A Statistically Motivated Likelihood for Track-Before-Detect

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***Abstract*— A theoretically sound likelihood function for passive sonar surveillance using a hydrophone array is presented. The likelihood is derived from first order principles along with the assumption that the source signal can be approximated as white Gaussian noise within the considered frequency band. The resulting likelihood is a nonlinear function of the delay-and-sum beamformer response and signal-to-noise ratio (SNR).**

Evaluation of the proposed likelihood function is done by using it in a Bernoulli filter based track-before-detect (TkBD) framework. As a reference, the same TkBD framework, but with another beamforming response based likelihood, is used. Results from Monte-Carlo simulations of two bearings-only tracking scenarios are presented. The results show that the TkBD framework with the proposed likelihood yields an approx. 10 seconds faster target detection for a target at an SNR of -27 dB, and a lower bearing tracking error. Compared to a classical detect-and-track target tracker, the TkBD framework with the proposed likelihood yields 4 dB to 5 dB detection gain.

I. INTRODUCTION

Underwater surveillance is traditionally carried out using towed hydroacoustic arrays and sonar [1]. In the passive sonar case, the surveillance is commonly based upon the conventional detect-and-track philosophy which is a three-step process [2, p. 16]: (i) beamforming is used to measure the signal energy in different directions, (ii) the measured energy is compared to a threshold to obtain detections, and (iii) target tracking methods are used to filter the detections and track the potential vessels.

A problem with the detect-and-track approach is that modern vessels often have a low acoustic signature with respect to the ambient noise, which makes the detection step difficult. Depending on the selected detection threshold, detections are either rare or indistinguishable from a large amount of clutter. Moreover, the detection step only uses data recorded in a single time slot, which means that information useful for detection is not accumulated over time. If instead the information is incoherently integrated over multiple time slots using a motion model of the target, then potential targets can be detected at a lower signal-to-noise ratio (SNR) [3]. This approach is commonly referred to as track-before-detect (TkBD) [4]. Different implementations and realizations of the TkBD concept can be found in [3], [5]–[7]. Since TkBD solves the detection and tracking problem jointly it is crucial to have not only an accurate model of the sound generated by the vessel, but also an accurate model of the vessels motion [8].

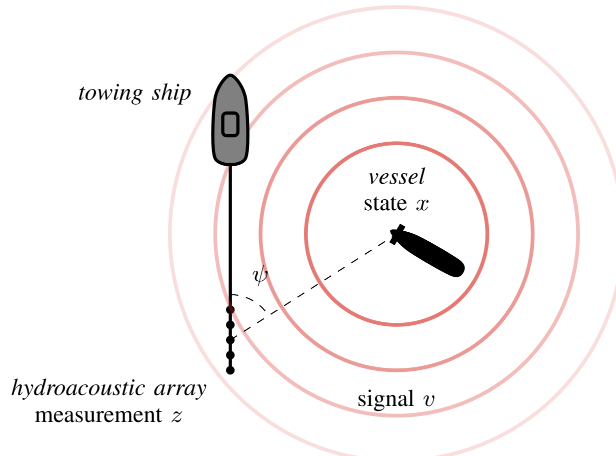

Fig. 1: The scenario considered in this paper. A hydroacoustic array is towed aft of a ship and measures the acoustic signals originating from an underwater vessel. The measurements are used in a TkBD framework to simultaneously track and detect the vessel.

Consider for instance the situation depicted in Fig. 1, showing a hydroacoustic array being towed behind a ship and an underwater vessel that emits an acoustic signal v. The task is to estimate the vessel state x, e.g., bearing ψ, using passive hydrophone measurements z of the signal v. To be able to estimate x it is necessary to model the relationship between z and x for the considered sound signal v. This relationship is typically specified via the likelihood function $\varphi(z|x, v)$, which describes the statistical distribution of z given the state x and source signal v. While established signal models exist for the active sonar and radar case [2, p. 457], [9, p. 323], the signal models and likelihood functions used in the passive sonar case often lack a solid theoretical foundation. This is likely owing to the complicated underwater environment, which makes it difficult to accurately model the properties of the measured sound signals.

Previous approaches have constructed the likelihood function by considering the beamforming response as the measurements. Broadly, two types of approaches have been considered before: (i) construction of a likelihood function through either assumptions of the statistics of the response [10], [11] or through some data fitting procedure [12], and (ii) construction of a likelihood function through a nonlinear function of the response, e.g., raising the response to some power to increase its contrast [13], [14]. Here, the choice

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of power has been ad-hoc. Hence, all currently proposed likelihoods more or less lack a theoretically sound connection to the statistical signal model, and their relation to the raw acoustic measurements is undermined.

In this paper a theoretically motivated likelihood function based on first order principles is proposed and evaluated. A theoretically motivated likelihood function may give several benefits, for instance: (i) better detection and tracking performance, (ii) better understanding of the behavior of the algorithm and the results, (iii) better ability to extend the model to include different acoustic phenomena, and (iv) insights into how one could create more robust algorithms.

Reproducible research: The code to reproduce the results in this paper is available at `https://gitlab.liu.se/coast/tkbd`.

II. TRACK-BEFORE-DETECT FILTERING

The problem considered in this paper is to find a suitable likelihood function $\varphi(z|x)$ that can be used for passive sonar surveillance of a single vessel. Next, to highlight how the likelihood function is used in a TkBD framework, the key equations for the Bernoulli filter based TkBD framework presented in [15] will be recapitulated. The Bernoulli filter based TkBD framework will later be used to evaluate the performance of the proposed likelihood function.

A. Algorithm Overview

The Bernoulli filter is a Bayesian filter that jointly estimates the probability that a target exists and the probability distribution of the target state x. Let $q_{k|k}$ ($q_{k+1|k}$) denote the estimated (predicted) target existence given the measurements $z_{1:k} = \{z_1, \dots, z_k\}$ at time slot k. Further, let $s_{k|k}(x)$ ($s_{k+1|k}(x)$) denote the estimated (predicted) distribution of the target state x given $z_{1:k}$. Like the Kalman filter and the particle filter, the Bernoulli filter recursively updates these quantities through a time update step and a measurement update step. In the Bernoulli filter, the prediction step is

$$q_{k+1|k} = p_b(1-q_{k|k}) + p_s q_{k|k}, \tag{1a}$$

$$s_{k+1|k}(x) = \frac{p_b(1-q_{k|k})b_{k+1|k}(x)}{q_{k+1|k}} + \frac{p_s q_{k|k}\int \pi_{k+1|k}(x|x')s_{k|k}(x')\,dx'}{q_{k+1|k}}. \tag{1b}$$

Here, $\pi_{k+1|k}(x|x')$ is the probability density function that describes the motion of the vessel and $b_{k+1|k}(x)$ is the birth density that describes where new vessels may appear. Further, p_b is the probability that a vessel appears, and $1-p_s$ is the probability that a vessel disappears. The measurement update, given the new measurements z_{k+1}, is

$$q_{k+1|k+1} = \frac{q_{k+1|k}I_{k+1}}{1-q_{k+1|k}+q_{k+1|k}I_{k+1}}, \tag{2a}$$

$$s_{k+1|k+1}(x) = \frac{\varphi_1(z_{k+1}|x)s_{k+1|k}(x)}{\int \varphi_1(z_{k+1}|x)s_{k+1|k}(x)\,dx}, \tag{2b}$$

where

$$I_{k+1} = \int \ell(z_{k+1}|x)s_{k+1|k}(x)\,dx, \tag{3a}$$

$$\ell(z|x) = \frac{\varphi_1(z|x)}{\varphi_0(z|x)}. \tag{3b}$$

Here, $\varphi_1(z|x)$ and $\varphi_0(z|x)$ are the likelihood functions for z given that there exists or does not exist a vessel with state x, respectively. As can be seen, the filtering recursions (1)–(2) depend on the likelihood functions $\varphi_1(z|x)$ and $\varphi_0(z|x)$. Hence, an accurate description of how the vessel state x relates to the hydrophone measurements z is essential.

B. Vessel Model

Consider a bearings only target tracking scenario. A simple model of a potential vessel is to describe it as a point with a bearing ψ and an angular rate ω. Moreover, to describe the relation between the measurements z and the power of the sound emitted by the vessel, the model also includes the SNR η in decibel (dB), at the array. Hence, the state at time slot k is

$$x_k = \begin{bmatrix}\psi_k & \omega_k & \eta_k\end{bmatrix}^T. \tag{4}$$

The state of the vessel is assumed to change according to

$$x_{k+1} = Fx_k + Gw_k, \tag{5}$$

where

$$F = \begin{bmatrix}1 & T & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\quad G = \begin{bmatrix}T^2/2 & 0\\ T & 0\\ 0 & T\end{bmatrix}, \tag{6}$$

and T is the time between two time slots. Further, w_k is white Gaussian noise with covariance

$$Q = \begin{bmatrix}0.001^2 & 0\\ 0 & 0.05^2\end{bmatrix}. \tag{7}$$

In other words, the bearing is assumed to be changing according to a constant velocity model, while the SNR is assumed to follow a random walk model. Hence, the probability density function relating the vessel states between two time instances is

$$\pi_{k+1|k}(x|x') = \mathcal{N}(x; Fx', GQG^T), \tag{8}$$

i.e., x follows a Gaussian distribution with mean Fx' and covariance GQG^T.

III. SIGNAL MODEL AND LIKELIHOOD FUNCTIONS

Consider an array with M hydrophones. Moreover, assume that there exists a vessel that emits a sound signal which is recorded in noise by each hydrophone. Depending on the bearing ψ_k to the vessel, the incoming sound signal $v_k(t)$ will be delayed with $\tau^{(m)}(x)$, due to the extra distance the sound needs to propagate to reach to hydrophone m compared to some reference microphone $m=1$. Thus, the sound signal recorded by the m:th hydrophone can be modeled as

$$y_{k,n}^{(m)} = v_k\left(nT_s + \tau^{(m)}(x_k)\right) + e_{k,n}^{(m)} \quad n = 0, 1, \dots, N-1, \tag{9}$$

where N is the number of samples collected during time slot k. Further, T_s is the sampling period and $e_{k,n}^{(m)}$ is the measurement noise. The measurement noise is assumed to be uncorrelated both in time and space and to follow a Gaussian distribution with variance σ_e^2. Let

$$y_k^{(m)} = \begin{bmatrix} y_{k,1}^{(m)} & y_{k,2}^{(m)} & \dots & y_{k,N}^{(m)} \end{bmatrix}^T, \tag{10}$$

denote the k:th batch of data recorded at hydrophone m. In the case that there is no vessel in the tracking frame at time slot k, the signal model is unchanged with the exception that $v_k(t) = 0$ for all t in (9).

Next, define the sound signal and the noise vector

$$v_k = \begin{bmatrix} v_k(0) & v_k(T_s) & \dots & v_k(T_s(N-1)) \end{bmatrix}^T, \tag{11}$$

and

$$e_k^{(m)} = \begin{bmatrix} e_{k,0}^{(m)} & e_{k,1}^{(m)} & \dots & e_{k,N-1}^{(m)} \end{bmatrix}^T, \tag{12}$$

respectively. Then, the measurement vector $y_k^{(m)}$ in (10) can, using trigonometric interpolation [16], be approximated as

$$y_k^{(m)} \approx W^* \Lambda(\tau^{(m)}) W v_k + e^{(m)}. \tag{13}$$

Here

$$[\Lambda(\tau)]_{n,l} = \begin{cases} \lambda_n(\tau) & \text{if } n = l \\ 0 & \text{otherwise} \end{cases}, \tag{14}$$

$$\lambda_n(\tau) = \begin{cases} \exp(-j2\pi n\tau/(NT_s)) & n < N/2, \\ \cos(\tau\pi/T_s) & n = N/2, \\ \exp(j2\pi(N-n)\tau/(NT_s)) & n > N/2, \end{cases} \tag{15}$$

is the fractional time delay operator and W the unitary discrete Fourier transform matrix.

A. Existing Approximate Likelihood Functions

Several publications, see e.g., [11], [12], [14], have suggested to use the beamforming response $B(\psi, z)$ to construct a likelihood function. Here, the measurements z is the concatenated vector of all hydrophone signals, i.e.,

$$z = \begin{bmatrix} y_k^{(1)T} & y_k^{(2)T} & \dots & y_k^{(M)T} \end{bmatrix}^T. \tag{16}$$

The conventional delay-and-sum beamformer evaluates $B(\psi, z)$ by reversing the delay at each hydrophone, and then examining the signal energy of the summed signal, i.e.,

$$\begin{aligned} B(\psi, z) &= \left\| \sum_{m=1}^{M} W^* \Lambda(-\tau^{(m)}(\psi)) W y^{(m)} \right\|_2^2 \\ &= \| U(\psi)(I_M \otimes W) z \|_2^2, \end{aligned} \tag{17}$$

where

$$U(\psi) = \begin{bmatrix} \Lambda(-\tau^{(1)})^* \, \Lambda(-\tau^{(2)})^* \dots \Lambda(-\tau^{(M)})^* \end{bmatrix}^*, \tag{18}$$

and I_M is the identity matrix of dimension M. Further, $\otimes$ denotes the Kronecker product. For a given z, $B(\psi, z)$ is usually precalculated for some set $\psi \in \Psi$ of linearly spaced bearing bins.

One of the proposed likelihood construction strategies compares the energy within the main lobe to the sum of energy outside the main lobe [14]. Assume that the $B(\psi, z)$ is evaluated for all $\psi \in \Psi$, and that the lobe width is L_ψ. Denote the bearing bins that are within the main lobe as $\Delta(\psi) = \{\psi' \in \Psi \text{ s.t. } |\psi - \psi'| < L_\psi\}$. Then, the likelihood functions suggested in [14] are defined as

$$\varphi_1(x, z) = \frac{1}{C|\Delta(\psi)|} \sum_{\alpha \in \Delta(\psi)} B(\alpha, z)^r, \tag{19a}$$

$$\varphi_0(x, z) = \frac{1}{C(|\Psi| - |\Delta(\psi)|)} \sum_{\alpha \in \Psi \setminus \Delta(\psi)} B(\alpha, z)^r, \tag{19b}$$

where $C = \sum_{\alpha \in \Psi} B(\alpha, z)^r$ is a normalization constant and r is an exponential factor to increase the contrast in the beamforming response. Note that the normalization with respect to the number of bearing bins in (19) were not included in [14]. Henceforth, this set of likelihood functions is referred to as the lobe width likelihood (LWL).

B. Proposed Likelihood Function

Next, a likelihood function will be derived directly from the signal model in (13). To get around the fact that the signal model depends on the unknown signal v_k, it will be assumed that v_k is Gaussian with covariance $\sigma_v^2 I_N$. For a given SNR η, it follows that $\sigma_v^2 = \sigma_e^2 10^{\eta/10}$.

Given the state x and sound signal v, the likelihood of the measurements z is given by

$$\varphi_1(z|x, v) = \mathcal{N}(z; (I_M \otimes W^*) U(\psi) W v, \sigma_e^2 I_{NM}). \tag{20}$$

Next, by marginalizing $\varphi_1(z|x, v)$ with respect to the unknown signal v, it holds that

$$\begin{aligned} \varphi_1(z|x) &= \int \varphi_1(z|x, v) \mathcal{N}(v; 0, \sigma_e^2 10^{\eta/10} I_N) \, dv \\ &= \mathcal{N}(z; 0, R), \end{aligned} \tag{21}$$

where

$$\begin{aligned} R = &\sigma_e^2 (I_M \otimes W^*)(I_{NM} + 10^{\eta/10} U^*(\psi) U(\psi)) \\ &(I_M \otimes W). \end{aligned} \tag{22}$$

Hence, the marginalized negative log likelihood is given by

$$-2 \log \varphi_1(z|x) = z^* R^{-1} z + \log|R| + NM \log(2\pi). \tag{23}$$

Using the matrix inversion lemma, it can be shown that

$$z^* R^{-1} z = \frac{\|z\|_2^2}{\sigma_e^2} - \frac{B(\psi, z)}{\sigma_e^2 (10^{-\eta/10} + M)}. \tag{24}$$

Further, it holds that

$$\log|R| = N \log(\sigma_e^2 M 10^{\eta/10} + \sigma_e^2) + N(M-1) \log(\sigma_e^2). \tag{25}$$

Thus, the final expression for the marginalized log likelihood becomes

$$\begin{aligned} -2 \log \varphi_1(z|x) = &\frac{\|z\|_2^2}{\sigma_e^2} - \frac{B(\psi, z)}{\sigma_e^2 (10^{-\eta/10} + M)} \\ &+ N \log(\sigma_e^2 M 10^{\eta/10} + \sigma_e^2) \\ &+ N(M-1) \log(\sigma_e^2) \\ &+ NM \log(2\pi), \end{aligned} \tag{26}$$

from which $\varphi(z|x)$ can be computed. The likelihood for the measurements z when there is no vessel can be found by letting $\eta \to -\infty$ in (26), which yields

$$-2\log\varphi_0(z|x) = \frac{\|z\|_2^2}{\sigma_e^2} + NM\log(2\pi\sigma_e^2). \qquad (27)$$

As seen in (26), the likelihood function takes the number of hydrophones M in the array, the number of samples N, and the SNR η into account. Henceforth, this set of likelihood functions is referred to as the white Gaussian signal model likelihoods (WGSML).

C. Connection to Existing Likelihood Functions

A noteworthy difference between the WGSML and the LWL is that the LWL functions do not require the SNR to be known or estimated, e.g., as a part of the vessel model. Implicitly, the LWL likelihood functions in (19) do estimate the SNR, since the functions compare the signal energy at the estimated target bearing to the surrounding environment.

Additionally, it can be seen that the beamformer response $B(\psi, z)$ naturally enters the WGSML, and does so without any prior assumption that the beamformer should have been used in the first place. Furthermore, the LWL [14] and other previously used likelihood functions [13] increases the contrast of the $B(\psi, z)$ by raising it some power. It is noteworthy that the WGSML also increases the contrast in $B(\psi, z)$, but does so by evaluating the exponential of the beamformer. Hence, the ad-hoc choice of power is no longer required, and the choice of nonlinear function is theoretically motivated.

IV. EVALUATION OF PROPOSED LIKELIHOOD FUNCTION

To evaluate the performance of the proposed likelihood function, i.e., the WGSML, the target tracking performance of the Bernoulli filter algorithm in (1)–(2) when using the proposed likelihood, is evaluated. The performance is compared to when using the LWL as the likelihood function in the Bernoulli filter algorithm, as well as a standard detect-and-track target tracking algorithm. The detect-and-track tracker uses a constant false alarm rate (CFAR) detector to construct detections from the beamformer response. The detections are then fed into a track score based track manager to evaluate the probability that there exists a vessel in the tracking frame, and a Kalman filter to estimate ψ_k and ω_k.

Two scenarios were used in the performance evaluation. The array used in the two scenarios is a linear array with 55 elements and a hydrophone spacing of 0.375 m. With this spacing and number of hydrophones, the lobe width is approximately $L_\psi = 2.12°$, which is a valid approximation for all bearings except for those close to the end fire directions [17, p. 237]. The beamformer $B(\psi, z)$ is evaluated over a linearly spaced set of 180 bearing bins between 0° and 180°. The time between two time slots is $T = 1$ s. Other simulation and the filter parameters used are found in Table I.

The ground truth vessel x movement is simulated using the motion model in (5). Here, the SNR η of the vessel

TABLE I: Parameters describing the simulation environment in Section IV and settings for the likelihood functions in Section III-A.

Param.	Value	Description
p_s	0.999	Survival probability
p_b	0.001	Birth probability
$\lvert\Psi\rvert$	180	Number of bearing bins
N	300	Number of samples from each hydrophone
T_s	0.250 ms	Sampling period
M	55	Number of hydrophones in the array
L_ψ	2.12 °	Main lobe width
T	1 s	Time between two consecutive time slots
r	5	The exponential factor in LWL

TABLE II: Thresholds γ for which the probability of false track confirmation, at any point in a simulation of 180 s without a vessel, is $\alpha = 10^{-3}$. The thresholds are based on data from 30 000 simulations without a vessel.

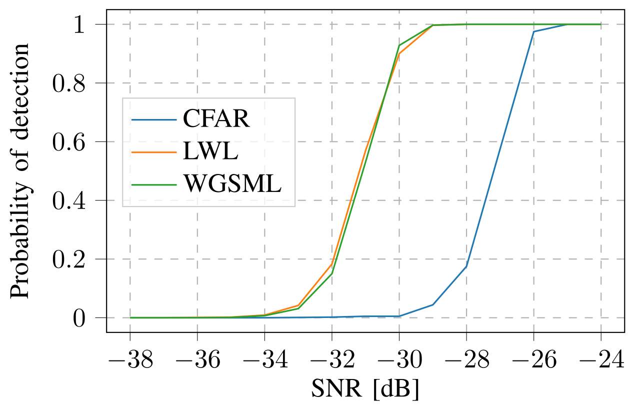
Method:	**WGSML**	**LWL**	**CFAR**
γ:	0.968	0.981	0.999

changes depending on the examined scenario. Hydroacoustic measurements $y^{(m)}$ are generated in accordance with the signal model (13). Further, the estimate $q_{k|k}$ is used for track confirmation, where the presence of a vessel is flagged when $q_{k|k}$ surpass some threshold γ. The threshold γ is determined by running the filters in a 180 s long simulation without any vessel, and finding for which γ the probability of a false track confirmation is α. If $q_{k|k}$ surpass γ at any time slot of the simulation, it is considered a false track confirmation. The thresholds, for $\alpha = 10^{-3}$, can be found in Table II. To ensure statistical significance in the thresholds, a total of 30 000 simulations are considered.

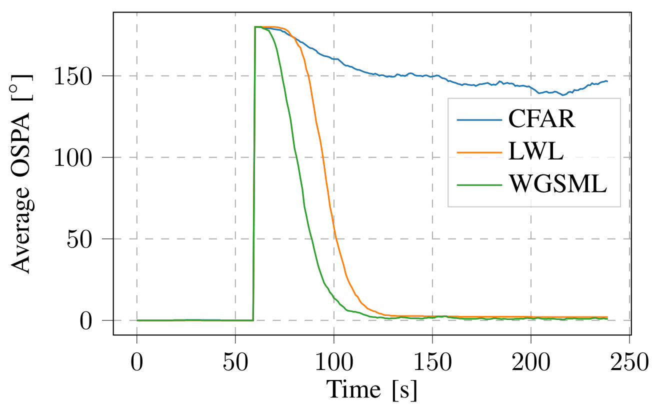
A. Scenario I

It is of interest to know the target detection capabilities of the different methods used for various SNRs. This is examined by simulating a target with a fixed SNR η, and observing the $q_{k|k}$ estimates, in a simulation that is run for 240 s. The target is introduced in the simulation after 60 s. A track confirmation is considered successful if the estimated target existence $q_{k|k}$ surpass the threshold γ and if the estimated target bearing $\psi_{k|k}$ differs at most one lobe width L_ψ from the true target bearing. The number of successful track confirmations are recorded over 1000 simulation for every examined SNR to estimate the probability of detection which is shown in Fig. 2a. From the figure, it is clear that there is no notable difference between the two considered TkBD methods. The methods can successfully detect a vessel with a 50 % probability when the SNR is -31 dB. For the CFAR method, the corresponding SNR is approximately -27 dB. Thus, it is clear that TkBD improves detection performance at lower SNRs, compared to classical detect-and-track methods.

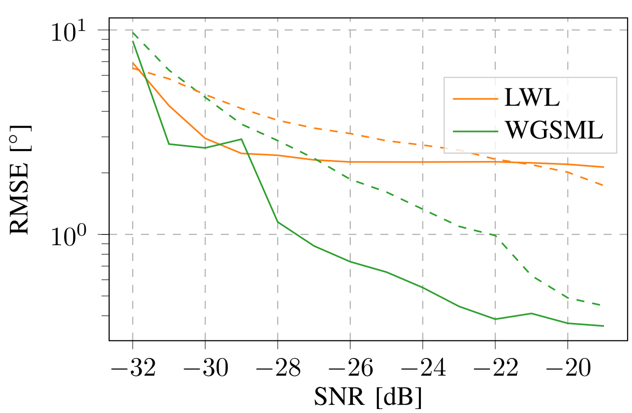
Additionally, it is also of interest to determine how fast each method is at detecting the vessel after it is introduced. To evaluate this, the optimal subpattern assignment (OSPA)



(a) Probability that the vessel is detected during the scenario.



(b) Average OSPA, with cutoff 180° and order 2, for a vessel with an SNR of -27 dB.



(c) RMSE in the bearing estimate. Dashed lines correspond to the standard deviation in the filter estimate.

Fig. 2: Results for Scenario I, evaluated using 1000 Monte-Carlo simulations. The scenario consists of a single vessel, for which its bearing changes according to (5) and has a constant SNR. The vessel is introduced at the 60 s mark.

[18] is used, which summarizes cardinality and state estimation errors in one metric. The average OSPA for a vessel at an SNR of -27 dB for every time slot is shown in Fig. 2b. From the figure, it is evident that the WGSML tracker outperforms the LWL in terms of time to detection. When the difference between the methods is the greatest, the LWL is lagging behind the WGSML by approximately 10 s. Moreover, given that the two methods have similar performance in terms of successful detections, as shown in Fig. 2a, indicates that the WGSML tracker is faster at signaling the presence of a vessel.

Lastly, the accuracy of the estimated bearing may also be a relevant factor, since it may differ between the two methods. This is due to the dependency on the lobe width in the LWL. To examine this, the root mean square error (RMSE) of the bearing estimate and the by the filter estimated standard deviation of the bearing estimate are evaluated at the moment of detection. These metrics are shown in Fig. 2c. From the figure, it can be seen that the two methods have comparable performance at lower SNRs. However, at higher SNRs, the RMSE of the LWL is approximately L_ψ, while the RMSE of the WGSML is considerably lower. This is likely due to the summation of the main lobe in the LWL, acting as smoothing of the response of beamformer $B(\psi, z)$ over ψ, which consequently limits the accuracy to the width of the main lobe. Meanwhile, the WGSML method does not require such a summation, which results in the lower RMSE at higher SNR.

B. Scenario II

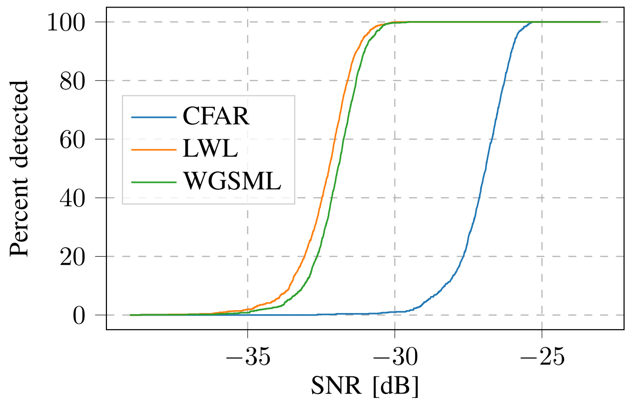
The second scenario considers a vessel moving towards the array. As the vessel approaches the array, the acoustic transmission loss decreases, which increases the SNR. Since the WGSML also estimates the SNR, unlike the LWL, a comparison in a changing SNR scenario is of interest.

The scenario considers a vessel that approaches the array at a speed of 2 m/s. Initially, the vessel is $r_1 = 6813$ m from the array. To relate the SNR to the current distance r_k, a simple ocean channel model [19, p. 130] is used to emulate the attenuation of the signal, as

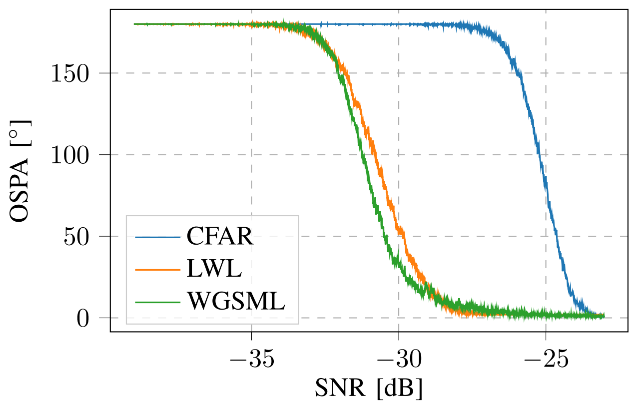
$$\eta_k = -10 \log_{10} \left(\frac{r_k}{r_0} \right)^{1.8}, \tag{28}$$

where $r_0 = 100$ m is the distance where the SNR is 0 dB. A total of 1000 Monte-Carlo simulations of this scenario are examined. The SNR of the first successful detection and the average OSPA for every time step can be seen in Fig. 3a and Fig. 3b, respectively. From the figures, it seems like the LWL tracker is able to make the first successful detection at slightly lower SNRs compared to the WGSML tracker. Again, the TkBD methods outperform the CFAR detect-and-track method by being able to detect targets with a 5 dB lower SNR.

While the first detection of the vessel can be made slightly earlier with the LWL method, it provides no significant benefit over the WGSML in terms of OSPA, as shown in Fig. 3b. This is an indication that the LWL is unable



(a) SNR at earliest detection.



(b) Average OSPA, with cutoff 180° and order 2, during the scenario.

Fig. 3: Results for Scenario II, evaluated using 1000 Monte-Carlo simulations. The scenario consists of a single vessel that is approaching the array. The percent detected indicates how many of the 1000 runs have had at least one successful detection at the considered SNR.

to make several consecutive successful detections after the initial detection at low an SNR.

V. Summary of Results and Conclusions

In this paper, we have introduced a theoretically sound likelihood function for passive sonar surveillance using a hydrophone array. The likelihood was derived from first principles via a measurement model for raw acoustic data. Our proposed likelihood model was compared to a likelihood model based on a common, yet ad-hoc, transformation of the beamformer response. These were used in a track-before-detect target tracker, and compared to a detect-and-track target tracker. Through simulation studies, it was shown that the proposed likelihood function yields a more accurate estimate of the bearing at higher SNRs, and enables earlier detection and tracking of the target. Compared to the classical detect-and-track method, the TkBD tracker with the proposed likelihood is able to detect the vessel at 4 dB to 5 dB lower SNR. This is lower than the expected gain of 6 dB [9, p. 318].

Future work will include adapting the proposed likelihood functions to work with real data, which likely will introduce the challenge of spatially correlated measurement noise.

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