipped with the servo valves and used to verify the results seen from simulation. As in previous analyses, the supply system is not within the scope of this work. It is assumed that a constant pressure is available.

6.2.1 System modelling

The nonlinear and linear models are presented in this section. Since the mechanical system is the same as the original system, the modelling procedure for the mechanical subsystem is referred to in chapter 3, section 3.1.1, equations 3.1 and 3.2. The idea is to study the system over the entire pressure range. This means that a heavy load is required, e.g. a parking scenario. The load is set to be a linear spring with the displacement of the rack, with stiffness C_{rw} . It can be seen as a tyre deflecting during stand-still, before it starts slipping. An accurate load model is not necessary for the scope of this study and the spring stiffness is set to an arbitrary number in order to ensure that the steering system works in the entire pressure region.

Nonlinear model

Figure 6.3 Representation of the pressure control system with servo valves for one chamber of the assistance cylinder.

Since both chambers of the assistance cylinder are controlled independently and are equal, it is sufficient to study one chamber. The system is illustrated in figure 6.3. The flows through the meter-in and the meter-out orifices are described by equations 6.1 and 6.2, respectively. The pressure derivative of the volume is described by equation 6.3 and is also affected by the motion of the

assistance cylinder x_{rw} .

$$q_{in} = C_q A_1 \sqrt{\frac{2}{\rho} (p_s - p)} \quad (6.1)$$

$$q_{out} = C_q A_2 \sqrt{\frac{2}{\rho} p} \quad (6.2)$$

$$q_{in} - q_{out} = A_p \dot{x}_{rw} + \frac{V}{\beta} \dot{p} \quad (6.3)$$

The opening area of each orifice is assumed to be linear to the spool displacement x_v with an underlap according to equations 6.4 and 6.5, where w is the area gradient and u is the underlap.

$$A_1 = w x_{v1} + u \quad (6.4)$$

$$A_2 = w x_{v2} + u \quad (6.5)$$

The valve spool motion is modelled with a second order dynamic as in equation 6.6, where U is the input signal, ω_v is the response and δ_v is the damping coefficient.

$$\ddot{x}_v = U \omega_v^2 - 2\delta_v \omega_v \dot{x}_v - \omega_v \dot{x}_v \quad (6.6)$$

A servo valve has an internal position control of the spool. In order to capture this behaviour and better resemble the real system a PI-controller is used to control the spool position according to equation 6.7, where the error $e(t)$ is the difference between the desired spool position and the actual position according to equation 6.8. The controller gains K_p and K_i are tuned for each valve to fit the measurements.

$$U(t) = K_p e(t) + K_i \int e(t) dt \quad (6.7)$$

$$e = x_{vref} - x_v \quad (6.8)$$

Linear model

The system equations are linearised and transformed into the Laplace domain. The flow equations and continuity equation are shown in equations 6.9 to 6.11, where s is the Laplace operator and Δ indicates a change in state variable.

$$\Delta q_{in} = K_{q1} \Delta x_{v1} + K_{c1} (\Delta p_s - \Delta p) \quad (6.9)$$

$$\Delta q_{out} = K_{q2} \Delta x_{v2} + K_{c2} (\Delta p) \quad (6.10)$$

$$\Delta q_{in} - \Delta q_{out} = A_p \Delta x_{rw} s + \frac{V}{\beta} \Delta s \quad (6.11)$$

The meter-in and meter-out valves are controlled simultaneously as shown in equation 6.12. This results in equations 6.9 to 6.11 being able to be formed into equation 6.13, with $\Delta q_l = A_p \Delta x_{rw}$ and hydraulic capacitance as $C_s = \frac{V}{\beta}$.

$$\Delta x_{v_2} = -\Delta x_{v_1} = -\Delta x_v \quad (6.12)$$

$$\begin{aligned} (K_{q_1} + K_{q_2}) \Delta x_v + K_{c_1} \Delta p_s - \Delta q_l = \\ (K_{c_1} + K_{c_2}) \Delta p + \frac{V}{\beta} \Delta p_s \\ \Rightarrow K_q \Delta x_v + \Delta q_l = (K_c + C_s s) \Delta p \end{aligned} \quad (6.13)$$

The derivatives are defined according to equations 6.14 to 6.19, where index 0 indicates the working point around which the system is linearised.

$$K_{q_1} = C_q w \sqrt{\frac{2}{\rho} (p_{s0} - p_0)} \quad (6.14)$$

$$K_{q_2} = C_q w \sqrt{\frac{2}{\rho} p_0} \quad (6.15)$$

$$K_{c_1} = \frac{C_q w x_{v_1 0} \sqrt{\frac{2}{\rho}}}{2 \sqrt{(p_{s0} - p_0)}} \quad (6.16)$$

$$K_{c_2} = \frac{C_q w x_{v_2 0} \sqrt{\frac{2}{\rho}}}{2 \sqrt{p_0}} \quad (6.17)$$

$$K_q = K_{q_1} + K_{q_2} \quad (6.18)$$

$$K_c = K_{c_1} + K_{c_2} \quad (6.19)$$

The spool motion is assumed to behave like a second order system for the linearised analysis according to equation 6.20.

$$\Delta x_v = \frac{\Delta x_{v_{ref}}}{\frac{s^2}{\omega_v^2} + 2 \frac{\delta_v}{\omega_v} s + 1} \quad (6.20)$$

6.2.2 Control

In order to control the static characteristic, the controller needs to be of integrator type. The valve dynamic also introduces some phase shift and in order to compensate for this a derivative part is also introduced in the controller. The system is strongly non-linear and to improve the performance, the controller is adaptive in that it compensates for the varying system dynamics. This can be seen from equation 6.13. The controller is implemented as a lead-lag filter as shown in equation 6.21, taking the difference between the reference pressure and measured pressure as input. A similar approach was used by Ramdén

(1999).

$$G_{ctrl}(s) = \frac{K_{p0}}{\hat{K}_q} \left(\frac{\frac{V}{\beta \hat{K}_c} s + 1}{\frac{V}{\beta \hat{K}_c} s + \gamma} \right) \left(\frac{\frac{1}{\omega_v} s + 1}{\frac{\alpha}{\omega_v} s + 1} \right) \quad (6.21)$$

The estimated system variables $\hat{K}_q$ and $\hat{K}_c$ are calculated according to the expression equations 6.14 to 6.19, and γ and α are tuned for good performance. In the subsequent analyses, it is assumed that $\hat{K}_q$ and $\hat{K}_c$ are perfectly estimated. With equations 6.13, 6.20 and 6.21, the pressure control loop for the closed-centre system with servo valves can be derived in the block diagram shown in figure 6.4. This loop constitutes the inner control loop of the power steering system, as will be shown.

Figure 6.4 Block diagram representation of the pressure control loop with servo valves.

The advantage of incorporating the system variables in the controller is seen from studying the loop gain transfer function of the pressure control loop. If γ and α are set to zero the loop gain transfer function is defined by equation 6.22. The working point dependent variables are cancelled and a pure integrator remains, ensuring the convergence to zero of the error.

$$A_g(s) = K_{p0} \frac{\frac{1}{\omega_v} s + 1}{\frac{s^2}{\omega_v^2} + \frac{2\delta_v}{\omega_v} s + 1} \frac{1}{C_s s} \quad (6.22)$$

The closed loop transfer function can also be derived from the block diagram. The effective force from the pressure that acts on the cylinder piston is the difference between the two chambers. By defining the load pressure $p_L = p_1 - p_2$, the transfer function can be defined as in equation 6.23. The reference pressure is changed for one chamber at a time. However, the pressure is affected by the motion of the piston, which occurs for both chambers simultaneously.

$$\Delta p_L = \Delta p_1 - \Delta p_2 = \underbrace{\frac{A_g(s)}{1 + A_g(s)}}_{G_c(s)} \Delta p_{ref} - \left(\underbrace{\frac{1}{(K_c + C_s s)(1 + G_o)}}_{\text{chamber 1, } = G_{q1}} + \underbrace{\frac{1}{(K_c + C_s s)(1 + G_o)}}_{\text{chamber 2, } = G_{q2}} \right) \Delta q_l \quad (6.23)$$

For a comparison between the original system and the closed-centre system to be valid, it is necessary to control the closed-centre system in a similar fashion to the original system. The original system is controlled by the Boost curve. The reference pressure of the servo valve closed-centre system on a linear form is therefore expressed as shown in equation 6.24, with K_{qoc} and K_{coc} being the flow gain and flow-pressure coefficient for the open-centre system, respectively. The derivation of the linear Boost curve was shown in chapter 3. Two different Boost curves are used: the curve as in the original system and a flatter curve. Both are shown in figure 6.5. A pre-filter is also applied to shape the response. Two cases are evaluated. One is a pre-filter generating open-centre dynamic, that is the nonlinear dynamic model of the open-centre valve from chapter 3. The other is a low-pass filter with 50 Hz break frequency. For good performance in the test rig, the mean pressure was raised by 50 bar in each cylinder and an offset in spool displacement of the valve was applied. This does not affect the assistance pressure, but might generate more friction.

$$\Delta p_{ref} = G_{filter}(s) \frac{1}{K_{coc}} (K_{qoc} (\Delta \theta_{sw} - R_T \Delta x_{rw}) K_T - A_p \Delta x_{rw} s) \quad (6.24)$$

Figure 6.5 Boost curves used for control of the closed-centre system with servo valves. The solid line shows the curve as in the original system. The dashed curve is an alternative used for comparison.

6.2.3 System analysis

With the linearised equations 6.23, 6.24 and the mechanical system with equations 3.53, 3.54 and 3.56 from chapter 3, the block diagram of the system can

be derived, shown in figure 6.6. The result resembles the original system, with the position control loop. The difference is the contribution from the servo valves. The pressure control loop with transfer function G_c constitutes an inner control loop, where the position control loop is the outer loop. There is also a feedback from the rack movement, which affects the pressure loop. The gain $\frac{K_{qoc}}{K_{coc}}$ depends on which of the two Boost curves is used. The filters G_1 and G_2 depend on which dynamic pre-filter is applied. For the case where the dynamic is defined as in the original system, then $G_1(s) = G_{K_q}(s)$ and $G_2(s) = G_{K_c}(s)$, see equation 3.51. In the case where the pre-filter with fixed break frequency is used, then $G_1(s) = 1$ and $G_2(s) = \frac{1}{1 + \frac{s}{\omega_b}}$, where ω_b is set to 50 Hz. The mechanical resonance from the steering wheel and column, G_{sw} , and the rack, G_{rw} , are defined by equations 6.25 and 6.26, respectively. It should be noted that the spring load C_{rw} is a part of the mechanical transfer function and is not treated as a disturbance.

$$G_{sw}(s) = \frac{1}{J_{sw}s^2 + b_{sw}s + K_t} \quad (6.25)$$

$$G_{rw}(s) = \frac{1}{M_{rw}s^2 + b_{rw}s + C_{rw} + K_t R_t^2} \quad (6.26)$$

Figure 6.6 Block diagram of the hydraulic power steering system with servo valves.

From the block diagram the closed loop transfer function from an applied steering wheel angle applied by the driver to a corresponding rack position is defined by equation 6.27. This function can be compared to the original system, equation 3.57 in chapter 3. The dynamic is affected by the inner pressure control loop. The damping can here be controlled but depends on how well the pressure control loop can follow the desired pressure. The disturbance sensitivity of the pressure loop, defined by transfer functions G_{q1} and G_{q2} , also

affects the damping.

$$\frac{R_T X_{rw}}{\theta_{sw}} = \frac{K_T R_T^2 + G_c A_p K_T K_{qoc} R_T / K_{coc}}{M_{rw} s^2 + \left(b_{rw} + \left(G_{q1} + G_{q2} + \frac{G_c}{K_{coc}} \right) A_p^2 \right) s + C_{rw} + K_T R_T^2 + \frac{G_c A_p K_{qoc} K_T R_T}{K_{coc}}} \quad (6.27)$$

In order to study stability for the different cases the loop gain transfer function $A_g(s)$ is defined by equation 6.28.

$$A_g(s) = \frac{R_T K_T K_{qoc} A_p G_c G_{rw}}{K_{coc} (1 + G_{rw} A_p^2 (G_{q1} + G_{q2}) s) + A_p^2 G_c G_{rw} s} \quad (6.28)$$

The frequency response of the loop gain is shown in Fig. 6.7 for zero load flow and with the steering wheel in the centre position. The two cases are shown for a very high torque level at 3.9 Nm, which corresponds to the highest gain level. In the first case the pre-filter of the controller is set to have the same first order dynamic as the open-centre valve. This is the dashed line. This results in a positive amplitude margin and a stable system. In the second case, the pre-filter is set at 50 Hz. This is much higher than the first case, where the response was about 5 Hz. This results in a negative amplitude margin and an unstable system.

Figure 6.7 Frequency plot of the open loop of the closed-centre power steering system. Solid curve represents with a pre-filter at 50 Hz, and dashed curve represents with a pre-filter with open-centre dynamic.

6.2.4 Simulation

The system is simulated in order to study its behaviour in the time domain. The basic model is the same as used when simulating the original system. The hydraulic system is, however, replaced with the model of the servo valves. To obtain a better resemblance to the actual system, i.e. the test rig, the load cylinder with force controller is also simulated. That model is detailed in chapter 7. Parameter values of the simulation model are found in table A.3 in the appendix.

6.2.5 Results

The results in this section present first the results from the pressure controller. The second part presents the results from the simulation and measurements of the different test cases. All results from the test rig have been sampled at a rate of 10 kHz.

Pressure control response

Figure 6.8 shows the responses of the pressure steps for the two different pre-filters used as well as without pre-filter. The top plot is without pre-filter. A high gain was chosen, which generated a fast response but some oscillations and long settling time. The time to reach 63% of the final value is about 2.2 ms. The settling time within 2% is about 16.5 ms. The oscillations can be adjusted for with the pre-filter, which is shown in the bottom plot. The faster curves are with pre-filter at 50 Hz and the slower curves with the pre-filter set to open-centre dynamic. With 50 Hz pre-filter the time to reach 63% of the final value is about 7 ms. The model, shown as dashed lines, is able to predict the behaviour of the pressure loop sufficiently well compared to measured results, shown as solid lines. The figure also shows the reference curves for each case, shown as dashed-dotted lines. There is a clear difference in bandwidth between the two cases with pre-filter. The time lag between the reference and actual pressure is not of importance here, but merely illustrates the time lag introduced by the valves.

Steering system behaviour

Figure 6.9 shows a comparison between the test rig and simulation for the open-centre system and closed-centre system with three cases. Figure 6.9a shows the assistive pressure, with a solid curve for measurement and a dashed curve for simulated results. Figure 6.9b shows the measured and simulated torsion bar torque. The top plots are for the open-centre system. The second top plots are for the closed-centre system with the pre-filter set to open-centre dynamic. The system is stable. The third top plots are for the system with 50 Hz pre-filter. As the pressure increases, instability finally occurs and the system returns to

Figure 6.8 Comparison between measured (solid) and simulated (dashed) pressure responses. The dashed-dotted line is the reference pressure for the respective case. In the top plot, the pressure response without pre-filter is shown. The lower plot shows the pressure response with pre-filter, where the faster curves represent the 50 Hz pre-filter and the slower curves the dynamic open-centre model as pre-filter.

a stable state as the pressure decreases again. The lower plots are the closed-centre system with 50 Hz pre-filter and the flatter Boost curve, shown as an alternative Boost curve in figure 6.5. The steering wheel angle was manually applied with an amplitude of 22-25° and no faster than 150°/s. In all cases, the simulation results also show the same behaviour.

6.2.6 Discussion

The results clearly show the behaviour and design aspects of the power steering system with electronically controlled closed-centre valves. While there is a fixed relation between the Boost curve and the response of the pressure control for the original system, there is no such relation in this case. Since the pressure controller can not be made fast enough to be negligible for the outer position control loop, its response must be designed and adapted to the rest of the system in order to preserve stability. The three bottom plots in figure 6.9, which are results from the closed-centre system, illustrate this. Comparing the second and third plots, where the system had the same static Boost curve but different pre-filter, it is seen that the system becomes unstable with the faster response. This might not be intuitively since a fast response might be necessary

Figure 6.9 Comparison between measured (solid) and simulated (dashed) pressure (a) and torsion bar torque (b). The top plots are the open-centre system while the other plots are the closed-centre system. The second top plots are with open-centre dynamic as pre-filter. The third top plots are with 50 Hz pre-filter. The bottom plots are with 50 Hz pre-filter and flat boost curve.

for a good steering feel. However, as touched on also in chapter 3, the response from the original valve is also slow at high load pressure and it is a way to ensure stability for Boost curve control. There is a compromise though. The response can not be too slow. Too slow response might generate poor steering feel and reduce the phase margin to such an extent that stability becomes an issue from that point of view. In the bottom plot, the Boost curve is much flatter. Also less damping was provided compared to the previous cases. Even though the response is faster here, stability is not an issue. Of course, the steering wheel torque is higher, but whether or not this is feasible or how the steering feel is affected is another discussion and not within the scope of this work. The result shows that everything is linked together. How the response of

the pressure control loop should be shaped depends on the system parameters, what Boost curve is used and the valve itself.

The two top plots should show similar results. A small discrepancy is, however, observed. The closed-centre system has a small tendency to oscillate here. There are probably several reasons for this. One might be the additional phase shift of the servo valves and the pressure control loop. This is shown in figure 6.8. Phase shift is also introduced from filtering of the rack position. The rack position is used to calculate the load but also to derive the velocity for calculation of the reference pressure. The derivation of the velocity requires quite a low break frequency of the filter. The closed-centre system also has slightly more friction in the assistance cylinder since the sum of the pressures is higher, as explained previously.

The method and control strategy for this system will be applied to the system with pressure control valves in the following sections.

6.3 Controlling the pressure

This section deals with an external software controller for the commercial pressure control valve modelled and analysed in chapter 5, section 5.2, and relates to research question 2). The reason for designing a controller is to increase and shape the performance, ensure steady-state error convergence to zero and to handle the effect of varying system properties. The control of fluid power systems is described by Eriksson (2010) as challenging due to several nonlinearities being present. These are nonlinear flow-pressure characteristic, hysteresis, dead-band and saturation. Classical PID control could still be effective if the performance specification is not too demanding (Burrows (2000)). An indirect adaptive controller for hydraulic servosystems was developed by Plummer and N. Vaughan (1996) in order to handle the nonlinearities. The controller responded well to changes in the plant parameters. In a review article by Edge (1997), several different control strategies are outlined. These include, among others, PID control, state-feedback control, adaptive control, nonlinear correction and robust control. A comparison of sliding mode control with state feedback control and PID control of a proportional valve was conducted by J. B. Gamble and N. D. Vaughan (1996). The sliding mode controller showed best performance and was derived with least design effort. It was also more robust against disturbances compared to the other approaches.

The results from chapter 5 are used to formulate and design the controller. The frequency response of the valve, shown in figure 5.40, revealed both a varying dynamic behaviour with working point and a low resonance frequency. In order to increase performance, the controller needs to address this. A pole placement approach is used by state feedback. Two controllers are designed and evaluated in simulation: a linear state feedback controller and a nonlinear state feedback controller. It is interesting to see how severe the variable plant dynamic is to the linear controller and what performance benefits there are

with a nonlinear controller. In simulation it is assumed that all states are measurable. In the test rig only two states are measurable: current and pressure. The other two states, spool velocity and position, are estimated in a simplified manner, where the simulation model runs in real time. It is then assumed that the calculated states are close enough to the actual states. This assumption is based on the accuracy of the model and is verified in the test rig.

6.3.1 Linear state feedback controller

There are several examples where linear control design has been successful for fluid power systems. Gunnarsson and Krus (1993) used LQG control design for position control of a hydraulic crane. They showed that with a simple linearised model it was possible to derive a controller that results in a well damped behaviour of the crane. The LQG method was also used for the velocity control of a hydrostatic transmission by Lennevi and Jan-Ove Palmberg (1995), where the effect of a variable system dynamic was also discussed. Eriks-son, Marcus Rösth, and Jan-Ove Palmberg (2009) used a LQ-design approach for MIMO-control (multiple-input multiple-output) of an individual metering system. Parts of the nonlinear behaviour were cancelled out by hardware by using compensators. It was shown that a simplified model was sufficient for the control design. Berg and Ivantysynova (1999) derived a linear cascaded controller with the LQG-LTR method for a secondary controlled hydrostatic drive. A linear inversion model of the servo valve was part of the controller to reach a high bandwidth of the swash plate control.

The idea here with this controller is to design it for the most critical working point and then verify the performance over the entire working region through simulation of the nonlinear model with controller. The design approach requires the model to be expressed in state-space form according to equation 6.29, see e.g. Glad and Ljung (2006), where u is the input signal, v is the disturbance and y is the output signal. The input signal is the voltage to the solenoid and the disturbance is the load flow.

$$\begin{aligned}\dot{\bar{x}} &= A\bar{x} + Bu + Fv \\ y &= C\bar{x}\end{aligned}\tag{6.29}$$

With the linearised system equations of the valve in chapter 5, section 5.2, the state-space vector is defined by equation 6.30. The matrices A , B , C and F are defined by equations 6.31 to 6.34.

$$\bar{x} = \begin{pmatrix} \Delta i \\ \Delta v \\ \Delta x \\ \Delta p \end{pmatrix}\tag{6.30}$$

$$A = \begin{pmatrix} -R/L & 0 & 0 & 0 \\ K_{fi}/m & -B/m & -K_e/m & -A_c/m \\ 0 & 1 & 0 & 0 \\ 0 & \beta A_c/V & \beta K_q/V & -\beta K_c/V \end{pmatrix} \quad (6.31)$$

$$B = \begin{pmatrix} 1/L \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (6.32) \quad F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \beta/V \end{pmatrix} \quad (6.33)$$

$$C = (0 \quad 0 \quad 0 \quad 1) \quad (6.34)$$

In order to make the error convergence to zero, an integrator must be introduced. This is done by adding an extra state to the state-space vector, which results in the system described by equation 6.35 or 6.36, where r is the reference signal.

$$\begin{aligned} \begin{bmatrix} \dot{\bar{x}} \\ \dot{w} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} F \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \\ y &= [C \quad 0] \begin{bmatrix} \bar{x} \\ w \end{bmatrix} \end{aligned} \quad (6.35)$$

$$\begin{aligned} \dot{\bar{x}}_i &= A_i \bar{x}_i + B_i u + F_i v + B_r r \\ \Rightarrow y &= C_i \bar{x}_i \end{aligned} \quad (6.36)$$

The control scheme can now be formulated as shown in figure 6.10, where L is a four dimensional row vector and K_i is a constant. A pre-filter is also added to the reference value in order to further shape the response, according to equation 6.37, where ω is the break frequency. It is implemented to avoid unnecessary phase shift for higher frequencies.

$$G_{pf}(s) = \frac{1 + \frac{0.01s}{\omega}}{1 + \frac{s}{\omega}} \quad (6.37)$$

The idea is to shape the response with the pre-filter so as to obtain the same behaviour as the open-centre valve, that is, fast at low load pressure and slow at high load pressure. As discussed in section 6.2, the response must be tuned with respect to the circumstances. In the evaluation of the controller the pre-filter is set with a break frequency of 100 Hz at low pressure with a decline to 5 Hz at 90 bar. As seen from the analysis of the original system in chapter 3, the valve adds damping to the motion of the rack. The disturbance flow q_l in figure 6.10 comes from the movement of the rack. If the controller is set to cancel the disturbance, the contribution to damping from the valve is also removed. This is undesirable. In addition to affecting the steering feel, damping is needed to increase the stability margins of the steering system position control loop. The reference value is therefore affected by the disturbance flow. For a positive flow the reference pressure is set to increase and for a negative flow it is set to

Figure 6.10 Linear state-space feedback control scheme.

decrease. The change is linear to the flow with an arbitrary constant for the evaluation of the controller.

The controller is designed for the working point with highest gain and slowest resonance, that is, point 5 in figure 5.40 in chapter 5. The L-vector and gain K_i are calculated with the help of MATLAB CONTROL SYSTEMS TOOLBOX by placing the poles of the system at a desired location. Manually selecting the poles gives the freedom to shape the response. One way of understanding where to place the poles is to look at the poles of the plant. They naturally vary with working point. The poles for the selected design point are defined in table 6.1. The pole location at zero corresponds to the integrator. The system is then dominated by a second order dynamic with very low damping and a first order dynamic that is somewhat faster. The idea is to keep the dynamic structure of

0
-8247.5
-15.5 +77i
-15.5 -77i
-54.8

Table 6.1 Eigenvalues for the selected design point of the linearised plant.

the plant but move the poles into a faster and more damped region. The poles of the closed loop system are placed according to table 6.2.

-600
-10000
-250 +10i
-250 -10i
-500

Table 6.2 Pole locations of the closed loop system.

When using the pressure control valves in an application it can be useful to

have the transfer function of the closed loop response. For Boost curve control of the power steering or position control, which are described later in sections 6.4 and 6.5, respectively, a change in reference pressure as well as a change in load flow can be present simultaneously. A change in load flow would come from the movement of the steering rack. The pressure response can be defined as in equation 6.38, with the transfer function from a reference pressure to the pressure, $G_c(s)$, as in equation 6.39 and from a disturbance flow to the pressure, $G_f(s)$, as in equation 6.40. Matrix I is the unity matrix. The dynamic of the closed loop is defined by the poles of the matrix A_{ci} in equation 6.41.

$$\Delta p = G_c(s)\Delta p_{ref} + G_f(s)\Delta q_l \quad (6.38)$$

$$G_c(s) = C_i(sI - A_{ci})^{-1}B_r \quad (6.39)$$

$$G_f(s) = C_i(sI - A_{ci})^{-1}F_i \quad (6.40)$$

$$A_{ci} = A_i - B_iL \quad (6.41)$$

Evaluation through simulation

The controller is evaluated with the help of simulating the nonlinear model for different working points. All simulations were conducted with a step time of $1 \cdot 10^{-4}$ s. Figure 6.11 shows the response with zero load flow for both high and low pressure. A step in reference pressure is applied from 20 bar to 30 bar, from 30 bar to 50 bar and back to 30 bar. From 20 bar to 30 bar the response has a small overshoot of 2.4%, 63% of the final value is reached within 16 ms and within 2% of the final value within 45 ms. From 30 bar to 50 bar the overshoot is 0.4%, 63% of the final value is reached within 19 ms and within 2% of the final value is reached within 40 ms. From 50 bar to 30 bar the undershoot is 1.2%, 63% of the final value is reached within 13 ms and within 2% of the final value is reached within 47 ms. For the last step the control signal is saturated. The response is slower at higher load pressure due to the setting of the pre-filter.

Figure 6.12 shows the result when a load flow is applied. The load flow is filtered with break frequency 2 Hz to be more realistic. At the lower pressure level the reference pressure change with flow gain is set to $1 \cdot 10^{10}$ Pa·s/m³ and for the higher pressure level to $5 \cdot 10^{10}$ Pa·s/m³.

Figure 6.11 Simulation results of pressure response with linear controller. Load flow is zero. a) and b) show the actual pressure (solid) and reference pressure (dashed) for low and high pressure levels, respectively. c) and d) show the corresponding control signals to the valve. e) and f) show the corresponding current through the solenoid.

Figure 6.12 Simulation results of pressure response with linear controller. A 2 Hz filtered load flow of 4 L/min is applied at 1.2 s and of -8 L/min at 1.6 s. a) and b) show the actual pressure (solid) and reference pressure (dashed) for low and high pressure levels, respectively. c) and d) show the corresponding control signals to the valve. e) and f) show the corresponding current through the solenoid.

6.3.2 Nonlinear state feedback controller

In the previous section, section 6.3.1, a linear controller was designed for the valve. Obviously, it is a compromise since the system dynamic varies with the working point and the same performance is not guaranteed over the entire working envelope. One way to handle the nonlinearities of the system is to use a nonlinear controller that transforms the system into a linear and controllable system using feedback. The method Exact linearisation via feedback, see e.g. Isidori (1995) or Glad and Ljung (2003), is investigated for the valve. When the linear system is derived, linear control design can be applied. Pole placement, as in section 6.3.1, is also used here to define the performance of the closed loop system.

Feedback linearisation for hydraulic systems has been investigated by several researchers. Hahn, Piepenbrink, and Leimbach (1994) derived a nonlinear controller for an electrohydraulic servo system. The efficiency and robustness against parameter uncertainties were showed through simulation. The same approach was also used by Chiriboga, Thein, and Misawa (1995) to control a load-sensing system. However, none of these authors have implemented the controller on a real system. This was done by Seo, Venugopal, and Kenné (2007). A feedback linearisation controller was derived and implemented for a rotational hydraulic drive. The controller was compared to a classic PID controller and it was shown that nonlinearities in the plant were effectively handled by the nonlinear controller, where the PID controller suffered from these variations.

The system is defined in the form shown in equation 6.42. The solenoid model is simplified to take only the nonlinear current-force relationship into account, while a linear inductor is considered. The hydro-mechanical subsystem is the simplified model from chapter 5. This gives the model for control design defined by equation 6.43.

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned}\tag{6.42}$$

$$\begin{aligned}\frac{di}{dt} &= -\frac{iR}{L} + g(x)u \\ \frac{dv}{dt} &= (F_s(i, x) - pA_c - Kx - Bv - f_0 \\ &\quad - F_{fin}(x, p_s, p) + F_{fout}(x, p))/m \\ \frac{dx}{dt} &= v \\ \frac{dp}{dt} &= (q_{in}(p_s, p, x) - q_{out}(p, x) + A_c v + q_l) \frac{\beta}{V} \\ g(x) &= 1/L \\ y &= p\end{aligned}\tag{6.43}$$

The system in equation 6.42 is transformed into the system in equation 6.44.

$$\begin{aligned}
 \dot{z}_1 &= z_2 \\
 \dot{z}_2 &= z_3 \\
 &\vdots \\
 \dot{z}_{r-1} &= z_r \\
 \dot{z}_r &= b(z) + a(z)u \\
 \dot{z}_{r+1} &= \psi_{r+1}(z, u) \\
 &\vdots \\
 \dot{z}_n &= \psi_n(z, u) \\
 y &= z_1
 \end{aligned} \tag{6.44}$$

Here, r is the relative degree and n the number of states. According to Isidori (1995), a system is said to have relative degree r at a point x_0 if the relations in 6.45 are fulfilled.

$$\begin{aligned}
 L_g L_f^k h(x) &= 0 \text{ for all } x \text{ in a neighborhood of } x_0 \text{ and all } k < r - 1 \\
 L_g L_f^{r-1} h(x_0) &\neq 0
 \end{aligned} \tag{6.45}$$

L is called the Lie derivative and is defined according to equations 6.46 and 6.47.

$$L_f = f_1 \frac{\partial}{\partial x_1} + \cdots + f_n \frac{\partial}{\partial x_n} \tag{6.46}$$

$$L_g = g_1 \frac{\partial}{\partial x_1} + \cdots + g_n \frac{\partial}{\partial x_n} \tag{6.47}$$

The new coordinates are defined as in equation 6.48.

$$z_i = \phi_i(x) \quad 1 \leq i \leq n \tag{6.48}$$

The coordinates are calculated with the help of the Lie derivatives as in 6.49.

$$\begin{pmatrix} \phi_1(x) \\ \phi_2(x) \\ \vdots \\ \vdots \\ \phi_r(x) \\ \phi_{r+1}(x) \\ \vdots \\ \vdots \\ \phi_n(x) \end{pmatrix} = \begin{pmatrix} h(x) \\ L_f h(x) \\ \vdots \\ \vdots \\ L_f^{r-1}(x) \\ \chi_{r+1}(x, u) \\ \vdots \\ \vdots \\ \chi_n(x, u) \end{pmatrix} \quad (6.49)$$

In this case, the relative degree is equal to the number of states and no extra functions except the Lie derivatives are needed. By setting the control signal u as in equation 6.50, with input signal v , the transformed system becomes as in 6.51. This system is linear and controllable.

$$u = \frac{1}{a(z)}(-b(z) + v) \quad (6.50)$$

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ &\vdots \\ &\vdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= r \\ y &= z_1 \end{aligned} \quad (6.51)$$

The Lie derivatives for the system in 6.43 are calculated using MATHEMATICA, resulting in some cumbersome expressions that are not listed here. The first step is to control the relative degree of the system. The system in 6.43 has a relative degree 3, that is less than the number of states. The transformation into linear coordinates is therefore in the form in 6.44. The system is only partially linear. There are different approaches to overcome this. One is to find another output, $c(x)$, so that the relative degree of the system becomes n . That output should fulfill 6.52.

$$L_g L_f^j c(x) = 0, \quad j = 0, \dots, n-2 \quad L_g L_f^{n-1} c(x) \neq 0 \quad (6.52)$$

Another way, which is chosen here, is simply to keep the structure in 6.44. As mentioned earlier, the system is partially linear with a linear subsystem of dimension r , in this case 3, which is responsible for the input-output behaviour

and a possibly nonlinear subsystem of dimension one. The controller is then evaluated with the nonlinear model of the valve derived in chapter 5. The feedback control is designed to place the poles of the linear system at desired locations. The control signal u remains as in equation 6.50. By using the original state vector x and the control law to place the poles, the control signal can be expressed as in equation 6.53, where K is the feedback gain vector.

$$u = \frac{-L_f^3 h(x) + \sum_{i=0}^2 K_i L_f^i h(x)}{L_g L_f^2 h(x)} \quad (6.53)$$

The new transformed system to be controlled can also be expressed in state space form, as in 6.54, with matrices A_z , B_z and C_z defined in equations 6.55, 6.56 and 6.57.

$$\begin{aligned} \dot{z} &= A_z z + B_z v \\ y &= C_z z \end{aligned} \quad (6.54)$$

$$A_z = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (6.55) \quad B_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (6.56)$$

$$C_z = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad (6.57)$$

Just as for the linear controller, an integrator is needed in order to handle the steady-state error. This is done in the same way by adding an extra state, w . The complete system becomes as in 6.58 or 6.59, with the reference value r .

$$\begin{aligned} \begin{pmatrix} \dot{z} \\ \dot{w} \end{pmatrix} &= \begin{pmatrix} A_z & 0 \\ C_z & 0 \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix} + \begin{pmatrix} B_z \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r \\ y &= \begin{pmatrix} C_z & 0 \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix} \end{aligned} \quad (6.58)$$

$$\begin{aligned} \Rightarrow \quad z_i &= A_{zi} z_i + B_{zi} v + B_r r \\ y &= C_{zi} z_i \end{aligned} \quad (6.59)$$

The system can be described by the block diagram in figure 6.13. Since the system is linearised, it can be presented in the Laplace domain. The same pre-filter, $G_{pf}(s)$, as in the linear case, is used. The reference value is Δp_{ref} and the output is the pressure Δp . The integrator gain C_i is added to the closed loop. The feedback gains K_i and C_i are calculated using the MATLAB CONTROL SYSTEMS TOOLBOX, based on the system in 6.59 and placing the poles as in table 6.3. The locations of the poles are chosen to match the response of the linear controller in order to compare the two controllers.

Implementation of simulation model

When implementing the controller, it is required to limit the allowed range of the spool stroke. Since the controller involves the inverse of the model, a

Figure 6.13 Block diagram representation of the feedback linearisation system. The dashed rectangle marks the linearised plant.

-220
-300
-350
-500

Table 6.3 Pole locations of the linearised closed loop system.

spool position of zero could lead to numerical issues. The controller therefore only accepts positive spool displacements and the minimum value was limited to above zero. The controller was also divided into a part for positive displacement and negative displacement, since the model differs depending on the spool position. Switches were used to change between the two controllers.

Evaluation through simulation

The controller is evaluated with the help of simulating the nonlinear model at the same working points as for the linear controller. All simulations were conducted with a step time of $1 \cdot 10^{-6}$ s. Figure 6.14 shows the response with zero load flow for both high and low pressure. A step in reference pressure is applied from 20 bar to 30 bar, from 30 bar to 50 bar and back to 30 bar. No overshoot is seen this time. From 20 bar to 30 bar the response reaches 63% of the final value within 16 ms and within 2% of the final value within 32 ms. From 30 bar to 50 bar, 63% of the final value is reached within 19 ms and within 2% of the final value is reached within 40 ms. From 50 bar to 30 bar, 63% of the final value is reached within 16 ms and within 2% of the final value is reached within 30 ms. As for the linear controller the control signal is saturated for the last step. The same pre-filter is also used, with slower dynamic at higher load pressure.

Figure 6.14 Simulation results of pressure response with the nonlinear controller. Load flow is zero. a) and b) show the actual pressure (solid) and reference pressure (dashed) for low and high pressure levels, respectively. c) and d) show the corresponding control signals to the valve. e) and f) show the corresponding current through the solenoid.

Figure 6.15 Simulation results of pressure response with the nonlinear controller. A 2 Hz filtered load flow of 4 L/min is applied at 1.2 s and of -8 L/min at 1.6 s. a) and b) show the actual pressure (solid) and reference pressure (dashed) for low and high pressure levels, respectively. c) and d) show the corresponding control signals to the valve. e) and f) show the corresponding current through the solenoid.

6.3.3 Discussion on controller strategy

Both the linear and the nonlinear controller provide for a good response of the pressure. The pre-filter was set to generate a faster response at low pressure and a slower response at high pressure. This is in accordance with the results from section 6.2. This is also advantageous since at low pressure there is more control energy available to boost the performance of the valve.

There is a compromise when designing the linear controller. It must be stable for all working points and is therefore designed for a point near the worst point from a stability view. The performance is therefore also as designed for at this point. At other points the performance might deteriorate. This is seen in figure 6.11 where there are small overshoots at the lower pressure levels. These are, however, very small and should not cause any issues. This will be seen in sections 6.4 and 6.5, where the controller is used in the applications. The nonlinear controller, on the other hand, generates a linear behaviour over the entire working range. This is at the cost of slightly more control energy. The nonlinear controller is also far more complicated and the expressions of the required derivatives are very large. This could be problematic when implementing the controller in a real-time computer. A much smaller time step is also necessary for the simulation and would probably also be the case in real time. The nonlinear controller also requires full knowledge of the plant and could therefore be more sensitive to parameter variations. In reality there is always some discrepancy between the model and the real plant. The linear controller should be more robust against parameter variations. However, this should be further investigated by alternating system parameters in the simulation. It could also be worth investigating possible simplifications of the nonlinear controller in order to facilitate implementation and to increase the step time.

The reason the linear controller is sufficient might be because the plant dynamic does not vary enough to cause any problems. The reason for this might be the valve's internal hydromechanical feedback loop. One way of improving the performance of the linear controller could be to use gain scheduling in order to meet the variable plant dynamic. With the derived model, the plant dynamics are known and can be cancelled out to some extent. The nonlinear controller also incorporates the load flow and handles disturbances somewhat better than the linear controller. One way to improve disturbance rejection of the linear controller is to make the reference pressure react to the movements of the load cylinder through a high-pass filter, but is anyway limited to the mechanical response of the valve.

Both controllers run a risk of saturating the control signal. This was not an issue in the test cases but could cause problems in other cases since none of the controllers is designed to handle saturation. The approach is instead to avoid, as far as possible, saturation of the control signal. One way to handle saturation would be to implement an anti-windup algorithm to prevent the integrative part of the controller increasing the control signal when the valve

is saturated.

Both controllers are sensitive to disturbances. This is because the disturbance acts directly on the output derivative. The nonlinear controller handles this better and also has to work less. The disturbance is part of the controller and can be estimated from the rack velocity. How the effect of the disturbance affects the application is investigated in sections 6.4 and 6.5. It was seen already in the generalised analysis of the close-centre power steering system that the disturbance affects the damping and should therefore not cause any control issues. It remains, however, to be seen how it affects the steering feel. The spikes seen in the control signal of the nonlinear controller comes from the fact that the controller is in practice an inverted model and when the spool position goes towards zero the control signal will increase.

The chosen control strategy is the linear one. The performance gain from the nonlinear controller, at this point, is not worth the effort of implementing it. The linear controller is much simpler and easier to implement and its performance is good over the entire working region. In the subsequent sections the controller is referred to as the linear controller.

6.4 Boost curve control with pressure control valves

Boost curve control of the power steering system is the most common form of control. It is the fundamental heritage of the system layout of the original system and is also widely used for electric power steering systems. In order to keep the structure and behaviour from the original system as far as possible, it is interesting to use the same control strategy for the closed-centre power steering system.

In this section the pressure control valves with the developed controller is investigated for use with Boost curve control of the closed-centre power steering system and the work relates to research question 4). The knowledge gained from the generalised analysis in section 6.2 will also be adapted in this case. The main idea is to keep the commercial valve from chapter 5 and limit the operating range of the power steering system to the allowed ranged of the valve for the sake of the analysis. The results from this section could then easily be extended to involve a valve with a symmetric flow range. The main analysis is aimed at investigating under what circumstances the valves can be used for Boost curve control. This includes how the stability of the power steering loop is affected, as well as closed loop performance. The work in this section relates to research question 4) in chapter 1.

The control scheme can be illustrated as shown in figure 6.16. The steering wheel torque is measured. This can be done in different ways, e.g. by adding a torque sensor or making use of the existing torsion bar. The latter is chosen here where the torsion bar torque is available. In the test rig (chapter 7) it

has been realised by attaching strain gauges to the torsion bar. The measured torque is fed to the Boost curve, which calculates the required assistive pressure. Two pressure control valves are needed that are controlled individually. The linear controller from section 6.3.1 is applied to each valve. From a control perspective, the pressure control constitutes an inner control loop and the position control of the steering system constitutes an outer control loop. This was seen from the generalised analysis in section 6.2 and will also be studied later in this section. Because of the Boost curve, the outer loop is already defined and limits what can be done. The degrees of freedom are in the inner loop, which must be tuned to reach the desired performance. The implications of this will be shown in the analysis.

Figure 6.16 *Illustration of the control scheme with Boost curve.*

The investigation is divided into several steps. The first step is to see if the original Boost curve, measured from the test rig, can be used and under what conditions. If the valves are fast enough, tuning with a pre-filter should be sufficient to ensure stability, as was shown in section 6.2. However, the servo valves used in section 6.2 are much faster than the pressure control valves (compare this with the results from section 6.3.1). If the valves do not meet the required response, it is also possible to increase damping to ensure stability, but

this will also alter the steering feel. The effect of this will be analysed. Hence, there are two parameters to tune, pre-filter break frequency and damping. The next step is to tune the Boost curve instead. The aim is to keep the damping as close as possible to the original system and the Boost curve is altered to give a stable system. The pre-filter break frequency will also be tuned for this case. Nonlinear simulation and linear model analysis will be applied to arrive at the conclusions.

6.4.1 System equations

The system equations of the electrohydraulic power steering system consists of already derived models. The mechanical system is the same as for the original system, which was defined in chapter 3, and the pressure control was defined in section 6.3.1. For convenience, the equations are rewritten here but directly in the Laplace domain, equations 6.60 and 6.61.

$$J_{sw}\Delta\theta_{sw}s^2 = \Delta T_d - K_T(\Delta\theta_{sw} - R_T\Delta x_{rw}) - b_{rw}\Delta\theta_{sw}s \quad (6.60)$$

$$\begin{aligned} M_{rw}\Delta x_{rw}s^2 &= \Delta p_L A_p + K_T(\Delta\theta_{sw} - R_T\Delta x_{rw}) R_T \\ &\quad - b_{rw}\Delta x_{rw}s - C_{rw}x_{rw} \end{aligned} \quad (6.61)$$

An external load is considered in the analysis. Both a vehicle model or a tyre model generate road wheel angle dependent forces. In a linear sense, it can be regarded as a spring with constant C_{rw} . This spring stiffness has a large impact on the behaviour of the system, where a stiffer spring reduces the gain. If a load was not present, the steering wheel torque would be zero. It is therefore justified to assume some kind of stiffness in the analysis. The load pressure Δp_L is defined as the difference between the two cylinder chambers as shown in equation 6.62. The response for one chamber is already defined in equation 6.38. It is assumed that the reference pressure is only changed for one chamber at a time but the movement of the rack affects both chambers, where chamber one is expanded and chamber two is compressed for a positive rack displacement.

$$\begin{aligned} \Delta p_L &= \Delta p_1 - \Delta p_2 = G_c(s)\Delta p_{ref} - 2G_f(s)\Delta q_l \\ &\quad - G_c(s)\Phi(\Delta p_{ref}, \Delta q_l) \end{aligned} \quad (6.62)$$

$$\Delta T_{tb} = K_t(\Delta\theta_{sw} - R_t\Delta x_{rw}) \quad (6.63)$$

The disturbance transfer function $G_f(s)$ will be different for the left and right cylinder chamber if the working points are different. The reference pressure, Δp_{ref} , is defined by the Boost curve, which takes the torsion bar torque as input. An exponential function can be used to represent the static Boost curve as in equation 6.64, a and b are chosen to obtain the desired curve. The torsion bar is defined in equation 6.63, as previously. The function $\Phi(p_{ref}, q_l)$ is an additional term to control the static characteristic of the valve and adds to

the damping of the rack movement. Different functions are possible, such as a linear function to the flow or one that also involves the pressure.

$$p_{ref} = K_{boost}(T_{tb}) = ae^{bT_{tb}} - ae^{-bT_{tb}} \quad (6.64)$$

6.4.2 Updating the controller

The pressure control valves have significantly more phase shift than the original open-centre valve. Since the pressure control loop constitutes an inner loop of the power steering position control loop, the additional phase might have a negative impact on stability. The closed loop of the pressure controller is dominated by two poles at around 40 Hz, which is seen from the closed loop pole location of the linear state feedback controller. One approach to improve this is to add a compensation filter outside the pressure loop. This filter basically moves the two dominating poles to give faster response. The compensation filter is on the form shown in equation 6.65.

$$G_{comp}(s) = \frac{a_0s^2 + a_1s + a_2}{b_0s^2 + b_1s + b_2} \quad (6.65)$$

The numerator is set to cancel the poles at 40 Hz, while the denominator is set to correspond to poles at 300 Hz. In order to avoid too large input signal, a low-pass filter is placed before the compensator filter. The break frequency of the low-pass filter can then be tuned according to section 6.2. An exponential function can be used to make the break frequency decay with the reference pressure, according to equation 6.66.

$$\omega_{pre} = 15e^{-0.01p_{ref}} + 5 \text{ [Hz]} \quad (6.66)$$

where p_{ref} is in unit Pa. The filter is limited to a lowest level at 5 Hz.

6.4.3 Simulation model and results

As in the previous implementations of the model, the Transmission Line Method is also used here. The layout of the simulation model is illustrated in figure 6.17, which shows the interaction between model components. The model is divided into submodels, such as Boost curve, valves, volumes, mechanical part of the steering system and load model. With the Transmission Line Method the interchange between TLM components is the characteristic impedance and wave variable and not pressure. However, for simplicity the pressure is used in the figure.

Four test cases are used to compare the electrohydraulic steering system to the original system and are as follows:

- 1) The vehicle model used as load, set with a constant speed of 70 kph. The steering wheel angle is a chirp signal up to 2 Hz and with 50° in amplitude. This resulted in a later acceleration of up to 3 m/s² and a flow rate of up to 3.6 L/min.

Figure 6.17 Layout of the simulation model implementation of the electrohydraulic power steering system.

- 2) A hysteresis model with maximum force at 2490 N is used as load. The steering wheel angle is a chirp signal up to 1 Hz and with 120° in amplitude. The flow rate reached 4 L/min.
- 3) The same hysteresis model is used with maximum force set at 7000 N. The steering wheel angle is a chirp signal up to 0.1 Hz and with 200° in amplitude which gave a flow rate of up to 0.8 L/min.
- 4) Weave test at 80 kph. The vehicle model used as load and the steering input is a sine wave at 0.25 Hz and $\pm 15^\circ$ angle.

Both the pre-filter break frequency and damping was tuned to ensure stability. First, the Boost curve from the original system is used and a comparison is made for the three test cases. A flatter Boost curve is then derived and implemented for both systems. It can be expressed by equation 6.67. By simply assuming a relation between load pressure and pump pressure, similar to what has been measured, a valve design can be derived by calculating the opening areas of the open-centre valve with the alternative Boost curve, according to equations 3.29 and 3.30. The two Boost curves are shown in figure 6.18. The first three tests are used to compare the two systems over time and for different load cases, with both the original Boost curve and the flatter one. The fourth test is to derive cross-plot for steering feel, where the results from the two Boost curves are plotted in the same figure.

$$p_{Boost} = 3.5e^{0.5T_{tb}} - 3.5e^{-0.5T_{tb}} \quad (6.67)$$

With the original Boost curve, the first three test cases are plotted in figures 6.19 to 6.21. Four subplots are shown: the steering wheel angle in a), the steering wheel torque in b), the torsion bar torque in c) and the load pressure in d). The damping function is tuned in and is defined by equation 6.68. The

Figure 6.18 Original Boost curve shown as dashed and alternative Boost curve shown as solid.

pre-filter was tuned and the function in the previous section was used.

$$\begin{aligned}\Phi(p, q_l) &= 2 \cdot 10^4 p_{ref} q_l \\ \max(\Phi) &= 5 \cdot 10^{10} \\ \min(\Phi) &= 1 \cdot 10^9\end{aligned}\tag{6.68}$$

The three cases with the alternative Boost curve are shown in figures 6.22 to 6.24. Since the Boost curve is much flatter, hence a lower gain, a different damping function could be used. This is shown in equation 6.69.

$$\begin{aligned}\Phi(p, q_l) &= 6 \cdot 10^3 p_{ref} q_l \\ \min(\Phi) &= 5 \cdot 10^9\end{aligned}\tag{6.69}$$

Case 4) is shown in figure 6.25. It shows both the original Boost curve and the alternative Boost curve and compares the open-centre system with the electrohydraulic closed-centre system.

Figure 6.19 Simulation results for case 1) with original Boost curve. Solid lines represents the EHPAS system and dashed lines the HPAS system.

Figure 6.20 Simulation results for case 2) with original Boost curve. Solid lines represents the EHPAS system and dashed lines the HPAS system.

Figure 6.21 Simulation results for case 3) with original Boost curve. Solid lines represents the EHPAS system and dashed lines the HPAS system.

Figure 6.22 Simulation results for case 1) with alternative Boost curve. Solid lines represents the EHPAS system and dashed lines the HPAS system.

Figure 6.23 Simulation results for case 2) with alternative Boost curve. Solid lines represents the EHPAS system and dashed lines the HPAS system.

Figure 6.24 Simulation results for case 3) with alternative Boost curve. Solid lines represents the EHPAS system and dashed line the HPAS system.

Figure 6.25 Steering wheel torque against steering wheel angle for case 4). a) is with original Boost curve and b) is with alternative Boost curve. Dashed lines represent the original system with open-centre valve and solid lines represent the electrohydraulic closed-centre system.

6.4.4 Analysis

In order to proceed with the analysis, the Boost curve and damping need to be linearised. The Boost curve, represented by an exponential function in equation 6.64, depends only on the torsion bar torque and becomes as shown in equation 6.70 when linearised, with the derivative defined in equation 6.71. Index 0 indicates the working point.

$$\Delta p_{ref} = \frac{dp_{ref}(T_{tb,0})}{dT_{tb}} \Delta T_{tb} = K_b \Delta T_{tb} \quad (6.70)$$

$$K_b = abe^{bT_{tb,0}} + abe^{-bT_{tb,0}} \quad (6.71)$$

The linearisation of the damping function is derived in equation 6.72, with derivatives in equation 6.73.

$$\Delta \Phi = \phi_q \Delta p_{ref} + \phi_p \Delta q_l \quad (6.72)$$

$$\begin{aligned} \phi_q &= \phi_{q,l,0} \\ \phi_p &= \phi_{p,ref,0} \end{aligned} \quad (6.73)$$

The expression for the load pressure, defined in equation 6.62, can now be expressed as shown in equation 6.74 by introducing the linearised Boost curve and damping.

$$\Delta p_L = G_c(s)(1 - \phi_q)\Delta p_{ref} - 2G_f(s)A_p\Delta x_{rw}s - G_c(s)\phi_p A_p\Delta x_{rw}s \quad (6.74)$$

With the linearised equations 6.60, 6.61, 6.63 and 6.74, a block diagram representing the system can be derived as shown in figure 6.26. Here is $G_c^*(s) = G_{pre}(s)G_{comp}(s)G_c(s)$ and $G_{rw}(s) = \frac{1}{M_{rw}s^2 + b_{rw}s + C_{rw}}$.

Figure 6.26 Block diagram of the electrohydraulic power steering system with pressure control valves.

From the block diagram, expressions for the loop gain transfer function $A_g(s)$, used for stability analysis, and closed loop transfer functions can be derived. These are shown in equations 6.75 and 6.76, respectively.

$$A_g(s) = \frac{R_T K_T K_b (1 - \phi_q) A_p G_c^*(s) G_{rw}(s)}{(1 + 2A_p^2 G_{rw}(s) G_f(s) s) + G_c^*(s) A_p^2 G_{rw}(s) \phi_p s} \quad (6.75)$$

$$\frac{R_T \Delta x_{rw}}{\Delta \theta_{sw}} = \frac{K_T R_T^2 + G_c^*(s) A_p K_T K_b (1 - \phi_q) R_T}{M_{rw} s^2 + (b_{rw} + 2G_f(s) A_p^2 + G_c^*(s) \phi_p A_p^2) s + C_{rw} + K_T R_T^2 + G_c^*(s) A_p K_T K_b (1 - \phi_q) R_T} \quad (6.76)$$

From the closed loop transfer function it is seen that both the transfer function from a disturbance to a pressure change of the valve controller, $G_f(s)$, and the parameter ϕ_p add to the damping of the rack motion. The stability is first examined for the original Boost curve at 3.9 Nm in figure 6.27. Figure 6.27a shows the frequency response for the two systems and it is seen that stability is maintained. However, the stability margin is significantly lower for the electrohydraulic system. Figures 6.27b shows the frequency response of the closed loop system. The two systems follow each other well up to about 10 Hz.

The case with the flatter alternative Boost curve is examined for a torque level at 8 Nm. The frequency response of the loop gain is shown in figure 6.28a, comparing the open-centre system to the electrohydraulic system. Stability is maintain but is lower for the electrohydraulic system, like in the previous case. The closed loop response is shown in figure 6.28b, where the two systems follow each other up to about 10 Hz. It is also interesting to study the effect

Figure 6.27 Stability plots for the case with the original Boost curve. Dashed lines represent the open-centre system and solid lines the electrohydraulic closed-centre system. Plot a) shows the loop gain frequency response and plots b) the closed loop response.

of the flow disturbance. Figure 6.28c shows the comparison of the closed loop responses, but with the dynamic flow disturbance removed from the electrohydraulic system. The response is much smoother and the two systems now follow each other up to about 30 Hz.

6.4.5 Discussion

The results show a clear trend that it is very challenging to achieve the desired performance under the circumstances in this analysis. Boost curve control is a restrictive strategy since it limits what can be done in terms of control. The main challenge is the additional phase shift of the pressure control valve. Compared to the open-centre valve, the pressure control valve has a phase shift of 360° and additional 90° with the integrator, while the latter only has 90° phase shift in total. While it was sufficient to tune the pressure response for the system with high performance servo valves in section 6.2, it was not for the system with the pressure control valves. The damping was also tuned to ensure stability.

For both cases with different Boost curves, the electrohydraulic steering system was stable. Seen was seen from both simulation and linear analysis. However, with the original Boost curve, the damping was set higher than the original system. By using a flatter Boost curve, the gain decreases and it was possible to use a lower damping level. It was also possible to tune the damping to have a similar characteristic to the open-centre system, which is also seen from the cross-plot steering wheel torque over steering wheel angle in figure 6.25. An issue here was a slightly jerky behaviour in the steering wheel torque. This is

Figure 6.28 Stability plots for the case with the alternative Boost curve. Dashed lines represent the open-centre system and solid lines the electrohydraulic closed-centre system. Plot a) shows the loop gain frequency response and plots b) the closed loop response. The effect of the dynamic flow disturbance is seen in plot c), where it is removed from the electrohydraulic system.

a result of the dynamic effect of the flow disturbance on pressure, which was difficult to handle. One way is to increase the slope of the static characteristic of the valve to such an extent that it masks the dynamic response of the disturbance, although this will also increase damping. The effect it has on the steering feel, however, must be investigated and related to how much damping is desired. The effect of the dynamic disturbance is also seen in figures 6.28b and 6.28c, where the latter figure shows a better closed loop response.

It should be noted that the results very much depend on the circumstances. Both the vehicle model and hysteresis model are simplified and their credibility is not verified. Research has shown that the whole system has an impact on the

steering feel, see e.g. Pfeffer (2006), and excluded dynamic and effects could have a positive or a negative influence on the results. This is not within the scope of this work and since the same load is used for both steering systems, it is possible to make a relative comparison and judgement.

6.5 Position control for autonomous steering

In this section position control of the steering rack with the help of the pressure control valves is analysed. As has been discussed and shown, the power steering system is a positioning follower where the driver sets the reference position. In this section, however, the driver is out of the picture and the control is undertaken solely by a computer. This is interesting when it comes to autonomous driving or active safety functions, such as collision avoidance, as discussed in chapter 4. For low speed manoeuvring it can be sufficient to only control the position of the rack, M. Rösth, J. Pohl, and J-O. Palmberg (2006). At high speed, however, it is the lateral position of the vehicle that is to be controlled. One way to do this is to use a nested controller, as is done by Tai, Hingwe, and Tomizuka (2004), where the inner controller is for the steering system. The linear state feedback controller for the valves, section 6.3.1, is considered here.

The system equations are presented first, followed by the design of the controller, results, and finally a brief discussion. The controller is designed in the linear frequency domain and later evaluated with the nonlinear simulation model of the complete system.

6.5.1 System equations

The system is the same steering system as in previous cases, with the difference that the driver's torque is non-existent. Since the pressure is being controlled by software, it can be assumed that the response is linear. The steering system is also linear and the system equations can be expressed directly in the Laplace domain by equations 6.77 and 6.78, where s is the Laplace operator.

$$J_{sw}s^2 = -K_T(\Delta\theta_{sw} - R_T\Delta x_{rw}) - b_{sw}\theta_{sw}s \quad (6.77)$$

$$M_{rw}s^2 = \Delta p_L A_p + K_T(\Delta\theta_{sw} - R_T\Delta x_{rw})R_T - b_{rw}\Delta x_{rw}s \quad (6.78)$$

The load pressure Δp_L is defined as the difference between the two cylinder chambers, as shown in equation 6.79. The response for one chamber is already defined in equation 6.38. It is assumed that the reference pressure is only changed for one chamber at a time but the movement of the rack affects both chambers, where chamber one is expanded and chamber two is compressed for a positive rack displacement.

$$\begin{aligned} \Delta p_L = \Delta p_1 - \Delta p_2 = G_c(s)\Delta p_{ref} - 2G_f(s)\Delta q_l = \\ G_c(s)\Delta p_{ref} - 2G_f(s)A_p\Delta x_{rw}s \end{aligned} \quad (6.79)$$

Equation 6.78 can now be updated according to equation 6.80. Damping can also be added from the valves. It is set to be the dominant contributor and varies with load pressure. For the calculation of the frequency response it is set to $b_{rw} = 4 \cdot 10^4$ Ns/m.

$$M_{rw} = A_p G_c(s) \Delta p_{ref} + K_T (\Delta \theta_{sw} - R_T \Delta x_{rw}) R_T - (b_{rw} + 2A_p^2 G_f(s)) \Delta x_{rw} s \quad (6.80)$$

Equations 6.77 and 6.80 can be combined to solve the transfer function from a change in load pressure to a change in rack displacement, as shown in equation 6.81. The frequency response is shown in figure 6.29. At low frequencies the plant can be approximated with a pure integrator. At around 5 Hz is the anti-resonance from the steering wheel and column is visible.

$$\Delta x_{rw} = \frac{A_p}{M_{rw} + (b_{rw} + 2A_p^2 G_f(s))s + K_T R_T^2 - \frac{K_T^2 R_T^2}{J_{sw} s^2 + b_{rw} s + K_T}} \Delta p_L \quad (6.81)$$

Figure 6.29 Frequency response of the plant, from a change in load pressure p_L to a change in rack displacement x_{rw} .

6.5.2 Control

The purpose of the controller is to follow a reference position of the steering rack. In order to avoid steady-state error, an integrator needs to be introduced into the controller. One way of doing so is presented by Krus and Gunnarsson

(1993) and is suitable for systems that can be approximated with a pure integrator. It is a PI-controller with a pre-filter that handles the overshoot from the integrative part and the system can be represented by the block diagram shown in figure 6.30, where $b_{rw}^* = b_{rw} + 2A_p^2 G_f(s)$ is the effective damping coefficient. The pre-filter for the pressure controller in section 6.3 is not used here. The shaping of the response is handled only by the outer position control loop.

Figure 6.30 Block diagram of the position controller.

The plant is approximated to equation 6.82, with K_x according to equation 6.83.

$$G_{plant}(s) = \frac{K_x}{s} \quad (6.82)$$

$$K_x = \frac{A_p K_T}{b_{rw} K_T + b_{sw} K_T R_T^2} \quad (6.83)$$

The PI-controller is defined as shown in equation 6.84, with proportional gain K .

$$G_{PI}(s) = \left(\frac{\omega_I}{s} + 1 \right) K \quad (6.84)$$

By introducing the parameters in equations 6.85, 6.86, 6.87 and with the pre-filter defined as in equation 6.88, the closed loop $G_{cl}(s)$ becomes as shown in equation 6.91. The response of the system is then defined by ω_a and δ_a . The control parameters of the PI-controller are calculated according to equations 6.89 and 6.90.

$$\omega_c = K K_x \quad (6.85)$$

$$\omega_a = \sqrt{\omega_c \omega_I} \quad (6.86)$$

$$\delta_a = \frac{\omega_a}{2\omega_I} \quad (6.87)$$

$$G_{pf}(s) = \frac{s/\omega_a + 1}{s/\omega_I + 1} \quad (6.88)$$

$$\omega_I = \frac{\omega_a}{2\delta_a} \quad (6.89)$$

$$K = \frac{2\delta_a \omega_a}{K_x} \quad (6.90)$$

$$G_{cl}(s) = \frac{s/\omega_a + 1}{\frac{s^2}{\omega_a^2} + \frac{2\delta_a}{\omega_a}s + 1} \quad (6.91)$$

The reference value to the pressure controller, calculated by the position controller, is the load pressure, that is, the difference between the left and right cylinder chamber. However, since each valve has its own controller, the reference load pressure is calculated to an individual reference pressure for each pressure controller. This is done according to equations 6.92 and 6.93. They are based on the desired load pressure and the sum of the cylinder pressures, which is calculated according to equation 6.94. In this way one chamber pressure is kept at 20 bar while the other chamber pressure is increased. Since the minimum pressure is larger than zero, the performance is increased somewhat and it gives room to control the static characteristic. The static characteristic of the pressure controller is defined by 6.95, and is linear to the pressure in the respective chamber. It is, however, limited to a minimum and a maximum value. This allows the damping of the rack to be controlled.

$$p_{1ref} = \frac{p_{sum} + p_L}{2} - D_q q_l \quad (6.92)$$

$$p_{2ref} = \frac{p_{sum} - p_L}{2} + D_q q_l \quad (6.93)$$

$$p_{sum} = |p_L| + 40 \cdot 10^5 \quad (6.94)$$

$$D_q = 7500p, \quad max = 7 \cdot 10^{10}, \quad min = 1 \cdot 10^{10} \quad (6.95)$$

6.5.3 Results from simulation

Simulation results of the position controller are shown in figures 6.31 and 6.32. In figure 6.31, the vehicle model, set at a constant speed of 70 kph, is used as load. The reference position to the controller is a sine sweep with a frequency up to 2 Hz. In figure 6.32 the load is a hysteresis model to represent the tyre-ground force during parking, that is, at very low speed. The reference position to the controller is a step input with a rate limiter set at ± 60 mm/s. The controller gains were set to $\omega = 8$ Hz and $\delta_a = 1.5$.

For both test cases the performance of the position controller is good. A small lag and a small overshoot are visible. The performance of the pressure control valves is also sufficient, with an even smaller lag. What is seen, though, are oscillations in rack velocity and torsion bar torque.

6.5.4 Analysis and discussion

The biggest challenge in achieving good performance is the effects from the two-mass system created from the rack and column attached through the torsion bar. When the rack starts to move, oscillations are initiated at the steering

Figure 6.31 Simulation results of position controller with vehicle model as load. Reference values are shown as dashed lines. Plot a) shows the rack position, plot b) shows the load pressure, plot c) shows the rack velocity and plot d) shows the torsion bar torque.

wheel. This causes the torsion bar to twist and generate a disturbing torque on the rack. The oscillating steering wheel tries to resist the movement of the rack, seen as an anti-resonance in the bode plot in figure 6.29. A higher gain would be necessary but that would also reduce the stability margins. However, as long as the input is slower than the anti-resonance, as is the case from the results, the anti-resonance should not cause any issues.

The results are also affected by the load, as seen from the results where the rack velocity oscillates more during the parking manoeuvre. The tyre load in that is set arbitrarily and how well it reflects a real tyre is not verified. For instance, more damping provided from the load would reduce the oscillations. It also depends on how much damping is provided from the valve and the controller gain. With a high gain the controller forces the rack to follow the reference better, which might cause the pressure and rack velocity to oscillate. A smaller gain makes the movement smoother but introduces more lag.

Figure 6.32 Simulation results of position controller with hysteresis model as load, which represent a parking manoeuvre. Reference values are shown as dashed lines. Plot a) shows the rack position, plot b) shows the load pressure, plot c) shows the rack velocity and plot d) shows the torsion bar torque.

6.6 Heavy truck steering system application

In this section the pressure control valves for use in a heavy truck power steering system are investigated. This relates to research question 6). With a few assumptions and using the framework derived for the previous analyses, it is possible to get an idea of how the approach with pressure control valves suit this application.

There are several differences between the steering system in a heavy truck and a passenger car that relate to the hydraulic system. The vehicle is much larger and heavier, which generates much higher loads on the steering system. The higher mass means that the inertia is larger, which obviously increases the phase shift from the mechanical system. Since the mechanical system is a part of the loop gain, it will have consequences for the stability margins. With the heavy loads, a high force from the hydraulic cylinder is required so the piston

area is much larger. The pump flow is slightly less than twice as high compared to a passenger car and the cylinder volumes are at least ten times larger. The torsion bar could also be stiffer. Other differences are more difficult to specify but the mechanical linkage is far more complex for a steering box than a rack and pinion system. This should generate more friction and damping. Naturally, the Boost curve is different and is set to be flatter than previous. The steering wheel is larger and consequently a higher torque is applied for the same for as in the passenger car. This leads to a flatter Boost curve with lower gain.

Relating the differences to the requirements of the valve, the most striking one is the much larger volumes while the pump flow is only less than twice as high. This should lead to a much slower response of the original open-centre valve. By assuming the open-centre valve to be a reference for performance the requirement on the pressure control valve is much lower than in the passenger car, but this also depends on the rest of the system dynamic. With the larger volumes, the pressure control valve should also be less prone to react to flow disturbances, which, was seen from the previous analysis, could have a negative effect on performance and steering feel.

The model is explained first, followed by an analysis of the open-centre valve. The controller for the pressure control valve needs to be tuned for the new system. The analysis of the complete system is then similar to previous cases, section 6.4, with simulation of the nonlinear system equations and a linear analysis in the frequency domain.

6.6.1 System modelling

Since all models are already derived at this point, only minor alterations and different parameters are needed. In reality the mechanical linkage is quite flexible and a higher order model than two degrees of freedom is needed, see M. Röstl, J. Pohl, and J-O. Palmberg (2006). However, for the sake of the analysis and see the trends for the application, the system is simplified into a system with two degrees of freedom, which makes it possible to re-use the models already derived. The model is defined from a steering wheel angle to a road wheel angle. In between is the cylinder piston moving in a linear motion. It is treated only as a gear, where the rotational motion of the column gives a linear motion of the piston, that turns in a rotational motion of the wheels through the steering box and linkage. The system equations are similar to as previous and are defined by equations 6.96 and 6.97.

$$J_{sw}\ddot{\theta}_{sw} = T_d - K_T(\theta_{sw} - R_T\theta_{rw}) - b_{sw}\dot{\theta}_{sw} - M_{f_{sw}} \quad (6.96)$$

$$J_{rw}\ddot{\theta}_{rw} = A_p p_L / R_l - K_T(\theta_{sw} - R_T\theta_{rw}) R_T - b_{rw}\dot{\theta}_{rw} - M_L - F_{f_{rw}}(p_L) / R_l \quad (6.97)$$

The total inertia of the road wheels is denoted J_{rw} and R_l transforms the linear motion of the piston to a corresponding rotational motion at the wheels. R_T is the total gear ratio of the steering system, while M_L is the load torque from

the tyre-ground interaction. Both the pressure dependent friction and column friction are set to be equal as in the previous case. The pressure is defined by the hydraulic system, which is the same model as derived in chapter 3. The static Boost curve is defined by equation 6.98 and is shown in figure 6.33.

$$p_{Boost} = 1.813e^{0.408T_{tb}} - 1.813e^{-0.408T_{tb}} \quad (6.98)$$

For an open-centre system the pump pressure is related to the load pressure,

Figure 6.33 Boost curve for heavy truck steering system.

that is, as the load pressure increases so will also the pump pressure. The pump pressure is not known here but by assuming a relation between the pump pressure and the load pressure, the opening areas of the open-centre valve can be calculated. The static pressure drop of a truck steering system valve is lower than in the passenger car. The pump pressure is set to be 2.5 bar higher than the load pressure, equation 6.99. The load pressure is already known from the assumed Boost curve.

$$p_{pump} = |p_{boost}| + 2.5 \text{ bar} \quad (6.99)$$

The opening areas can now be calculated according to equations 3.29 and 3.30 and this completes the model of the open-centre system. The remaining parameters are found in the appendix.

When it comes to the electrohydraulic pressure control valves, it is justified to set a higher possible flow rate at the tank port. Recall that the examined valve has an asymmetric flow rate, which makes it not suitable for the application. However, it can be modified in order to have a symmetric flow rate and such

valves exists on the market. The allowed range is therefore extended to allow a higher flow rate at the tank port.

6.6.2 Response of open-centre system

It is interesting to study the pressure response of the open-centre valve since it is a benchmark for the pressure control valves. There are a few factors that should tend to make the response faster and other to make it slower compared to the passenger car system. A larger flow rate and a flatter Boost curve tends to increase the response while larger volumes decreases the response. The response is shown in figure 6.34 at zero load flow for torque levels 1, 6 and 10 Nm. The bandwidth, defined at -3 dB from the static level, is 80, 9 and 1.7 Hz correspondingly. The response is thus slower than in the passenger car case, mainly due to the larger volumes. It is therefore possible that the response of the pressure control valve can be lower than previous but must of course handle a larger flow rate.

Figure 6.34 Pressure response of the open-centre valve at zero load flow with centred piston.

6.6.3 Controller design

Next step is to re-design the controller for the valves. The same linear controller approach will be used. The same linear model from section 6.3.1 is still valid

here. Only the size of the volume changes. The same placement of the poles of the closed loop system can not be kept since it would require a much larger control energy, which is limited. The poles are instead placed as shown in table 6.4. The performance of the controller is seen in figure 6.35. The response time, defined as time to reach 63% of final value, is 30 ms. When the flow disturbance enters the controller has to do a lot of work, but the flow rate is high and the influence on the pressure is minor. A linear static characteristic was set with rate $1 \cdot 10^{10} \text{ Pa}\cdot\text{s}/\text{m}^3$.

-500
-9000
-80
-81
-400

Table 6.4 Pole location of the closed loop system.

Figure 6.35 Simulation result of the pressure controller. Plot a) shows the reference pressure (dashed) and actual pressure (solid). Plot b) shows the control signal. A disturbance flow of 10 L/min is entering the volume at 1.2 s and leaving the volume 1.6 s. The flow disturbance is low-pass filtered with a break frequency of 2 Hz.

6.6.4 Simulation of complete system

The electrohydraulic was tested by simulation for a steering wheel angle as a chirp signal up to 1 Hz and 50° in amplitude. The load flow reach 6.5 L/min. A vehicle model was used as load and all parameters are found in the appendix. The result of the simulation is seen in figure 6.36. The damping function was

on the same form as previous and is shown in equation 6.100. A low-pass filter with the break frequency at 1 Hz was used in conjunction with the damping function in order to break up the dependency on the pressure. The break frequency of the pre-filter was used as shown in equation 6.101 and was limited to a lowest level at 2 Hz.

$$\begin{aligned}\Phi(p, q_l) &= 1 \cdot 10^4 p_{ref} q_l \\ \min(\Phi) &= 1 \cdot 10^{11} \\ \min(\Phi) &= 1 \cdot 10^9\end{aligned}\tag{6.100}$$

$$\omega_{pre} = 15e^{-0.05p_{ref}} + 2 \text{ [Hz]}\tag{6.101}$$

Figure 6.36 Simulation results for a truck steering system and vehicle model. Solid lines represents the EHPAS system and dashed lines the HPAS system.

6.6.5 Linear analysis

The linear analysis becomes identical to the passenger car case in section 6.4. The loop gain and closed loop transfer functions are only updated to fit the new system parameters in equations 6.102 and 6.103. The resulting loop gain frequency response and closed loop frequency response at 10 Nm are shown in figure 6.37. Both systems are stable and the electrohydraulic system follows the open-centre system up to about 20 Hz.

$$A_g(s) = \frac{R_T K_T K_b (1 - \phi_q) A_p / R_l G_c^*(s) G_{rw}(s)}{(1 + 2A_p^2 / R_l^2 G_{rw}(s) G_f(s)s) + G_c^*(s) A_p^2 / R_l^2 G_{rw}(s) \phi_p s} \quad (6.102)$$

$$\frac{R_T \Delta x_{rw}}{\Delta \theta_{sw}} = \frac{K_T R_T^2 + G_c^*(s) A_p / R_l K_T K_b (1 - \phi_q) R_T}{J_{rw} s^2 + (b_{rw} + 2G_f(s) A_p^2 / R_l^2 + G_c^*(s) \phi_p A_p^2 / R_l^2) s + C_{rw} + K_T R_T^2 + G_c^*(s) A_p / R_l K_T K_b (1 - \phi_q) R_T} \quad (6.103)$$

Figure 6.37 Stability plots for the truck steering system. Dashed lines represent the open-centre system and solid lines the electrohydraulic closed-centre system. Plot a) shows the loop gain frequency response and plots b) the closed loop response.

6.6.6 Discussion

At this point the model of the truck steering system is very simplified, but it is possible to see some trends in the behaviour. The larger volumes of the assistance cylinder means that the pressure control valve can not reach the same performance as previous. A larger valve could, on the other hand, increase the performance. The larger volumes also mean that the open-centre valve has a slow response in pressure. It should therefore be possible to set the break frequency of the pre-filter at a lower value than in the passenger car system. The larger volumes also makes the valve less prone to react to disturbances in

the flow. The steering torque is much smoother in the simulation than what was seen before. The larger inertia of the steering system should also help to smoothen any disturbance. At the same time, the larger inertia introduces more phase shift that reduces the stability margins.

6.7 Implementation and verification

This section aims at implementing the developed controllers and discuss some issues that were encountered and how they were addressed. The aim is not to illustrate a fully functional solution but to verify the ideas. As will be seen, one of the major concerns was the restrictions in the hardware. The system is described in section 7.2, chapter 7.

6.7.1 Pressure controller

The linear state feedback controller was chosen for the evaluation of the applications. A state feedback controller relies on the existent measurements of the state, which is not the case here. The solenoid current and pressure are easily obtained, but this is not the case for the spool position and velocity. During the measurements of the valve characteristics, a position transducer was indeed installed on the valve. This caused a high leakage flow rate, which was not an issue during the specific test but would not be very practical in the steering system test rig. A state observer is therefore needed to estimate the spool position and velocity.

The initial approach was to derive a model with very good accuracy, which could also be use as state observer. With a feedback loop the estimated state would convergence towards the actual state. Since the valve is nonlinear, however, a nonlinear state observer is necessary, for example, the Extended Kalman filter. This requires that the Jacobian is calculated at each time step.

It turned to be crucial to keep the time step as low as possible in order to minimise the time lag in the system. The model, as it was initially implemented for the simulations, was to complex to run at the desired time step in real time. Some alterations were needed with the objective to try and reduce the computational time. The jet stream angle was set to fixed values, that is, on the pump side $\cos(\delta) = 0.7$ and on the tank side $\cos(\delta) = 1$. The jet stream angle changes more dramatically for large spool displacements and since the valves would only work in a small region in the test rig, they can be kept constant. The nonlinear characteristic of the discharge coefficient and current-force relationship of the solenoid were kept but implemented with look-up tables to avoid the many trigonometric functions. Since the current was measurable, it was directly fed to the corresponding look-up table to obtain the force. The lowest possible time step with this configuration and two valve models implemented was 0.1 ms, which was not possible with the initial model. With additional controllers implemented in the real-time computer, the lowest possible time step

was instead 0.2 ms. A state observer was never implemented. The required Jacobian would be complex and would require several additional look-up tables, that would increase the time step further. It was therefore decided to only use the model to estimate the states and that the increased time step would have a greater negative impact on the result than the estimation error. A linear state estimator is also a possibility, but the linearised states are only valid for an infinitely small change in input signal. It was also believed that the nonlinear model would do a better job alone.

Another contributor to the time lag is the conversion to a PWM signal. The valve requires a PWM frequency in the range 100 - 200 Hz. This is required in order to obtain the dither effect, which minimises the stick-slip friction and improves the performance. However, this also means that the control signal only updates every 5 ms, which adds a great deal of time lag in the system. The result from this can be verified from simulation by adding a time lag to the model. The results is shown in figure 6.38, with a clearly unstable system. The figure shows also the result with a 0.2 ms time lag. In this case, the control signal is still oscillatory but the pressure is still controlled. A possible solution is to increase the PWM frequency above the time step of the real-time computer. This requires that the dither frequency is superimposed on the control signal to the valve. However, the intended PWM frequency has been purposely chosen and the valve has undergone extensive testing in the factory to obtain best possible performance. The time required to find a new set of parameters with equally good performance, was just not simply possible within the time frame of this work. The PWM frequency was set to 100 kHz with a dither frequency at 200 Hz and 10% duty cycle.

A concern with the valve is the fact that high pressure acts on the magnet core where it is in contact with the spool, see chapter 5. It was seen from observations how the magnet core and spool could separate and the valve is no longer controllable. The valve was never designed for this application but it is likely that this effects is easily altered. However, it constitutes a restriction for the tests in this work and possible effects of this will be seen.

Measurement results and discussion

Slightly slower pole placements, compared to simulation, were set and the locations are defined in table 6.5. The resulting pressure response for a filtered step from 30 bar to 50 bar and back is shown in figure 6.39a, and from 50 to 70 bar and back in figure 6.39b. The corresponding solenoid currents are shown figures 6.39c and 6.39d, respectively. The step increase and step decline are measured for two different data sets but are plotted in the same figure for convenience. The time step was 0.1 ms and a FIR filter at 200 Hz was used for the currents. The supply pressure was set to 150 bar. This was used to filter the effects of the dither frequency.

The response for a pressure increase is quite good and not far from what was seen when simulating the model with controller. The desired pole locations

Figure 6.38 Simulation of linear state space controller with an introduced time lag. Plots a), b) and c) show the control signal, solenoid current and pressure with a 5 ms time lag. Plots d), e) and f) show the same signals with a 0.2 ms time lag.

-400
-10000
-200+10i
-200-10i
-350

Table 6.5 Pole locations of the linearised closed loop system.

are set to obtain a somewhat slower response. A faster response would give an unstable system. The response for a pressure decrease, however, is poor. This is most likely caused by a separation of the magnet core and spool. The current is very oscillatory and behaves similar as shown if figure 6.38. This indicates that the time lag is somewhat too long and might also be a reason to why the pole locations would have to be set to obtain a slower response. Another concern was the lack of repeatability. The performance of the valve with the higher PWM frequency has not been optimised and there is likely that some friction still remains. If this could be improved, it is possible that also the performance could be improved. Another possible improvement could come from the implementation of a more accurate state observer. The indication from

Figure 6.39 Results from the pressure controller. Plots a) and b) show the pressure for both chambers (solid and dashed lines). The faster curves represents the reference pressure (also solid). Plots c) and d) show the corresponding solenoid currents.

the results, however, are positive and verifies that a linear state feedback controller is a possible solution. Future work would involve an Extended Kalman filter, or possibly a linear filter, to better estimate the states, reduce the time lag, and reduce friction by optimising PWM and dither frequencies. These are beside the alternation done to the valve. Due to this behaviour, other control strategies could not be confirmed at this stage.

7

Hardware-in-the-loop simulation

Hardware-in-the-loop, HWIL, is the simulation technique where a part of the software implemented simulation model is replaced by actual hardware. This has the advantage of being able to test parts of the system without going for the full prototype, especially for large systems. As described by Sannelius (1999), there are basically two approaches. One is where new components are tested as hardware and the rest of the system is simulated. The other approach is to simulate the new component while the system is tested as hardware. The first approach is adopted in this work, which can be illustrated by figure 7.1.

Figure 7.1 *General approach of hardware-in-the-loop simulation where the vehicle is implemented as a simulation model and the hardware consists of the steering system.*

Another classification of HWIL simulation is related to where in the system the transformation from hardware to simulation takes place. For hydraulic systems it can take place in the hydraulic line where pressure and flow is transformed. An example of this is explained in the work by Ramdén (1999). Here is a valve tested and the load simulated by controlling pressure and flow. Sannelius (1999) also presents how HWIL simulation is used for testing of hydro-

static transmissions. In this work it is the force and velocity that is transformed between the hardware and simulation.

The simulation is divided so the steering system constitutes the hardware and the vehicle is represented by the simulation model. There are two main purposes with the test rig in this work. First is to develop a simulation model, described in chapter 3. With the test rig at hands the system under study can be isolated and different parts can easily be tested, compared to a system that is in the vehicle. The other purpose is to have a more realistic environment for the investigation and evaluation of the closed-centre steering system. A pure simulation model is always just a reflection of the reality and possible issues that might have been overseen in the simulation may arise in the actual system. A human can also be integrated in the loop with HWIL simulation. One of the most important aspects of the steering system is the steering feel. This is a subjective measure and a qualitative judgement of the closed-centre system is possible with the test rig.

The work consists of two parts. For the first part the test rig was equipped with fast acting servo valves for a general study of a closed-centre power steering system. For the second part the servo valves were replaced by the pressure control valves described in chapter 5, section 5.2. Several controllers were developed and implemented. A force controller takes care of providing the right amount of load on the steering system. The servo valves require a controller to control the pressure. This is also the case for the pressure control valves.

This chapter gives an overview of the hardware involved in the test rig. The force controller is also derived and analysed. The analyses of the controllers for the servo valves and pressure control valves have already been outlined in chapter 6.

7.1 Servo valves

7.1.1 Hardware description

A schematic of the test rig is shown in figure 7.2 and a photograph is figure 7.3. The test rig is built up around a rack-and-pinion steering system. A fixed displacement bent-axis pump with size 5 cc/rev driven by an electric motor is used to supply the open-centre valve with a constant flow. It is not an original pump used in vehicles but since the pump is excluded from the analyses, it is only of interest to deliver a constant flow. Several sensors are used to measure the pump pressure as well as both chamber pressures. To measure the torque applied by the driver, strain gauges are attached directly to the torsion bar. This means that the actual torsion bar torque is measured instead of the steering wheel torque and an additional torsion bar is not needed. Since the torsion bar torque is linear to the twist of the torsion bar, hence the twist of the open-centre valve, the measurement of the torque gives a direct measure of the valve actuation. The steering wheel angle and rack position are

Figure 7.2 Schematic description of the test rig.

measured with potentiometers. A weight of 20 kg is attached to one end of the rack to give the system the right inertia. At the other end of the rack, a load cylinder is attached to generate the load experienced by the steering system due to the tyre-ground interaction. Between the load cylinder and the rack, a force sensor measures the force applied by the load cylinder and is used in a closed loop controller. The load cylinder is an identical rack from another steering system. One concern about using such a cylinder is the amount of friction generated from the internal sealings. This is discussed below in section 7.3. A Moog servo valve is used to control the load cylinder.

Parallel to the original system, four high performance 4/3 servo valves from Moog control the meter-in and meter-out flow to each chamber of the assistance cylinder. The approximate bandwidth is 200 Hz at 10% stroke. These valves are over-dimensioned for the application and the result is that they work with very small movements, which should be beneficial from a performance point of view. They are also arranged in such a way that each valve deliver double the flow, i.e. the two load ports are connected to a single load port. All servo valves use an external supply system of 200 bar. Argus valves are used to switch between the original system and the servo valve system. The Argus valves are placed in

the line to each cylinder chamber. The reason for keeping the original system is to always have the reference system available for a qualitative comparison between the open-centre and closed-centre systems. A potograph of the Moog servo valves is shown in figure 7.4.

Figure 7.3 *Photograph of the test rig.*

Figure 7.4 *Photograph of the Moog servo valve pack installed in the test rig.*

7.2 Pressure control valves

7.2.1 Hardware description

Figure 7.5 Photograph of the Hydac valves installed in the test rig.

In this part of the work the servo valves have been replaced with two pressure control valves, one for each chamber of the assistance cylinder, as shown in figure 7.5. They use a constant pressure supply of 200 bar. A small accumulator of 1 L is placed in the supply line right before the valves. This will help to keep the supply at a more even level when sudden flow demands are required by the valves. The valves are controlled with a 200 Hz PWM duty cycle.

7.3 Force control

The force controller's task is to generate the right load on the steering rack depending on the driving scenario. The reference load is calculated from a software implemented model, which typically is a vehicle model. For the design and analysis of the force controller, a mathematical model is derived and is described below. The model considers the load cylinder with hydraulic system and controller. It is attached to a moving mass through a very stiff spring. The moving mass is positioned controlled and represents the motion of the steering rack, which in the test rig is attached to the load cylinder. The analysis is based on the schematic description of the system in figure 7.6.

Figure 7.6 Layout of the force controller of the load cylinder. The actual force between rack and load cylinder is measured and fed to the controller. The force is controlled with a servo valve.

7.3.1 Mathematical description

The model consists of three parts: the servo valve, the cylinder volumes and the moving mass of the cylinder. The model has been somewhat simplified, but as will be shown, it is accurate enough to predict the behaviour of the system.

Servo valve

The servo valve is modelled as four variable orifices and a second order dynamics for the movement of the spool. The four orifices connects the supply port p and the tank port t to the respective load port a and b . The orifice flows are describe by equation 7.1 to 7.6, where C_q is the flow coefficient and ρ is the oil density.

$$q_{pa} = C_q A(x_v) \sqrt{\frac{2}{\rho} (p_s - p_a)} \quad (7.1)$$

$$q_{bt} = C_q A(x_v) \sqrt{\frac{2}{\rho} (p_b - p_t)} \quad (7.2)$$

$$q_{pb} = C_q A(-x_v) \sqrt{\frac{2}{\rho} (p_s - p_b)} \quad (7.3)$$

$$q_{at} = C_q A(-x_v) \sqrt{\frac{2}{\rho} (p_a - p_t)} \quad (7.4)$$

$$q_a = q_{pa} - q_{at} \quad (7.5)$$

$$q_b = -q_{bt} + q_{pb} \quad (7.6)$$

The opening area A is assumed to linear with spool displacement and is modelled according to equation 7.7 and 7.8, where w is the area gradient and u is the underlap of the area opening.

$$A(x_v) = wx_v + u \quad (7.7)$$

$$A(-x_v) = |wx_v - u| \quad (7.8)$$

$$(7.9)$$

The displacement x_v of the spool is described by the differential equation 7.10 where U is the control signal to the valve, ω_v is the response of the valve and δ_v is the damping coefficient.

$$\ddot{x}_v = U\omega_v^2 - 2\omega_v\delta_v\dot{x}_v - \omega_v^2x_v \quad (7.10)$$

Cylinder volumes

The cylinder volumes are modelled with the continuity equation according to equation 7.11 and 7.12, where A_p is the piston area, x_p is the piston position, V is the cylinder volume and β is the bulkmodulus.

$$q_a - A_p\dot{x}_p = \frac{V}{\beta}\dot{p}_a \quad (7.11)$$

$$q_b + A_p\dot{x}_p = \frac{V}{\beta}\dot{p}_b \quad (7.12)$$

Cylinder motion

The motion of the cylinder is modelled with the help of Newton's second law according to equation 7.13, where m is the mass of the piston and rod, b is the viscous friction coefficient, $F_f(p_{sum})$ is the static friction as described in chapter 3 and F_L is the external load acting on the cylinder which is to be controlled.

$$m\ddot{x}_p = p_aA_p - p_bA_p - b\dot{x}_p - F_f(p_{sum}) - F_L \quad (7.13)$$

7.3.2 Controller design

For the design of the controller the system can be somewhat simplified and analysed in the frequency domain. It is assumed that there is no underlap of the spool and since the cylinder is symmetrical the flow is equal on both chambers defined by equation 7.14.

$$q_a = q_b = q \quad (7.14)$$

From the flow equations the relation in equation 7.15 is valid and the load pressure p_L is defined according to equation 7.16.

$$p_s = p_a + p_b \quad (7.15)$$

$$p_L = p_a - p_b \quad (7.16)$$

The flow equation for the servo valve can now be rewritten in terms of the load pressure as in equation 7.17.

$$q = C_q A(x_v) \sqrt{\frac{1}{\rho}(p_s - p_L)} \quad (7.17)$$

The same is also true for the continuity equation, which results in equation 7.18.

$$q - A_p \dot{x}_p = \frac{V}{2\beta} \dot{p}_L \quad (7.18)$$

In order to analyse the system in the frequency domain the system equations need to be linearised and transformed into the Laplace domain. This results in equation 7.19 to 7.23, where s is the Laplace operator and Δ -sign indicates a small change in the system state variable. The control signal U is calculated by multiplying the error in force to a controller $G_{reg}(s)$.

$$\Delta q = K_q \Delta x_v - K_c \Delta p_L \quad (7.19)$$

$$\Delta q - A_p \Delta x_p s = \frac{V}{2\beta} \Delta p_L s \quad (7.20)$$

$$\Delta x_v = \frac{\Delta U}{\frac{s^2}{\omega_v^2} + \frac{2\delta_v}{\omega_v} s + 1} \quad (7.21)$$

$$m \Delta x_p s^2 = \Delta p_L A_p - \Delta F_L - b \Delta x_p s \quad (7.22)$$

$$\Delta U = G_{reg}(s)(\Delta F_{ref} - \Delta F_L) \quad (7.23)$$

The derivatives are defined according to equation 7.24 and 7.25, where index 0 represents the working point for which the system is linearised.

$$K_q = \frac{\partial q}{\partial x_v} = C_q w \sqrt{\frac{1}{\rho}(p_{s0} - p_{L0})} \quad (7.24)$$

$$K_c = -\frac{\partial q}{\partial p_L} = \frac{C_q w x_{v0} \sqrt{\frac{1}{\rho}}}{2\sqrt{p_{s0} - p_{L0}}} \quad (7.25)$$

From the linearised system equations a block diagram can be derived that represents the control loop. This block diagram is shown in figure 7.7. From

Figure 7.7 Block diagram representation of the force control loop.

the block diagram the transfer function of the loop gain can be defined according to equation 7.26.

$$A_g(s) = G_{reg}(s) \frac{K_q A_p}{\left(\frac{s^2}{\omega_v^2} + \frac{2\delta_v}{\omega_v} s + 1\right) \left(K_c + \frac{V}{2\beta} s\right)} \quad (7.26)$$

The controller $G_{reg}(s)$ is designed as a lead-lag filter according to equation 7.27.

$$G_{reg}(s) = K_p \frac{1 + \tau_i s}{\gamma + \tau_i s} \frac{1 + \tau_d s}{1 + \alpha \tau_d s} \quad (7.27)$$

The controller parameters are defined in table 7.1. The complete system with

Parameter	Value
K_p	$2.5 \cdot 10^{-5}$
τ_i	0.24
γ	0.001
τ_d	0.003
α	0.2

Table 7.1 Controller parameter values.

controller can now be studied for stability by plotting the frequency response of the loop gain $A_g(s)$. The result will vary depending on the working point. In this case the system is studied for the worst point, which is for a very small spool displacement and a large pressure drop. Figure 7.8 shows the frequency response with system parameter values defined in table A.5 in the appendix. The amplitude margin is at this point 8.11 dB at 303 Hz and the phase margin is 49.1° at 178 Hz. The controller is then evaluated by simulating the nonlinear model. The results are shown in the next section.

7.3.3 Results

Figure 7.9 shows the results of the simulation of the force controller where the rack is moving with an increased frequency up to 2 Hz and an amplitude of 3 mm. The controller is able to follow the reference value with satisfying result. At the turning points there is a "bump" in the actual force. Figure 7.10 shows the results from the test rig, where figure 7.10a shows the rack position applied and figure 7.10b shows the corresponding reference and measured force. A proportional gain K_p of $0.7 \cdot 10^{-5}$ was used in the figure. The other control parameters were the same as in the simulations. The result is similar to the simulation and the measured force shows the same behaviour at the turning points. This is believed to be caused by the friction in the load cylinder and is analysed in detail below.

Figure 7.8 Frequency plot of loop gain of the force controller system.

Figure 7.9 Simulation results of the force controller model. Dashed line is reference force and solid line is actual force.

7.3.4 Discussion

The force controller worked satisfyingly. What was noticed, however, is a small "bump" in force at the turning points. This has an effect on the steering wheel torque but since it is always present the situation is equal regardless of what hydraulic system is evaluated. Interesting is that the simulation also shows the same behaviour, which indicates that the reason to this effect is caught by the model. The hypothesis is that it is the static friction in the load cylinder that

Figure 7.10 Comparison of reference and actual force of the force controller.

causes this. This can be easily tested by simply removing the static friction from the model of the load cylinder. The viscous friction is still present. The result is shown in figure 7.11, where the resulting force of the controller is compared for the case with and without friction. It is seen that without friction the behaviour is much better, which confirms the hypothesis. It is difficult to overcome this in reality though. One way is to increase the controller gains but they are limited by the stability margins of the system. Another way would simply be to remove the sealings. A leakage over the cylinder piston is not an issue in a laboratory environment.

Figure 7.11 *Simulation results of the force controller model. Solid line represents the results when a cylinder with friction is used. Dashed line represents the results when a cylinder without static friction is used.*

8

Discussion

The biggest difference between the open-centre valve from the original system and the electrohydraulic pressure control valves with closed-centre, used in this work, is the much higher phase shift. The open-centre valve is attached to and controlled by the twisting of the torsion bar, which in turn is a result of the torque applied by the driver. This means that the controlling of the valve is done directly by the driver and the only lag that occurs comes from the oil compressibility in the volumes. This is not the case for the electrohydraulic valve. There are several parts that all add to the phase shift. The valve is activated based on measurements of the torsion bar torque. The signal is processed and sent to the solenoid. The force is increased with a certain lag and then acts on the inertia of the moving mass. When the mass has been moved, a flow can enter the volume and the pressure starts to change.

Since the pressure control loop constitutes the inner control of the power steering system, the additional phase shift has a negative impact on stability and must be handled. This is most severe for Boost curve control, where the outer control loop is already defined with a high gain at high pressure levels. Preferably, a fast valve gives little additional phase shift in the critical region and the performance of the pressure control loop can be controlled with a pre-filter and set to be the dominating dynamic. In this way stability can be maintained. This was seen and also verified in the test rig, from the analysis in section 6.2, chapter 6. In this analysis high-response servo valves were used to control the pressure.

If the valves are too slow, ensuring stability with a pre-filter will not be sufficient. The pre-filter also adds to the phase-shift. This was the case for the pressure control valves, which were analysed with Boost curve control in section 6.4, chapter 6. The performance and analysis of the system very much depend on the circumstances given by the system properties and driving scenarios. Another approach to ensure stability is also to tweak the damping. Since the pressure is controlled from a computer, any desired pressure can be set. Allowing the pressure change with flow will affect the damping. An in-

creased damping will increase stability margins but also affect the steering feel. However, the critical region is high pressure, which occurs during parking or low-speed manoeuvring, and steering feel is less important in this region. It is, though, fair to conclude that the additional phase shift gives a limitation in possible performance variations. The best solution is simply to use a flatter Boost curve that has a lower gain. The performance of the valve then only affects the steering feel. One way to analyse this is to look at the hysteresis curve, steering wheel torque over steering wheel angle, where it was seen that the electrohydraulic closed-centre system can be tuned to have similar performance to the open-centre system if the Boost gain is kept low enough.

The challenge with Boost curve control leads to the question of if this is the proper control strategy at all. Boost curve control is an inherent solution of the original open-centre system but has clear restrictions. It can be seen as a pure proportional controller with very high gain when the load increases. Other approaches exist where the Boost curve is more or less moved out from the position control loop of the power steering system, see e.g. Birk (2010) or Wolfgang Kemmetmüller, Steffen Müller, and Andreas Kugi (2007). This, however, is a different topic and not analysed in this work. Which control strategy is used must ultimately be chosen by the vehicle manufacturer. The approach in this work has been that the manufacturer wants to keep the fundamental behaviour from the original system.

Position control with electrohydraulic pressure control valves, analysed in section 6.5, chapter 6, gives a greater degree of freedom to design the outer position control loop. It seems that the requirements on the valve are less strict and the results show that the controller is able to follow the reference signal with little time lag. This could also be seen as an indication of whether other control strategies other than Boost curve control could be used. Such a control strategy could be based on a position control loop, where the reference position is set by the driver's steering wheel angle. Position control with the power steering system is an important subject and the near future will see autonomous vehicles on the roads. The results show that it is possible to achieve with pressure control valves. Another important feature is that steering feel is not an issue if the driver is only a passenger.

An accurate model of the different parts of the system has been very useful during the work. Both the model of the power steering system and the pressure control valve model were validated against measurements and the models show a very good agreement with the measurements. The models have played an important role in control design, in analysing the system, and in verifying the concept. Since the models have a high accuracy, when combining the models to generate the concept with closed-centre pressure control valves, high confidence in the results can be maintained.

The modelling procedure for the valve stresses the importance of studying and including the nonlinear behaviour of the pressure-flow characteristic and flow forces. It turned out that assumptions like a constant discharge coefficient

and jet stream angle were not valid. The reason is most likely the low flow level, while the stroke was quite large, which means the flow is in the mixed region of laminar and turbulent flow. The weak mechanical spring of the valve also means that the induced flow forces dominate the characteristic's behaviour, which in turn is defined by, among other things, the discharge coefficient and jet stream angle.

An external controller for the pressure control valve was designed. This was required in order to increase the performance and to ensure that the reference value was followed. A state feedback approach was investigated. The reason was the ability to place the closed loop poles at a location where the desired performance can be obtained. This requires knowledge of the states and two of the states, namely current and pressure, were measured. Both a linear and a nonlinear controller were developed and analysed. The linear controller is a compromise since it does not consider the variations in system dynamics and characteristic and was designed for a working point near the worst case. The nonlinear controller, on the other hand, cancels the nonlinearities of the plant and transforms the system into a linear, controllable system. The requirement is a good knowledge of the plant. Basically, it involves the inverse of the plant and is something that needs to be considered during the design process in order to avoid numerical issues.

Both controllers performed well. This was under the assumption that all states were obtainable. The nonlinear controller showed best performance and it was also better at handling flow disturbances. The linear controller had to do more work when a flow disturbance appeared. However, the nonlinear controller is far more complex to implement and it is left for future work to investigate possible simplifications and robustness against parameter variations. The linear controller was therefore chosen to be implemented with Boost curve control and position control. It was also possible to run the model with a larger time step, which is beneficial since it reduces the computational time. It would, however, be interesting to see if any performance gains can be obtained for the applications with the nonlinear controller. With higher flow rates, the valve shows a less linear characteristic. Higher flow rates occur, for example, in the truck steering system. This is more difficult to handle for the linear controller and the nonlinear controller would most likely do a better job here. When it comes to the flow disturbances, it was clear that they had a negative impact on the steering wheel torque. One way to handle this was, as mentioned previously, to make the static characteristic flatter, which increases the damping. How much the driver would actually notice the torque disturbance or how much damping can be applied is a question related to steering feel and not treated here. Improving the performance of the controller was also possible by adding a compensation filter at the reference pressure. With the state feedback controller the valve can be seen as linear and since the closed loop pole locations are known, a compensation filter can compensate for the phase shift.

The valve has never been intended for the applications studied in this work,

while the analysis and results show how the valve can be used. It is possible that small alterations to the valve could improve performance. The valve has a very flat static characteristic. In order to achieve that with a single-stage valve, the gain of the valve's hydromechanical control loop needs to be high. This is achieved with a relatively large pressure sensing area and a weak spring. A large pressure area requires a large solenoid, which tends to become slow when its size increases. A weak spring means that the static characteristic is dominated by the flow forces, which could result in a nonlinear and somewhat unpredictable characteristic, as was seen in this case. Since an external controller is used, the valve itself does not require a flat characteristic, but rather a linear characteristic would be more useful and easier to control. This could be achieved by using a stiffer spring that would instead dominate over the flow forces. A smaller pressure sensing area could also be used and then a smaller and faster solenoid could be used. The damping restrictor of the valve could also be designed to actually replace the low mechanical resonance with a very high hydraulic resonance. The valve would then have three dominating poles instead of four, as is the case now. It is possible that such a valve already exists on the market.

For the truck steering system case, the biggest difference from the valve's perspective is the larger cylinder volumes and flow rate. With the larger cylinder volumes the response of the original open-centre valve becomes slower. This gives an indication that the requirement on the response of the pressure control valve is lower than in the passenger car case. Larger volumes also means that the valve is less prone to react to flow disturbances, which was seen from the analysis in section 6.6, chapter 6. The performance could be further increased if a pilot-operated pressure control valve was used. The performance of the investigated valve is limited by the available control energy and relatively low flow rate. Larger volumes evidently require a higher flow rate. However, a pilot-operated valve is more complex and expensive, and a single-stage valve is therefore to be preferred. Single-stage valves with a higher flow rate already exists on the market. The flatter Boost curve of the truck steering system reduces the gain of the power steering control loop, which increases the stability margins. There is also more friction in the steering system compared to a passenger car, if a steering box is assumed and compared to a rack and pinion system. Friction will decrease the impact of the flow disturbance on steering wheel torque.

Variations in temperature have been ignored during this work. For each test, the temperature was maintained at a constant working level. However, road vehicles operate in a wide range of temperatures and the control loop must handle this. A robust controller will be able to do this and it is left for future work to investigate this further.

9

Conclusions

Relating to the research questions in chapter 1, the following conclusions can be drawn:

- 1) The electrohydraulic pressure control valve showed a nonlinear static characteristic. In order to capture this behaviour, it was necessary to consider the flow-pressure characteristic and the transition from laminar to turbulent flow. It was also necessary to consider the behaviour of the flow forces, especially the variation of the jet stream angle with spool stroke.
- 2) A state feedback controller is a feasible solution to control and increase the performance of the valve. A linear controller can be designed with respect to the worst operating condition. A nonlinear feedback linearisation controller is also a feasible solution that eliminates the nonlinear characteristic but is more complex to implement.
- 3) The main challenge with Boost curve control is the high gain at high pressure levels. Since the pressure control loop constitutes the inner loop of the power steering control loop, the additional phase shift from the valve has a great impact on stability. This can be addressed by tuning the response of the valve, alone or in combination with increasing the damping. This depends on the initial performance of the valve.
- 4) The electrohydraulic pressure control valves studied in this work adds a greater limitation in possible Boost curves than the original system. If a steep Boost curve is desired, higher damping must be added to the power steering control loop in order to ensure stability. This affects steering feel but only at high torque levels, where the steering feel is less important. A flatter Boost curve gives a greater possibility to tune the system to the desired performance. For example, a lower damping can be set. However, a lower damping will make the dynamic response to a flow disturbance of the valve more apparent and there exists a compromise.

- 5) Using the pressure control valves for position control of the steering rack offers a promising solution in realising autonomous driving. The pressure control loop constitutes the inner loop and can be treated as linear for the design of the outer position control loop. A PI-controller with a pre-filter resulted in good performance.
- 6) A truck steering system has several properties that could reduce the dynamic requirements of the electrohydraulic pressure control valve. The cylinder volumes are much larger than in a passenger car, while the maximum flow rate is also larger but not to the same extent. The larger volumes mean that the original pressure loop is slower, which gives an indication that the dynamic response of the pressure control valve can also be lower. Larger volumes also make the valve less prone to react to flow disturbances and the larger mass of the vehicle restricts the steering demand to a lower level than in a passenger car.

A

Parameter values

A.1 Parameters for HPAS system simulation

Parameter	Value	Unit
J_{sw}	0.053	kgm ²
M_{rw}	30	kg
K_t	60.73	Nm/rad
R_t	154	rad/m
b_{sw}	0.23	Nm/s
b_{rw}	$1.05 \cdot 10^4$	Ns/m
C_{rw}	$3.86 \cdot 10^6$	N/m
A_p	$8.26 \cdot 10^{-4}$	m ²
V	$1.02 \cdot 10^{-4}$	m ³
V_s	$2 \cdot 10^{-4}$	m ³
C_q	0.67	-
ρ	850	kg/m ³
β	$8.7 \cdot 10^8$	Pa
q_p	9	L/min
F_f	0.3	Nm
m_{veh}	1657	kg
J_{veh}	4875	kgm ²
C_{α_f}	$2.05 \cdot 10^5$	N/rad
C_{α_r}	$1.06 \cdot 10^6$	N/rad
l_f	0.7	m
l_r	2	m
p_t	0.055	m

Table A.1 Table of parameter values

A.2 Parameter values for commercial pressure control valve

Parameter	Value	Unit	Parameter	Value	Unit
p_{f11}	-69.18	$A/(W_b^3 m^5)$	p_{fs23}	9.16	$N/(A^4 x)$
p_{f12}	335.12	$A/(W_b^3 m^4)$	p_{fs24}	14.69	N/A^4
p_{f13}	-449.62	$A/(W_b^3 m^3)$	p_{fs31}	-12.83	$N/(A^3 x^3)$
p_{f14}	99.99	$A/(W_b^3 m^2)$	p_{fs32}	47.77	$N/(A^3 x^2)$
p_{f15}	36.65	$A/(W_b^3 m)$	p_{fs33}	-17.55	$N/(A^3 x)$
p_{f16}	324.00	A/W_b^3	p_{fs34}	-41.77	N/A^3
p_{f21}	15.23	$A/(W_b^2 m^5)$	p_{fs41}	10.76	$N/(A^2 x^3)$
p_{f22}	-72.02	$A/(W_b^2 m^4)$	p_{fs42}	-40.61	$N/(A^2 x^2)$
p_{f23}	87.96	$A/(W_b^2 m^3)$	p_{fs43}	23.38	$N/(A^2 x)$
p_{f24}	-7.55	$A/(W_b^2 m^2)$	p_{fs44}	47.33	N/A^2
p_{f25}	-5.17	$A/(W_b^2 m)$	m	0.04	kg
p_{f26}	-31.69	A/W_b^2	A_c	$6.362 \cdot 10^{-5}$	m^2
p_{f31}	2.13	$A/(W_b m^2)$	A_{cc}	$6.048 \cdot 10^{-5}$	m^2
p_{f32}	-9.48	$A/(W_b m)$	r	$1.5 \cdot 10^{-3}$	m
p_{f33}	19.14	A/W_b	h_{gap}	$1.5 \cdot 10^{-5}$	m
p_{d1}	-0.0004	A/V^3	K	0.5	N/mm
p_{d2}	-0.0003	A/V^2	f_0	2	N
p_{d3}	0.0472	A/V	B	4.47	Ns/m
p_{d4}	0.0059	A	$B_{simplified}$	223.6	Ns/m
p_{fs11}	-1.06	$N/(A^5 m^3)$	V	$1 \cdot 10^{-4}$	m^3
p_{fs12}	4.30	$N/(A^5 m^2)$	V_c	$4.7 \cdot 10^{-11}$	m^3
p_{fs13}	-2.30	$N/(A^5 m)$	ρ	900	kg/m ³
p_{fs14}	-1.55	N/A^5	β	$1 \cdot 10^9$	Pa
p_{fs21}	6.07	$N/(A^4 m^3)$	ν	$46 \cdot 10^{-6}$	m^2/s
p_{fs22}	-23.45	$N/(A^4 m^2)$	K_d	$1.24 \cdot 10^{-11}$	m^5/Ns

Table A.2 Parameter values for commercial pressure control valve simulation model.

A.3 Parameters for steering system with servo valves simulation

Parameter	Value	Unit
J_{sw}	0.053	kgm^2
M_{rw}	30	kg
K_t	60.73	Nm/rad
R_t	154	rad/m
b_{sw}	0.23	Nm/s
b_{rw}	$1.05 \cdot 10^4$	Ns/m
C_{rw}	$3.86 \cdot 10^6$	N/m
A_p	$8.26 \cdot 10^{-4}$	m^2
V_p	$1.02 \cdot 10^{-4}$	m^3
C_q	0.67	-
ρ	850	kg/m^3
β	$8.7 \cdot 10^8$	Pa
q_p	9	l/min
F_f	0.3	Nm
ω_v	150	Hz
δ_v	0.7	-
w	$6 \cdot 10^{-5}$	m^2/U

Table A.3 Table of parameter values

A.4 Parameters for truck system simulation

Parameter	Value	Unit
J_{sw}	0.09	kgm^2
J_{rw}	16	kg^2
K_t	126	Nm/rad
R_t	21	rad/m
R_l	17.7	rad/m
b_{sw}	1.8	Nm/s
b_{rw}	2500	Nms
C_{rw}	$2 \cdot 10^6$	Nm/rad
A_p	0.0092	m^2
V	$1 \cdot 10^{-3}$	m^3
V_s	$0.5 \cdot 10^{-3}$	m^3
C_q	0.67	-
ρ	900	kg/m^3
β	$1 \cdot 10^9$	Pa
q_p	16	L/min
F_f	0.3	Nm
m_{veh}	8000	kg
J_{veh}	$2.5 \cdot 10^4$	kgm^2
C_{α_f}	$3.3 \cdot 10^5$	N/rad
C_{α_r}	$2 \cdot 10^5$	N/rad
l_f	0.9	m
l_r	2.8	m
p_t	0.075	m

Table A.4 Table of parameter values

A.5 Parameters for force controller simulation model

Parameter	Value	Unit
A_p	$8.26 \cdot 10^{-4}$	m^2
C_q	0.67	-
m	10	kg
b	5000	Ns/m
p_s	200	bar
w	$1.1 \cdot 10^{-5}$	m
V	$1.02 \cdot 10^{-4}$	m^3
u	$1 \cdot 10^{-7}$	m^2
β	$8.67 \cdot 10^8$	Pa
δ_v	0.7	-
ω_v	200	Hz
ρ	850	kg/m^3

Table A.5 System parameter values for force controller simulation model.

Bibliography

- Ackermann, J. and T. Bunte (1997). “Yaw disturbance attenuation by robust decoupling of car steering”. In: *Control Eng. Practice* 5.8, pp. 1131–1136.
- Ackermann, J., T. Bunte, and D. Odenthal (1999). “Advantages of active steering for vehicle dynamics control”. In: *32nd International Symposium on Automotive Technology and Automation*.
- Ackermann, Jürgen (1998). “Active Steering for Better Safety, Handling and Comfort”. In: *Proceedings of Advances in Vehicle Control and Safety*.
- Ackermann, Jürgen and Dirk Odenthal (1999). “Damping of vehicle roll dynamics by gain scheduled active steering”. In: *Proceedings of European Control Conference*.
- Adams, F.J. (1983). “Power Steering ‘Road Feel’”. In: *SAE Paper 830998*.
- Åman, Rafael, Heikki Handroos, and Tero Eskola (2008). “Computationally efficient two-regime flow orifice model for real-time simulation”. In: *Simulation Modelling Practice and Theory* 16, pp. 945–961.
- Amirante, R., G. Del Vescovo, and A. Lippolis (2006). “Flow forces analysis of an open center hydraulic directional control valve sliding spool”. In: *Energy Conversion and Management* 47, pp. 114–131.
- Audi Dynamic Steering (2011). <http://www.audi-technology-portal.de/en/chassis/wheel-suspension-steering/dynamic-steering>.
- Auslander, D.M. (1968). “Distributed System Simulation With Bilateral Delay-Line Models”. In: *Journal of Fluids Engineering* 90.2, pp. 195–200.
- Badawy, Aly, Dieter Fehlings, et al. (2004). “Development of a New Concept of Electrically Powered Hydraulic Steering”. In: *SAE 2004 Automotive Dynamics, Stability and Control Conference and Exhibition*. 2004-01-2070. USA.
- Badawy, Aly, Jeff Zuraski, et al. (1999). “Modeling and Analysis of an Electric Power Steering System”. In: *SAE International Congress and Exposition, Steering and Suspension Technology Symposium*. 1999-01-0399. Detroit, Michigan.
- Baxter, John (1988). “Analysis of Stiffness and Feel for a Power-Assisted Rack and Pinion Steering Gear”. In: *SAE Technical Paper 880706*.

- Berg, H. and M. Ivantysynova (1999). "Design and testing of a robust linear controller for secondary controlled hydraulic drive". In: *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems & Control Engineering* 213.5, pp. 375–386.
- Birk, Wolfgang (2010). "Method for the control of vehicle steering and vehicle behaviour". WO 2010/144049 A1.
- Bohm, Jürgen (2007). "Method for steering a vehicle by means of a superimposed steering system". US 2007/0162205.
- Bootz, A. (2006). "Evolutionary Energy Saving Concepts for Hydraulic Power Steering Systems". In: *5th International Fluid Power Conference*.
- Bootz, A., Michael Brander, and B. Stoffel (2003). "Efficiency Analysis of Hydraulic Power Steering Systems". In: *The Eighth Scandinavian International Conference on Fluid Power, SICFP'03*. Tampere, Finland.
- Borghi, M., M. Milani, and R. Paoluzzi (2000). "Stationary flow force Analysis on compensated spool valves". In: *International Journal of Fluid Power* 1.1, pp. 17–25.
- Borutzky, W., B. Barnard, and J. Thoma (2002). "An orifice flow model for laminar and turbulent conditions". In: *Simulation Modelling Practice and Theory* 10, pp. 141–152.
- Branciforte, M. et al. (2011). "ANN and Non-Integer Order Modeling of ABS Solenoid Valves". In: *IEEE Transactions on Control Systems Technology* 19.3, pp. 628–635.
- Burrows, C. R. (2000). "Fluid power systems-some research issues". In: *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 214.1, pp. 203–220.
- Chapman, Stephen J. (1998). *Electric Machinery Fundamentals*. 3rd. McGraw-Hill.
- Chiriboga, Jorge, May-Win L. Thein, and Eduardo A. Misawa (1995). "Input-output feedback linearization control of a load-sensing hydraulic servo system". In: *the 4th IEEE Conference on Control Applications*.
- Dasgupta, K. and J. Watton (2005). "Dynamic analysis of proportional solenoid controlled piloted relief valve by bondgraph". In: *Simulation Modelling Practice and Theory* 13, pp. 21–38.
- Duffy, James J. (1988). "Variable assist power steering system using electronic pressure control". US 4760892.
- Edge, K. A. (1997). "The control of fluid power systems - responding to the challenges". In: *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems & Control Engineering* 211.2, pp. 91–110.
- Eidehall, Andreas et al. (2007). "Towards autonomous collision avoidance by steering". In: *IEEE Transactions on Intelligent Transportation Systems* 8, pp. 84–94.
- Ellman, Akso and Robert Piché (1996). "A modified orifice flow formula for numerical simulation of fluid power systems". In: *Fluid Power Systems and Technology* 3, pp. 59–63.

- Enache, N. Minoiu et al. (2009). "Driver steering assistance for lane departure avoidance". In: *Control Engineering Practice* 17, pp. 624–651.
- Erhard, M., J. Weber, and G. Schoppel (2013). "Geometrical Design and Operability Verification of a Proportional Pressure Relief Valve". In: *The 13th Scandinavian International Conference on Fluid Power, SICFP2013*. Linköping, Sweden.
- Eriksson, Björn (2010). "Mobile Fluid Power Systems Design". PhD thesis. Linköping University.
- Eriksson, Björn, Marcus Rösth, and Jan-Ove Palmberg (2009). "Energy saving utilizing LQ-technique design". In: *the Seventh International Conference on Fluid Power Transmission and Control, ICFP'09*. Hangzhou, China, pp. 224–229.
- Forbes, James E., Steven M. Baird, and Thomas W. Weisgerber (1987). "Electrohydraulic Power Steering - An Advanced System for Unique Applications". In: *International Congress and Exposition*. 870574. Detroit, Michigan, USA.
- Gamble, J. B. and N. D. Vaughan (1996). "Comparison of sliding mode control with state feedback and PID control applied to a proportional valve". In: *Transactions of the ASME Journal of Dynamic Systems, Measurements and Control* 118.3, pp. 434–438.
- Genta, Giancarlo and Lorenzo Morello (2009). *The Automotive Chassis Volume 1: Component Design*. Springer.
- Gessat, Jochen (2007). "Electrically Powered Hydraulic Steering Systems for Light Commercial Vehicles". In: *SAE 2007 Commercial Vehicle Engineering Congress and Exhibition*. 2007-01-4197.
- Gillespie, Thomas D. (1992). *Fundamentals of vehicle dynamics*. Society of Automotive Engineers.
- Glad, Torkel and Lennart Ljung (2003). *Reglerteori*. Studentlitteratur.
- Glad, Torkel and Lennart Ljung (2006). *Reglerteknik*. Studentlitteratur.
- Gunnarsson, Svante and Petter Krus (1993). "LQG control of a flexible mechanical system using hydraulic actuators". In: *the 12th World Congress, IFAC*.
- Hahn, H., A. Piepenbrink, and K.-D. Leimbach (1994). "Input/output linearization control of an electro servo-hydraulic actuator". In: *the Third IEEE Conference on Control Applications*.
- Harrer, Manfred, Peter E. Pfeffer, and Nigel D. Johnston (2006). "Steering Feel - Objective Assessment of Passenger Cars Analysis of Steering Feel and Vehicle Handling". In: *Fisista World Automotive Congress*.
- iHSA, intelligent Hydraulic Steering Assist. <http://www.td-steering.com/intelligent-hydraulic-steering-assist.html>.
- Inaguma, Yoshiharu, Keiji Suzuki, and Kyouosuke Haga (1996). "An Energy Saving Technique in an Electro-Hydraulic Power Steering (EHPS) System". In: *SAE Paper 960934*.
- Isidori, Alberto (1995). *Nonlinear Control Systems*. Springer-Verlag.

- Jaksch, Friedrich O. (1979). "Driver-Vehicle Interaction with Respect to Steering Controllability". In: *SAE Technical Paper*.
- Keller, Christoph G. et al. (2011). "Active Pedestrian Safety by Automatic Braking and Evasive Steering". In: *IEEE Transactions on Intelligent Transportation Systems* 12.4, pp. 1292–1304.
- Kemmettmüller, Wolfgang, Steffen Müller, and Andreas Kugi (2007). "Mathematical Modeling and Nonlinear Controller Design for a Novel Electrohydraulic Power-Steering System". In: *IEEE/ASME Transactions on Mechatronics* 12.1, pp. 85–97.
- Koehn, Philip and Michael Eckrich (2004). "Active Steering - The BMW Approach Towards Modern Steering Technology". In: *SAE Steering and Suspension Technology Symposium*. 2004-01-1105. Detroit, Michigan, USA.
- Krus, Petter and Svante Gunnarsson (1993). "Adaptive control of a hydraulic crane using on-line identification". In: *the Third Scandinavian International Conference on Fluid Power*. Linköping, Sweden.
- Lennevi, Jerker and Jan-Ove Palmberg (1995). "Application and implementation of LQ design method for the velocity control of hydrostatic transmissions". In: *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems & Control Engineering* 209.4, pp. 255–268.
- Leutner, Wilfried (2005). "Valve". US 2005/0258384 A1.
- Lexus Variable Gear Ratio Steering, VGRS. http://210.211.49.136/technology_explorer/variable_gear_ratio_steering_VGRS.asp?model=All.
- McCann, Roy and Anh Le (2008). "Gain Scheduling in Commercial Vehicles with Electrohydraulic Power Steering". In: *SAE Paper 2008-01-2703*.
- McCann, Roy and Son Nguyen (2004). "Jackknife Avoidance in Large Trucks Using Active Front Steering". In: *SAE Paper 2004-01-2639*.
- Merritt, Herbert E. (1967). *Hydraulic Control Systems*. John Wiley & Sons, USA.
- Mesiti, Domenico and Renzo Avenatti (1995). "Power steering device for motor-vehicles". 0665157A1.
- Morton, Christopher, Volker Pickert, and Matthew Armstrong (2014). "Self-Alignment Torque as a Source of Energy Recovery for Hybrid Electric Trucks". In: *IEEE Transactions on Vehicular Systems* 63.1, pp. 62–71. DOI: 10.1109/TVT.2013.2271049.
- Müller, S., A. Kugi, and W. Kemmettmüller (2005). "Analysis of a closed-center hydraulic power steering system for full steer-by-wire functionality and low fuel consumption". In: *The Ninth Scandinavian International Conference on Fluid Power, SICFP'052*.
- Neureder, U. (2002). "Investigation into Steering Wheel Nibble". In: *Institution of Mechanical Engineers, Journal of Automobile Engineering* 216, pp. 297–277.
- Norman, Kenneth D. (1984). "Objective Evaluation of On-Center Handling Performance". In: *SAE Technical paper 840069*.

- Palmberg, Jan-Ove (1983). *Tryckstyrning (in Swedish)*. Tech. rep. Division of fluid and mechanical engineering systems, Linköping University.
- Palmberg, Jan-Ove, Bo Andersson, and Christer Malmros (1983). “Analysis and synthesis of hydraulic pressure regulators”. In: *Applied Control & Identification, IASTED*. Copenhagen, Denmark.
- Pawlak, Andrzej M., David W. Graber, and Dennis C. Eckhardt (1994). “Magnetic power steering assist system - Magnasteer”. In: *SAE International Congress and Exposition*. 940867.
- Pfeffer, Peter E. (2006). “Interaction of Vehicle and Steering System Regarding On-Center Handling”. PhD thesis. Department of Mechanical Engineering, University of Bath.
- Pfeffer, Peter E. and Manfred Harrer (2008). “On-Centre Steering Wheel Torque Characteristics during Steady State Cornering”. In: *SAE World Congress & Exhibition*.
- Pfeffer, Peter E., Manfred Harrer, et al. (2006). “Modelling of a Hydraulic Steering System”. In: *Fisita World Automotive Congress*.
- Pfeffer, Peter E., D. Nigel Johnston, et al. (2005). “Energy Consumption of Electro-Hydraulic Steering Systems”. In: *SAE 2005 World Congress & Exhibition*.
- Plummer, A.R. and N.D. Vaughan (1996). “Robust Adaptive control for hydraulic servosystems”. In: *Journal of dynamic systems, measurements and control* 118, pp. 237–234.
- Pluschke, Albrecht and Thomas Stehli (2001). “Hydraulic power steering with a closed center”. US 6250416 B1.
- Post, J.W. and E.H. Law (1996). “Modeling, Characterization and Simulation of Automobile Power Steering Systems for the Prediction of On-Center Handling”. In: *SAE International Congress & Exposition*. 960178. USA.
- Ramdén, Christer (1999). “Experimental methods in valve characteristic assessment”. PhD thesis. Linköping University.
- Roos, Fredrik (2005). *DESIGN AND THEORETICAL EVALUATION OF ELECTRIC POWER STEERING IN HEAVY VEHICLES*. Tech. rep. Department of Machine Design, Royal Institute of Technology.
- Rösth, M. and J-O. Palmberg (2007). “Robust design of power steering system with emphasis on chattering phenomena”. In: *The 10th Scandinavian International Conference on Fluid Power, SICFP’07*.
- Rösth, M., J. Pohl, and J-O. Palmberg (2003). “Active Pinion - A Cost Effective Solution for Enabling Steering Intervention in Road Vehicles”. In: *The Bath Workshop on Power Transmission & Motion Control, PTMC’03*. Bath, United Kingdom.
- Rösth, M., J. Pohl, and J-O. Palmberg (2006). “Increased Hydraulic Power Assisted Steering Functionality Using the Active Pinion Concept”. In: *The 5th International Fluid Power Conference, IFK 2006*. Aachen, Germany.

- Rösth, Marcus (2007). “Hydraulic Power Steering System Design in Road Vehicles; Analysis, Testing and Enhanced Functionality”. PhD thesis. Division of Fluid and Mechanical Engineering Systems, Linköping University.
- Rothhämel, Malte (2010). “Capturing Steering Feel - A step towards implementation of active steering in heavy vehicles”. PhD thesis. Royal Institute of Technology.
- Rothhämel, Malte (2011). “Anordning för aktiv styrning av ett lastfordon och styrinrättning med sådan anordning”. SE 534469 C2.
- Sannelius, Mikael (1999). “On Complex Hydrostatic Transmissions”. PhD thesis. Linköping University.
- Schorn, Matthias et al. (2006). “Nonlinear Trajectory Control for Automatic Steering of a Collision Avoiding Vehicle”. In: *American Control Conference*.
- Segel, Leonard (1964). “An investigation of automobile handling as implemented by a variable-steering automobile”. In: *Human factors* 6.4, pp. 333–341.
- Segel, Leonard (1966). “On the Lateral Stability and Control of the Automobile as Influenced by the Dynamics of the Steering System”. In: *ASME Journal of Engineering for Industry* 88.3, pp. 283–295.
- Seo, Jaho, Ravinder Venugopal, and Jean-Pierre Kenné (2007). “Feedback linearization based control of a rotational hydraulic drive”. In: *Control Engineering Practice* 15, pp. 1495–1507.
- Stone, Richard and Jeffrey K. Ball (2004). *Automotive Engineering Fundamentals*. SAE International.
- Suzuki, Keiji et al. (1995). “Integrated Electro-Hydraulic Power Steering System with Low Electric Energy Consumption”. In: *SAE International Congress and Exposition*. 950580. USA.
- Tai, Meihua, Pushkar Hingwe, and Masayoshi Tomizuka (2004). “Modeling and Control of Steering System of Heavy Vehicles for Automated Highway Systems”. In: *IEEE/ASME Transactions on Mechatronics* 9, pp. 609–618.
- Tjønnås, Johannes and Tor A. Johansen (2010). “Stabilization of Automotive Vehicles Using Active Steering and Adaptive Brake Control Allocation”. In: *IEEE Transaction on Control Systems Technology* 18.3, pp. 545–558.
- Ueda, Etsuhiro et al. (2002). “The Development of Detailed Steering Model for On-Center Handling Simulation”. In: *International Symposium on Advanced Vehicle Control (AVEC)*, pp. 657–662.
- Vaughan, N.D. and J.B. Gamble (1996). “The Modeling and Simulation of a Proportional Solenoid Valve”. In: *Transactions of the ASME Journal of Dynamic Systems, Measurements and Control* 118, pp. 120–125.
- Volvo Dynamic Steering (2013). http://www.volvogroup.com/group/global/en-gb/newsmedia/corpnews/_layouts/cwp.internet.volvocom/newsitem.aspx?news.itemid=139174&news.language=en-gb.
- Walker, Paul D., Bo Zhu, and Nong Zhang (2014). “Nonlinear Modeling and Analysis of Direct Acting Solenoid Valves for Clutch Control”. In: *Transac-*

- tions of the ASME Journal of Dynamic Systems, Measurements and Control 136.
- Wellenzohn, Michael (2008). "Improved Fuel Consumption through Steering Assist with Power on Demand". In: *SAE Technical Paper 2008-21-0046*.
- Wenzel, Craig E., Thomas J. Stobbs, and James R. Ward (1996). "Variable assist power steering system with electro-hydraulic working pressure and directional pressure control". US 5553683.
- Wiesel, Urs et al. (2009). "A method for evaluating the fuel saving potential of different hybrid steering system configurations in heavy vehicles". In: *VDI-Berichte 2068*, pp. 167–188.
- Williams, D E (2009). "Synthetic torque feedback to improve heavy vehicle drivability". In: *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering* 223, pp. 1517–1527.
- Wong, J.Y. (2001). *Theory of ground vehicles*. Third. John Wiley & Sons, Inc.
- Wu, Duqiang, Richard Burton, and Greg Schoenau (2002). "An empirical discharge coefficient model for orifice flow". In: *International Journal of Fluid Power* 3.3, pp. 13–18. DOI: 10.1080/14399776.2002.10781143.
- Yu, Wentao and William Szabela (2007). "Energetic Efficiency Analysis and Performance Evaluation for a Closed Center Steering System". In: *SAE 2007 Commercial Vehicle Engineering Congress & Exhibition*. 2007-01-4217.
- Zaremba, A. and R.I. Davis (1995). "Dynamic Analysis and Stability of a Power Assist Steering System". In: *Proceedings of the American Control Conference*. Seattle, Washington.