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Effect of Energy Harvesting on Stable Throughput in Cooperative Relay Systems

Nikolaos Pappas, Marios Kountouris, Jeongho Jeon, Anthony Ephremides, Apostolos Traganitis

Abstract—In this paper, the impact of energy constraints on a two-hop network with a source, a relay and a destination under random medium access is studied. A collision channel with erasures is considered, and the source and the relay nodes have energy harvesting capabilities and an unlimited battery to store the harvested energy. Additionally, the source and the relay node have external traffic arrivals and the relay forwards a fraction of the source node’s traffic to the destination; the cooperation is performed at the network level. An inner and an outer bound of the stability region for a given transmission probability vector are obtained. Then, the closure of the inner and the outer bound is obtained separately and they turn out to be identical. This work is not only a step in connecting information theory and networking, by studying the maximum stable throughput region metric but also it taps the relatively unexplored and important domain of energy harvesting and assesses the effect of that on this important measure.

I. INTRODUCTION

Taking advantage of renewable energy resources from the environment, also known as energy harvesting, enables unattended operability of infrastructure-less wireless networks. There are various forms of energy that can be harvested, including thermal, solar, acoustic, wind, and even ambient radio power [2]. Energy harvesting is recently seen as a promising feature for wireless networks regarding self-sustainability and also efficiency. This permits long-term operation of distributed wireless communication systems, such as sensor networks, without the need for regular maintenance. However, the additional functionality of energy harvesting in wireless networks introduces several changes and calls for assessment of the system long-term performance such as in terms of the throughput and stability. The ideal scenario is to make the energy limitations transparent to the network.

Among distributed communication protocols, we are particularly interested in ALOHA, a simple random access scheme in which transmission attempts are performed randomly, independently, and distributively [3]. In [4], the capability of energy harvesting was first introduced in the analysis of the slotted ALOHA for a simple setting as an initial step to understand its impact on the achievable stability region. Recently, this result has been generalized in [5] by taking into account the multi-packet reception capability at the receiver and finite capacity batteries at the energy harvesting sources. In [6], a cognitive access protocol was studied for the scenario where the higher priority primary source is powered by harvesting energy whereas the lower priority secondary source is assumed to have a reliable power supply.

Cooperative communication is one of key technologies to achieve coverage extension and throughput enhancement in wireless networks [7]. In this work, we consider packet-level cooperation rather at the physical layer, in which a relay node takes responsibility of packet delivery for those it could overhear and successfully decode from the transmissions by source node [8]–[10]. A key difference between physical-layer and network-layer cooperation is that the latter can capture the bursty nature of traffic. The impact of network-level cooperation in an energy harvesting network with a pure relay (without its own traffic) under scheduled access (time division multiple access in a controlled manner) was studied in [11].

A major limitation of Information Theory is the inability to handle bursty traffic and queueing delay. In the communications networks bursty traffic and delay are central and indispensable concepts. The information theoretic capacity region is derived under the assumption of saturated queues. However, under stochastic and bursty traffic arrivals, the maximum stable throughput or stability region becomes a meaningful and relevant measure of rates in packets per slot in wireless networks. Thus, the maximum stable throughput is an important performance measure, akin to information theoretic capacity, but simpler to track and analyze and more appropriate for systems with sources that generate signals randomly in time. Understanding the relationship between information-theoretic capacity and stability region has received considerable attention in recent years and some progress has been made primarily for multiple access channels [12].

The characterization of random access stability for bursty traffic is a challenging problem even without energy harvesting [13]–[15]. Additionally, for a network with more than three users (interacting queues), the exact characterization of the stability region is not known. This is because each node transmits and, thereby, interferes with the others only when its queue is non-empty. Such queues are said to be interacting
with each other in the sense that the service process of one
depends on the status of the others. The analysis for the case
with energy harvesting becomes significantly more challenging
because the service process of a node depends not only on the
status of its own queue and battery, but also on the status of
the other node’s queue and battery.

In this paper, we study the impact of energy constraints on
a two-hop network with a source, a relay and a destination
under random medium access as shown in Fig. 1. We assume
a collision channel with erasures. Both the source and the
relay node have external traffic arrivals. The relay forwards a
fraction of the source node’s traffic to the destination and the
cooperation is performed at the network level. In addition, both
source and relay nodes have energy harvesting capabilities
and an unlimited battery to store the harvested energy. We
provide necessary and sufficient conditions for the stability of
the considered network as shown in Fig. 1. We first obtain an
inner and an outer bound of the stability region for a given
transmission probability vector. We then take the closure of
the inner and the outer bound separately over all feasible
transmission probability vectors. Interestingly, it turns out
that the bounds are tight in terms of the closure, as also
stated in [5]. This study provides insights on designing a
relay-assisted network under energy constraints. When the
aggregate charging rate is above one and the source and the
relay lie in the intermediate traffic regime, the system has
identical performance with that of a network without energy
constraints, meaning that in that regime the energy limitations
are transparent to the network operation. In this paper we
focus on a simple network, as mentioned earlier, more realistic
and complex systems are impossible to analyze, primarily
due to the difficulty in tracking interacting queue. However,
insights can still be obtained, even from simple models.
This work provides a step in connecting information theory
and networking, by studying the maximum stable throughput
region metric. Further, it taps the relatively unexplored and
important domain of energy harvesting and assesses the effect
of that on this important measure.

The rest of this paper is organized as follows. In Section II,
we define the stability region, describe the channel model, and
explain the packet arrival and energy harvesting models. In
Section III, we present inner and outer bounds on the stability
region as well as the closure of the stability region. The proofs
of our results are given in IV and V. Finally, we conclude our
work in Section VI.

II. SYSTEM MODEL

We consider a time-slotted system in which the nodes
randomly access a common receiver and both source and relay
nodes are powered from randomly time-varying renewable
energy sources, as shown in Fig. 1. Each node stores the
harvested energy in a battery of unlimited capacity. We denote
with \( S, R, \) and \( D, \) the source, the relay and the destination,
respectively. Packet traffic originates from both \( S \) and \( R, \)
and because of the wireless broadcast nature, \( R \) may receive some
of the packets transmitted from \( S, \) which in turn can be relayed
to \( D. \) The packets from \( S \) that fail to be received by \( D \) but are
successfully received by \( R \) are relayed by \( R. \) A half-duplex
constraint is imposed here, i.e. \( R \) can overhear \( S \) only when it
is idle.

Each node has an infinite size buffer for storing incoming
packets and the transmission of each packet occupies one
time slot. Node \( R \) has separate queues for the exogenous
arrivals and the endogenous arrivals being relayed through \( R. \)
Nevertheless, we can let \( R \) have a single queue and merge all
arrivals into a single queue as the achievable stable throughput
region is not affected [16]. This is due to the fact that the link
quality between \( R \) and \( D \) is independent of which packet is
selected for transmission.

The packet arrival and energy harvesting processes at \( S \)
and \( R \) are assumed to be Bernoulli with rates \( \lambda_S, \delta_S \) and \( \lambda_R, \delta_R \),
respectively, and are independent of each other. \( Q_i \) and
\( B_i, i = S, R, \) denote the steady state number of packets and
energy units in the queue and the energy source at node \( i, \)
respectively. Furthermore, a node \( i \) is called active if both its
packet queue and its battery are nonempty at the same time,
which is denoted by the event \( A_i = \{ \{ B_i \neq 0 \} \cap \{ Q_i \neq 0 \} \} \)
and idle otherwise (denoted by \( \overline{A_i} \) ). In each time slot, nodes
\( S \) and \( R \) attempt to transmit with probabilities \( q_S \) and \( q_R \),
respectively, whenever they are active. Decisions on trans-
mission are made independently among the nodes and each
transmission consumes one energy unit. We assume a collision
channel with erasures in which if both \( S \) and \( R \) transmit at
the same time slot, a collision occurs and both transmissions
fail. The probability that a packet transmitted by node \( i \) is
successfully decoded at node \( j(\neq i) \) is denoted by \( p_{ij} \),
which is the probability that the signal-to-noise ratio (SNR) over
the specified link exceeds a certain threshold for successful
decoding. These erasure/outage probabilities capture the effect
of random fading at the physical layer. The probabilities \( p_{SD}, \)
\( p_{RD}, \) and \( p_{SR} \) denote the success probabilities over the link
\( S – D, R – D, \) and \( S – R, \) respectively. We also assume that
node \( R \) has a better channel to \( D \) than \( S, \) i.e. \( p_{RD} > p_{SD}. \)

The cooperation is performed at the protocol (network)
level as follows: when \( S \) transmits a packet, if \( D \) decodes it
successfully, it sends an ACK and the packet exits the network;
if \( D \) fails to decode the packet but \( R \) does, then \( R \) sends an
ACK and takes over the responsibility of delivering the packet
to \( D \) by placing it in its queue. If neither \( D \) nor \( R \) decode (or if
\( R \) does not store the packet), the packet remains in \( S \)’s queue
for retransmission. The ACKs are assumed to be error-free,
instantaneous, and broadcast to all relevant nodes.

The average service rate for the source node is given by

\[
\mu_S = \left[ q_S \left( 1 – q_R \right) \Pr (B_S \neq 0, A_R) + q_S \Pr( B_S \neq 0, \overline{A_R}) \right] \times [ p_{SD} + (1 – p_{SD}) p_{SR}],
\]

and for the relay is given by

\[
\mu_R = \left[ q_R \left( 1 – q_S \right) \Pr (B_R \neq 0, A_S) + q_R \Pr(B_R \neq 0, \overline{A_S}) \right] \times p_{RD},
\]

Denote by \( Q_i^t \) the length of queue \( i \) at the beginning of time
slot \( t \). Based on the definition in [15], the queue is said to be
The closure of the stability region is defined by
\[
\mathcal{L}(\delta_S, \delta_R) \triangleq \bigcup_{(q_S, q_R) \in [0,1]^2} \mathcal{L}(q_S, q_R, \delta_S, \delta_R).
\] (8)

The following theorem describes the closure of the stability region for the network we consider.

**Theorem III.1.** If \( \delta_S + \delta_R \geq 1 \), the closure of the stability region, \( \mathcal{L}(\delta_S, \delta_R) \), is illustrated in Fig. 3 and is described by three parts. (i) The line segment AB, where \( x_A = 0 \), \( y_A = \delta_R p_{RD} \) and \( x_B = (1 - \delta_R)^2 [ p_{SD} + (1 - p_{SD}) p_{SR} ] \), \( y_B = \delta_R^2 p_{RD} - (1 - \delta_R)^2 (1 - p_{SD}) p_{SR} \). (ii) the curve from B to C which is described by (6). (iii) the line segment CD where \( x_C = \delta_S^2 [ p_{SD} + (1 - p_{SD}) p_{SR} ] \), \( y_C = (1 - \delta_S)^2 p_{RD} - \delta_S (1 - p_{SD}) p_{SR} \), and \( x_D = \min \left\{ (1 - \delta_S)^2 p_{SD} + (1 - p_{SD}) p_{SR} \right\} \), \( y_D = 0 \). If \( \delta_S + \delta_R < 1 \), the closure of the stability region, \( \mathcal{L}(\delta_S, \delta_R) \), is illustrated in Fig. 4 and is described by the line segments EF and FG, where \( x_E = 0 \), \( y_E = \delta_R p_{RD} \), \( x_F = \delta_S (1 - \delta_R) [ p_{SD} + (1 - p_{SD}) p_{SR} ] \), \( y_F = \delta_R (1 - \delta_S) p_{RD} - \delta_S (1 - \delta_R) (1 - p_{SD}) p_{SR} \). \( x_G \) is given by (9) and \( y_G = 0 \).

**Proof.** The proof is given in Section V. \( \square \)

**Remark 1:** Regarding the stability region \( \mathcal{L}(q_S, q_R, \delta_S, \delta_R) \), we only have inner and outer bounds and not the exact expression, however for the closure \( \mathcal{L}(\delta_S, \delta_R) \), we do have the exact characterization.

**Remark 2:** If \( \delta_S + \delta_R < 1 \) as depicted in Fig. 4, the closure of the stability region has a linear behavior, which is an indication that the performance of the system is affected by the low energy harvesting rates. Furthermore when \( \delta_S + \delta_R > 1 \), when the arrival rate at the source or the relay is at high arrival rate regime then the performance is affected by the energy harvesting rate and is depicted by the linear segments in Fig. 3. The interesting case is when both the arrival rates \( \lambda_S \) and \( \lambda_R \) lie in the intermediate arrival rate regime, then the performance is identical to the relay network without energy limitations and the closure of the stability region has a non-linear behavior.

**IV. Analysis**

To derive the stability condition for the queue in the relay node, we need to calculate the total arrival rate. There are two independent arrival processes at the relay: the exogenous traffic with arrival rate \( \lambda_R \) and the endogenous traffic from \( S \). Denote by \( S_A \) the event that \( S \) transmits a packet and the packet leaves the queue, then
\[
\Pr(S_A) = [1 - q_R \Pr(A_R)] [p_{SD} + (1 - p_{SD}) p_{SR}].
\] (10)

Among the packets that depart from the queue of \( S \), some will exit the network because they are decoded by the destination directly, and some will be relayed by \( R \). Denote by \( S_B \) the event that the transmitted packet from \( S \) will be relayed from \( R \), then
\[
\Pr(S_B) = [1 - q_R \Pr(A_R)] (1 - p_{SD}) p_{SR}.
\] (11)
The conditional probability that a transmitted packet from $S$ is relayed by $R$ given that the transmitted packet exits node $S$’s queue is given by

$$
\Pr(S_B|S_A) = \frac{(1 - p_{SD})p_{SR}}{p_{SD} + (1 - p_{SD})p_{SR}}.
$$

The arrival rate from the source to the relay is

$$
\lambda_{S \rightarrow R} = \Pr(S_B|S_A)\lambda_S.
$$

Note that the average arrival rate from the source to the relay, $\lambda_{S \rightarrow R}$, does not depend on the transmission probabilities or the energy harvesting rates, it depends only from the success probabilities $p_{SD}$ and $p_{SR}$.

The total arrival rate at the relay node is given by

$$
\lambda_{R,\text{total}} = \lambda_R + \frac{(1 - p_{SD})p_{SR}}{p_{SD} + (1 - p_{SD})p_{SR}}\lambda_S.
$$

### A. Sufficient Conditions

A queue is considered saturated if in each time slot there is always a packet to transmit, i.e. the queue is never empty. Assuming saturated queues for the source and the relay node, the saturated throughput for the source node is given by (15) and for the relay is given by (16).

Each node transmits with probability $q_i$, $i = S, R$, whenever its battery is not empty and each transmission demands one energy packet. Each energy queue $i$ is then decoupled and forms a discrete-time $M/M/1$ queue with input rate $\delta_i$ and service rate $q_i$, thus the probability the energy queue to be empty is given by

$$
\Pr(B_i \neq 0) = \min\left(\frac{\delta_i}{q_i}, 1\right).
$$

Then, after some calculations, we obtain that the saturated throughput for the source is

$$
\mu_S^s = \min(\delta_S, q_S) [1 - \min(\delta_R, q_R)] [p_{SD} + (1 - p_{SD})p_{SR}],
$$

and for the relay is

$$
\mu_R^s = \min(\delta_R, q_R) [1 - \min(\delta_S, q_S)] [p_{RD} + (1 - p_{SD})p_{SR}].
$$

The sufficient conditions ($\mathcal{R}_{inner}$) for the stability are obtained by $\lambda_{S} < \mu_S^s$ and $\lambda_{R,\text{total}} < \mu_R^s$ and are given by (3), in Proposition III.1.

The saturated throughput that we obtained in this subsection is an inner bound of the stability region, since the assumption that the source and the relay have always packets to transmit leads to lower achievable rates in terms of packet per slot [12], [18], [19].

### B. Necessary Conditions

The average service rates for the source and the relay are given by (1) and (2), respectively. The average service rate of each queue depends on the status of its own energy and also the queue size and the energy statuses of the other queues. This coupling between the queues (both packet and energy) results in a four dimensional Markov chain which makes the analysis cumbersome. Therefore, the stochastic dominant technique [14] is essential in order to decouple the interaction between the queues, and thus to characterize the stability region. Thus, we first construct parallel dominant systems in which one of the nodes transmits dummy packets when its packet queue is empty. Note that even in the dominant system a node cannot transmit if the energy source is empty (because even the dummy packet consumes one energy unit).

We consider the first hypothetical system in which the source node transmits dummy packets when its queue is empty.
After changing (17) and (22) into (1), the service rate for the source becomes

$$\mu_S = \min(\delta_S, q_S) \left[ 1 - \frac{\lambda_R + (1 - p_{SD}) p_{SR} \lambda_S}{1 - \min(\delta_S, q_S) p_{RD}} \right] \frac{p_{SD} + (1 - p_{SD}) p_{SR}}{}.$$  

(23)

The queue in $S$ is stable if $\lambda_S < \mu_S$ and after some manipulations we obtain

$$\left[ 1 + \frac{\min(\delta_S, q_S) (1 - p_{SD}) p_{SR}}{1 - \min(\delta_S, q_S) p_{RD}} \right] \lambda_S + \frac{\min(\delta_S, q_S) [p_{SD} + (1 - p_{SD}) p_{SR}]}{1 - \min(\delta_S, q_S) p_{RD}} \lambda_R < \min(\delta_S, q_S) [p_{SD} + (1 - p_{SD}) p_{SR}].$$

(24)

The derived stability conditions from the first hypothetical system are summarized in (4).

In the second hypothetical system, the relay node transmits dummy packets and all the other assumptions remain intact. Thus, the average service rate for the source given by (1) becomes

$$\mu_S = \{q_S (1 - q_R) \Pr(B_S \neq 0, B_R \neq 0) + q_S \Pr(B_S = 0, B_R = 0)\} \times [p_{SD} + (1 - p_{SD}) p_{SR}],$$

which is equal to saturated throughput of the source and is given by

$$\mu_S = \min(\delta_S, q_S) \left[ 1 - \min(\delta_R, q_R) \right] [p_{SD} + (1 - p_{SD}) p_{SR}].$$

(26)

From Loyne’s theorem, the queue in source is stable if $\lambda_S < \mu_S$, thus

$$\lambda_S < \min(\delta_S, q_S) \left[ 1 - \min(\delta_R, q_R) \right] [p_{SD} + (1 - p_{SD}) p_{SR}].$$

(27)

The average number of packets per active slot for $S$ is $q_S \left[ 1 - \min(\delta_R, q_R) \right] [p_{SD} + (1 - p_{SD}) p_{SR}].$ The fraction of active slots for the source $S$ is

$$\Pr(B_S \neq 0, Q_S \neq 0) = \frac{\lambda_S}{q_S \left[ 1 - \min(\delta_R, q_R) \right] [p_{SD} + (1 - p_{SD}) p_{SR}]}.$$  

(28)

After replacing from (17) and (28) into (2), the service rate for the relay is

$$\mu_R = \min(\delta_R, q_R) \left[ 1 - \frac{\lambda_S}{1 - \min(\delta_R, q_R) \left[ p_{SD} + (1 - p_{SD}) p_{SR} \right]} \right] p_{RD}.$$  

(29)

The queue in the relay node $R$ is stable if $\lambda_{R, \text{total}} < \mu_R$ and after some manipulations we obtain

$$\lambda_R + \frac{[1 - \min(\delta_R, q_R) \left[ (1 - p_{SD}) p_{SR} \right] + \min(\delta_R, q_R) p_{RD}}{[1 - \min(\delta_R, q_R) \left[ p_{SD} + (1 - p_{SD}) p_{SR} \right]} \lambda_S < \min(\delta_R, q_R) p_{RD}.$$  

(30)

The derived stability conditions from the second hypothetical system are given by (5).

An important observation made in [14] is that the stability conditions obtained by using the stochastic dominance technique are not merely sufficient conditions for the stability of
\[ x_G = \min \left\{ \frac{(1 - \delta_S)\delta_R P_{RD} [P_{SD} + (1 - P_{SD})P_{SR}]}{(1 - P_{SD})P_{SR}}, \frac{\delta_S(1 - \delta_S)P_{RD} [P_{SD} + (1 - P_{SD})P_{SR}]}{(1 - \delta_S)P_{RD} + \delta_S(1 - P_{SD})P_{SR}} \right\} \] (9)

\[ \lambda_R \]

\[ \delta_S + \delta_R \geq 1 \]

Fig. 3: The closure of the stability region for \( \delta_S + \delta_R \geq 1 \).

\[ \delta_S + \delta_R < 1 \]

Fig. 4: The closure of the stability region for \( \delta_S + \delta_R < 1 \).

the original system but are sufficient and necessary conditions. However, the indistinguishability argument does not apply to our problem. In a system with batteries, the dummy packet transmissions affect the dynamics of the batteries. For example, there are instants when a node is no more able to transmit in the hypothetical system because of the lack of energy, while it is able to transmit in the original system, thus it may result to a better chance of success for the other node.

The obtained stability conditions are necessary conditions of the original system and are summarized in Proposition III.2.

V. PROOF OF THEOREM III.1

In this section we will derive the closure of the outer of the stability region defined in Proposition III.2. The closure will be obtained over all the feasible transmission probability vectors \((q_S, q_R) \in [0, 1]^2\) and is defined by (8). After that we will show that the closure of the outer bound can be achieved.

An interesting observation is that the outer bound in Proposition III.2 does not depend on \( \delta_i, i = S, R \) for \( q_S \leq \delta_S \) and \( q_R \leq \delta_R \). Furthermore, if \( q_i \) is increased over \( \delta_i \) for \( i = S, R \) has no effect because the value of \( \min(\delta_i, q_i) \) is bounded by \( \delta_i \).

In order to obtain the closure we have to solve two optimization problems. By replacing \( \lambda_S \) by \( x \) and \( \lambda_R \) by \( y \), the optimization problems are \([P_1]\) and \([P_2]\).

\[
[P_1] \quad \max_{q_S} \quad x = \frac{q_S(1 - q_S) [P_{SD} + (1 - P_{SD})P_{SR}] P_{RD} - q_S - [P_{SD} + (1 - P_{SD})P_{SR}] (1 - q_S)P_{RD} + q_S(1 - P_{SD})P_{SR}}{(1 - q_S)P_{RD} + q_S(1 - P_{SD})P_{SR}} y
\] (31)

subject to

\[
(q_S, q_R) \in [0, \delta_S] \times [0, \delta_R] \quad (x, y) \in [0, 1]^2
\] (32)

and

\[
[P_2] \quad \max_{q_R} \quad y = q_R P_{RD} - \frac{1 - (P_{SD})P_{SR}}{[P_{SD} + (1 - P_{SD})P_{SR}]} x - \frac{q_R P_{RD}}{(1 - q_R)P_{RD} + q_S(1 - P_{SD})P_{SR}} x
\] (35)

subject to

\[
q_S(1 - q_R) [P_{SD} + (1 - P_{SD})P_{SR}] \quad (q_S, q_R) \in [0, \delta_S] \times [0, \delta_R] \quad (x, y) \in [0, 1]^2
\] (36)

In order to solve \([P_2]\), the differentiation of \( y \) with respect to \( q_R \) gives

\[
\frac{dy}{dq_R} = P_{RD} \left[ 1 - \frac{x}{(1 - q_R)^2 [P_{SD} + (1 - P_{SD})P_{SR}]} \right].
\] (39)

The second derivative is negative since

\[
\frac{d^2y}{dq_R^2} = -\frac{2P_{RD}x}{(1 - q_R)^3 [P_{SD} + (1 - P_{SD})P_{SR}]} < 0,
\] (40)

thus, the objective function is concave with respect to \( y \). Solving the equation \( \frac{dy}{dq_R} = 0 \), we obtain the maximizing \( q^*_R \) where

\[
q^*_R = 1 - \sqrt{\frac{x}{P_{SD} + (1 - P_{SD})P_{SR}}},
\] (41)

the corresponding maximum value of the objective function is

\[
y^* = P_{RD} - 2P_{RD} \sqrt{\frac{x}{P_{SD} + (1 - P_{SD})P_{SR}}} + \frac{x}{P_{SD} + (1 - P_{SD})P_{SR}} [P_{RD} - (1 - P_{SD})P_{SR}].
\] (42)

Suppose that \( q^*_R \in (0, \delta_R) \), then

\[
(1 - \delta_R)^2 [P_{SD} + (1 - P_{SD})P_{SR}] < x < [P_{SD} + (1 - P_{SD})P_{SR}],
\] (43)
\[ \mu_S^* = \{ q_S(1-q_R) \Pr(B_S \neq 0, B_R \neq 0) + q_S \Pr(B_S \neq 0, B_R = 0) \} [p_{SD} + (1-p_{SD})p_{SR}] \tag{15} \]

\[ \mu_R^* = \{ q_R(1-q_S) \Pr(B_S \neq 0, B_R \neq 0) + q_R \Pr(B_S = 0, B_R \neq 0) \} p_{RD} \tag{16} \]

but since \( x < q_S(1-q_R)[p_{SD} + (1-p_{SD})p_{SR}] \) and \( q_S \in [0, \delta_S] \) then

\[ x \leq \delta_S^2 [p_{SD} + (1-p_{SD})p_{SR}] \tag{44}. \]

We need to find the intersection of (43) and (44). If \( \delta_S + \delta_R < 1 \) the intersection is an empty set, if \( \delta_S + \delta_R \geq 1 \) then

\[ (1-\delta_R)^2 [p_{SD} + (1-p_{SD})p_{SR}] < x \leq \delta_S^2 [p_{SD} + (1-p_{SD})p_{SR}] \tag{45}. \]

Suppose that the \( q_R^* = 0 \) or \( q_S^* = \delta_R \), which is the case that \( x \) lies outside (45). If \( x \) lies outside (45) on the right side then \( \frac{dy}{dq_R} \) is always non-positive and \( y \) is a non-increasing function of \( q_R \), thus \( q_R^* = 0 \) and \( y^* < 0 \).

On the other hand if \( x \) is on the left side \( x \leq (1-\delta_R)^2 [p_{SD} + (1-p_{SD})p_{SR}] \), then \( \frac{dy}{dq_R} \) is always non-negative and thus \( y \) is a non-decreasing function \( q_R \), hence \( q_R^* = \delta_R \) and the maximum objective function is

\[ y^* = \delta_R p_{RD} - \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}} x - \frac{(1-\delta_R) [p_{SD} + (1-p_{SD})p_{SR}]}{x} \tag{46}. \]

for \( x \leq (1-\delta_R)^2 [p_{SD} + (1-p_{SD})p_{SR}] \). \( \tag{47} \)

The initial constraint of the \([P_2]\) should also hold for \( q_R^* = \delta_R \), we have an additional condition \( x \leq \delta_S(1-\delta_R) [p_{SD} + (1-p_{SD})p_{SR}] \).

Summarizing, the closure obtained by \([P_2]\), If \( \delta_S + \delta_R < 1 \) then

\[ y^* = \delta_R p_{RD} - \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}} x - \frac{(1-\delta_R) [p_{SD} + (1-p_{SD})p_{SR}]}{x} \tag{48}. \]

for \( x \leq \delta_S(1-\delta_R) [p_{SD} + (1-p_{SD})p_{SR}] \). \( \tag{49} \)

If \( \delta_S + \delta_R \geq 1 \) then \( y^* \) is given by (50), where \( x_1 = (1-\delta_R)^2 [p_{SD} + (1-p_{SD})p_{SR}] \) and \( x_2 = \delta_S^2 [p_{SD} + (1-p_{SD})p_{SR}] \).

Following the same methodology we obtain the solution to the \([P_1]\), If \( \delta_S + \delta_R < 1 \) then the closure is

\[ \frac{x}{\delta_S [p_{SD} + (1-p_{SD})p_{SR}]} + \frac{(1-p_{SD})p_{SR} x}{(1-\delta_R) [p_{SD} + (1-p_{SD})p_{SR}]} + \frac{y}{(1-\delta_S)p_{RD}} = 1, \tag{51} \]

for \( \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}} x + y \leq p_{RD}(1-\delta_R) \delta_S \).

If \( \delta_S + \delta_R \geq 1 \) then if \( p_{RD}(1-\delta)^2 \leq \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}} x + y \leq p_{RD} \delta_S^2 \) the closure is

\[ \sqrt{\frac{p_{SR}(1-p_{SD})x}{p_{RD} (p_{SD} + (1-p_{SD})p_{SR})} + \frac{y}{p_{RD}}} = 1. \tag{52} \]

If \( \frac{(1-p_{SD})p_{SR}}{p_{SD} + (1-p_{SD})p_{SR}} x + y \leq p_{RD}(1-\delta_S)^2 \) the closure is

\[ \frac{x}{\delta_S [p_{SD} + (1-p_{SD})p_{SR}]} + \frac{(1-\delta_S)p_{RD} x}{(1-\delta_R) [p_{SD} + (1-p_{SD})p_{SR}]} + \frac{y}{(1-\delta_S)p_{RD}} = 1. \tag{53} \]

In the previous sections, we obtained the closure of the outer bound of the stability region, which we show below that this closure is achievable. The sufficient conditions for stability for fixed transmission probabilities are given in Proposition III.1. For \( q_S \leq \delta_S \) and \( q_R \leq \delta_R \) we have that the saturated throughput for the source and the relay are given by

\[ \mu_S^* = q_S(1-q_R) [p_{SD} + (1-p_{SD})p_{SR}] \], \( \mu_R^* = q_R(1-q_S) p_{RD} \).

From (54) we obtain that

\[ q_S = \frac{\mu_S^*}{(1-q_R) [p_{SD} + (1-p_{SD})p_{SR}]} \]. \( \tag{56} \)

By substituting (56) into (55) we have

\[ \mu_R^* = q_R \left[ 1 - \frac{\mu_S^*}{(1-q_R) [p_{SD} + (1-p_{SD})p_{SR}]} \right] p_{RD}. \] \( \tag{57} \)

After replacing \( \mu_R^* \) with \( \lambda_{R_{\text{total}}} = \lambda_R + \frac{p_{SR}(1-p_{SD})x}{p_{SD} + (1-p_{SD})p_{SR}} \lambda_S \) and \( \mu_S^* \) with \( \lambda_S \), then it is identical with the expression in (35) which corresponds to the outer bound. As a result, a point of the saturated throughput can be controlled to any point on the boundary of \( R \) which is given in Proposition III.2. The same argument holds for \( \mu_S^* \) and the expression in (31).

The previous concludes that the closure of the outer bound of the stability region is achievable.

**VI. CONCLUSIONS**

In this paper, we studied the effect of energy constraints on a relay-aided wireless network in which both source and relay have energy harvesting capabilities. The source and the relay nodes also have external arrivals and network-level cooperation is employed, i.e. the relay forwards a fraction of the source’s traffic to the destination.

We derived necessary and sufficient conditions for stability of the above cooperative communication scenario and we also obtained the exact maximum stable throughput region. Interestingly, the closure of the inner and the outer bound is
\[
y' = \begin{cases} 
    P_{RD} - 2P_{RD} \sqrt{\frac{P_{SD} + (1 - P_{SD})P_{SR}}{(1 - P_{SD})P_{SR}}} x + P_{SD} + (1 - P_{SD})P_{SR}\left[P_{RD} - (1 - P_{SD})P_{SR}\right] & \text{if } x_1 < x \leq x_2 \\
    \delta_R P_{RD} - \frac{P_{SD} + (1 - P_{SD})P_{SR}}{(1 - P_{SD})P_{SR}} x & \text{if } x \leq x_1 
\end{cases}
\] 

identical. A key insight of this work with an impact on the design of relay-assisted networks with energy limitations is as follows: when the aggregate charging rate is above one and both the source and the relay lie in the intermediate traffic regime, then the system has identical performance with the network without energy constraints. Otherwise stated, in the above setting, the energy limitations are transparent to the network operation, which is also demonstrated by the non-linear behavior of the bound of the maximum stable throughput region.

This work provides a step in connecting information theory and networking by studying the stable throughput region metric. Additionally, it sheds light on the relatively unexplored and important domain of energy harvesting and assesses the effect of that on this important measure.

Future work will include the characterization of the stable throughput region using multi-packet reception instead of the erasure channel with collisions.

REFERENCES


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