Estimation and Compensation of Load-Dependent Position Error in a Hybrid Stepper Motor

Anton Ronquist and Birger Winroth
Estimering och kompensering av lastberoende positionsfel i en elektrisk stegmotor

Hybrid stepper motors are a common type of electric motor used throughout industry thanks to its low-cost, high torque at low speed and open loop positioning capabilities. However, a closed loop control is often required for industrial applications with high precision requirements. The closed loop control can also be used to lower the power consumption of the motor and ensure that stalls are avoided. It is quite common to utilise a large and costly position encoder or resolver to feedback the position signal to the control logic. This thesis has explored the possibility of using a low-cost position sensor based on Hall elements. Additionally, a sensorless estimation algorithm, using only stator winding measurements, has been investigated both as a competitive alternative and as a possible complement to the position sensor. The thesis work summarises and discusses previous research attempts to adequately measure or estimate and control the hybrid stepper motors position and load angle without using a typical encoder or resolver. Qualitative results have been produced through simulations prior to implementation and experimental testing.

The readings from the position sensor is subject to noise, owing to its resolution and construction. The position signal has been successfully filtered, improving its accuracy from 0.56° to 0.25°. The output from the sensorless estimation algorithm is subject to non-linear errors caused by errors in phase voltage measurements and processing of velocity changes. However, the dynamics are reliable at constant speeds and could be used for position control.

Keywords: Adaptive Notch Filter, Back-EMF, Electric Motor Position Control, Error Modelling, Field-Oriented Control, Hybrid Stepper Motor, Kalman Filter, Load Angle, Magnetic Rotary Position Sensor, Sensorless Control, Sliding Window Discrete Fourier Transform, Stall Detection and Prediction.
Abstract

Hybrid stepper motors are a common type of electric motor used throughout industry thanks to its low-cost, high torque at low speed and open loop positioning capabilities. However, a closed loop control is often required for industrial applications with high precision requirements. The closed loop control can also be used to lower the power consumption of the motor and ensure that stalls are avoided. It is quite common to utilise a large and costly position encoder or resolver to feedback the position signal to the control logic. This thesis has explored the possibility of using a low-cost position sensor based on Hall elements. Additionally, a sensorless estimation algorithm, using only stator winding measurements, has been investigated both as a competitive alternative and as a possible complement to the position sensor. The thesis work summarises and discusses previous research attempts to adequately measure or estimate and control the hybrid stepper motors position and load angle without using a typical encoder or resolver. Qualitative results have been produced through simulations prior to implementation and experimental testing.

The readings from the position sensor is subject to noise, owing to its resolution and construction. The position signal has been successfully filtered, improving its accuracy from $0.56^\circ$ to $0.25^\circ$. The output from the sensorless estimation algorithm is subject to non-linear errors caused by errors in phase voltage measurements and processing of velocity changes. However, the dynamics are reliable at constant speeds and could be used for position control.
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Anton Ronquist och Birger Winroth
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<tbody>
<tr>
<td>ABB</td>
<td>ASEA Brown Boveri</td>
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<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
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<tr>
<td>ANF</td>
<td>Adaptive Notch-Filter</td>
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<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
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<td>EKF</td>
<td>Extended Kalman Filter</td>
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<td>EMF</td>
<td>Electromotive Force</td>
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<td>FOC</td>
<td>Field-Oriented Control</td>
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<td>FPGA</td>
<td>Field-Programmable Gate Array</td>
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<tr>
<td>IC</td>
<td>Integrated Circuit</td>
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<tr>
<td>I²C</td>
<td>Inter-Integrated Circuit</td>
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<tr>
<td>LPF</td>
<td>Low-Pass Filter</td>
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<tr>
<td>LSQR</td>
<td>Least-Squares</td>
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<tr>
<td>MOSFET</td>
<td>Metal Oxide Semiconductor Field Effect Transistor</td>
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<tr>
<td>MRPS</td>
<td>Magnetic Rotary Position Sensor</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional, Integral, Differential (Regulator)</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse-Width Modulation</td>
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<tr>
<td>SPI</td>
<td>Serial Peripheral Interface</td>
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<tr>
<td>SDFT</td>
<td>Sliding (Window) Discrete Fourier Transformation</td>
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## Nomenclature

<table>
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<th>Notation</th>
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<tr>
<td>$e_a$</td>
<td>Back-EMF induced in winding A [V]</td>
</tr>
<tr>
<td>$e_b$</td>
<td>Back-EMF induced in winding B [V]</td>
</tr>
<tr>
<td>$i_a$</td>
<td>Current in winding A [A]</td>
</tr>
<tr>
<td>$i_b$</td>
<td>Current in winding B [A]</td>
</tr>
<tr>
<td>$i_d$</td>
<td>Phase current expressed in dq-reference frame [A]</td>
</tr>
<tr>
<td>$i_q$</td>
<td>Phase current expressed in dq-reference frame [A]</td>
</tr>
<tr>
<td>$i_s$</td>
<td>Current resultant in dq-frame [A]</td>
</tr>
<tr>
<td>$J$</td>
<td>Rotor inertia [$kgm^2$]</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Magnitude of detent torque [Nm]</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Motor Torque constant [Nm/A]</td>
</tr>
<tr>
<td>$L$</td>
<td>Self-inductance in windings A and B [H]</td>
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<tr>
<td>$L_{d,q}$</td>
<td>Self-inductance expressed in dq-reference frame [H]</td>
</tr>
<tr>
<td>$N_r/p$</td>
<td>Number of rotor teeth/Number of poles</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance in windings A and B [$\Omega$]</td>
</tr>
<tr>
<td>$u_a$</td>
<td>Terminal voltage over winding A [V]</td>
</tr>
<tr>
<td>$u_b$</td>
<td>Terminal voltage over winding B [V]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Viscous friction constant [Nms$^2$/°]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Load angle [°]</td>
</tr>
<tr>
<td>$\Psi_r$</td>
<td>Rotor flux linkage [Wb]</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Electromagnetic torque [Nm]</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>Load torque [Nm]</td>
</tr>
<tr>
<td>$\theta_e$</td>
<td>Electrical angle [°]</td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>Rotor angle [°]</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>Electrical angular velocity [°/s]</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Rotor angular velocity [°/s]</td>
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Bold font denotes that the variable is a vector.
Introduction

This report details and analyzes the methods and results of a research project that has been carried out during a period of 20 weeks in the spring of 2016. The research project constitutes the master’s thesis work for two master’s students enrolled at Linköping University. This project has been conducted at ABB Corporate Research Center in Västerås, Sweden.

1.1 Purpose

The purpose of this research project is to investigate and develop the use of a low cost solution to increase the accuracy of a controlled stepper motor. It would thereafter be possible to produce a stepper motor with numerous applications within industrial systems.

1.2 Problem Definition ”Estimation and Compensation of Load-Dependent Position Error in a Hybrid Stepper Motor”

The goal of the control system is to estimate and compensate the angular position error of a stepper motor at any given time during normal operation. Normal operation will be defined as any setup or input signal that does not exceed the stepper motor’s maximum torque and current.

The angular error, or position error, is the difference between the reference angular position and the angular position of the rotor. The control system makes use of phase currents to create an electrical field angle. If the electromagnetic torque would be the only torque acting on the system, this electrical angle would
correspond directly to the reference angle.

The position error is to a large degree due to the so-called load angle when the motor is positioned by an open-loop controller. The load angle results from applying an external torque to the stepper motor, causing the magnetic rotor to be out of phase with the electrical field. The external torque can be the result of accelerating an object, gravity acting on a mass or attempting to rotate a fixed stationary object, amongst other sources. Different working conditions are considered.

The aim of this thesis project is to investigate three research questions:

- Can the load angle be estimated without using additional sensors?
- Can the rotor position be measured with a simpler sensor than an encoder or resolver without a significant loss of accuracy?
- Can the position error be compensated by using position feedback control?

1.3 Scope

In order to compensate for the position error caused by the load angle in a stepper motor, the load angle must either be measured or estimated. Both estimation and compensation of the load-dependent position error are within the scope of this thesis. The emphasis of the report is on the estimation of the position without adding large and costly hardware. The functionality of the produced estimates are ultimately evaluated by their applicability within a feedback controller.

The project has been limited to managing the feedback signals and the control structure and therefore does not investigate the effect of using different stepper motors or different stepper motor architectures. For practical reasons, the tests will be performed using the same stepper motor.

1.4 Method

Current research proposes a variety of solutions to the problems at hand. Position feedback solutions with costly encoders exist whereas there seems to be a lack of solutions based on feedback from low-cost Hall element sensors. The research regarding load angle estimation by "sensorless" observer solutions is quite rich and the solutions seem to impose many interesting and challenging compromises. Efforts have been made to find and summarise the existing approaches presented by the research field. In parallel with the research summary, solutions have been implemented and their respective performances are evaluated and compared in a series of tests defined in Chapter 7. Simulations have been used to evaluate the potential of different solutions before implementation and experimental testing. The results of the simulations are presented in Chapter 6. The experimental results are presented and discussed in Chapter 8. The complete experimental setup is described in Appendix A.
1.5 Scientific Contribution

Firstly, the scientific contribution of this thesis is the summary and discussion of previous research attempts to adequately measure or estimate the hybrid stepper motors position and load angle without using a typical encoder or resolver. Secondly, the evaluation of different such methods and their performance are evaluated both in simulations and experimentally. The most interesting result relating to this thesis problem definition is how a feedback controller performs when the position signal is provided from one of the evaluated solutions and not from an encoder. Conclusions and suggestions for future work are collected at the end of the thesis in Chapter 9.

1.6 Individual Contributions

In order to produce a more thorough investigation as well as a clearer presentation of the thesis, the work has been distributed evenly between the two authoring master’s students. Each student has been given the responsibility for different research topics and different parts of the control structure. Special attention has been given to dividing the different approaches to rotor position estimation between the two authors.
Introduction to Stepper Motors and Position Errors

This chapter provides context for the hybrid stepper motor and its load dependent position error. A dynamic model and the working principle of the hybrid stepper motor are presented in chapter 3.

2.1 Background to Stepper Motors in Industrial Applications

The control of rotary motors has long been one of the most well-explored areas of research in modern industries. Synchronous machines in particular can be used in a wide array of situations and purposes. Recently, however, the need for low cost machines has brought about an interest in other types of electrical machines. The stepper motor is a very common low cost machine with discrete angular positions and relative good torque and motion performance. It is commonly used wherever small rotary motions are needed and the angular accuracy is of little concern. In more complex industrial applications, however, demands on angular accuracy at varying speeds are commonplace. A focus of recent studies of stepper motors has therefore been to increase the accuracy of this type of machines.

The load angle relates to the static error which arises when a motor is subjected to a load torque. The research done on stepper motors has mostly not been concerned with this type of problem. It is however significantly detrimental to the accuracy of stepper motors, especially considering that the problem can become fully additive with the number of motors used within an application.
2.2 Load Angle and Position Error

Load angle control has been a problem pertaining to synchronous machines such as the hybrid stepper motor. Figure 2.1 illustrates the problem by showing the effect a change in load torque has on the position error due to the fact that the load angle increases in order to meet the torque demand. The position error becomes 1.7° when 0.85 Nm is applied in this case. This position error might have severe effects in the final application. The position error needs to be either measured or observed, or both, in order to be compensated properly.

![Stepper Motor Position Error due to Load Angle](image)

*Figure 2.1: A stepper motor is turned out of position when loaded. The position error becomes 1.7° when 0.85 Nm is applied in this case. This position error might have severe effects in the final application. The position error needs to be either measured or observed, or both, in order to be compensated properly.*

2.3 Existing Position Control Solutions

Open-loop control systems can achieve a high-degree of position control by designing the stepper motor to produce large restoring torque [Fitzgerald et al., 2003]. The most common way to do this is to always operate the motor with its rated winding currents. This results in significant energy losses and heating of the motor which deprive the motor of its efficiency. The position error caused
by the load torque can not be compensated completely by only increasing the magnitude of the current, there is therefore a need for feedback control.

Previous implementations of position and speed control of stepper motors have utilized additional hardware, e.g. encoders or resolvers, to produce a feedback controlled stepper motor capable of much higher accuracy. Any additional hardware adds to the cost of the motor, which contradicts the purpose of these types of motors. It is therefore of interest to better explore the possibility of using low cost components to control stepper motors.

There are also a variety of "sensorless" approaches where either current or back-EMF measurements are utilized by an observer in order to estimate the present load angle. Some of these approaches are presented in section 4.1

### 2.4 Low Cost Position Control

By gathering and evaluating several different solutions, a clear and objective view of the research field has been established and several solutions have been evaluated. With different solutions being best for different points of operation, it is preferable to alter between the estimation and compensation strategies most suitable for the present operating point.

The low cost solutions presented in this thesis may make it possible to operate the hybrid stepper motor with good accuracy and lower winding currents at the same time. Not only does this improve the accuracy and precision of the hybrid stepper motor to a low cost, but also makes it possible to improve the energy efficiency.
Model of a Hybrid Stepper Motor

This chapter presents the construction of a two-phase hybrid stepper motor with 50 poles as well as the associated stepper motor model. The chapter also explains the dynamic chain from control of the phase currents, to electromagnetic torque and ultimately to rotor motion.

**Figure 3.1:** The interior of a two phase hybrid stepper motor [Dolly1010, 2011]. This motor has eight windings and 50 rotor teeth.

**Figure 3.2:** Schematic of a two phase hybrid stepper motor. This motor has four windings and 15 pole pairs.
3.1 Stepper Motor Model

The hybrid stepper motor is made up of certain distinguishable components arranged in a specific layout. The outer layer of the stepper contains eight electromagnets spread out evenly around the central rotor wheel, as can be seen in Figure 3.1 and Figure 3.3. The central rotor is a solid metal piece with several teeth. There are usually 50 teeth in total. The motor works by attracting and repulsing teeth using the electromagnets. In a hybrid stepper motor, the rotor is a permanent magnet and is moved by exciting a single electromagnet (one phase) or a pair of electromagnets (two phase) in turn. The effect is complemented by a minimal reluctance effect, where the rotor is attracted to a position where the space between the teeth and the electromagnet is minimised. The rotor does not line up against all of the electromagnets at the same time, as is shown in Figure 3.2. By varying the magnitude and direction of the winding currents, the rotor is continuously attracted in the desired direction. A "step" occurs whenever a rotor tooth moves slightly to align itself to an electromagnet tooth [Fitzgerald et al., 2003, chapter 8].

It is possible to decrease the step size of the hybrid stepper motor by using a control logic called microstepping. As opposed to fully exciting each phase in turn, as described previously, microstepping involves transitioning between each phase shift. That is, the current references are defined by sinusoidal signals displaced 90 electrical degrees from each other. For most time instances, then, both phases are excited to a certain degree. The result is that the electric position vector can be placed between two teeth. The resolution of the motor has therefore been increased. The upper limit to this resolution is then given by the smallest increment which the digital controller can handle. Although this control logic does add some complexity to the system, the principle is quite simple and can greatly improve the performance of the stepper motor in various applications [Fitzgerald et al., 2003, chapter 8]. Microstepping is being used in this thesis if no other control logic is specified.

The dynamic model of a stepper motor is formed by the system of Equations (3.1)-(3.5). The equations are general and combine models used in several articles, e.g. [Zribi and Chiasson, 1991], [Bendjedia et al., 2012] and [Wale and Pollock, 1999]. The model’s components together with common assumptions and approximations are presented in this section. A resulting non-linear state space model is presented at the end of the section.

\[
\begin{align*}
u_a(t) &= R i_a(t) + L\frac{d i_a(t)}{dt} + e_a(i_a(t), i_b(t)) \\
u_b(t) &= R i_b(t) + L\frac{d i_b(t)}{dt} + e_b(i_a(t), i_b(t)) \\
\frac{d \theta_r(t)}{dt} &= \frac{1}{J} \left( (\tau_e(t) - \tau_l(t)) - K_d \sin(N_r \theta_r(t)) - \beta \omega_r \right) \\
\tau_e(t) &= -K_m i_a(t) \sin(N_r \theta_r(t)) + K_m i_b(t) \cos(N_r(t) \theta_r(t)) \\
\frac{d \theta_r(t)}{dt} &= \omega_r(t)
\end{align*}
\]
The constants in the model are the winding resistances $R$, the motor torque constant $K_m$, the magnitude of the detent torque $K_d$, the rotor inertia $J$, the viscous friction constant $\beta$ and the number of rotor teeth $N_r$, which is the same as the number of rotor poles $p$.

The variables $u_a(t)$ and $u_b(t)$ are the terminal voltages. The stepper motor can be controlled by alternating the currents or the terminal voltages. The common implementation involves a fixed voltage control, where the terminal voltages can only attain predefined voltage levels which are all below or equal to the rated values. For each pulse in a PWM-signal, the current requires some time to reach the corresponding magnitude. The faster the motor is rotating, the shorter the pulses become. Past a certain point, the winding currents cannot change fast enough and will result in an unreasonably weak produced torque. Another control method involves current control, where the voltages are allowed to be increased past rated values in order to produce a faster current response [Manea, 2009]. The motor in this project is assumed to operate at relatively low speeds. Therefore, fixed voltage control will be used and the voltages $u_a(t)$ and $u_b(t)$ will only attain predefined discrete values. Furthermore, microstepping produces very little noise in fixed open loop voltage control [Manea, 2009], which is beneficial in various position estimation problems. Additional details about the current controller implemented in this thesis are available in Appendix A.4.

The alternating currents $i_a(t)$ and $i_b(t)$ are the phase currents which give rise to the torque and position change produced by the stepper motor. The phase currents are therefore also the control signals of the system.

The electromagnetic torque given by $\tau_e(t)$ is the torque produced by the motor. The load torque, $\tau_l(t)$, is the torque applied by an external system. Both of these are considered to be time-dependent in order to properly describe a dynamic physical system. Do note that there are additional sources of torque loss other than the load torque, i.e. viscous friction and detent torque.

The self-inductance of the windings, $L$, are often considered to be constant. However, Butcher et al. [2014] argue that the inductance has some slight position and current dependencies. The position dependency surfaces due to a varying air gap distance in the motor and is of less importance in hybrid stepper motors. The current dependency is caused by the relation between the flux linkage and the reluctance of the magnetic circuit, which is current dependent. The current dependency is assumed to be negligible in the scope of this model.

The detent torque is normally not considered in dynamic model equations for hybrid stepper motors as the magnitude of the detent torque $K_d$ is generally less than 10% of the holding torque [Balakrishnan et al., 2013]. The detent torque is therefore neglected in this model. Additionally, the detent torque can be viewed as a minor disturbance that will get compensated in a torque load control loop.

The angle $\theta_r(t)$ denotes the rotor angle, and the phase of the electrical field is denoted by $\theta_e(t)$. These angles are related by the number of rotor poles

$$\theta_e(t) = N_r \theta_r(t) \quad (3.6)$$
Furthermore, the electrical angular velocity is given by

\[
\frac{d\theta_e(t)}{dt} = \omega_e(t) = N_r \omega_r(t) \tag{3.7}
\]

[Schweid et al., 1995b].

According to Schweid et al. [1995a], the back-EMF:s \(e_{a,b}(i_a(t), i_b(t))\) are given by

\[
e_a(i_a(t), i_b(t)) = -K_m \omega_r(t) \sin(N_r \theta_r(t))
\]

\[
e_b(i_a(t), i_b(t)) = K_m \omega_r(t) \cos(N_r \theta_r(t)) \tag{3.8}
\]

Zribi and Chiasson [1991] manage to construct a nonlinear transformation matrix that results in a nonlinear controller form. Using state feedback linearization, the system then becomes linearized. Although elegant, the transformation is computationally intensive in and of itself and assumes that all state variables are directly measurable. It is therefore not a reasonable simplification for any system which requires an observer.

The equations can be represented by a state-space system where the states are chosen as

\[
\mathbf{x}^T = \begin{bmatrix} i_a(t) & i_b(t) & \omega_r(t) & \theta_r(t) \end{bmatrix}^T \tag{3.9}
\]

By expressing equation (3.1)-(3.2) as derivatives of the phase currents, the following equations are obtained

\[
\frac{di_a(t)}{dt} = \frac{1}{L} \left( u_a(t) - R i_a(t) + K_m \omega_r(t) \sin(N_r \theta_r(t)) \right) \tag{3.10}
\]

\[
\frac{di_b(t)}{dt} = \frac{1}{L} \left( u_b(t) - R i_b(t) - K_m \omega_r(t) \cos(N_r \theta_r(t)) \right) \tag{3.11}
\]

Thus, the system can be represented in the non-linear state space form

\[
\begin{align*}
\frac{di_a(t)}{dt} &= \frac{1}{L} \left( u_a(t) - R i_a(t) + K_m \omega_r(t) \sin(N_r \theta_r(t)) \right) \\
\frac{di_b(t)}{dt} &= \frac{1}{L} \left( u_b(t) - R i_b(t) - K_m \omega_r(t) \cos(N_r \theta_r(t)) \right) \\
\frac{d\omega_r(t)}{dt} &= \frac{1}{L} \left( -K_m i_a(t) \sin(N_r \theta_r(t)) + K_m i_b(t) \cos(N_r \theta_r(t)) - \beta \omega_r(t) - \tau_l(t) \right) \\
\frac{d\theta_r(t)}{dt} &= \omega_r(t)
\end{align*} \tag{3.12}
\]

where also (3.4) has been substituted into (3.3).
3.2 \textit{ab-} and \textit{dq-}Reference Frames

A detailed description of the dq0-transformation, also called Clarke- and Park transformation, for a synchronous motor is given by Fitzgerald et al. [2003]. The dq0 reference frame is placed on and rotates with the rotor. The coordinate in line with the rotor is called "direct" and the perpendicular term leading by 90° electrical is called the "quadrature". The zero is only used in three-phase electrical systems and is not used in association with the hybrid stepper motor. Wonhee et al. [2011] provide the dq-transformation

$$\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_e(t)) & \sin(\theta_e(t)) \\
-\sin(\theta_e(t)) & \cos(\theta_e(t))
\end{bmatrix} \begin{bmatrix}
i_a(t) \\
i_b(t)
\end{bmatrix}$$

(3.13)

applicable to the currents of the two phase stepper motor. This transform and its inverse transform (3.14) are often used when applying Field-Oriented Control (FOC), further discussed in Section 5.2.

$$\begin{bmatrix}
i_a(t) \\
i_b(t)
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_e(t)) & -\sin(\theta_e(t)) \\
\sin(\theta_e(t)) & \cos(\theta_e(t))
\end{bmatrix} \begin{bmatrix}
i_q(t) \\
i_d(t)
\end{bmatrix}$$

(3.14)

Note that a position sensor or a good estimate of the electrical position is necessary in order to implement the dq-transform [Fitzgerald et al., 2003].

3.3 Load Angle and Electromagnetic Torque Relation

The load angle in any synchronous machine, such as the hybrid stepper motor, is necessary for the electric machine to produce electromagnetic torque in order to meet the torque requirements of the application. The load angle can however significantly contribute to the motor's angular position error and will potentially decrease the accuracy and precision of the final application. For example, a robot with several stepper motor joints will have poor precision at the point of the final

\textit{Figure 3.3: Disassembled hybrid stepper motor. (ABB)}
tool if this error is not compensated. The position error needs to be either measured or estimated, or both, in order to be compensated properly. It is therefore vital to understand the relationship between the load angle and the torque of the hybrid stepper motor in order to be able to control and compensate the position error. Figure 3.4 shows the position error due to applying an external torque on the stepper motor. The increase in external torque needs to be compensated by an increase in load angle from the system. This load angle makes up the added position error which is the concern of this thesis.

**Figure 3.4:** A stepper motor is turned out of position when loaded. The position error becomes 1.7°, mechanical degrees, when 0.85 Nm is applied at standstill in this case. This position error might have severe effects in the final application.
3.3 Load Angle and Electromagnetic Torque Relation

![Diagram of rotor flux and current](Image)

**Figure 3.5:** Rotor flux $\Psi_r$ and current resultant $i_s$ in the rotor fixed dq-reference frame. Load angle $\gamma$ in inertial reference frame. The working principle is illustrated with two stator poles and two rotor teeth respectively.

The produced torque from a stepper motor is given, according to Vas [1998], by the cross product between the stator flux linkage space vector $\Psi_s$ and the stator current resultant space vector $i_s$ by

$$\tau_e = \Psi_s \times i_s \quad (3.15)$$

If saturation is neglected, the stator flux linkage space vector $\Psi_s$ is formed by the sum of the permanent-magnet rotor flux $\Psi_r$ and the two stator flux linkages. Derammelaere et al. [2014] use the dq-reference frame to express the produced electromagnetic torque

$$\tau_e = (\Psi_r + i_dL_d + i_qL_q) \times i_s \quad (3.16)$$

Note that the expression for the electromagnetic torque was previously expressed in the ab-reference frame by Equation (3.4). By performing the cross-product in Equation (3.16), the electromagnetic torque can be expressed as a function of the current resultant $i_s$, the rotor flux $\Psi_r$ and the load angle $\gamma$ where $\gamma$ is defined as the angle between $i_s$ and $\Psi_r$,

$$\tau_e = \Psi_r i_s \sin(\gamma) + \frac{L_d - L_q}{2} i_s^2 \sin(2\gamma) \quad (3.17)$$
The first term of Equation 3.17 is dominating the second term when the phase inductances are assumed to be constant and equally large for the two phases. This assumption is common when concerning motion control and used by Derameelaere et al. [2014] amongst others. Thus the electromagnetic torque is principally a function of the flux linkage, the resultant of the phase currents and the load angle

\[ \tau_e \propto \Psi_i i_s \sin(\gamma) \]  

(3.18)

The electromagnetic torque increases with the load angle to meet the load torque until the load angle is 90°. After that point, the torque decreases and the motor might lose synchronism. Another alternative to meet the demands of the load torque would be to increase the magnitudes of the phase currents. That solution, and why it has not been used in this thesis, is discussed in Section 5.1. Figure 3.5 shows a generalised picture of one full step of the hybrid stepper motor and illustrates its torque-producing components.
This chapter will describe some of the approaches to estimating the load angle and measure the position in order to be able to compensate for position errors. Each approach is explained along with some potential problems which need to be well considered before any solution is implemented.

### 4.1 Load Angle Estimation using an Observer

Since it is of interest to know the load angle in an application online, a first attempt might be to invert Equation (3.18)

\[ \gamma \approx \arcsin\left(\frac{\tau}{\Psi r i_s}\right) \]  

(4.1)

This approach would require knowledge of both the load torque and the orientation of the rotor, which indirectly is the sought information when one wants to find the load angle. It is therefore not a useful method of calculating the load angle in a real system.

It is clear that information about the current rotor position needs to be added to the system in order to determine the position error. Adding extra hardware in the form of sensors is however not desired. Sensors add extra costs to the product and are susceptible to weather and wear. Therefore, sensorless observers have been the topic of several research articles, e.g. [Wale and Pollock, 1999]. In a sensorless approach, the position or speed of the rotor is indirectly measured by observing another signal, such as the back-EMF or phase currents. Current and voltage sensors are often a component of driver blocks and so does not necessitate any extensive addition of hardware.

There are in general two physical properties which can be used as a sensorless position or speed observer of an AC machine: fundamental excitation or signal
injection [Harnefors and Nee, 2000]. The principle of fundamental excitation refers to the manner in which a provided machine affects the stator-side currents and voltages during runtime. The voltages and currents could therefore naturally contain information which relates to the position or speed of the rotor. In many cases, processing or sampling these signals implies an estimation of the back-EMF, which does contain rotor information according to Equation (3.8). The other approach is to inject certain test signals into the stator. The purpose of this is to detect discrepancies in the construction of the rotor. For example, the positioning of winding coils and the permanent magnet creates a slight difference in the inductance in the d and q directions, which depends on the current orientation of the rotor. A high-frequency injected signal could then be introduced to evaluate these inductances [Harnefors and Nee, 2000].

4.1.1 Estimation Based on Estimated Back-EMF

Derammelaere et al. [2014] suggest a solution using the back-EMF induced in the stator from the movement of the rotor. The back-EMF can be defined by

$$e_s = C \frac{d\Psi_r}{dt} \quad (4.2)$$

where $C$ is a constant. The derivation implies a $\frac{\pi}{2}$ phase difference between $e_s$ and $\Psi_r$, see Figure 4.1. That is, the angle between $e_s$ and $i_s$ is $\frac{\pi}{2} - \gamma$. The back-EMF and the stator currents can therefore be used to derive the load angle $\gamma$. This is an interesting solution, especially as it does not require information about the load torque, only good knowledge of the electrical components used in the motor. The back-EMF can theoretically be found by sampling the phase voltages when the phase currents decays to zero during the intervals at which the current setpoints are also zero [Derammelaere et al., 2014]. There will during this instance be nothing contributing to the phase voltages besides the induced back-EMF. However, Derammelaere et al. [2014] point out that at higher speeds, where this window is too short for the current to decay fully, or when using a high inductance, where the current decays slower, this state might not be occurring and the back-EMF cannot be sampled correctly. Derammelaere et al. [2014] suggest that the back-EMF instead be calculated using measurements of the phase currents and the phase voltages. The back-EMF relates to these variables through Equations (3.1) and (3.2). However, the current measurements are inescapably noisy and taking the derivative of a noisy signal would introduce undue errors. Instead of applying a filter which would add to the time delay of the calculations, Derammelaere et al. [2014] propose the use of the Fourier transform of these equations:

$$E_s(j\omega) = U_s(j\omega_e) - j\omega_e L_s I_s(j\omega_e) - R_s I_s(j\omega_e) \quad (4.3)$$

The estimated load angle, $\hat{\gamma}$, is then given by:

$$\hat{\gamma} = \frac{\pi}{2} - (\angle E_s - \angle I_s) \quad (4.4)$$

Using Fourier transformations is not always computationally efficient. However, provided that the currents which are used are sinusoidal in steady state,
4.1 Load Angle Estimation using an Observer

Figure 4.1: Displaying the back-EMF vector $e_s$ being perpendicular to the flux vector $\Psi_r$.

One would only need to consider one full period of samples when calculating the Fourier transformation. This is the case when microstepping so the calculations can be made much more efficient than just a general case. Derammelaere et al. [2014] suggest the use of a Sliding-Discrete-Fourier-Transform (SDFT) to make the calculation more effective.

Using SDFT, the Fourier transform of a particular sinusoidal signal is given by [Derammelaere et al., 2014]:

$$X_h(k) = \left(X_h(k-1) + x(k) - x(k-N)\right)e^{jh(2\pi/N)} \quad (4.5)$$

This function can be constructed in Simulink according to Figure 4.2. SDFT will be used twice in the algorithm to calculate the Fourier transform of both $I_s$ and $U_s$.

### 4.1.2 Potential Problems with Back-EMF methods

As shown in Equation (3.8), the produced back-EMF is inherently proportional to the speed of the rotor. At low speeds, then, the losses due to resistive voltage drops and voltage drops over the MOSFETs become relatively very large. The reliability of the back-EMF measurements therefore deteriorates at lower speeds. At zero speeds, no back-EMF measurements can be done.

An issue arises from the assumption in the SDFT algorithm that the period of the currents are constant. The current signals’ period is dependent on the rotational speed of the reference signal. It is therefore necessary to wait a full
4.1.3 Estimation Utilizing Injected Signals

Generally, the aim of injecting signals into the stator is to detect the rotor magnetic saliency. The saliency is position dependent and can be caused by the rotor structure or magnetic saturation. Given that the signal which is injected has an adequately high frequency, this property can produce an effect on the added signal and the response from the system will therefore be position dependent and measurable [Zhao, 2013]. In order to derive a speed and position estimation, this method of injecting a signal requires a demodulation process and the result needs to be interpreted through a dedicated estimation function [el Murr et al., 2008].

The demodulation process depends on the method of injecting signals. The injected signal can be constructed following different schemes, each of which results in slightly different mathematics and different performances. Zhao [2013] and [el Murr et al., 2008] describe two methods of injecting signals: a rotating injection, in both the \( \dot{d} \)- and \( \dot{q} \)-currents, and a single-phase pulsating injection into the estimated \( \hat{d} \)-axis. Provided that an added voltage is virtually the same as an added current, these two cases can be written as [Zhao, 2013]:

\[
\text{Rotating voltage: } v_c^s = V_c [\cos(\omega_c t) \sin(\omega_c t)]^T \\
\text{Pulsating voltage: } v_c = V_c [\cos(\omega_c t) 0]^T
\]

where the superscript "s" denotes that the given voltage vector is in the \( a \) and \( b \) coordinate system, the lack of such superscript denoting a vector in the \( d, q \) domain, \( V_c \) denotes the amplitude of the injected voltage and \( \omega_c \) denotes the carrier frequency. Note that \( V_c \) is relatively large in order to produce a fast response.
and that the frequency $\omega_c$ is normally much higher than the natural operating frequency of the motor.

In order to simplify the interpretation of the response, Zhao [2013] simplifies the motor model. Assuming that the motor is operated in the low speed region, the back-EMF voltage component can be neglected. Furthermore, the resistive voltage drops can be neglected as the injected current magnitude is relatively small. The original motor equations (3.1) and (3.2) can then be simplified as

$$u_s = L_s \frac{di_s}{dt}$$  \hspace{1cm} (4.8)

Assuming that cross saturation and spatial harmonics are neglected, the inductance matrix can be simplified as [Zhao, 2013]:

$$L_s = \begin{bmatrix} L'_d & 0 \\ 0 & L'_q \end{bmatrix} = \begin{bmatrix} \frac{\Delta \Psi_d}{\Delta i_d} & 0 \\ 0 & \frac{\Delta \Psi_q}{\Delta i_q} \end{bmatrix}$$  \hspace{1cm} (4.9)

These injected signals can then be demodulated and passed through a low-pass filter (LPF) [Zhao, 2013]:

$$\epsilon_{rot} = \text{LPF} \left( \frac{V_c}{2\omega_c L'_d L'_q} \left[ -(L'_q + L'_d) \sin(2\omega_c t - 2\hat{\theta}_e) + (L'_q - L'_d) \sin(2\hat{\theta}_e) \right] \right)$$

$$= \frac{(L'_q - L'_d)}{2\omega_c L'_d L'_q} \sin 2\hat{\theta}_e = K_{rot} \sin 2\hat{\theta}_e$$  \hspace{1cm} (4.10)

$$\epsilon_{pulse} = \text{LPF} \left( \hat{i}_q \sin \omega_c t \right) = \frac{(L'_q - L'_d)}{4\omega_c L'_d L'_q} \sin 2\hat{\theta}_e$$

where $\epsilon$ contains the position information and ideally denotes $\theta_e - \hat{\theta}_e$. The information parameter, $\epsilon$, approximately equals this value if $\theta \approx \hat{\theta}$. The variable $\hat{\theta}_e$ is an estimate of the actual rotor position in the electrical field.

Using this value, an estimator can be designed to calculate the rotor’s position and speed. Harnefors and Nee [2000] and Zhao [2013] suggest a phase-locked loop estimator. Using this estimator, the position and speed estimates of the rotor can be updated by [Harnefors and Nee, 2000]

$$\dot{\hat{\omega}_r} = g_1 \epsilon$$

$$\dot{\hat{\theta}} = \hat{\omega}_r + g_2 \epsilon$$  \hspace{1cm} (4.12) \hspace{1cm} (4.13)

where $g_1$ and $g_2$ are gain parameters. Using this algorithm with appropriate tuning, the estimates can be driven to their true values. Harnefors and Nee [2000] provide a stability analysis of this method, which proves that the algorithm can be asymptotically stable if the gain parameters are chosen appropriately.

### 4.1.4 Choice of Sensorless Estimation Algorithm

Although both the estimated back-EMF and the injected signal methods are interesting to evaluate further in a physical setup, the scope of this thesis is limited
to investigating one approach. One possible development from this thesis is a combination of a load angle estimation and a filtered low-cost position sensor. It is therefore attractive to select a good complement to the position sensor.

The position sensor is sampled at a given set interval. Therefore, the position sensor achieves a higher position accuracy at lower rotational velocities where the position difference between samples is lower. As a result, a compound load angle estimation system would have a wider effective range if this sensor operated together with an algorithm that performed better at higher rotational velocities. The magnitude of the induced back-EMF becomes greater the faster the motor rotates. This will increase the accuracy and precision of the estimated back-EMF method. Furthermore, one of the assumptions for the injected signal method is that the back-EMF can be neglected. The back-EMF would therefore act as noise on the measured signals, decreasing the performance of this method at higher rotational velocities.

Therefore, the back-EMF SDFT algorithm is investigated in this thesis. The injected signal method makes up an interesting stepping stone for further research, but is not explored further within the scope of this thesis.
4.2 Position Feedback from a Magnetic Rotary Position Sensor (MRPS)

Angular position feedback together with proper feedback control is an effective way of eliminating position errors in many rotating machines and servo applications. Typically, an optical encoder or an electromagnetic resolver is used in applications with high demands on accuracy and precision. Optical encoders and resolvers can however be quite costly which defeats the purpose of choosing a low cost hybrid stepper motor for certain applications. It is therefore of great interest to explore the possibility of obtaining the angular position feedback signal from a low cost Magnetic Rotary Position Sensor (MRPS) instead.

4.2.1 MRPS Working Principle

The MRPS is an incremental encoder which utilizes several Hall effect sensors mounted in quadrature and parallel to a permanent magnet mounted on the rotor of the subject machine. The magnetic flux of the permanent magnet gives rise to a voltage potential across each respective Hall element. Knowledge of the potential across each Hall element in the MRPS is then used to determine the orientation of the permanent magnet and, thereby, also the orientation of the rotor [Qi et al., 2011]. Figure 4.3 shows a MRPS mounted on a hybrid stepper motor. The magnet is mounted on the rear shaft of the stepper motor and the Hall effect sensors are attached to the integrated circuit board. Figure 4.4 illustrates the working principle of the angular position measurement. Four Hall effect sensors are fitted in the MRPS used throughout this thesis.

![Figure 4.3: MRPS and rotor magnet mounted on the rear of a stepper motor. (ABB)](image1)

Only two Hall effect sensors are needed to determine the angular position, but more sensors can be added to reduce the variance of the position signal. When the rotor and the mounted magnet are spinning, two sinusoidal voltages will appear...
Determination of Load Angle and Rotor Position

across two orthogonal Hall effect sensors.

\[ V_a = V_0 + V \sin(\theta_r) \] (4.14)

\[ V_b = V_0 + V \cos(\theta_r) \] (4.15)

Hence, the angular position expressed in mechanical degrees \( \theta_r \) can be calculated by setting the DC offset \( V_0 \) equal to zero and then applying simple geometry

\[ \theta_r = \arctan \left( \frac{V_a}{V_b} \right) = \arctan \left( \frac{V \sin(\theta_r)}{V \cos(\theta_r)} \right) \] (4.16)

The present control error, i.e. the position error, is then obtained by comparing the measured angular position to the reference position.

The position signal of the MRPS can be retrieved by either an incremental square wave interface or through Serial Peripheral Interface (SPI), which provides an absolute position. The incremental interface is preferred when small delay is critical and interfaces such as SPI or Inter-Integrated Circuit (I\(^2\)C) are too slow. The SPI has been regarded fast enough and has been used to retrieve the position signal from the MRPS throughout this thesis work. The SPI-clock was run at 2.5 MHz and therefore the position was retrieved at a rate of 156 kHz. Since the Speedgoat operates at 20 kHz, it is possible to oversample the position signal by a factor of at least four times, effectively low-pass filtering it. Acquiring the position at 20 kHz may however still be frequent enough to do sufficient oversampling in Simulink on the Speedgoat CPU. The maximum internal resolution available from the used MRPS is 14-bit which implies a mechanical angular resolution of \( \frac{360^\circ}{2^{14}} = 0.022^\circ \).

### 4.2.2 Measurement Errors and Low-Pass Filtering of the MRPS Signal

The MRPS is simpler and generally has a lower cost than optical encoders and resolvers. Unfortunately, the measurement obtained from the MRPS may be subject to several potential error sources such as amplitude imbalance, non-orthogonal phase, DC offset and unwanted harmonics [Qi et al., 2011]. The root causes to these problems may be improper mounting of the Hall effect sensors or an imperfect magnet etc. Regardless of the cause of the error, the measurement error appears as noise and non-linearity errors in the position signal [Lozanova and Roumenin, 2010].

Figure 4.5 shows the difference between the measurement signal obtained from a MRPS and a position signal provided by a high-performance absolute encoder. In the left plot the raw signal from the MRPS has been used and the right plot shows the same comparison after a 3\(^{rd}\) order Butterworth filter with a 50 Hz cutoff frequency has been applied to the MRPS signal. In these tests, a hybrid stepper motor was revolved several revolutions at 300 rpm. The noise level is clearly reduced but the low-pass filter has also imposed a delay on the signal. The non-linearity error is still present in the signal and according to the figure, the MRPS position signal still have an accuracy of less than 0.8\(^\circ\). The size of
4.2 Position Feedback from a Magnetic Rotary Position Sensor (MRPS)

Figure 4.5: The left plot displays a polar diagram of the MRPS error when the raw output rotary position signal is compared to an optical encoder while the motor is driven at 300 rpm. It can be observed that the MRPS signal is both noisy and irregular over the full revolution. In the right figure, the same signal has been low-pass filtered with a 3rd order Butterworth filter with a cutoff frequency of 50 Hz. One can notice a delay in the filtered signal imposed by the Butterworth filter, visible as a phase difference between the two plots.

the position error is the radial distance between the blue circle and the red dots. Since the non-linearity error appears regularly over one revolution, its frequency must have a linear relationship to the motors operating speed. Hence, the signal has to be adaptively filtered to be usable in any application with relatively high position demands. Filtering the signal will however always impose a delay in the measurement which could degrade the performance of the final application.

4.2.3 MRPS Non-Linearity Error Compensation

Various position sensor designs utilizing magnetic field orientation exist and many research papers focus on the physical design of the sensor. That means arranging the Hall elements in clever ways [Lozanova and Roumenin, 2010], using polynomial curve-fitting techniques [Weinhoffer et al., 1993] or even neural networks [Oliver et al., 2006] in order to reduce the noise and compensate for the non-linearity as well as potential cross-talk effects. Qi et al. [2011] suggest to create a look-up table where the irregular or noisy signal can be mapped to a more accurate reference measurement, then use the look-up table to translate the MPRS signal during operation. Kondraske and Ramaawamy [1986] make use of a look-up table as well and also propose a universal and automated linearization and calibration scheme. The look-up table is then formed by mapping the Hall sensor output to the output of a reference position transducer during an initialisation phase. Since the mapping procedure is automated, Kondraske and Ramaawamy [1986] foresee a cost-effective very-large-scale integration (VLSI) realisation of their device.
Figure 4.6: Single sided amplitude spectrum of the error signal, measured as the difference between the raw MRPS signal and the output from the optical encoder at 300 rpm. A speed of 300 rpm is the same as a revolving frequency of 5 Hz, clearly visible as one of the spectrum peaks. The second harmonics has the highest amplitude and is potentially containing the non-linearity error. The first and fourth harmonic is also visible in the amplitude spectrum.

Since the hybrid stepper motor has good accuracy in open loop operation when unloaded, the MRPS signal may only need to be mapped onto the open loop position provided by the unloaded stepper motor. Thus, no additional hardware would be needed.

Online compensation of the MRPS errors is addressed by Ji-Won et al. [2015] whom apply a recursive least square method. The method is somewhat advanced and the implementation on the real system, not purely simulations, is not quite clear from the description provided by the aforementioned authors.

In the recent work by Lara et al. [2016], the authors are able to improve the accuracy of a MRPS position signal from 0.8° to 0.2° by using an algorithm based on a 5th order polynomial approximation of the non-linearity error and a phase-locked loop.

Another method is presented by Jung and Nam [2011] who make use of an adaptive notch filter (ANF) and a phase-locked loop. The authors’ application of interest is a permanent-magnet synchronous motor with six poles. The measured position signal contains large third order harmonics which appear to contain the non-linearity error of the signal. The authors effectively manage to suppress the
third-harmonics by using a speed adaptive notch-filter combined with a phase-locked loop. The adaptive notch-filter eliminates the harmonics from the phase-locked loop input and the phase-locked loop output is used for generating the harmonic reference signals that are required by the notch filter. The authors describe this method as easy to implement and insensitive to disturbances from stator currents.

Figure 4.6 shows the single sided amplitude spectrum of the error signal, measured as the difference between the raw MRPS signal and the output from the optical encoder at 300 rpm. A speed of 300 rpm is the same as a revolving frequency of 5 Hz, clearly visible as one of the spectrum peaks. The second harmonic has the highest amplitude and potentially contains the non-linearity error. The fourth harmonic is also visible in the amplitude spectrum. The findings of Jung and Nam [2011] encourage the attempt to remove the non-linearity error by using a speed adaptive notch filter.

The interesting methods for compensating or suppressing harmonic errors can be summarised in the list below. Post-processing and simulation experiments have been performed, and is presented in chapter appendix 6, in order to deduce a suitable solution for this thesis work with regards to performance and method complexity. The conclusion from the research study and the offline tests is that the adaptive notch filter has potential to be a generic solution that can yield sufficient performance. Since first modelling and then compensation of the error has provided good results for both Ji-Won et al. [2015] and Lara et al. [2016], it has been motivated to try such a method as well. The modelling method which has been tested is based on modelling the MRPS error by using a sum of sine and cosine functions with coefficients determined by a least-square minimization.

- Look-up table [Kondraske and Ramaawamy, 1986] and [Qi et al., 2011].
- Recursive least square method [Ji-Won et al., 2015].
- Polynomial approximation [Lara et al., 2016].
- Iterative optimization algorithm [Lara and Chandra, 2014].
- Adaptive notch filter and phase locked loop [Jung and Nam, 2011].

### 4.2.4 Adaptive Notch filter

A notch filter is a deep and narrow frequency band-stop filter. The adaptive notch filter can update its stop band frequency online and is ideally suitable for removing specific frequency components of a signals spectrum. The design of the adaptive notch filter implemented in this thesis work is inspired by the notch filter used by Liu et al. [2012] to suppress vibration in a magnetically suspended flywheel.

The ideal notch frequency can be the 2\(^{nd}\) harmonics of the rotational speed. The rotational speed can either be approximated as the desired rotational speed or measured speed using the MRPS position signal. By measuring the time between two equal position values, \(t_{rev}\), the rotational speed can be calculated in
rpm according to

\[ \hat{\omega}_r = \frac{60}{t_{rev}} \]  

(4.17)

This method works well when the desired motion involves several revolutions and no changes in direction. Figure 4.7 shows the performance of the speed measurement tracking the reference speed. It is worth noting that the reference speed is realised practically perfect when the motor is not under any external load and revolves at constant speed. The speed measurement is saturated, to a magnitude less than 500, so as not to yield any infinite estimates of the speed. It is a good property for the measurement to rather overestimate the speed than to underestimate it. An overestimate will result in a higher notch frequency and would then be less likely to disrupt the important information in the signal.

The adaptive notch-filter designed by Liu et al. [2012] subtracts a signal \( c(t) \), which is updated online, from the signal subject to the filtering \( p(t) \) and produces the filtered signal \( f(t) \)

\[ f(t) = p(t) - c(t) \]  

(4.18)

![Figure 4.7: Performance of the speed tracking measurement used to select the notch frequency of interest online.](image-url)
where

\[ c(t) = \epsilon \cdot \left[ \sin\left(\frac{4\pi}{60} \hat{\omega}_r t\right) \cos\left(\frac{4\pi}{60} \hat{\omega}_r t\right) \right] \cdot \left[ \int \sin\left(\frac{4\pi}{60} \hat{\omega}_r t\right) \cdot f(t) dt \right] \cdot \left[ \int \cos\left(\frac{4\pi}{60} \hat{\omega}_r t\right) \cdot f(t) dt \right] \] (4.19)

when the objective is to band-stop the 2\textsuperscript{nd} harmonics of the rotational speed. It is important to note that \( c(t = 0) = 0 \) and therefore \( f(t = 0) = p(t = 0) \). The notch filter can be tuned by only one parameter, \( \epsilon \). A smaller \( \epsilon \) will create a deeper and narrower frequency notch and vice versa, i.e. it will suppress a smaller frequency band and do so to a greater extent. The same method can be used to attempt to notch any desired frequency component of a signal.

### 4.2.5 Least Squares (LSQR) Modelling and Compensation of the MRPS Error

Since the frequency analysis of the deviation between the MRPS and the encoder output shows significant peaks at the first, second and the fourth harmonic of the speed and since the error is reappearing sinusoidally for each revolution, the following modelling of the error is suggested. Model the error \( \epsilon_{modelled}(\theta_r(t)) \) as a sum of sine and cosine functions of the angular position

\[
\epsilon_{modelled}(\theta_r(t)) = c_0 + a_1 \sin(\theta_r) + b_1 \cos(\theta_r) + a_2 \sin(2\theta_r) + b_2 \cos(2\theta_r) + a_4 \sin(4\theta_r) + b_4 \cos(4\theta_r)
\] (4.20)

The constant and the coefficients \( c_0, a_1, b_1, a_2, b_2, a_4 \) and \( b_4 \) are then determined by the least-squares (LSQR) solution to the data from a recorded training session

\[
\begin{bmatrix}
1 & \sin(\theta_r(1)) & \cos(\theta_r(1)) & \sin(2\theta_r(1)) & \cdots & \cos(4\theta_r(1)) \\
1 & \sin(\theta_r(2)) & \cos(\theta_r(2)) & \sin(2\theta_r(2)) & \cdots & \cos(4\theta_r(2)) \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \sin(\theta_r(e)) & \cos(\theta_r(e)) & \sin(2\theta_r(e)) & \cdots & \cos(4\theta_r(e)) \\
\end{bmatrix} \begin{bmatrix}
c_0 \\
a_1 \\
b_1 \\
a_2 \\
b_2 \\
b_4
\end{bmatrix} = \begin{bmatrix}
\epsilon_{true}(1) \\
\epsilon_{true}(2) \\
\vdots \\
\epsilon_{true}(e)
\end{bmatrix}
\] (4.21)

where \( e \) is the end value of the acquired session. The coefficients from the least-square calculation are then inserted into Equation (4.20). The modelled error \( \epsilon_{modelled}(\theta_r(t)) \) can now be subtracted from the position signal from the MRPS. The position signal from the MRPS has thus been compensated and thereby improved. One drawback using this method on a large scale is the training sessions necessary to form a “true” reference to be able to form the true MRPS error. Data for the training session may be either the output from a reference encoder or the open loop position when the motor is rotating unloaded at a constant speed. Section 6.2.2 presents the simulation results from performing the suggested error compensation offline. This method might also suffer if the error is not only position dependent but also speed dependent.
4.3 Filter Utilizing Combined State Estimation and Position Feedback

As discussed in the two previous sections, different estimation and measurement methods may be suitable for different operating conditions. The authors of this thesis has therefore been excited about evaluating the potential of combining the information yielded from the different methods in a filter solution. This section will therefore discuss the potential of combining a sensorless estimation of the position with the MRPS measurements of the same position. A first approach might be to use the MRPS signal at standstill and low speeds and then switch to the sensorless approach when the operating speed increases. Another even more appealing approach is to combine the different methods in a Kalman filter solution. Figure 4.8 illustrates the concept of combining the previously described position estimation and measurement approaches in a Kalman filter to produce an even better estimate of the position.

![Kalman filter diagram](image)

**Figure 4.8:** Kalman filter utilizing an internal model, observer estimations and measurements to determine a good estimation of the rotors mechanical and electrical position $\theta_r(t)$, $\theta_e(t)$ as well as mechanical and electrical speed $\omega_r(t)$, $\omega_e(t)$.

4.3.1 Switching Filter

A straightforward approach might be to implement a switching filter. The switching filter may use the filtered MRPS position signal at low speed and standstill, and then switch to use the sensorless position estimate at speeds higher than a certain threshold. In order for the filter to not switch to often close to the threshold, a hysteresis switching threshold can be implemented. This filter is a simpler alternative to the Kalman filter solution. The quality of the solutions depends on the quality of the individual estimates for different operating points.
4.3.2 Extended Kalman Filter (EKF)

The Extended Kalman Filter (EKF) is a recursive predictive filter which can use the non-linear state-space model (3.12) for prediction as well as the sensorless estimation and the MRPS feedback for correction of the position. Since the Kalman filter is a recursive algorithm it is suitable for online operation. The EKF-algorithm used in this thesis is based on the presentation of Gustafsson et al. [2010, Chapter 8]. The dynamics of the hybrid stepper motor system is defined by Equation (4.22a)-(4.22d). Equation (4.22a) describes the dynamic model, Equation (4.22b) is the measurement equation, \( w(t) \) is the process noise and \( v(t) \) is the measurement noise. The initial estimates and covariances of the states are provided through \( x_0 \) and \( P_0 \).

\[
\begin{align*}
x(t + 1) &= A(t)x(t) + Bu(t) + w(t) \quad (4.22a) \\
y(t) &= Cx(t) + v(t) \quad (4.22b) \\
w(t) &\sim \mathcal{N}(0, Q(t)), \quad v(t) \sim \mathcal{N}(0, R(t)) \quad (4.22c) \\
x_0 &= E(x(t_0)), \quad P_0 = \text{Cov}(x(t_0)) \quad (4.22d)
\end{align*}
\]

Equation (3.12) of the system model in Chapter 3 is used to form the necessary state-space motion model in continuous time. The vector containing the states of interest is

\[
x^T = \begin{bmatrix} i_a(t) & i_b(t) & \omega_r(t) & \theta_r(t) \end{bmatrix}^T
\]

The state dynamics are governed by the following non-linear state space model

\[
\dot{x}(t) = \begin{bmatrix}
-\frac{R}{L} & 0 & K_m \sin(\theta_e(t)) & 0 \\
0 & -\frac{R}{L} & -K_m \cos(\theta_e(t)) & 0 \\
-\frac{K_m}{J} \sin(\theta_e(t)) & \frac{K_m}{J} \cos(\theta_e(t)) & -\beta & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
i_a(t) \\
i_b(t) \\
\omega_r(t) \\
\theta_r(t)
\end{bmatrix}
+ \begin{bmatrix}
1 \quad 0 \quad 0 \quad 0 \\
0 \quad 1 \quad 0 \quad 0 \\
0 \quad 0 \quad 1 \quad 0 \\
0 \quad 0 \quad 0 \quad 1
\end{bmatrix}
\begin{bmatrix}
u_a(t) \\
u_b(t)
\end{bmatrix}
+ \begin{bmatrix}
t_l(t)
\end{bmatrix}
\]

where the terminal voltages \( u_a(t) \) and \( u_b(t) \) are inputs and the load torque \( \tau_l(t) \) is modelled as a disturbance acting on the system. The electrical position may be added as an extra, linearly dependent, state \( \dot{\theta}_e = N \omega_r \). The vector \( y(t) \) contains the measured currents and the measured rotor position. The rotational speed is not measured directly. Therefore \( C \) is given in the measurement equation by

\[
y(t) = Cx(t) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
x(t)
\]

The initial states will be assumed to be zero, except the rotor position which will have to be estimated, possibly provided by the MRPS

\[
x_0 \in (0 \ 0 \ 0 \ [0, 360^\circ])^T
\]
Bendjedia et al. [2012] use the following covariance matrices to model the noises

\[
Q(t) = \begin{bmatrix}
10^{-4} & 0 & 0 & 0 \\
0 & 10^{-4} & 0 & 0 \\
0 & 0 & 10^{-3} & 0 \\
0 & 0 & 0 & 10^{-6}
\end{bmatrix} \quad R(t) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{4.26}
\]

and use the covariance matrix

\[
P_0 = \begin{bmatrix}
10^{-4} & 0 & 0 & 0 \\
0 & 10^{-4} & 0 & 0 \\
0 & 0 & 10^0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{4.27}
\]

for the initial estimation. These values may be used as initial estimates but will most likely change.

It is important to note that all the states, except the speed, are available as measurements from the current sensors and the MRPS. These measurements may however have been subject to additional filtering before entering this final filter. The covariance \(R(t)\) of the measurement noise \(v(t)\) describes how reliable the measurement information is [Gustafsson et al., 2010]. This means the weighting between the estimation and the measurement could be changed online with different covariance \(R(t)\) for different operating points, i.e. different speeds and loads. An interesting approach is to model the noise and non-linearity error of the MRPS measurement through \(v(t)\). The model noise \(w(t)\) is used to model uncertainties of the model, a special emphasis may be put on the part of the model where the load torque comes in as a disturbance.

The above presented continuous time model needs to be discretised and it is commonplace to only consider the mechanical system [Bendjedia et al., 2012], i.e. the two lower Equations of the state space representation in (4.23), when designing and implementing an extended Kalman filter. The reason for this is that the electrical dynamics are usually much faster, almost instant, compared to the mechanical dynamics.
This chapter will outline some of the approaches to controlling the position and compensating the load angle in a hybrid stepper motor when the position and load angle is known or accurately estimated. The explicit time dependencies have been omitted in parts of the chapter for enhanced readability. All speeds, positions, torques and currents may be written as functions of time.

### 5.1 Open-Loop Control of the Hybrid Stepper Motor

Stepper motors are primarily used to realise position demands but can also be used to meet speed demands in an application. To control the position of the hybrid stepper motor’s rotor and the attached application, the stepper motor torque is controlled. Since the stepper motor torque is a function of the winding currents according to (3.4), utilizing an inverter, i.e. a current controller, to control the winding currents is necessary. The desired reference position is thus translated into desired winding current according to

\[ i_a = I_0 \cos(N_r \theta_{ref}) \]  
\[ i_b = I_0 \sin(N_r \theta_{ref}) \]  

Inserting these currents into the electromagnetic torque Equation (3.4) gives

\[ \tau_e = -K_m I_0 \cos(N_r \theta_{ref}) \sin(\theta_e) + K_m I_0 \sin(N_r \theta_{ref}) \cos(\theta_e) = K_m I_0 \sin(N_r \theta_{ref} - \theta_e) \]  

where the trigonometric identity \( \sin(\theta - \psi) = \sin(\theta) \cos(\psi) - \cos(\theta) \sin(\psi) \) has been used. Thus, by setting the current references according to Equations (5.1) and (5.2), the produced torque will turn the unloaded stepper motor’s rotor to
the electrical reference position. Since sine has its smallest value for a zero angle, this position is also stable. This is how an open-loop controller can select the appropriate winding currents. When a motor is subject to an external load, the controller also has to account and be able to compensate for that as well.

One approach might be to increase the magnitude of the phase currents. This will produce a larger torque according to Equation (5.3) and therefore decrease the size of the position error. But, from Equation (3.16) it is obvious that the torque is zero when the load angle is zero, and therefore zero load angle and thus a small position error can not be achieved by only increasing the magnitude of the phase currents since it would require infinitely large currents. A large current would also tend to make the system unstable. The following section will describe feedback control strategies and discuss why the hybrid stepper motor can operate with smaller winding currents, and therefore lower power consumption, if the load angle is allowed to be larger during motion.

5.2 Closed Loop, Field-Oriented Control (FOC)

Field Oriented Control (FOC), sometimes also known as Vector Control, applies a zero direct current and a controlled quadrature current, considering the $dq$-reference frame described in Section 3.2 [Fitzgerald et al., 2003]. The direct and quadrature currents are then transformed into reference currents $i_{a,ref}$ and $i_{b,ref}$ for the two phases using the inverse $dq$-transform (3.14). The field oriented controller may vary both the load angle and the amplitude of the current, thus it can run the motor with larger load angle and reduce the current - making the motor run more energy efficiently. So far the Field-Oriented Controllers all work the same. There are however differences in how the magnitude of the desired quadrature current is determined and what load angle to aim for. A larger load angle gives rise to a larger torque but smaller stability margins.

Wonhee et al. [2011] use PID control with additional velocity feed-forward to derive a desired torque

$$
\tau_{e,ref} = kp(\theta_{r,ref} - \theta_r) + k_i \int_0^t (\theta_{r,ref} - \theta_r) \, d\tau + k_D (\dot{\theta}_{r,ref} - \dot{\theta}_r) + B\dot{\omega}_{ref} + J \ddot{\omega}_{ref} \quad (5.4)
$$

The desired torque is then fulfilled by setting the phase currents according to

$$
i_{a,ref} = \frac{\tau_{e,ref}}{K_m} \cos(N_r \theta_{r,ref}), \quad i_{b,ref} = \frac{\tau_{e,ref}}{K_m} \sin(N_r \theta_{r,ref}) \quad (5.5)
$$

This choice of phase currents $i_a$ and $i_b$ are then proven to be the same as applying a desired direct and quadrature current according to

$$
i_{d,ref} = 0, \quad i_{q,ref} = \frac{\tau_{e,ref}}{K_m} \quad (5.6)
$$
Their solution was implemented on an Atmega128 processor and utilized position feedback from an optical encoder. The solution has accuracy better than 0.3° in the performance tests consisting of a ramp response without any applied load. The steady state error converges to zero. The position ripples are claimed to be unavoidable and possibly stem from encoder coupling effect, the PWM driver noise, modeling uncertainty, non-ideal sinusoidal flux distribution or the detent torque. A similar control strategy is used for microstepping position control in the aforementioned authors’ most recent work [Wonhee et al., 2016] where the experimentally verified position tracking accuracy is better than 0.05°. The position feedback signal was provided by an incremental optical encoder and quadrature signals to obtain a resolution of 32000 pulse/rev in the experiments.

Bendjedia et al. [2012] considers only the mechanical system and relies on good estimations of the rotor position and load torque through an EKF. The discretized quadrature current is then formed by a direct intervention of the reference position and another term to compensate the load torque

\[ i_{q,\text{ref}}(k) = -K_\omega \dot{\omega}(k) - K_\theta \dot{\theta}(k) + K_{\theta,\text{ref}} \theta_{\text{ref}}(k) + K_L \hat{\tau}_l(k) \] (5.7)

The coefficients \( K_\omega \) and \( K_\theta \) are computed by using a canonical form method to impose poles in the closed-loop system.

Zaky et al. [2012] designs and implements a gain-scheduling adaptive PI controller for FOC speed tracking of a hybrid stepper motor. First, the speed error is formed

\[ e_\omega(t) = \omega_{r,\text{ref}}(t) - \omega_r(t) \] (5.8)

When the speed error is large, there is a need for better control effort through a high proportional gain. When the speed error is small, it is desired to have a high integral gain to overcome the steady state error. The suggested solution utilize proportional and integral gains \( K_p(t) \) and \( K_i(t) \) that are adapting according to the magnitude of the speed error \( e_\omega(t) \), in the PI control of the quadrature current

\[ i_{q,\text{ref}}(t) = K_p(t)e_\omega(t) + K_i(t) \int e_\omega(t) d\tau \] (5.9)

The desired behaviour of the proportional gain is realised by

\[ K_p(t) = K_{p,\text{max}} - \left( K_{p,\text{max}} - K_{p,\text{min}} \right) e^{-ke_\omega(t)} \] (5.10)

where \( k \) is the rate at which \( K_p(t) \) varies between its maximum and minimum value. A large speed error will reduce the second term and therefore set \( K_p(t) \) to maximum. A small speed error will make the gain approach its minimum. The desired behaviour of the integral gain is formed in a similar fashion

\[ K_i(t) = K_{i,\text{max}} e^{-ke_\omega(t)} \] (5.11)

The integral gain is only large if the speed error is small. The maximum and minimum of the respective gains can then be selected according to, for example
Ziegler-Nichols tuning rules. The simulations and experimental results show that the system has a fast dynamic response as well as good steady-state performance for the speed tracking. A similar gain-scheduling strategy with an adaptive PI-controller may therefore be a good option also for position control. That possibility is however not discussed by the aforementioned authors.

5.3 Complete Motion Control System

The block diagram in Figure 5.1 illustrates a top view of the hybrid steppers motor’s complete position control system. User input is provided through an operator PC, the control error of the rotor position is then formed. The rotor position is either measured by the MRPS or estimated by an observer. Additional signal processing may be applied by filters in order to improve the estimates. A Field-Oriented Position controller is used to determine the suitable reference current for each of the two phases. The realisation of the reference currents is performed by a current controller controlling the on and off times of the MOSFET:s on the two H-bridges. The current controller consists of a PI-controller in series with a PWM-generator. A safety block provides interlock and dead time compensation functionality to protect the H-bridges from undesired current shoot through. The inverted currents enter the armature of the hybrid stepper motor which produce torque and motion.

**Figure 5.1:** Schematic and illustrative top view of the hybrid steppers motor’s motion control system.
Simulation and Offline Results

To demonstrate the performance of the proposed schemes and solutions, several simulation studies were conducted using MATLAB/SIMULINK. The results and qualitative analyses are presented in this chapter.

6.1 Sensorless Observer in Simulation

The back-EMF algorithm was evaluated in Simulink using a ready-made hybrid stepper motor model block provided by the Simscape powerlib library. The purpose was to investigate the feasibility of using the algorithm in a physical system. That is, to investigate the correctness of the result, computational effort as well as to investigate what type of sensor inputs would be required.

The conclusion from the simulation results below is that whilst the profile of the estimated load angle is promising during some working conditions, it suffers from primarily two different sources of error. The first one relates to the sampling of the phase voltages, which causes the magnitude of the calculated $U_s$ to be smaller. The other error source appears to be inherent to the current implementation of the SDFT algorithm, causing an offset which is dependent on the rotor speed.

Figure 6.1 shows the result of the estimation process during a small simulated time period. Halfway through the simulation, a small load torque is immediately applied to the stepper motor. The estimation algorithm is running continuously throughout the simulation.

A problem of the SDFT algorithm, which is presented in Section 4.1.2, is that it cannot produce a viable new estimation shortly after a velocity change. This can be seen in Figure 6.1 as an irregularity at the beginning, where the motor is speeding up. The estimation, however, quickly starts following an oscillatory behaviour that is a lot more reminiscent of the motor’s physical behaviour. It is
worth noting fast transients are to be expected as the motor starts rotating. These transients also help create such a response. It is therefore concluded that the time delay before a new useful estimation can be produced is in fact small, and likely does not impair the functionality of the algorithm in a physical setup.

![Comparison between actual and estimated load angle](image)

**Figure 6.1:** Simulation of the load angle estimation algorithm based on back-EMF estimation. The SDFT algorithm has the same sampling rate here as the PWM-modulator, 1 MHz. The stepper motor is running at a constant 100 rpm. The oscillations shown are an expected result from the motor settling at a certain position relative to the electrical field. Adding a torque causes the motor to settle at a new relative position.

Although all signals are naturally easily obtained in the Simulink environment, not all sensor signals are available in the physical setup. The phase voltages, which defines the value of $U_s$, is not measured by the driver boards, whereas the bus voltage is. Therefore, this problem has been solved by estimating the phase voltage using the switching logic of the PWM-generator and the measured bus voltage. Simulations of the load angle estimation algorithm have shown that this method of estimating the phase voltage results in no discernible loss of precision. Derammelaere et al. [2014] point out that this is a reasonable method of measuring the phase voltages as well, although non-linearities should be accounted for in order to provide a more correct phase voltage estimation. These non-linearities could be caused by, amongst other sources, from turn-on/turn-off
times and resistive voltage drops [Hejny and Lorenz, n.d.]. However, simulations have shown that these effects are negligible for the operating region which this thesis focuses on, see Figure 6.2. Therefore, these non-linearities have been assumed to be negligible in the implemented load angle estimation function.

Another issue arises when the sample rate of the load angle estimation function is lower than the switching frequency of the PWM-generator. The SDFT algorithm described in Section 4.1.1 assumes that the sampling succeeds at capturing the dynamics of the measured signal. This causes no problems of implementing an SDFT calculation of the phase currents, as the sinusoidal frequency is relatively small compared to the operation frequency of Simulink. However, a low sampling rate will miss some of the faster PWM-pulses. This results in a lower magnitude of the resulting first harmonic fourier-component $U_s$. The phase of $U_s$, however, tends to be unaffected by this effect, because the phase voltage has the same fundamental frequency as the much slower current. This magnitude error causes an error when estimating the back-EMF transform $E_s$, which is cal-
culated from $I_s$ and $U_s$ according to Equation 4.3. As a result, the estimated load angle is lowered by an unknown offset and any changes to the estimation are affected by an unknown scaling factor. An example of these effects is provided in Figure 6.3. These effects in a physical environment are also shown in Figure 8.5.

![Comparison between actual and estimated load angle](image)

**Figure 6.3:** Simulation of the load angle estimation algorithm based on back-EMF estimation, with 20 kHz sampling rate of the SDFT algorithm. The stepper motor is running at a constant 100 rpm. The simulink program in the physical tests will be running at 20 kHz. This causes an undersampling of the phase voltage measurements, resulting in an offset of the estimated load angle as well as worse dynamic properties.

There are also a dependency of the algorithm on past velocity values. For this reason, the algorithm needs to be re-initiated after a velocity change. However, the current implementation of this process adds a static offset to the actual load angle. This can be seen in Figure 6.4. On the other hand, failing to reset the algorithm causes an unknown offset which could cause the load angle to attain any value, even negative values. This can be seen in Figure 6.5, where a negative load angle is achieved at a rotational velocity of 200 rpm.
Figure 6.4: Simulation of the load angle estimation algorithm based on back-EMF estimation. The SDFT algorithm has the same sampling rate here as the PWM-modulator, 1 MHz. The stepper motor is subjected to no load torque. An error in the current implementation of the algorithm causes the estimated load angle to be subjected to an increasing static offset after velocity changes, placing the load angle estimate at no torque close to zero.
Simulation and Offline Results

Figure 6.5: Simulation of the load angle estimation algorithm based on back-EMF estimation with no reset. The SDFT algorithm has the same sampling rate here as the PWM-modulator, 1 MHz. The stepper motor is subjected to no load torque. The steady-state value of the estimated load angle is dependent nonlinearly on past velocity changes.
6.2 Offline Treatment of the MRPS Signal

These post filtering experiments were conducted in order to qualitatively deduce whether and to what extent it is possible to suppress errors and improve the quality of the MRPS-signal through the use of filters.

6.2.1 Notch and Low-Pass Filtering of the MRPS

An oversampling filter combined with a notch filter is the filter of interest and has been evaluated in this section. The filtering has been applied to sets of data acquired while driving the motor using a commercial stepper motor driver.

Figure 6.6 contains subplots with the unfiltered MRPS signal and the error compared to an optical encoder as well as their respective frequency spectrum. The error is between ±0.40° and composed of noise and non-linearity errors.

![Figure 6.6: Frequency analysis of the acquired MRPS signal and the error compared to an optical encoder. Three distinct peaks, the first, the second and the third harmonics of the rotational speed are visible in the error's spectrum.](image)

Figure 6.7 shows the effect of oversampling the error signal starting at $t = 4$. Each "oversampled" sample is the arithmetic mean of the last six samples. The signal is therefore not decimated in this test. The error before oversampling has a peak of about 0.4° whereas the peak magnitude of the error after oversampling is 0.3°. The noise seems to have been effectively eliminated. The fundamental
frequency content of the signal is practically unaltered. When oversampling the acquired MRPS signal, one must carefully consider how to oversample that signal correctly, using common unwrap and wrap functions, at the points where the signal alternates from 360° to 0°.

![Error(t), Oversampling (4 times sliding) start at t = 4](image)

**Figure 6.7:** Oversampling the error signal starting at $t = 4$. The error before oversampling has a peak of about 0.4° whereas the peak magnitude of the error after oversampling is 0.3°. The noise seems effectively eliminated. The fundamental frequency content of the signal is practically unaltered.

Figure 6.8 shows the results of applying a notch filter with notch frequency 10 Hz and 3 dB bandwidth of 3 Hz to the error signal in order to further reduce the error. The experiment shows that the notch filter combined with the oversampling can reduce the error from about ±0.40° to less than ±0.15°. The frequency spectrum plots clearly show that the second harmonics have been effectively suppressed. The final filter must however also be speed adaptive to effectively identify and suppress the second harmonics. Suppressing the first or fourth harmonic instead of the second harmonic does not reduce the error at all.

It’s worth noting that in these experiments executed in MATLAB/Simulink, the post treatment has been performed on the error signal and not on the noisy position signal from the MRPS. The conclusion is however that it is worthwhile to design and implement an oversampling filter and a speed adaptive notch filter as well as evaluating them in a physical experimental setup. The SPI-clock in
Figure 6.8: A notch filter with notch frequency 10 Hz and a 3 dB bandwidth of 3 Hz has been applied to the signal to further reduce the error. The experiment show that the notch filter combined with the oversampling can reduce the error from about 0.40° to less than 0.15°. The frequency spectrum plots clearly shows that the second harmonic has been effectively suppressed.
the communication between the Speedgoat and the MRPS runs at 2.5 MHz and therefore the position, a 16-bit data transfer, was read out at a rate of 156 kHz. Since the Simulink controller operates at 20 kHz on the Speedgoat CPU, it is possible to over-sample the position signal by at least four times at FPGA-level, effectively low-pass filtering it, without loss in update rate from the Speedgoat CPU point of view.

Throughout the thesis, there was continuous discussion with the MRPS supplier who provided a filter functionality with similar expected effect as the oversampling, i.e. reduce the noise. This filter functionality was provided as an additional hardware module which can be the subject for further experimental tests.

### 6.2.2 Modelling and Compensation of the Non-linearity Error Using Least-Squares Fitting

Section 4.2.5 discussed how the MRPS error compared to an optical encoder can be modelled and compensated. Finding the LSQR coefficients to Equation 4.21 using data from a session with more than 30000 data points sampled at 20 kHz while the motor was rotating at 100 rpm, 200 rpm and 300 rpm respectively resulted in the coefficients in the table. The SPI-interface was used to retrieve the absolute position signal from the MRPS. Table 6.1 The calculated LSQR coefficients are very similar except for the modelling of the constant error $c_0$. The constant error will however depend on the size of the error at the beginning of the data set used for the LSQR fitting. To sync the MRPS and the Encoder, the Encoder signal is digitally altered to start at the same angular position as the MRPS. The modelling of the offset error is therefore omitted in the following offline test. The offset error may be modelled as zero in the final solutions as well since the effect is comparably small. Since the other coefficients doesn’t vary with the speed, it can be concluded that the error is only dependent on the position. The coefficients found when revolving at 100 rpm is the coefficients that have been used to compensate the MRPS signal at 200 rpm and 300 rpm also. The results of performing the compensation offline are displayed in Figure 6.9, Figure 6.10 and Figure 6.11 respectively. The modelled error $\epsilon_{\text{modelled}}(\theta_r(t))$ has been subtracted from the MRPS output. The error compared to a reference encoder is visibly reduced in all tests. There is however a bias error left in the test where $c_0 = 0$ is not a perfect modelling of the offset error. The maximum average error (ex-
explained in Section 7.2) is reduced from $0.49^\circ$ to between $0.16^\circ$ and $0.21^\circ$ in the offline experiments. The frequency analysis displayed in the figures show that the harmonic errors have been effectively removed. Experimental test results of the LSQR-compensation are presented in Section 8.3.
Figure 6.9: The non-linear error, appearing with frequency of the $1^{st}$, $2^{nd}$ and the $4^{th}$ harmonics of the speed, which is 100 rpm in the test displayed in the figure. The error has been modelled as sine and cosine functions of the first, second and fourth order of the position. The respective coefficients have been determined using a least square minimization. Removing the constructed error signal from the raw output of the MRPS reduces the maximum average error from $0.46^\circ$ to about $0.16^\circ$. The frequency analysis show that the harmonic errors have been effectively suppressed.
Figure 6.10: The non-linear error, appearing with frequency of the $1^{st}$, $2^{nd}$ and the $4^{th}$ harmonics of the speed, which is 200 rpm in the test displayed in the figure. The error has been modelled as sine and cosine functions of the first, second and fourth order of the position. The respective coefficients have been determined using a least square minimization. Removing the constructed error signal from the raw output of the MRPS reduces the maximum average error from $0.47^\circ$ to about $0.18^\circ$. The frequency analysis show that the harmonic errors have been effectively suppressed.
Figure 6.11: The non-linear error, appearing with frequency of the $1^{st}$, $2^{nd}$ and the $4^{th}$ harmonics of the speed, which is 300 rpm in the test displayed in the figure. The error has been modelled as sine and cosine functions of the first, second and fourth order of the position. The respective coefficients have been determined using a least square minimization. Removing the constructed error signal from the raw output of the MRPS reduces the maximum average error from 0.49° to about 0.21°. The frequency analysis show that the harmonic errors have been effectively suppressed.
6.3 Field-Oriented Control (FOC) in Simulation

The FOC suggested by Wonhee et al. [2011] was implemented in Simulink and tested in simulation before attempting to apply the controller to the real system. Figure 6.15 shows the Simulink scheme for the FOC. The simulated hybrid stepper motor parameters were chosen according to the specifications of the physical stepper motor in the experimental setup Section A.1. The structure of the current controller used in the simulations is the same as was used in the final experimental setup. The controller parameters may however differ in the fine tuned final experimental setup. The sample time for the simulation was 50 $\mu$s and chosen to coincide with the control loop frequency used in the Speedgoat setup. This sampling time was therefore used also by the PWM-generator and the simulated stepper motor which is not the case in the final setup.

To test the qualitative performance of the FOC regarding position control and compensation for external load torque, the hybrid stepper motor is commanded to follow a position profile. The position feedback signal contains added white noise of magnitude comparable to that of the MRPS. The position profile consists of a ramp, starting at time 0.3 s, then revolving with 240 rpm and finally stopping 1620° away from the initial position. During the motion, an external load of 0.4 Nm is applied to the motor at time 1 s. The results of the test is displayed in Figure 6.12 and a zoom in on the phase currents at the time when the load is applied is shown in Figure 6.13. The position error is largest 1.15° when starting the movement and the motor is turned no more than 0.77° out of position when the external load is applied. The implemented FOC will regulate the phase currents in order to meet the torque demand as well as produce rotary motion. The phase current looks roughly sinusoidal and the peak amplitudes are notably higher after the external load has been applied. The current references are limited within the controller. This limitation also limits the maximum electromagnetic torque that can be delivered from the hybrid stepper motor.

Since it was shown in Section 6.2.2 that the MRPS error can be modelled as a sum of sine and cosine terms. This error model was used to simulate the expected performance from combining FOC together with the untreated MRPS position signal. The results are shown in Figure 6.14. It’s clear from the figure that the position performance will be poor if the untreated MRPS signal is used as feedback together with the FOC. The shape of the phase currents are similar to the previous experiment.

The qualitative conclusion from the simulation results is that the implemented FOC can fulfill position demands and compensate for external loads if harmonic errors can be effectively removed from the feedback signal.
Simulation and Offline Results

Figure 6.12: FOC performance when following a 240 rpm ramp profile. The feedback signal is only disturbed by added white noise. The motor is loaded with 0.4 Nm at time 1 s. The position error is largest 1.15° when starting the movement and the motor is turned no more than 0.77° out of position when the external load is applied. The steady state error is zero.
Figure 6.13: The implemented FOC will regulate the phase currents in order to meet the torque demand as well as produce rotary motion. The phase current looks roughly sinusoidal and the peak amplitudes are notably higher after the external load has been applied.
6 Simulation and Offline Results

**Figure 6.14:** The simulated performance of combining FOC together with the untreated MRPS position signal. The positioning performance is obviously poor and the MRPS needs to be treated to produce a good enough position feedback signal.

**Figure 6.15:** Simulink realisation of the FOC. Position field-oriented controller parameters for the simulation tests: \([K_p, K_i, K_d, N_{filter}, B, J, T_s] = [4, 8, 0.0050, 8000, 8e−6, 0, 5e−5]\)
Experimental Test Cases

The purpose of the testing is to provide an objective platform on which the different position estimates and measurements as well as the control systems can be evaluated. The different test cases provide the data which will form the basis for the final conclusions. Therefore, the cases should be different enough to investigate different parts of the work profile of the stepper motor.

Another concern in designing these test cases are the physical limitations in the lab environment. For example, the equipments should be rather isolated without large moving parts for safety reasons. There are also limitations to both the mechanical components and the resolution and speed of the reference encoder.

The experimental setup is presented and discussed in Appendix A. This chapter will discuss the test procedure based on the assumption that only the components presented in Appendix A are used. A quick overview of the setup is given in Figure 7.1.

The output from the optical encoder is often used and considered as the ground truth in this type of performance tests. The encoder has an absolute resolution of 25 bits, $2^{25} = 33554432$ positions per revolution which is finer than 0.000011°. The torque meter used is also of relative high performance type. The motor can be loaded by a hysteresis brake which is attached at the end of the test bench.

7.1 Sensorless Position Estimation Performance Test

In Section 4.1.1, a sensorless position estimation algorithm which outputs an estimation of the load angle is presented. The actual load angle at a given time instance is given by the difference between the applied electrical field and the measured rotor position. These tests aim to compare the estimated load angle to a calculated true value. The reliability of the algorithm is evaluated by running
Experimental Test Cases

Figure 7.1: Test equipment for the experimental tests. (ABB)

the tests at different operating points and during acceleration phases. Furthermore, the tests aim to determine the correctness of the result during different well-known system events, such as step-loss, no load or changing torque.

- Drive the motor at 100 rpm at no load and compare the estimated load angle output to a calculated value based on the output of the encoder.

- Drive the motor at 100 rpm with a load of 0.1 Nm and compare the estimated load angle output to a calculated value based on the output of the encoder.

- Drive the motor at 400 rpm at no load and compare the estimated load angle output to a calculated value based on the output of the encoder.

- Increase the motor speed linearly over 5 seconds from 100 rpm to 400 rpm at no load and compare the estimated load angle output to a calculated value based on the output of the encoder.

- Drive the motor at 100 rpm starting with no load, then adding a 0.15 Nm load after 5 seconds and compare the estimated load angle output to a calculated value based on the output of the encoder.

- Drive the motor at 100 rpm, increasing the torque from zero Nm until a step-loss is reached. After a step loss is reached, the torque is decreased until normal rotation is attained. Compare the estimated load angle profile to the applied load torque.

- Drive the motor at 300 rpm, increasing the torque from zero Nm until a step-loss is reached. After a step loss is reached, the torque is decreased until normal rotation is attained. Compare the estimated load angle profile to the applied load torque.
7.2 Filtered MRPS Performance Test

In these tests, the outputs of the filtered MRPS position signal are compared to the readouts of an optical encoder. The output from the optical encoder is considered as the ground truth in these performance tests. The effects of oversampling as well as adaptive notch filtering is evaluated for both constant and varying speed as well as dynamic operation according to the bullet list below. A good performance metric for the error suppression is the maximum average error calculated as the average of the largest positive and the largest negative deviation, the error $\epsilon$ between the MRPS and the encoder outputs.

$$\bar{\epsilon}_{\text{max}} = \frac{\max(\epsilon) - \min(\epsilon)}{2}$$

This metric is accompanied by the normal arithmetic mean of the deviation to deduce if the filtered signal has attained some offset due to the expected small delay stemming from the filtering.

- Drive the motor at 100 rpm and compare the MRPS output to that of the encoder.
- Drive the motor at 100 rpm and compare the oversampled/low-pass filtered MRPS output to that of the encoder.
- Drive the motor at 100 rpm and compare the 2$^{nd}$ harmonics notch filtered MRPS output to that of the encoder.
- Drive the motor at 100 rpm and compare the oversampled and notch filtered MRPS output to that of the encoder.
- Drive the motor at 100 rpm and compare the oversampled and twice notch filtered, at both the 1$^{st}$ and 2$^{nd}$ harmonics of the speed, MRPS output to that of the encoder.
- Alternate the rotor position by hand and compare the oversampled and notch filtered MRPS output to that of the encoder.
- Drive the motor at 300 and compare the MRPS output as well as the LSQR-compensated MRPS signal to output of the encoder. The coefficients for the LSQR-compensation shall be the ones calculated for the motor driven at 100 rpm.
- Drive the motor at 100 rpm, accelerate it to 200 rpm and then brake the motor until stall. Compare the MRPS output as well as the LSQR-compensated MRPS signal to that of the encoder.

7.3 Motion Control and Load Angle Compensation Tests

This sections describes tests where the performance of the motion control with different feedback signals is evaluated. The intention is for the tests to show how
well the FOC strategy can meet the position demands and compensate for position errors caused by the necessary load angle when loaded with an external load.

- Apply a load torque of 0.85 Nm to an open loop position controlled hybrid stepper motor and study the response. The load angle is then approximated by the resulting position error.

- Perform a position step response followed by applying a load torque when the encoder is used for position feedback. Study the response.

- Perform a position step response followed by applying a load torque when the unfiltered MRPS is used for position feedback. Study the response.

- Perform a position step response followed by applying a load torque when the filtered MRPS is used for position feedback. Study the response.
This chapter will present the experimental test results for each solution. Prior to implementing the proposed solutions, several simulations were carried out in advance. The results and findings from the simulations are given in Chapter 6. To get the most complete understanding of the final results, it is suggested to study the simulation results prior to reading the experimental results.

8.1 Test Setup and Procedure

Refer to Appendix A for a detailed description of the experimental setup. The tests performed have been specified in Chapter 7.

8.2 Sensorless Load Angle Estimation Results

The performance, demands and assumptions of the back-EMF estimation algorithm has been investigated using a simulation platform. The simulations show that the algorithm approaches constant values for constant speed and torque. The estimated load angle shows similar qualities to the measured load angle. However, it is clear that the estimation is subject to additional errors during speed or torque changes. The errors can be related to the phase voltage measurements and unresolved issues with the SDF algorithm during speed changes.

The produced load angle estimate is adequately close to the true load angle when the motor has managed to start and remain at a given rotational velocity. This is shown in Figure 8.1, where the motor is running at a continuous 100 rpm. It is important to note that this starting velocity has to be large enough for a sizable back-EMF to be produced. At close to zero rotational velocity, no reasonable estimate will be produced.
Figure 8.1: Constant rotation at 100 rpm with no load torque. The average value of the measured torque is 0.0052, which is likely due to friction losses. The average measured load angle is 16.7528 electrical degrees, whereas the average estimated load angle is 5.3402 electrical degrees. This difference translates into 0.23 mechanical degrees difference.
Adding a load torque will theoretically increase the load angle by a certain amount. The magnitude of the load angle increase is difficult to predict offline. The analysis of this increase is therefore limited to comparing the magnitude of the change to the noisy measured load angle and the direction of the change in load angle as dictated by the increase in torque.

Figure 8.2 shows the estimated load angle compared to the measured load angle at a small load torque, 0.1 Nm. It is clear that the measured and estimated load angles have been increased slightly with the addition of a load torque. It can therefore be assumed that running the motor at this small load torque produces a reasonable change in the produced load angle estimate.

*Figure 8.2: Constant rotation at 100 rpm with 0.1 Nm load torque. The average measured torque is 0.1014 Nm. The average measured load angle is 17.4012 electrical degrees, with the average of the estimate being 9.0782 electrical degrees.*

Increasing the rotational velocity introduces additional friction losses. Friction will act on the system in much the same way as an added load torque would. Additionally, increasing the rotational velocity requires that the current can be switched rapidly. Due to the current rise time being fixed, the current could become unable to reach the maximum specified current when the motor operates at very high speeds. Then, since the maximum current is effectively lowered, the load angle will be increased even further.
This is shown in Figure 8.3, where the load angle has increased dramatically compared to Figure 8.1 and Figure 8.2. In comparison, the estimated load angle has been increased, but not by as much as to follow the actual load angle.

![Graph showing load torque and estimated load angle](image)

**Figure 8.3**: Constant rotation at 400 rpm. The average measured torque is 0.0077 Nm. The average measured load angle is 80.8078 electrical degrees, whereas the average estimated load is 18.5465 electrical degrees. This is a sizable difference, causing a 1.24 mechanical degrees difference.

This small change is also made clear by viewing the acceleration from 100 rpm to 400 rpm. In Figure 8.4, it is apparent that the load angle estimation algorithm is subjected to a much smaller increase than the measured load angle.

This issue likely have multiple sources. One problem relates to the phase voltage measurements. It is possible that the phase voltage estimation fails to account for a large back-EMF. This effect of using estimated phase voltages as opposed to measured voltages is simulated, as shown in Figure 6.2. However, there could be a difference in result between this simulation and the physical setup.

Also, the SDFT algorithm is running at a lower frequency than the switching logic of the FPGA board, causing a lower magnitude of the estimated $U_s$ vector. There were indications of this issue in simulation as well, as shown in Figure 6.3. This issue causes a negative offset as well as a gain error. It is reasonable to assume that the faster phase voltage dynamics at higher rotational velocities cause
8.2 Sensorless Load Angle Estimation Results

Figure 8.4: Acceleration from 100 rpm to 400 rpm at no load. The starting and end results are roughly equal to the individual test results presented in Figure 8.1 and Figure 8.3 respectively.
the issue of undersampling to become more prominent. The error would therefore reasonably become larger at faster speeds.

Another issue relates to the constant offset introduced when the algorithm is reset after a velocity change. This was shown in simulation as well, as depicted in Figure 6.4. The cause of this issue is not apparent. It would be reasonable, however, to assume that extra control logic needs to be implemented to handle the feedback principle shown in Figure 4.2 during velocity changes.

In Figure 8.5, a load torque is applied during execution. Here, only the problem of measuring the phase voltage will pose an issue. It is clear that the estimated load angle does show the same profile as the actual load angle, although the difference between the two is increased after the torque step.

![Applied Load Torque](image1)

![Sensorless Load Angle Estimation](image2)

**Figure 8.5:** Constant rotation at 100 rpm. Load is set to increase from 0 Nm to about 0.3 Nm after some time. The average of the measured torque after the step is 0.3153 Nm. The average of the measured load angle is 33.2785 electrical degrees after the step, whereas the average of the estimated load angle is 17.1295 electrical degrees.

It can be seen in Figure 8.5 that the load angle follows the changing torque closely. This is to be expected, as the true load angle is related to the applied torque through Equation 3.17, where the electromagnetic torque in question is the additional torque required to counteract the applied load torque. As Equation 3.17 is sinusoidal in nature, the profile of the torque and the load angle
should be similar for small values of $\gamma$.

The following two tests investigate the behaviour of the estimation process as the motor experiences a stall. A stall occurs when the load angle increases to about 110 electrical degrees. Past this point, the rotor fails to move a step to the next position. Here, the load torque is applied using a brake and the reaction is therefore that the rotor stops after stalling. The load angle will then start changing rapidly at the speed of the rotating electric field.

Figure 8.6 shows the stall occurring while the motor is rotating at 100 rpm. The estimated load angle does show some attractive properties. Again, the estimated load angle is subjected to an offset as the torque increases. During the step loss, no discernible information can be gathered from the measured phase currents and voltages. Therefore, the estimated load angle assumes a random value during the step loss. After normal rotation resumes, the estimated load angle quickly regains a new reasonable value.

![Applied Load Torque](image1)

![Sensorless Load Angle Estimation](image2)

**Figure 8.6:** The stepper motor rotates at a constant 100 rpm. A continuously increasing torque is applied until the rotor is pulled out of step and stops. The torque is then lowered until normal rotation resumes.

In Figure 8.7, the same behaviour can be seen as that in Figure 8.6. However, the dynamics are much smaller. Note that the load angle after the step loss is nonsensical as the rotation could not be resumed by just lowering the torque. The problem with the low dynamics are likely due to the previously described
Figure 8.7: The stepper motor rotates at a constant 300 rpm. A continuously increasing torque is applied until the rotor is pulled out of step and stops. The rotor is unable to start at 300 rpm from standstill, even at no load torque.
8.3 Filtered MRPS Results

Filtering of the position signal was at first performed offline, with causal methods. The promising results of the post filtering are presented in Chapter 6 Section 6.2.1. This section will present the results of the tests specified in Chapter 7, Section 7.2. The general noise filtering and the notch filter implementations in the real-time system differ from the offline filtering. They address the same issues with general noise and 1st as well as 2nd harmonics non-linearity error. The 1st harmonics component in the error was more significant in the experimental tests than in the previously acquired data sets used for the offline tests. The low-pass or oversampling filter must consider the position signals’ abrupt changes from 360° to 0° every revolution, which can be handled by unwrap and remainder functions in Simulink. The notch filter must effectively find the frequency of the 1st and 2nd harmonics and apply the correct frequency notch. The speed has been estimated according to the description in Section 4.2.4.

Firstly, the output of the unfiltered MRPS position signal is compared to the output of an optical encoder in Figure 8.8 when the motor is driven unloaded at 100 rpm. The maximum average error is 0.61°. The difference between the both signals, considered as the MRPS error, is plotted in the middle plot. The maximum average error is 0.61°. As described earlier, the deviation has some noise and is subject to disturbance with frequency equal to the 1st and 2nd harmonics of the speed. Since the speed during the tests were 100 rpm, the first and second harmonics are found at frequencies of \( \frac{100}{60} = 1.67 \) Hz and \( 2 \times \frac{100}{60} = 3.33 \) Hz. The frequency spectrum of the deviation is clearly visible as the first and second peak. Figure 8.9 provides an upper full range plot and a zoomed in plot of the deviation in the middle. The deviation is considered as the MRPS position error. The smaller this position error is, the better. The deviation has been zoomed in to provide a better view of the noise addressed by the low-pass oversampling filter. The lower subplot displays the amplitude spectrum of the deviation. Removing the first and second peak and the highest frequency components is expected to result in a reduced error. The small general noise is visible as blurriness of the signal. The relatively high resonance in the signal is due to common hybrid stepper motor oscillation during motion. The longer oscillation (about one period is visible in the middle subplot) is the harmonics of the speed. The highest frequency component may be due to resonance in the mechanical movement of the hybrid stepper motor. This movement may be detected by the encoder and not by the MRPS or vice versa. Another possible reason might be that the encoder or the MRPS updates its position more often than the other. The latter possibility has been investigated and does not seem likely.

The results from the oversampling filter test are displayed in Figure 8.10. The deviation between the filtered MRPS and the encoder is not visible in the uppermost subplot. The deviation i.e. the error is therefore plotted in the middle zoomed in subplot. The oversampling has reduced some of the general noise, but it has introduced a constant offset of 0.12° in the difference. The constant offset stems from the signal delay that the oversampling generates. The deviation signal is now narrower and more distinct. The average maximum error is lowered from
0.63° to 0.58°. Since the offset is larger than the improvement on the average maximum error, the overall effect of this oversampling filter is far from perfect.

The lower subplot displays the amplitude spectrum of the deviation which is not changed notably for the displayed frequencies. The frequency spectrum has not changed notably for the displayed frequencies containing the information about the true position.

The results from the notch filtering tests are shown in Figure 8.11. The unfiltered and the 2\textsuperscript{nd} harmonics notch filtered MRPS position are compared with the encoder signal in the uppermost subplots. The deviation between the two signals when not applying and when applying the notch filter respectively, are shown in the middle plots. It’s clearly visible that the second harmonics is suppressed. This is also indicated by the frequency spectrum displayed in the bottom plots where the 2\textsuperscript{nd} harmonics peak is considerably smaller after the filtering. The second harmonics have been dampened by the notch filter.

Figure 8.12 shows the unfiltered and the treated MRPS position compared with the encoder signal in the uppermost subplots. In this test, the MRPS signal is both low-pass and notch filtered. The deviation is shown in the middle plots. The maximum average error has been lowered from 0.61° to 0.47°. This shows that applying both filters to the signal gives a smaller error than only applying one of the filters.

Figure 8.13 displays the effect of oversampling and then notch filtering both the 1\textsuperscript{st} and the 2\textsuperscript{nd} harmonics. The maximum average error has been lowered from 0.63° to 0.30°. The error has been biased with a constant 0.15° due to the delay imposed by the filtering. The filter parameters where $\epsilon_1 = 0.0040$, $\epsilon_2 = 0.0087$ and the oversampling was done over 10 samples. Since a 0.12° error offset was introduced by the low-pass filtering, one can deduce that the notch filtering induces an additional 0.03° in this case. The major part of the remaining error is suspected to originate from the, yet to be explained, faster oscillations in the deviation signal.

To clearly see the effects of the oversampling and notch filtering, a zoomed in plot of the encoder, the MRPS and the filtered MRPS position signal is shown in Figure 8.14. One can see that the oversampled and notch filtered signal is smoother than the raw MRPS signal, but it is delayed compared to the encoder and raw MRPS position signal.

The results of the test when the rotor position has been altered by hand are presented in Figure 8.15. The effects of the oversampling remain but there is no visible positive effect of the notch filtering. The maximum average error is actually increased from 0.451° to 0.454°. This is no surprise since there is no movement with constant speed in the motion profile. The estimation of the speed therefore works bad and the notch filter is unable to track and suppress the correct parts of the frequency spectrum. A good property is that the notch filtering doesn’t destroy the fundamental position from the MRPS.
Figure 8.8: The unfiltered MRPS signal is compared to the output of an optical encoder in the upper subplot. The difference between the both signals, considered as the MRPS error, is plotted in the middle plot. The maximum average error is $0.61^\circ$. The deviation shows noise and is subject to disturbance with frequency peaks at the 1st and 2nd harmonics of the speed. Revolving at 100 rpm results in harmonics frequencies of $\frac{100}{60} = 1.67$ Hz and $2 \cdot \frac{100}{60} = 3.33$ Hz
Figure 8.9: The unfiltered MRPS position is compared with the encoder signal in the uppermost subplot. The deviation between the two signals is plotted and focused in the middle subplot. Ideally, the error is zero at all times. The lower subplot displays the amplitude spectrum of the deviation. Removing the first and second peak and the highest frequency components is expected to result in a smaller error. The highest frequency component may be due to resonance in the mechanical movement of the hybrid stepper motor. This movement may be detected by the encoder and not by the MRPS or vice versa.
Figure 8.10: The oversampled, effectively, low-pass filtered MRPS position is compared with the encoder signal in the uppermost subplot. The deviation between the two signals is plotted in the middle subplot for both the unfiltered and the low-pass filtered case respectively. The oversampling have reduced some of the general noise, but it has introduced a constant offset of 0.12° in the difference. The constant offset stems from the slight signal delay that the oversampling entails. The deviation signal is now narrower and more distinct. The average maximum error is lowered from 0.63° to 0.58°. The lower subplot displays the amplitude spectrum of the deviation which is not changed notably for the displayed frequencies.
Figure 8.11: The unfiltered and the notch filtered MRPS position are compared with the encoder signal in the uppermost subplots. The deviation between the two signals when not and when applying the notch filter to the 2\textsuperscript{nd} harmonics of the speed are shown in the middle plots. It is clearly visible that the 2\textsuperscript{nd} harmonics is suppressed. This is also indicated by the frequency spectrum displayed in the bottom plots. The second harmonics have been dampened by the notch filter. The maximum average error has been lowered from 0.61° to 0.50°.
8.3 Filtered MRPS Results

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Figure 8.12: The unfiltered and the treated MRPS position are compared with the encoder signal in the uppermost subplots. In this test, the MRPS signal has been oversampled and the 2\textsuperscript{nd} harmonics of the speed has been notch filtered. The deviation is shown in the middle plots. The maximum average error has been lowered from 0.61° to 0.47°. This shows that applying both filters to the signal gives a smaller error than applying only any of the filters. \( \epsilon = 0.0087 \) and the oversampling was done over 10 samples.
Figure 8.13: The unfiltered and the treated MRPS position is compared with the encoder signal in the uppermost subplots. In this test, the MRPS signal is both low-pass and two times notch filtered. Both the first and second harmonics have been notch filtered. The errors are shown in the middle plots. The maximum average error has been lowered from 0.63° to 0.30°. The error has been biased with a constant 0.15° due to the delay imposed by the filtering. \( \epsilon_1 = 0.0040, \epsilon_2 = 0.0087 \) where the notch filter parameters for the 1\(^{st}\) and 2\(^{nd}\) harmonics notch filters respectively. The oversampling was done over 10 samples.
Figure 8.14: The effects of the oversampling and notch filtering is visible as the treated position signal is smoother and more similar to the encoder output, but slightly delayed compared to the raw MRPS and the encoder.
Figure 8.15: The results of the test when the rotor position has been altered by hand. The notch filtering results in no improvement in the position measurement at this low speed and ad hoc motion profile.
Figure 8.16 shows the results to the test where the motor is driven at 300 rpm. The MRPS output and the LSQR-compensated MRPS signal are compared to the output of the encoder in the uppermost subplots. The coefficients used for the LSQR-compensation are calculated for the motor driven at 100 rpm, refer to Section 6.2.2. The maximum average error after the acceleration phase ending at \( t = 1.5 \) s is reduced from 0.56° to 0.25°.

Figure 8.17 shows the results to the test where the motor is driven at 100 rpm, then accelerated to 200 rpm and then incrementally braked until stall. The MRPS output as well as the LSQR-compensated MRPS signal are compared to the output...
of the encoder. In this test, the maximum average error is a poor performance metric. It is however clearly visible that the error is smaller both during motion and at motor stall when the LSQR-compensation is applied to the MRPS signal. The frequency peaks subject to the compensation are also effectively removed. It can

\[ \text{Figure 8.17: Results to the test where the motor is driven at 100 rpm, then accelerated to 200 rpm and then braked until stall. The incremental braking of the signal starts at } t = 7 \text{ s. The motor stalls at } t = 25 \text{ s. The MRPS output as well as the LSQR-compensated MRPS signal are compared to that of the encoder.} \]

be concluded that the LSQR-modelling and compensation of the harmonic errors can be used to improve the performance of the MRPS-signal. The performance is better than what is accomplished with the notch-filter and the solution functions work well during constant speed, during acceleration and when external torque is applied. The drawback of the method is still the training session necessary to find the coefficients to be able to properly model the harmonic errors.
This section presents the results of the tests specified in Section 7.3.

An external load of 0.85 Nm is applied to the open loop position controlled hybrid stepper motor after 9.5 seconds in Figure 8.18. The rotor of the motor is thereby turned 1.7° mechanical, 85° electrical degrees out of desired position. This position error thus form the load angle. These test results illustrates one of the drawbacks with open loop control of the stepper motors, that its position performance suffers when subject to external loads. The following tests aim to illustrate potential performance improvements from using closed loop control with special emphasis on methods that do not add large and costly hardware.

Figure 8.18: The open loop controlled hybrid stepper motor is turned out of position when loaded at standstill. The position error becomes 1.7° when 0.85 Nm is applied in this case. The resulting load angle is the same as the position error.

The remainder of the specified experimental tests for the field-oriented position controller performance was never conducted due to problems pertaining to the current controller’s poor performance at low rotational speeds, further described in Appendix A.4. This limitation made it impossible to rotate slowly and smoothly which was necessary for the position control. The root cause of the problem was not found within the time frame of the thesis work.
This chapter concludes the work presented by this thesis and proposes future work related to the topic of position estimation and control for hybrid stepper motors.

During the course of this thesis, the wide research fields of hybrid stepper motor feedback control, sensorless positioning and Hall element based position sensors have been summarised and a complete feedback control structure has been presented. A current control system, which is necessary for stepper motor control, has been constructed in a real life setup. Special care and attention have been given to the hardware surrounding a two-phase inverter as well as setting up good data acquisition functionality.

The conclusions are drawn based on simulation results, presented in Chapter 6, and physical experiments which are described in more detail in Chapter 8. These chapters contain further discussion concerning the relevance and reliability of the results. These chapters should be reviewed for a more complete background to the conclusions.

9.1 Conclusions

Methods involving a sensorless position estimation algorithm and a low cost magnetic position sensor have been evaluated as to their ability to improve the rotor precision. This research has shown that it is indeed possible to implement and can be expected to result in improved positioning.

The physical system has been constructed complete with the necessary drivers needed to control the motor. It has been found that the measured electrical values in this driver could be enough for a sensorless estimation algorithm. Also, a magnetic rotary position sensor has been connected to the computer system. This
sensor’s output value needs appropriate compensation in order to produce a position signal good enough to be used together with a feedback position controller.

Sensorless load angle estimation, based on the SDFT method of estimating and interpreting the back-EMF, can deliver a load angle estimate which shows promising dynamics during constant speed. The estimated load angle can therefore reasonably be used for position control at close to constant speeds. It could also be used for stall detection, if the offset has been measured at that load and speed beforehand. The error sources have been either identified or suggested and their respective effects on the calculations have been described.

Low-pass filtering, adaptive notch filtering of the $1^{st}$ and $2^{nd}$ harmonics of the speed can be used in combination to improve the quality of the position signal from the MRPS. The maximum average error is lowered from $0.63^\circ$ to $0.30^\circ$. The filtering imposes a delay on the signal which results in a constant offset of $0.15^\circ$ from the true position when moving. The delay, and thus the offset, may be made smaller by implementing the suggested filters in faster lower-level electronics. Another approach is to first model the error as a function of the rotor position, then subtract the modelled error from the MRPS signal, effectively removing the error compared to an encoder. The maximum average error is lowered from $0.56^\circ$ to $0.25^\circ$. One drawback with the second method is that it requires a training session where reference equipment, or possibly the open loop position when unloaded, is used to derive the model of the errors.

9.2 Discussion

In this section, the conclusions made in Section 9.1 and the results acquired in Chapter 8 is compared to the theoretical background provided in Chapter 3 and to the goal of the thesis, given in Section 1.2. The method is discussed and the work is put in a social and ethical context.

9.2.1 Analysis of the Acquired Results

The properties of the stepper motor appear to be largely in agreement with the theoretical background presented in Chapter 3. An appropriate amount of control of the stepper motor has been achieved in this project both in respect to speed and reference following. It is therefore reasonable to assume that the simplifications and assumptions were largely valid.

However, the physical motor suffers from detrimental effects at velocities below 50 rpm. This is caused by the driver board’s inability to supply the necessary phase currents demanded by the reference signal. This is likely not an issue with the presented motor model but with the implementation of the driver logic.

The estimated load angle based on back-EMF performs poorly in a lot of circumstances. With the maximum amount of calculations at constant speed, the method does perform well in simulation. It can therefore be concluded that the theoretical method presented in Section 4.1.1 could perform as expected but that additional considerations have to be made in order to implement it in a physi-
9.2 Discussion

For example, a method of re-initialising the algorithm needs to be defined.

In the case of the MRPS, the implemented functions and results seem to be in agreement with the theory presented in Section 4.2. Therefore, the reliability and usability of the MRPS can be deemed to be high.

The simulation results verified that the suggested FOC may work well if the feedback signal is good enough. For the MRPS this means suppressing the errors. The FOC was never evaluated experimentally. However, there already exists numerous commercial stepper motor drivers for closed loop position control. The scientific contribution of evaluating yet another position controller is small, but the potential of using Hall element based sensors has proven to be very promising and is undoubtedly a frontier research topic!

9.2.2 Analysis of the Method

The method for this thesis is presented in Section 1.4. In short, the work focused on:

- Identifying the current research trends presented in articles and summarise these findings.
- Choosing methods based on viability and usefulness in the project.
- Constructing a simulation of the proposed solutions to evaluate the expected performance.
- Applying the solutions to the physical setup and gather results.

Naturally, this method contains certain flaws. Although the research provided a good overview of the project at hand, it would have likely been better to attempt simple open loop control strategies first to familiarise oneself with the system. Also, it would have provided the authors with the opportunity to identify the problems which the literary research should aim to solve. However, the complete physical setup and control logic had to be constructed in parallel during the course of the project, which essentially made this introduction to the project impossible.

To that end, it would have been reasonable to purchase a ready-made driver board which would offer all the relevant information signals. This would have saved time during the project as well as allowing more focus to the added estimation approaches.

Being a complex system, it is only natural that slightly different results could be had with a different setup, different software functions or different measuring equipment. The obtained results should however be indicative of what kind of performance to expect. The performance of each method should still be the same provided that they are configured properly.

The validity of the study is high. It is apparent that the results are either what is to be expected from the theory or what can be shown in simulation. It is therefore unlikely that the conclusions are drawn on wildly erroneous calculations or measurements, although the existence of such problems cannot be disproven.
The sources used in this project have largely been gathered from scholarly sources, such as conference papers. As such, most of them present new methods on their own with a background based on other articles. Furthermore, most articles are published in connection with the IEEE standard. These articles can therefore be deemed reliable and presenting research relevant to the topic.

However, the sources have been treated as more reliable if several sources are in agreement. Therefore, several sources are referenced whenever possible, so as to lend credibility to the presented argument. In some circumstances, however, only one work can be referenced as the exact method in question is presented in that paper. For example, this is the case with the article written by Derammelaere et al. [2014] and the back-EMF estimation algorithm. In these instances, careful consideration has been given to discerning the purpose, results, language and bibliography of the article.

9.2.3 The Thesis Work in a Social and Ethical Context

The social and ethical impact of this thesis work is arguably indirect. The ethical consequences depend heavily on the applications which the hybrid stepper motor is a part of. The motor could often be substituted for another one and therefore does not enable any certain technology. The hybrid stepper motor does however enable rotary motion at a lower cost where positioning is of less importance. The results of this thesis may enable the use of hybrid stepper motors in applications with higher demands on position. The work may therefore contribute with more and better applications to a lower price to the society. The suggested feedback controller does not only improve the positioning but also improves the energy efficiency of the applications where this motor is included. Typical open loop controllers always use maximum phase current, whereas the feedback control may only use precisely the power necessary to meet torque and motion demands. The solution therefore effectively helps conserving our modern lifestyle while limiting our environmental impact.

9.3 Future Work

Future work should include addressing the issues described in this report. It is also of interest to look into alternative methods and hardware to see if an increase in performance is achieved.

The sensorless estimation algorithm can be improved by increasing the frequency with which the SDFT algorithm executes. By bringing the algorithm closer to the switching frequency of the control board, a more accurate estimation can be achieved. The sensorless method can also be further improved by investigating methods to account for velocity changes and re-initialising the SDFT algorithm.

In addition, it is of interest to investigate other sensorless algorithms which are not dependent on a constant speed in the same way as the SDFT algorithm. Although the resulting precision might be different, it could be advantageous to continuously being able to control the rotation speed. Similarly, it is interesting
to explore other filters for the position sensor that do not impose as much of a delay as the current filter.

Further, it is an appealing opportunity to combine the sensorless observer and the filtered MRPS solutions. The performance of such a method should then be compared to each individual method for a range of operating points. It is of interest to research the suitability of the proposed Kalman filter as a method to achieving this combined sensor solution.

The FOC algorithm should be investigated further experimentally. The performance of the control algorithm when using the encoder, an estimated position or filtered MRPS can then be compared to each other.

After the performance of the implemented methods has been verified, moving the control and position sensor system onto an embedded system could reasonably present an increase in performance. This would allow all the necessary logic to operate at a high frequency which should result in good system dynamics. Furthermore, an embedded system can be designed to be more reliable and more energy as well as cost efficient than a manually assembled test rig.
Appendix
Experimental Setup

This appendix describes the experimental setups used during this thesis work in order for the work to be reproducible. Great care has however been taken to respect corporate confidential information and thus some information, such as specific brands and model numbers, has intentionally been omitted. Figure A.1 shows the available laboratory setup. The setup includes the stepper motor to be controlled, an operator PC for the operator to send speed and position references, a MRPS, a Speedgoat real-time target machine and two driver boards.

Figure A.1: The laboratory setup of the stepper motor, MRPS, reference interface, Speedgoat and driver board. The necessary control algorithms and filters is implemented using the Speedgoat and compatible IO-modules. (ABB)
A Experimental Setup

A.1 Hardware

Naturally, a hybrid stepper motor has been part of the hardware since the component itself is the concern of this thesis. The hybrid stepper motor used in the tests has a resistance of 1 Ω per phase and an inductance of 1.6 mH per phase. Its rated voltage is 3 Volt and its rated current is 3 Ampere.

A special designed driver board is used containing a full three-phase inverter bridge i.e. six metal oxide semiconductor field effect transistors (MOSFET:s) with turn on and turn of times of 33 ns and 35 ns respectively. The dead-time imposed by the safety logic is at least 10 times higher than these values to ensure shoot-through protection. In addition, the driver board contains two current sensors, one bus voltage sensor and two 12-bit ADC:s. Figure A.2 displays the used type of driver board. Four of the MOSFET:s, one current sensor and one ADC is used to realise one one-phase H-bridge. Two driver board pieces is used in the final setup necessary to drive the two-phase hybrid stepper motor.

![Figure A.2: Single drive inverter board containing six MOSFET:s, two current sensors, one bus voltage sensor and two 12-bit ADC:s. Four MOSFET:s are used in one one-phase H-bridge. (ABB)](image)

The rotary position sensors used for feedback control is the MRPS described in section 4.2.1. Cables, connectors, solder tin and project boards which are commonplace in electronic projects have been used to connect the different modules.

A.2 Test Bench

The lab test bench is composed of a stepper motor, controlled by the Speedgoat, which is connected to a braking system and an encoder. The braking system can effectively apply torque to the mechanical shaft at any normal speed. The encoder makes up one possibility of measuring the rotary position. Additionally, the lab setup offers the possibility of adding objects onto the shaft or fixing the shaft and thus keeping it stationary.
A.3 Speedgoat Real-Time Target Machine

The Speedgoat controller offers the ability to program an FPGA pulse-width modulated (PWM) generator necessary for the current controller from a Simulink model on a computer. There is also a CPU where the position and speed controller has been implemented as well as parts of the current controller. The clock frequency of the FPGA is up to 100 MHz and the default value is 33 MHz.

A.4 Current Controller

Fitzgerald et al. [2003, 10.3.2] provides a brief description of a PWM voltage source inverter realised with an H-bridge similar to the solution constructed and used throughout this thesis work. Each phase of the stepper motor is supplied with power from an H-bridge on a driver board. The MOSFET:s of the H-bridge is turned on and off according to a three-level PWM scheme realised by an FPGA implementation of a PWM-generator in the Speedgoat. The PWM-generator is fed with a reference signal from a PI current controller realised in the Speedgoat CPU. The PI current controller has to be provided with current reference and a current measurement signals as inputs. The current reference signal can either be user-defined or provided by a speed or position controller. Ultimately, the speed or position reference must be provided by an operator. Shoot-through protection through interlock and dead-time compensation for the open and closing times of the MOSFET:s was realised by a software safety component in series after the PWM-generator.

Figure A.3 shows the reference following accomplished by the current controller. This current controller consists of a PI-controller in Simulink with $K_p = 10$ and $K_i = 10000$ in series with a PWM-generator on an FPGA. The two phase currents realised by the PWM-controlled H-bridges follows their respective reference signals well. The plotted signals are acquired while the motor is revolved at 120 rpm, i.e. 100 Hz electrical frequency and the amplitude of the reference currents peak at 2 A. The performance is equally good for higher speeds up to 300 Hz. At higher speeds the currents does not have time to fully overcome the inductance and meet the references at the peaks. The operation of the motor is however still smooth but will provide less torque. The current controller does not perform well at speeds lower than 50 rpm. This is displayed by Figure A.4. The driver board is abruptly shut of during operation. There might be some violation regarding over current, under voltage or high temperature causing the safety component of the driver board to shut of the outputs to the MOSFET:s. This problem remains unsolved at the end of the thesis work.

The electronics project board with the two driver boards connected along with optocouplers and RC-filters are shown in Figure A.5. The MRPS receives PWM from the Speedgoat which in turn receives current- and voltage measurements from the sensors and which communicates through SPI. The driver boards are connected to multiple power supplies.

Figure A.6 shows the electronics project board, the motor test bench and the
Figure A.3: Current controller performance at 100 Hz, corresponding to 120 rpm stepper motor rotational speed. Bus voltage is 24 V and the peak current is 2 A.
Figure A.4: Current controller performance at 4 Hz, corresponding to 4.8 rpm stepper motor rotational speed. Bus voltage is 24 V and the peak current is 2 A. The driver board is abruptly shut off during operation. There might be some violation regarding over current, under voltage or high temperature causing the safety component of the driver board to shut off the outputs to the MOSFET:s. This problem remains unsolved at the end of the thesis work.
Figure A.5: The electronics project board with the two driver boards connected along with optocouplers and RC-filters. (ABB)
A.4 Current Controller

Speedgoat used throughout this thesis work.

Figure A.6: The electronics project board, the motor test bench and the Speedgoat used throughout this thesis work. (ABB)
A.5 Hybrid Stepper Motor Controller Top View

Figure A.7 displays the final top view of the user interface for the hybrid stepper motor controller, including the current controller, designed in Simulink.

*Figure A.7: Top view of the hybrid stepper motor controller, including the current controller, designed in Simulink.*


