Linköping Studies in Science and Technology
Licentiate Thesis No. 1963

Data-driven Modeling of Robotic Manipulators – Efficiency Aspects

Stefanie A. Zimmermann

LINKÖPING UNIVERSITY

Linköping studies in science and technology. Licentiate Thesis
No. 1963

Data-driven Modeling of Robotic Manipulators – Efficiency Aspects

Stefanie A. Zimmermann

LINKÖPING
UNIVERSITY

This is a Swedish Licentiate's Thesis.

Swedish postgraduate education leads to a Doctor's degree and/or a Licentiate's degree.
A Doctor's Degree comprises 240 ECTS credits (4 years of full-time studies).
A Licentiate's degree comprises 120 ECTS credits,
of which at least 60 ECTS credits constitute a Licentiate's thesis.

Linköping studies in science and technology. Licentiate Thesis
No. 1963

Data-driven Modeling of Robotic Manipulators – Efficiency Aspects

Stefanie A. Zimmermann

stefanie.zimmermann@liu.se
www.control.isy.liu.se
Department of Electrical Engineering
Linköping University
SE-581 83 Linköping
Sweden

ISBN 978-91-8075-197-1(Print) 978-91-8075-198-8 (PDF) ISSN 0280-7971

Printed by LiU-Tryck, Linköping, Sweden 2023

"Curiosity is the key to problem solving."
– Galileo Galilei

Abstract

Robotic manipulators are used for industrial automation and play an important role in manufacturing industry. Increasing performance requirements such as high operating speed and motion accuracy conflict with demands on heavy payloads and light-weight design with reduced structural stiffness. The motion control system is a key factor for dealing with these requirements, particularly for increasing the robot performance, improving safety and reducing power consumption. Most industrial robot control systems rely on current and angular position measurements from the motors, meaning that the actual controlled variable, that is the position of the robot's end-effector, needs to be calculated using a model. Therefore, the mathematical model used for motion control must accurately describe the system's dynamic behavior. Based on physics equations, the model contains unknown parameters that are usually identified from experimental data. This identification is a challenging problem, since the equations are nonlinear in the parameters, the system is highly resonant and experiments can only be done in closed loop with a controller.

Assuming a real robot is available for experiments, data-driven identification is common in order to obtain the most accurate description of the real system's behavior. The method applied in this thesis estimates the dynamic stiffness parameters by matching the model's frequency response function to the system's frequency response, which is obtained from measurements done with the closed-loop robot system. The main focus of this thesis are strategies for increasing the process efficiency such that the time it takes to do the experiments is reduced, while the quality of the model is maintained or improved. Two strategies related to experiment design are presented: First, the number of quasi-static robot configurations for data collection is decreased by choosing the most informative configurations from a set of candidates. Second, less data-demanding methods for estimating the system's frequency response are considered. The effectiveness of the presented approaches is demonstrated both in simulation and with real data.

If no robot is available for experiments, e.g. in the development phase, a model must be built based on specification data of components and other information available to the designer, such as CAD data. This thesis contains a modeling approach that derives a high-fidelity robot model of low order (lumped parameter model with few degrees of freedom) by combining results from test-rig measurements of isolated components with carefully reduced finite element models of the robot's structural parts.

Populärvetenskaplig sammanfattning

Robotmanipulatorer används för industriell automation och de spelar en viktig roll inom tillverkningsindustrin. Ökande prestandakrav som hög hastighet och noggrannhet hos robotens rörelse står i konflikt med trenden att bygga lättviktsrobotar som kan hantera tunga laster och som samtidigt är säkra för att jobba nära människor. Robotens styrsystem är en nyckelfaktor för att hantera dessa krav, särskilt för att öka robotens prestanda, förbättra säkerheten och minska strömförbrukningen. I de flesta tillämpningar är styrsystemets uppgift att säkerställa att robotens hand gör den önskade rörelsen, d.v.s. att handens position och hastighet motsvarar användarprogrammet. Positionen och hastigheten hos robotens hand är inte mätbara med sensorerna som är inbyggda i vanliga industriella robotar, vilket gör att de måste beräknas med hjälp av en matematisk modell. Denna modell måste beskriva det komplicerade sambandet mellan robotarmens rörelser och de motorer som orsakar rörelsen. Modellen är baserad på fysikaliska samband och innehåller okända parametrar som vanligtvis tas fram med hjälp av mätdata.

Det som mäts är position och moment hos robotens alla motorer och det som är eftersökt är parametrarna relaterat till robotens styvhet. Metoden som används i denna avhandling tar fram styvhetsparametrarna genom att matcha modellens frekvenssvarsfunktion med det uppmätta frekvenssvaret för den verkliga roboten. Huvudfokus är strategier för att öka processeffektiviteten så att tiden det tar att utföra mätningarna minskar, samtidigt som modellens kvalitet bibehålls eller förbättras. Två strategier presenteras: Den första minskar antalet robotkonfigurationer för mätdatainsamling genom att välja de mest informativa konfigurationerna från ett antal kandidater. Den andra strategin bygger på mindre datakrävande metoder för att skatta robotens frekvenssvar. Effektiviteten av de presenterade strategierna visas både i simulering och med verklig mätdata.

Att få fram en bra matematisk modell är svårt om ingen robot är tillgänglig för mätningar, t.ex. i utvecklingsfasen av en ny robot. I så fall måste en modell byggas baserat på specifikationsdata för komponenter, t.ex. leverantörens information om växellådans styvhet, eller materialegenskaper för robotens strukturdelar. Styvheten av robotens strukturdelar kan beskrivas mycket noggrant med den så kallade finita element-metoden som delar strukturen i små delar och kombinerar ekvationerna för varje del till ett stort ekvationssystem. Detta ekvationssystem måste reduceras för att vara användbart i styrsystemets robotmodell. Denna avhandling innehåller ett modelleringssätt där man får fram en noggrann robotmodell genom att kombinera en reducerad styvhetsbeskrivning av robotens strukturdelar med specifikationsdata för komponenter.

Acknowledgments

I want to thank my supervisors Assoc. Prof. Martin Enqvist and Prof. Svante Gunnarsson from the division of Automatic Control, as well as Stig Moberg, Ph.D., and Mikael Norrlöf, Ph.D., from ABB Robotics for continuous support and inspiring discussions in our regular meetings. A special thanks goes to Stig, who has also given me practical support with measurements and has been available for answering questions related to robot control and implementation. I would like to thank Svante for his positive attitude and motivational influence, and especially for his physical presence at the beginning of my time in Linköping, during the Corona pandemic. In his role as head of department, I would like to thank Martin for enabling an opportunity-rich research environment and friendly atmosphere in the group.

Gratefully acknowledged is the research funding by the Vinnova competence center LINK-SIC, that has also enabled a smooth collaboration with ABB. Being able to do measurements at ABB's facilities and using the collected data has given me unique opportunities and has been a great enrichment to my research.

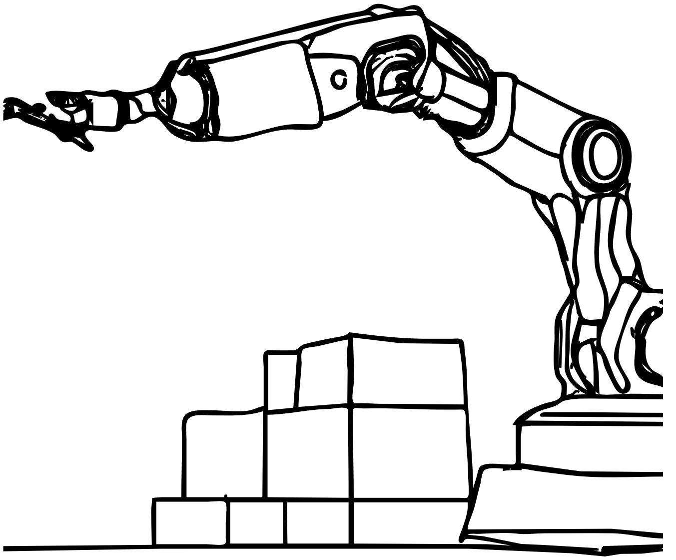
Stefanie Zimmermann
Linköping, May 2023

Contents

II Publications

1

Introduction



1.1 Research motivation

Most industrial robot motion control systems rely on torque input (or actually current input controlled by the drive system) and primary angular measurements from the motors. This means that the actual controlled variable, the position of the manipulator's end-effector, needs to be calculated using a model. In the simplest case the model is a static function, the direct kinematic model, but to achieve performance in more than a static scenario a dynamic model is needed. By considering data from the design phase, including component specifications and CAD models, it is possible to create detailed models that can be reduced to a model of a chosen complexity while keeping the essential characteristics. While certain information such as rigid body parameters can be easily obtained or considered as known, other parameters such as elasticity, damping and friction need to be identified from measurement data. An efficient modeling and parameter identification method makes it possible to faster reach the market with new robot types. It also makes it possible to more efficiently perform innovation related to the motion control since the models also can be used in simulations reducing the need for prototypes in development.

Research goals are related to the efficiency of data-driven modeling and the identification of robotic manipulators and can be formulated as,

1. Reduce the risk of mechanical wear or damage due to identification experiments,

2. Reduce the time it takes to do the experiments,

3. Maintain or improve the quality of the parametric robot models while satisfying 1 and 2.

The main driver for wear or damage related to today's identification experiments are high excitation amplitudes. Therefore, a long-term goal of this research is to investigate if such high amplitudes are needed in order to obtain informative data and thus achieve a high model quality. Risk for mechanical wear can also be addressed by reducing the need for experiments in general, either by replacing them with simulations, or by limiting the amount of data that is collected. This latter motivation is how Goal 1 is considered in this thesis, leaving explicit research on amplitude minimization for future work and focusing on research Goals 2 and 3. A method for improving the robot model based on an experiment at a customer site could be the ultimate goal for this research project, providing a significant value in many industrial applications. Therefore, the time it takes to do the experiments needs to be decreased compared to state-of-the-art methods. Thus, the main goal of this work is to consider the efficiency of the modeling and identification, still giving high quality models as a result.

1.2 Thesis outline

Part I of the thesis serves as an introduction to modeling of robotic manipulators and to frequency-domain system identification methods, aiming to show how the publications in Part II relate to previous research and to each other.

Chapter 2 summarizes basic concepts that are common for modeling of robotic manipulators. Differently detailed model structures for a complete robot model are introduced. Furthermore, aspects of modeling the three main components are addressed, i.e. the transmission and friction occurring in the joints, as well as the structural mechanics of the robot's links.

Chapter 3 discusses the data-driven estimation of a parametric robot model in frequency-domain. The gray-box idea used in this work is introduced and the handling of nonlinearities, in particular friction, is addressed. Since the presented identification method is based on input-output measurement data, techniques for FRF estimation are re-called in the second part of the chapter. Many choices need to be made during the identification, and a selection is named to conclude the chapter.

Chapter 4 is about experiment design for frequency-domain identification of robotic manipulators. The design of the excitation signal that is used for data collection is addressed. Furthermore, the problem of selecting the best robot configurations from a set of candidates is formulated. The chapter is concluded by a brief discussion about the exploitation of available experiment time aiming to collect the most informative data and to obtain the most accurate identification result.

Chapter 5 provides a concluding summary as well as ideas for future work.

Part II of the thesis consists of a collection of publications, presenting the main research results.

Paper A introduces a multibody modeling approach that includes flexible body descriptions. These descriptions are obtained from the corresponding Finite Element model of the structural parts by reducing the number of their degrees of freedom and introducing interface points. It is shown that such a flexible link manipulator model, which is purely based on development data and component specifications, is suitable for an accurate description of the dynamics of the robot. Validation experiments in time and frequency domain are presented showing accurate model performance compared to behavior of the real robot.

Paper B builds on the method for improved experiment design that was suggested in Wernholt and Löfberg [2007]. Aiming to find the best manipulator configurations for data collection experiments, the information content of a set of candidates is estimated and the optimal combination of robot configurations is obtained. Compared to computing the information matrix analytically based on noise assumptions and a known controller as suggested in Wernholt and Löfberg [2007], Paper B proposes to use high-fidelity simulations for estimating the uncertainty of the FRF estimates, which is then used to estimate the information matrix of each candidate configuration. The results of the presented simulation study validate that a realistic estimate of the uncertainty of the frequency response function (FRF) estimate is crucial for successful experiment design.

Paper C completes the work of Paper B by an experimental validation with measurements from a medium size industrial robot. It is shown that the experiment design is improved by the method in terms of data-efficiency and parameter accuracy. A significantly shorter time is needed for conducting data collection experiments, if only the best robot configurations resulting from the optimization algorithm are used. Furthermore, the average standard deviation of the parameter estimate is reduced compared to random experiment configurations.

Paper D deals with the estimation of nonparametric FRFs as an intermediate step in a parametric identification. Obtaining the FRF estimate from input-output data is conventionally done by averaging over different periods of the frequency domain signals. Since averaging techniques require long measurements and many experiments, local parametric modeling methods have been developed [Pintelon et al., 2010]. In Paper D, these local methods are adapted and applied for estimating the FRFs of a 6-axis robotic manipulator, which is a nonlinear MIMO system operating in closed loop. The resulting FRFs are analyzed in an experimental study and compared to the estimates obtained by averaging techniques. It is shown that the choice of parametrization in local modeling methods has impact on the FRF quality, and that a full MIMO parametrization gives more accurate FRF estimates compared to simpler parametrizations. It is also shown that considering the reference signal by following a Joint Input-Output approach improves the estimate significantly compared to methods assuming just the measured input and output data. The paper furthermore presents results of the second step, i.e. the parametric model identification based on the different FRF estimates. Based on these results it is concluded that a trade-off needs to be made between FRF quality and amount of estimation data, i.e. experiment time.

1.3 Contributions

In line with the publications introduced above, the main contributions of this thesis are:

- Manipulator modeling based on development and specification data:

  A low order flexible link description is proposed targeting the construction of a highly accurate multibody model of the complete manipulator that is purely based on development data. Experimental results validating the approach are presented and a high model accuracy is demonstrated.

- Improved experiment design by an optimized choice of manipulator configurations:

  Based on a realistic simulation study it is concluded that a data-based estimate of FRF uncertainty is crucial for successful experiment design. The study is completed by an experimental validation showing that the amount of data needed for system identification can be significantly decreased by improved experiment design, while the quality of the resulting robot model is improved.

- Evaluation of data efficiency in FRF estimation using a local modeling approach:

  Local modeling methods and traditional averaging techniques are reviewed and experimentally compared. The impact of the parametrization that is chosen for the local models is experimentally analyzed, showing that a full MIMO parametrization is beneficial compared to simpler parametrizations. The experimental study furthermore shows that a Joint-Input-Output approach allows more accurate FRF estimation when using local modeling methods than estimates assuming an open-loop system.

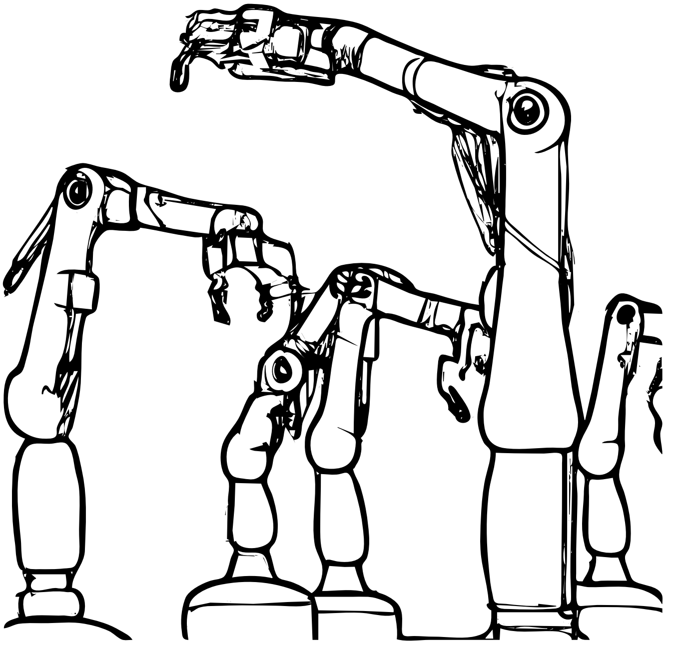
The results of thorough simulation studies, both for the nonparametric FRF estimation and for the parametric identification, add value compared to previous work and academic literature. The simulation setup includes a realistic nonlinear robot model where friction as well as other nonlinear effects in actuators and sensors are considered, such as torque- and resolver ripple (Gutt et al. [1996], Hanselman [1990]). The main appeal of the thesis are the results obtained with real measurement data. A medium size industrial manipulator was used for experimental validation of the above mentioned concepts.

Part I

Methods

2

Modeling



Dynamic models of robot manipulators describe the relation between the actuation and acting contact forces, and the resulting acceleration and motion trajectories [Siciliano and Khatib, 2016]. Building a dynamic model is useful for purposes of the mechanical design process, as well as for simulation, control analysis, and real-time control. The quality of the dynamic model is of high importance, especially if the control system purely relies on a model, which is often the case for state-of-the-art industrial robots. Modeling robotic manipulators is a challenging problem, since the behavior is nonlinear with respect to the rigid body dynamics, and since complex physical phenomena such as friction, torque and resolver ripple, transmission backlash, hysteresis, and nonlinear stiffness are present. Furthermore, the mechanical structure of the manipulator, as well as the transmission are elastic, especially for modern light-weight robot types.

A very thorough overview of the field of modeling of robot manipulators is given in Zimmermann [2018]. Basic literature like Siciliano and Khatib [2016] and Sciavicco et al. [2010] give comprehensive summaries of relevant topics related to the modeling of robot manipulators. This chapter will therefore be based on the above mentioned sources. Various model structures that are commonly used to describe the dynamic behavior of robotic manipulators are outlined. Furthermore, the model structure that is used in the scope of this thesis will be presented and assumptions will be stated.

2.1 Motivation for accurate robot models

The user of a robot usually generates a trajectory describing the desired movement of the tool. This trajectory needs to be transformed to corresponding motor input signals in order to realize the commanded movement. Most industrial robot control systems rely on current and angular position measurements from the motors, meaning that the actual controlled variable needs to be calculated using a dynamic model. This model must accurately describe the dynamic behavior of the system in order to enable high control performance.

Figure 2.1 shows the control framework of an industrial robot, using both feedback and feedforward loops. The feedforward controller commonly uses the known trajectory and the inverse dynamics model for generating a signal known as *computed torque* [Spong et al., 2020]. Due to model errors and disturbances, there will be a non-zero tracking error, which is handled by the feedback controller.

2.2 Modeling of robotic manipulators

A mechanical system can often be seen as the sum of a number of more or less stiff elements. Such a system is called a Multibody System and is characterized by two distinguishing features [Bauchau, 2011]: The overall motions of the system's bodies are finite and large, and all bodies are connected by mechanical joints that impose restrictions on their relative motion. Depending on the intended use of

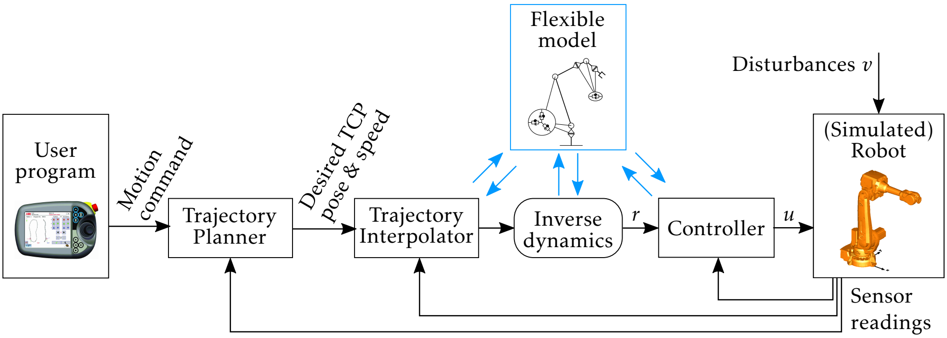


Figure 2.1: *Control framework of an industrial robot.*

the dynamic model, the bodies can be assumed as rigid, while being rigidly or elastically connected (see below), or elastically deformable (see Sec. 2.6).

Rigid body model The basic model of a robotic manipulator as the one shown in Figure 2.2a consists of a kinematic chain of rigid bodies (the robot's links) which are connected by rigid joints. A body is considered as rigid, if the distance between any two of the body's particles remains constant at all times and all configurations. The motion of a rigid body in space can therefore be fully described by six generalized coordinates (translation and rotation). A rigid body is usually described by its mass m, its center of mass defined by x_{cog}, y_{cog}, z_{cog}, and its inertia tensor with respect to the center of mass described by I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}.

Using the simple rigid body approach to model a manipulator, the joints are also modeled as entirely rigid, and the mass and inertia of the actuators and gearboxes are added to the corresponding link parameters. Thus, rigid body models possess in principle as many degrees of freedom (DOFs) as they are defined by the joints constraining the motion of the links.

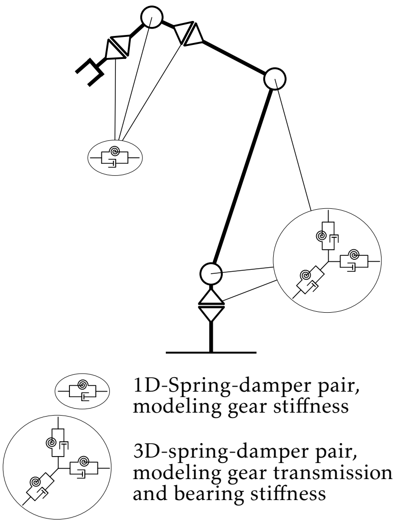
The dynamic equations of a rigid body robot model are

$$M_a \, \ddot{q}_a + c_a + g_a + \tau_{f\,a} = \tau_a \tag{2.1}$$

where q_a is the vector of joint angles, $M_a = M(q_a)$ is the inertia matrix, $c_a = c(q_a, \dot{q}_a)$ is the velocity dependent torque, containing centrifugal and Coriolis effects, $g_a = g(q_a)$ is the gravity torque, $\tau_{f\,a} = \tau_f(\dot{q}_a)$ the friction torque, and τ_a the vector of applied torques. In the following, a realization of q_a is called *configuration* of the robot.

Flexible joint model Taking into account the increasing operational speed of contemporary robots, and the demand for high payload capacity with lightweight arm structures, a more detailed and realistic model of the robot system is needed. Instead of assuming massless coupling between the links, the joints are modeled according to physics. The basic model of robotic joints consists of two inertias

(a)



(b)

Figure 2.2: *Photo of a robotic manipulator (a) and illustration of the extended flexible joint model (b).*

representing motor and gearbox, as well as the gear ratio. Additionally, a more or less sophisticated friction model can be added (see Sec. 2.5). Models known as lumped parameter models add elastic coupling, i.e. they add DOFs, to a rigid body model using so-called flexible joints [Bauchau, 2011]. Flexible joint models are based on the assumption that the main elasticity occurs in the drive chain between the motors and the links. Flexible joints are often modeled as linear torsional springs (see e.g. Tomei [1991], de Luca [2000]), or torsional spring-damper pairs [Sweet and Good, 1985]. The model parameters such as spring or damping constants are either assumed based on the data of the respective component or can be estimated from experiments in order to adapt them to reality.

Extended flexible joint model In the scope of this thesis, a 6-axes industrial robot is considered (see Figure 2.2a), and a model structure similar to the one presented in Öhr et al. [2006] is used for modeling it. This model structure extends the rigid body model (2.1) by adding different kinds of flexibility to the joint models. In order to take into account the transmission stiffness, spring-damper pairs acting in the direction of motion are introduced in the joint model. The spring is tensed by the difference between the angle on arm-side of the joint and the angle on motor-side. The gearbox torque is a function of this difference, which is nonlinear for commonly used robotic gears, see e.g. Seyfferth et al. [1995]. In addition to the six spring-damper pairs modeling the transmission flexibility of the six joints, two more spring-damper pairs acting orthogonal to the transmission are introduced in joints 1 to 3 for taking into account flexibility that occurs in the bearings and the link structure. Since the loading around the wrist joints 4 to 6 is comparably low, and for keeping the number of parameters as low as possible, these joints are modeled with one-dimensional spring-damper pairs. A schematic drawing of such a lumped parameter model structure is shown in Figure 2.2b.

The angular motion between the rigid bodies due to elastic effects that act perpendicular to the transmission direction is described by the additional variables q_e. The index a indicates an expression on arm side of the gearbox, the index m an expression on motor side. The model dynamics are then expressed by the following set of differential equations:

$$\begin{aligned} M_m \ddot{q}_m + \tau_{fm} + r_g \tau_g &= \tau \\ M_{ae} \begin{bmatrix} \ddot{q}_a \\ \ddot{q}_e \end{bmatrix} + c_{ae} + g_{ae} &= \begin{bmatrix} \tau_g \\ \tau_e \end{bmatrix} \\ k_g \cdot (r_g q_m - q_a) + d_g \cdot (r_g \dot{q}_m - \dot{q}_a) &= \tau_g \\ -k_e q_e - d_e \dot{q}_e &= \tau_e \end{aligned} \tag{2.2}$$

where $M_{ae} = M(q_a, q_e)$ is the inertia matrix, $c_{ae} = c(q_a, q_e, \dot{q}_a, \dot{q}_e)$ is the velocity dependent torque, $g_{ae} = g(q_a, q_e)$ is the gravity torque, q_m is the vector of motor angles, $M_m = \operatorname{diag}(J_{m1}, ..., J_{m6})$ is the matrix of motor inertias, k_g and d_g are the joint stiffness and damping constants in the direction of transmission, k_e and d_e are the stiffness and damping constants perpendicular to the direction of transmission, r_g is the matrix of inverse gear ratios, and $\tau_{fm} = \tau_f(\dot{q}_m)$ is the motor friction. Choosing the state vector $x = [q_m, q_a, q_e, \dot{q}_m, \dot{q}_a, \dot{q}_e]^T$, the applied torque $\tau = u$ as input, and the motor angular velocity $\dot{q}_m$ as output results in the following state space model:

$$\begin{aligned} \dot{x} = f(x, u, \theta) &= \begin{bmatrix} \dot{q}_m \\ \dot{q}_a \\ \dot{q}_e \\ M_m^{-1} \cdot (u - \tau_{fm} - r_g \tau_g) \\ M_{ae}^{-1} \left(\begin{bmatrix} \tau_g \\ \tau_e \end{bmatrix} - c_{ae} - g_{ae} \right) \end{bmatrix} \\ y = h(x, u, \theta) &= \dot{q}_m \end{aligned} \tag{2.3}$$

This model contains $\dim(q_m) + \dim(q_e)$ unknown stiffness parameters that need to be estimated from data. These parameters are collected in the vector θ.

2.3 Linearization and frequency response function

Assume that the system is excited by a signal that is a small perturbation around a stationary operating point $(x_0^{(i)}, u_0^{(i)})$, called *configuration*. Then, the nonlinear system (2.3) can be linearized at each configuration i such that

$$\begin{aligned} 0 &= f(x_0^{(i)}, u_0^{(i)}, \theta) \\ y_0^{(i)} &= h(x_0^{(i)}, u_0^{(i)}, \theta) \end{aligned} \tag{2.4}$$

Note that the linearized dynamics depend on the parameter vector θ. The linearized dynamics is obtained from a first-order Taylor series expansion. At each configuration i

$$\begin{aligned}\Delta\dot{x} &= A^{(i)}(\theta)\Delta x + B^{(i)}(\theta)\Delta u,\\ \Delta y &= C^{(i)}(\theta)\Delta x + D^{(i)}(\theta)\Delta u\end{aligned} \tag{2.5}$$

where $\Delta x = x - x_0^{(i)}$, $\Delta u = u - u_0^{(i)}$, $\Delta y = y - y_0^{(i)}$, and where the matrices $A^{(i)}(\theta)$, $B^{(i)}(\theta)$, $C^{(i)}(\theta)$, $D^{(i)}(\theta)$ are functions of the parameters θ defined by

$$\begin{aligned} A^{(i)}(\theta) &= \frac{\partial f}{\partial x}(x_0^{(i)}, u_0^{(i)}, \theta), \;\; B^{(i)}(\theta) = \frac{\partial f}{\partial u}(x_0^{(i)}, u_0^{(i)}, \theta),\\ C^{(i)}(\theta) &= \frac{\partial h}{\partial x}(x_0^{(i)}, u_0^{(i)}, \theta), \;\; D^{(i)}(\theta) = \frac{\partial h}{\partial u}(x_0^{(i)}, u_0^{(i)}, \theta).\end{aligned} \tag{2.6}$$

This state-space description leads to the parametric model FRF $G^{(i)}(\theta)$:

$$G^{(i)}(\theta) = C^{(i)}(\theta)\left(sI - A^{(i)}(\theta)\right)^{-1} B^{(i)}(\theta) + D^{(i)}(\theta) \tag{2.7}$$

For taking into account the variations around the operating point, a statistical linearization approach is used for linearizing the system around each robot configuration [Crandall, 2004].

2.4 Transmission modeling

The dynamic behavior of a robotic manipulator is highly influenced by the joints that connect its links (see e.g. Abele et al. [2011]). Joint flexibilities are common in current industrial robots, when harmonic drives or compact gears are used. Such components have gained wide acceptance because of their compact design, light weight and high reduction ratios, but they also introduce considerable flexibilities in the drivetrain [Seyfferth et al., 1995].

In order to achieve the goal of a high transmission stiffness, and finally a high positioning accuracy, two kinds of rigidity must be taken into account [Nab, 2013]: The torsional rigidity, which describes the ability of a part to resist deformation under torque loads, and the bending moment rigidity, which describes the ability of a part to resist deformation under moment loads. A low rigidity, i.e. high flexibility in the robot's joints, occurs due to elasticity in material, as well as play between components. Backlash, soft- and windup-zones are characteristic for the transmission stiffness curve of robotic applications where highly variable payloads and changing motion directions are common [Schempf, 1990]. The resulting characteristic of a robotic joint is therefore nonlinear with respect to the motor motion, making it difficult to model and to identify from experimental data.

In order to model nonlinear transmission stiffness, (2.2) can be adapted by a nonlinear function $\tau_{nls} = \tau_{nls}(r_g q_m - q_a)$ replacing the linear relation $k_g\,(r_g q_m - q_a)$.

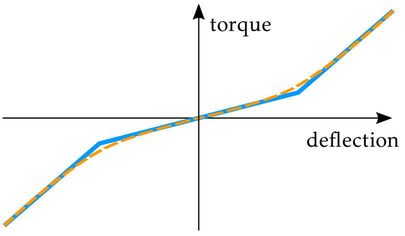


Figure 2.3: *Nonlinear transmission stiffness.*

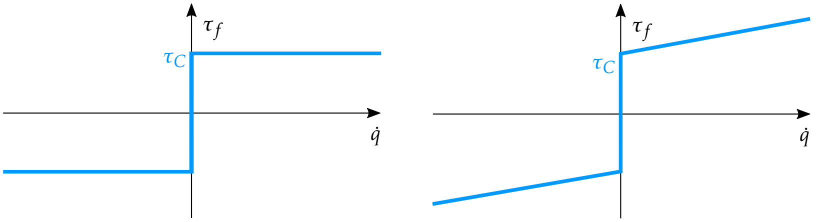
An example of such a nonlinear function is a piece-wise constant function as is shown in Figure 2.3.

An early but still common proposal of a more detailed transmission model is for example given in Good et al. [1985]: A stiffening spring characteristic is used for the gear model, as well as viscous damping with additional Coulomb friction and current limiters for the motor.

2.5 Friction modeling

A common design for the drive mechanism of a robotic joint is the combination of electrical actuators with high transmission gear ratios. These gears usually have a considerable amount of friction, which can have a significant impact on the structure's dynamic performance. Friction effects that occur in robotic joints are particularly critical, since they can induce "large positioning errors, stick-slip motions, and limit cycles" [Bona and Indri, 2005]. Due to the physical complexity of friction phenomena, investigations within the field of dynamic modeling have not focused on the friction models available from the experimental and theoretical work of tribology [Armstrong-Hélouvry et al., 1994]. Most friction models used in engineering are empirical, and based on observations and interpretations. Many friction models of different complexity have been proposed in literature: A general overview is, for example, provided in Olsson et al. [1998], or Bona and Indri [2005], and a summary of common friction models that are used for automatic control can be found in Egeland and Gravdahl [2002].

The classical friction models, such as the Coulomb-model (with or without viscous friction), are static models. These models describe a static relationship between the friction forces, acting during a relative motion at constant or slightly changing velocities. They explain neither hysteretic behavior when studying friction for nonstationary velocities nor variations in the break-away force with the experimental condition (e.g. temperature) nor the presliding behavior [Canudas de Wit et al., 1995]. In classical models, the system stiction (presliding) is captured by the static break-away force, which has to be overcome in order to initiate the motion. In order to capture the complex frictional behavior involving stick-slip motion and presliding hysteresis, more sophisticated dynamic friction models are required.



Coulomb friction model. Coulomb-viscous friction model.

Figure 2.4: *Friction models.*

Static friction models The Coulomb friction model is the most basic proposal, modeling the friction force (or torque) proportional to the load, with direction opposite to the motion. The friction torque τ_f assumes a constant value τ_C, which is independent of the magnitude of the velocity $\dot{q}$, and the contact area. It is described by

$$\tau_f = \tau_C \cdot \mathrm{sign}(\dot{q}), \;\; \dot{q} \neq 0 \tag{2.8}$$

and sketched in the left half of Figure 2.4. It should be emphasized that the Coulomb friction model does not specify the friction force for zero velocity. For simplifying simulation, (2.8) is often approximated by the continuous function:

$$\tau_f = \tau_C \cdot \tanh(\beta \dot{q}), \;\; \beta >> 1. \tag{2.9}$$

A viscous friction model assumes that the friction torque is not only dependent on the direction of the velocity, but also on its magnitude. This allows taking into account hydrodynamic effects occurring in fluid lubricated contacts between solids. A linear model of viscous friction is expressed by $\tau_f = \tau_v \dot{q}$, where the constant of proportionality τ_v depends on lubricant viscosity, loading and contact geometry. Viscous friction is often combined with Coulomb friction as

$$\tau_f = \tau_C \cdot \mathrm{sign}(\dot{q}) + \tau_v \dot{q}, \;\; \dot{q} \neq 0 \tag{2.10}$$

The right half of Figure 2.4 shows the Coulomb-viscous friction model, which is considered as the most commonly used friction model in engineering [Egeland and Gravdahl, 2002], and which is used in the scope of this thesis.

Dynamic friction models Dynamic friction models describe complex frictional behavior in terms of differential equations. They involve stick-slip motion and presliding hysteresis, while providing a smooth transition through zero velocity without discontinuities typical for static friction models. Examples are the model proposed by Dahl [Dahl, 1968], the LuGre (Lund-Grenoble) friction model [Canudas de Wit et al., 1995], and the two-state dynamic friction model with elasto-plasticity (2SEP-Model) proposed by Ruderman and Bertram [2011].

2.6 Flexible link models

A lumped parameter model that concentrates the flexibilities of the robot in its joints might be sufficient for many applications, and is because of its simplicity often the favored choice. However, if high accuracy is required models of much higher level of detail are necessary. The sensitivity analysis presented in Zimmermann [2018] shows that the total flexibility of a modern robot manipulator is distributed among its joints and links. A realistic model for a medium-size or large industrial robot would therefore include link flexibility, i.e. some description for the flexibility distributed over the different bodies. Robotic systems with flexible links are "continuous dynamical systems characterized by an infinite number of degrees of freedom and are governed by nonlinear, coupled, ordinary, and partial differential equations" [Theodore and Ghosal, 1995]. Due to the fact, that the exact solution of such a mathematical model is not feasibly practical, methods that capture the most essential flexibilities, using a finite number of parameters, are required. A lot of research effort has been spent in the field of modeling robot manipulators with flexible links, see Dwivedy and Eberhard [2006] for a review. The three different approaches to derive finite dimensional flexible link models that can be found in literature are described below.

Finite element models describe the link deflection with a large number of DOFs. The solution domain is divided into a finite number of sub-domains called Finite Elements. Within each element, a small number of continuous functions are formulated, while continuity of the solution across elements is ensured. Due to their high accuracy, finite element models are commonly used in the mechanical design of robotic systems. Due to their complexity, they are rarely used to develop models suitable for dynamic simulation and control. Examples applying the finite element method for robot modeling are given in Du et al. [1996], Jonker [1990], Hardeman et al. [2006]. Recent attempts for making the approach more efficient are, e.g., My et al. [2019], Li et al. [2021].

Assumed modes models are derived from the partial differential equation formulation by modal truncation. This description requires to select suitable link boundary conditions, which makes this conceptually simple approach challenging. The assumed modes approach is used in the context of dynamic substructuring and component mode synthesis. As an example, Nicosia et al. [1996] derive an approximate finite dimensional model of a two-link flexible robot arm based on the Ritz expansion method.

Lumped parameter models divide each flexible link into a finite number sub-links, which are assumed to be rigid. The sub-links are coupled by pseudojoints and flexibility is modeled as springs that restrict the motion of each pseudojoint. The lumped parameter approach is a straight-forward way to develop a model that is simple enough to be used for real-time control and for dynamic simulation of a manipulator. However, the method is rarely used because of the difficulty in determining the spring constants of the pseudojoints. Early references presenting

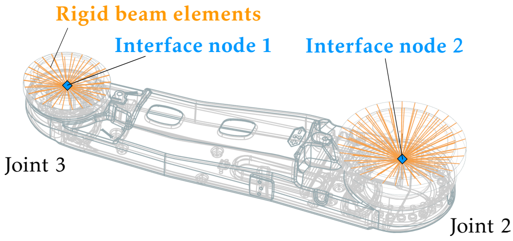


Figure 2.5: *Finite element model of the robot's lower arm structure with definition of two interface nodes.*

the concept of lumped parameter flexible link models are, e.g., Khalil and Gautier [2000], Yoshikawa and Hosoda [1991].

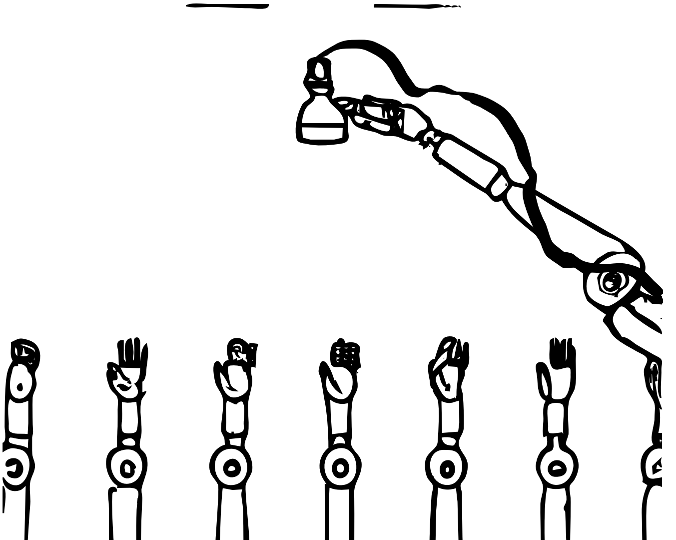
The flexible joint approach presented in Sec. 2.2 can be combined with the idea of modeling flexible links as proposed in Paper A. The introduced *Flex-Model* uses component specification data to estimate models for the gearbox and bearings. A friction model is identified with the help of data from test-rig measurements of isolated components. The structural flexibilites of the links are described by order-reduced finite element models. Taking the full finite element formulation of each link as a starting point, the number of DOFs is reduced in order to efficiently include the flexible link description in the multibody model of the complete robot. Frequently employed techniques to reduce a system's number of DOFs are the Guyan reduction method [Guyan, 1965, Irons, 1965], and the Craig-Bampton method [Craig and Bampton, 1968], which are used in Paper A. When reducing the finite element description of the robot links, interface nodes are defined, which make it possible to connect the flexible components to other parts of the multibody model, e.g. the gearboxes. Figure 2.5 illustrates an example of how the interface nodes can be defined.

The approach proposed in Paper A is appealing since the manipulator model is based on information that is available to the designer during development, without having a prototype available. Only specification data of the joint components is needed, together with straight-forward test-rig measurements of isolated components. The geometrical data of the manipulator structure is known from the CAD model which is available to the designer, and the flexible link description can be derived from the finite element model, which is usually built during development.

The main drawback of the model structures presented in Sec. 2.2 is the fact that the model parameters such as stiffness and damping constants need to be estimated from experimental data. Furthermore, the contribution from each individual element of the robot is hard to deduce, since elasticity from multiple elements are often lumped together. A parameter estimation for fitting the model's flexible behavior is not needed for the *Flex-Model* proposed in Paper A, making the approach especially attractive during development, or as initialization for methods aiming to further improve the accuracy of a lumped parameter model. Because the structural dynamics of the links are considered, a high-fidelity robot model can be obtained that accurately describes the real robot's dynamic behavior.

3

System identification



"Identification is a powerful technique for building accurate models of complex systems from noisy data" [Pintelon and Schoukens, 2012]. Assuming that the approximate physics of the system are known and the goal is to find a parametric gray-box model, system identification generally consists of three tasks:

1. Model construction based on physical laws (see Chapter 2)
2. Experiment design (see Chapter 4)
3. Estimation of model parameters from data (addressed in this chapter)

Considering a frequency-domain identification, Step 3 might be divided into

3a. Estimation of the system's FRF from measurement data (see Sec. 3.2)

3b. Estimation of model parameters based on the estimated FRF (see Sec. 3.1)

The system identification workflow is visualized in Figure 3.1.

Many methods have been developed for system identification, both for specific applications and for general purpose. The identification of a robotic manipulator is associated with many challenging problems for traditional methods, since the system is multivariable, nonlinear, unstable, and oscillatory. However, a lot about the system is known in advance. The robot dynamics, for example, can be modeled quite accurately using known mechanical and electrical relations. These dynamic equations have characteristic features that can be exploited during identification.

This chapter gives background information about Step 3 of the procedure above and introduces the concepts and methods that are used in the scope of this work. Since identification in robotics is a much studied problem, the purpose of this chapter is to re-call a selection of research results related to the identification of kinematics and rigid body dynamics, flexibilities, and nonlinearities. The second part of this chapter summarizes available methods for estimating nonparametric FRFs from noisy measurement data.

3.1 Identification of a parametric robot model

The goal is to identify the parameters of a dynamic robot model from experimental data such that the model behaves as similarly as possible to the real system. Model structures as outlined in Chapter 2 are considered, i.e. nonlinear parametric models in continuous time formulation.

3.1.1 Related work

Experimental techniques for identifying robots involve estimating their dynamic parameters based on measurements of motion and torque. If the position (including velocity and acceleration) of all DOFs can be measured, the dynamic model can be represented as a set of equations linear w.r.t. the unknown dynamic parameters. In this case, a straight-forward linear least squares identification procedure can be used to identify the model parameters. See e.g. Kozlowski [1998]

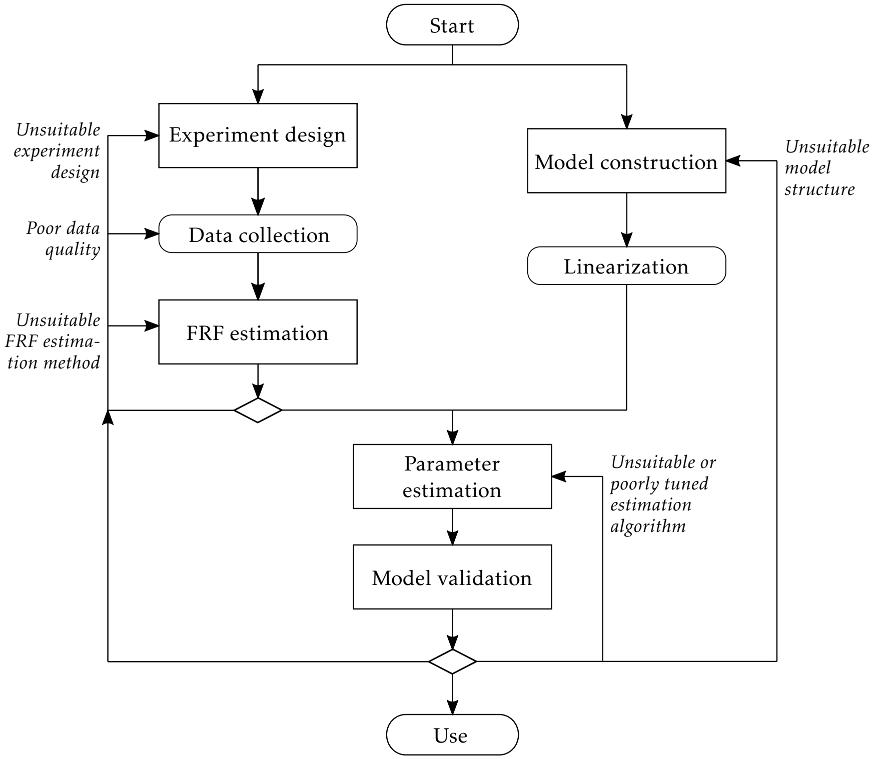


Figure 3.1: *Procedure for frequency-domain identification of a gray-box model.*

and Khalil and Dombre [2002] for a general overview of the parameter identification methods for rigid robot models. Assuming that the robot model can be formulated as a linear set of equations w.r.t. the dynamic parameters, the trajectories performed for data collection can be optimized, see e.g. Armstrong [1987], Swevers et al. [1996].

However, for industrial robots, only motor position and torque data are typically available. The dynamics of additional DOFs due to flexibilities are not directly measurable, making the linear least squares technique inappropriate for flexible robot models. Of course, additional sensors for measuring the elastic deformations can be applied, such as acceleration sensors (see e.g. Pham et al. [2002]), link position and/or velocity sensors (see e.g. Tsaprounis and Aspragathos [2000], Huang [2003]), or additional joint torque sensors (see e.g. Albu-Schaffer and Hirzinger [2001]). However, from a practical point of view, the use of additional sensors for identification is not preferred due to high cost or feasibility issues. Without measurements from additional sensors, the elastic deformations can be obtained from the equations of motion by solving a non-linear optimization problem. This is a challenging task and it cannot be excluded that a local minimum is obtained. Good guesses for the initial parameter values are therefore essential for such methods in order to succeed.

System identification theory offers various algorithms for estimating state space or transfer function models from a limited number of input and output measurements, such as prediction-error methods or subspace techniques. In the scope of this thesis, an identification approach in frequency domain is considered. Frequency domain methods are well established in mechanical analysis, for example related to modal analysis. The purpose here is to estimate the parameters of a flexible dynamic manipulator model with emphasis on stiffness identification. Since the stiffness characteristics can be conveniently described by the system's frequency response, the domain is advantageous. Another motivation for the choice to work in frequency domain is that a lot of data can be handled in compressed form. Several experiments can be combined to a consistent system description in frequency domain. Further discussion about the choice between frequency domain and time domain identification can be found in e.g. [Pintelon and Schoukens, 2012, p. 522 ff.].

3.1.2 Gray-box approach

Assuming that the goal is to identify a parametric robot model such as the examples in Section 2.2, the total set of parameters can be classified in three categories:

- Static and geometric parameters θ_{stat}:
  This subset contains the masses of all robot components, the lengths and the inertia data. These parameters can be gained with high accuracy from the CAD model of the manipulator, which is available to the robot manufacturer.

- Quasi-static, separately measurable parameters θ_{char}:
  These parameters are also called characteristic curve parameters and can often be estimated from specific measurements of isolated components in test rig experiments. Separating components from the robot assembly and measuring them individually can be laborious or expensive. In this case where test rig measurements are unfavorable, parameters θ_{char} can still be identified separately from each other by experiments without highly dynamic excitation. The most common representative of this parameter subset are friction parameters.

- Dynamic parameters θ_{dyn}:
  These parameters determine the dynamic behavior of the robot and can practically not be estimated separately. Parameters of this subset are usually highly coupled and dependent on each other. As an example, the transmission parameters of Axis 3 of a 6-axis manipulator (the 'elbow') are very difficult to separate from the stiffness behavior of the robot's lower arm and the Axis 2 gear stiffness, since dynamic excitation of Axis 3 always also excites the dynamics of the other components. The parameters θ_{dyn} can be identified simultaneously from data, if the system is dynamically excited and some form of optimization algorithm is applied.

The gray-box identification approach that is used in this work assumes that only the parameter subset θ_{dyn} must be identified by a sophisticated method. This approach exploits the fact that many parameters can be estimated independently from each other, or can be provided by the robot manufacturer. In the scope of this work, the set of θ_{stat} is assumed to be known, while the friction parameters are identified separately in advance to the actual identification, see Sec. 3.1.4.

3.1.3 Handling of nonlinearities

For industrial robots, the following nonlinearities are common and must be dealt with during identification [Moberg et al., 2014, Saupe and Knoblach, 2015]:

- Configuration dependent mass distribution (nonlinearities associated with the rigid body dynamics)
- Gear load constraints
- Friction of motor bearings and gear transmission
- Nonlinearities caused by motor torque ripple and sensor position ripple
- Nonlinear transmission stiffness

For dealing with nonlinearities associated with the rigid body dynamics, we design the experiments such that large deviations from the initial configuration are avoided. We aim for quasi-static experiments, since then the Coriolis forces can be neglected due to only low velocities, and the gravity torques can be assumed to act like constant disturbances. Dynamic gear load constraints must be taken into account during experiment design. Overloading of the gearboxes risking mechanical failure is prevented by the controller which applies a saturation if the torque limit is exceeded. In order to avoid the occurrence of these nonlinearities introduced by the controller, careful design of the commanded motor torque is required, while providing a high excitation level.

Friction affects the resonances in the FRF estimate by acting as a damping force. For reducing the effects of friction, the excitation needs to be designed such that the low velocity regime is avoided, i.e. the resulting mean of the exciting motor torque should be well above the Coulomb friction level.

Nonlinearities that are caused by the drive train torque ripple and the sensor position ripple are treated as input nonlinearities (see Figure 3.2) and are compensated offline in a separate step from the actual identification. Even though the position resolvers have high resolution, severe noise can occur in the velocity signals which usually are computed by numerical differentiation. Careful selection of numerical methods can solve this issue.

The stiffness of commonly used gears is deflection dependent, and the deflection in turn depends on the chosen excitation. The FRF also depends on the excitation, meaning that resonances can be shifted by choosing either a weak excitation (leading to low deflections) or a strong excitation (leading to high deflections).

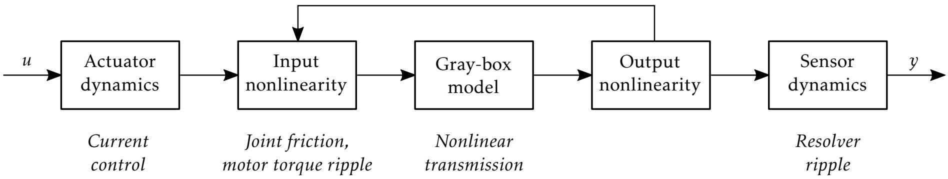


Figure 3.2: *Nonlinearities of a robot system (adapted from Moberg et al. [2014]).*

The summary above makes clear that dealing with nonlinearities is strongly related to experiment design. Carefully chosen robot configurations and excitation signals are crucial for collecting informative data that allows to identify the system's nonlinearities. Chapter 4 is therefore dedicated to experiment design for frequency domain identification of robotic manipulators.

3.1.4 Identification of friction

Using the gray-box approach introduced in Sec. 3.1.2, the rigid body parameters are assumed to be known, while the a priori identification of the parameters of the friction model is still needed. The following paragraphs give an idea about the available methods for separate friction identification. It should be mentioned that methods to simultaneously identify friction and the rigid body dynamics have been developed, see e.g. Grotjahn et al. [2001].

The basic procedure is to run each joint with constant velocity in the steady state regime. Then, the corresponding motor input represents the torque required to keep the system in motion against the friction torque. There is a lot of literature on this type of identification, see e.g. Swevers et al. [1997], Kozlowski [1998], Grotjahn et al. [2001], Gautier and Poignet [2000].

In the scope of this thesis, the Coulomb-viscous friction model (2.10) is used, resulting in two additional parameters per joint that need to be estimated. Even though this simple friction model does not correctly describe dynamic friction (see Armstrong-Hélouvry et al. [1994]), it compensates for the major frictional effects, assuming a high amplitude in the motor torque excitation signal. For identification of the friction parameters, applied torque and joint movements are recorded, and the parameters are estimated using linear regression. This identification of the friction torque is usually done in advance to the stiffness identification, such that τ in (2.2) can be replaced by $\tau - \tau_f$.

Different methods have been proposed for identifying more complex friction models, as for example the LuGre friction model. Hensen et al. [2002] propose a frequency domain identification technique in the presliding regime that linearizes the dynamic LuGre model in the stiction regime in order to obtain a linear second-order description. The proposed technique allows to estimate both the stiffness and damping of the presliding behavior from a measured FRF. Another identification technique based on FRFs is described in Ruderman and Bertram [2011] by means of a two-state dynamic friction model with elasto-plasticity.

3.1.5 Parameter estimation in frequency-domain

The overall goal is to identify the stiffness parameters of a nonlinear robot model (2.3), which are a subset of θ_{dyn} and are collected in the parameter vector called θ. The frequency-domain procedure that was proposed in Wernholt and Moberg [2008a] is used and developed in the scope of this work, following the previously introduced gray-box approach. The method is based on linearization as described in Sec. 2.3. The optimal parameters $\hat{\theta}$ are obtained by minimizing the weighted logarithmic error between the measured FRFs and the FRFs of the linearized gray-box model (2.7). The method can be summarized in three steps (see Moberg et al. [2014] for more details):

1. Estimate non-parametric FRFs $\hat{G}^{(i)}(\omega)$ in a number of robot configurations i based on measurement data, see Sec. 3.2.

2. Select a model structure and linearize the gray-box model (2.3) in each of the robot configurations to get parametric FRFs $G^{(i)}(\omega, \theta)$, see Sec. 2.3.

3. Compute the error

$$\mathcal{E}^{(i)}(\omega_l, \theta) = \log \operatorname{vec}(\hat{G}^{(i)}(\omega_l)) - \log \operatorname{vec}(G^{(i)}(\omega_l, \theta)) \tag{3.1}$$

for all frequencies ω_l and obtain the parameter vector by solving

$$\hat{\theta} = \arg\min_{\theta} \sum_{i \in Q_c} \sum_{l=1}^{N_f} \left[\mathcal{E}^{(i)}(\omega_l, \theta)\right]^T W^{(i)}(\omega_l)\, \mathcal{E}^{(i)}(\omega_l, \theta) \tag{3.2}$$

where $W^{(i)}(\omega)$ is a weighting matrix, and N_f the number of frequencies.

The optimization problem is solved using the function *fminunc* in MATLAB, which finds a local minimum. In order to find the global minimum, a good initial parameter vector is important, which can be gained from e.g. the modeling approach presented in Paper A. Furthermore, the problem is solved for a number of random perturbations around the initial guess.

The weighting matrix $W^{(i)}$ would optimally be the covariance matrix of the FRF estimate, which is usually unknown. If available, an estimate can be used instead, but it was shown that "rough user-defined weights work much better than the theoretically optimal weights" [Wernholt and Moberg, 2008a]. If selected manually, the weights are defined such that they reflect where the best model performance is required, i.e. favoring frequencies and/or configurations that are most important and critical for the application. For control purposes, for example, the frequency region around the first resonance is usually required to be most accurate.

The logarithmic least squares criterion (3.1) is used because of the large dynamic range of the highly resonant robot system. From a theoretical perspective, this criterion is not optimal since it gives inconsistent estimates. In practice, this is of minor importance if a good signal-to-noise ratio can be ensured. Furthermore, this criterion has improved numerical stability as well as robustness with respect to outliers in the measurement data [Pintelon and Schoukens, 2012, p. 208 f.].

3.2 Estimation of frequency response functions

The method described in the previous section requires an estimate of the system's FRF in order to estimate the parameters of the gray-box robot model. The goal of this section is therefore to estimate the $n_y \times n_u$ nonparametric time-discrete FRF $\hat{G}^{(i)}(\omega)$ of the robot system in configuration i from experimental data. Motor torque and motor acceleration are defined as the system's input and output, in accordance with the sensors that are commonly available in industrial robots. The current of each motor is measured, and a simple linear relation between the current and the motor torque τ_m is used since the motor dynamics are much faster compared to the dynamics of the manipulator arm. The angular position q_m of the six actuators is measured by resolvers and numerically differentiated to become angular acceleration. All motors are excited simultaneously with a speed reference signal and the motor torque time series is recorded based on current measurements. In the scope of this thesis, an orthogonal random phase multisine is used as excitation signal, given certain amplitude constraints. This signal was suggested by Dobrowiecki and Schoukens [2007] for multivariable nonparametric FRF estimation. In this section, it is assumed that the optimal excitation signal is known and that informative data is available. How the excitation is chosen is described in Sec. 4.2.

3.2.1 Basics on discrete time systems in frequency domain

Consider the setting in Figure 3.3, where $u(t)$ is the plant input (dimension n_u) and $y(t)$ is the measured output (dimension n_y), corrupted by measurement noise $v(t)$. The goal is to obtain a non-parametric estimate of the system's transfer function $G(\Omega)$, as well as the noise covariance matrix $C_V = \mathrm{Cov}(V)$. Since the open-loop system is unstable, data needs to be collected in closed loop, i.e. the robot operates together with a controller. The input to the controller is the difference between the reference signal $r(t)$ and the measured output $y(t)$.

A limited number of discrete time data points $y(kT_s), k = 1, 2, ..., N$ are collected, where the time interval T_s is the sampling period and N the total number of samples. It is common to consider the Discrete Fourier Transform (DFT), de-

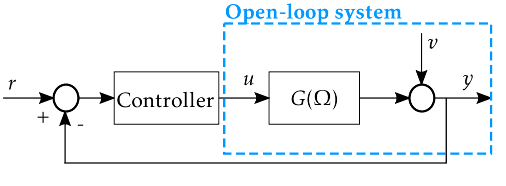


Figure 3.3: *Linear closed-loop system.*

fined as

$$Y(\omega_k) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} y(nT_s) e^{-j\omega_k T_s n} \tag{3.3}$$

with the DFT frequencies

$$\omega_k = k\frac{2\pi}{NT_s}, \; k = 1, 2, \ldots, N. \tag{3.4}$$

The DFTs of input and output, $U(\omega_k)$ and $Y(\omega_k)$, are related as

$$Y(\omega_k) = G(e^{j\omega_k T_s})U(\omega_k) + V(\omega_k) \tag{3.5}$$

where $G(e^{j\omega_k T})$ is the $n_y \times n_u$ transfer function of the system and $V(\omega_k)$ the noise DFT. In the following, the complex variable $\Omega_k = e^{j\omega_k T_s} = e^{j2\pi k/N}$ is used.

$G(\Omega_k)$ contains $n_y \cdot n_u$ unknown transfer functions, i.e. $n_e \geq n_u$ experiments are needed for estimating $G(\Omega_k)$ in the noise-free case of (3.5) and by use of classical FRF estimation techniques. The data vectors from n_e experiments are collected into matrices (bold-face) where each column corresponds to one experiment. Eq. (3.5) then becomes

$$\mathbf{Y}(\omega_k) = G(\Omega_k)\mathbf{U}(\omega_k) \tag{3.6}$$

where $\mathbf{Y}(\omega_k)$ and $\mathbf{U}(\omega_k)$ have the dimensions $n_y \times n_e$ and $n_u \times n_e$, respectively. A basic method for obtaining the estimate $\hat{G}(\Omega_k)$ is the H_1 estimator (see e.g Guillaume et al. [1996]):

$$\hat{G}^{H_1}(\Omega_k) = \mathbf{Y}(\omega_k)\mathbf{U}^H(\omega_k)\left[\mathbf{U}(\omega_k)\mathbf{U}^H(\omega_k)\right]^{-1} \tag{3.7}$$

Other methods are summarized in e.g. Ljung [1999] or Pintelon and Schoukens [2012], which are also recommended as basic references and for detailed theory.

3.2.2 Closed-loop identification

The system of an industrial robot is unstable and open-loop experiments are therefore not feasible. This section gives a brief introduction of the field of closed-loop system identification. The following closed-loop system is considered:

$$\begin{aligned} y(t) &= G_0 \cdot u(t) + v(t) \\ u(t) &= r(t) - F_y \cdot y(t) \end{aligned} \tag{3.8}$$

where F_y denotes the controller and G_0 is the plant. The reference signal $r(t)$ is typically considered as the excitation signal.

Closed-loop identification is challenging, because there will be correlation between the input signal at time t and past values of the noise signal $v(t)$. Furthermore, output nonlinearities perturb the input via the feedback. Because of this, several methods that work well in open loop will give biased estimates in closed loop. Furthermore, a closed-loop experiment may be non-informative even if the input is persistently exciting.

The available methods for closed-loop identification can be categorized as following [Gustavsson et al., 1977]:

- The direct approach
- The indirect approach
- The joint input-output approach (including two-stage/projection method)

The theoretically interested reader is referred to, e.g., Ljung [1999], Forssell [1999] or Forssell and Ljung [1999] for a detailed discussion on the topic of closed-loop identification. A brief summary of the main ideas is given below.

The direct approach applies a standard identification method using the input $u(t)$ and the output $y(t)$ in the same way as for an open-loop system. Any possible feedback is ignored and the reference signal $r(t)$ is not used. The bias in the estimate of G_0 will be small in frequency regions where either (or all) of the following holds: The noise model is accurate (or flexible enough), the feedback contribution is small, the signal-to-noise ratio is high. Since the latter can be realized for a robotic system [Wernholt, 2007] the direct approach is the primary choice in this work.

The indirect approach assumes that the reference signal $r(t)$ is measured and that the controller is known. The system is identified by a two-step process: First, a model for the closed-loop system from the reference signal to output signal $y(t)$ is estimated, which is an open-loop problem. In a second step, an estimate of the open-loop transfer function is computed by making use of the known controller.

The joint input-output (JIO) approach considers $y(t)$ and $u(t)$ jointly as outputs of a system that is driven by $r(t)$. That is, models that describe how both $u(t)$ and $y(t)$ depend on $r(t)$ are estimated (open-loop problems). Working with the complete model makes it possible to consider the fact that the noise on the u-channel is correlated with the noise on the y-channel. The classic method for spectral analysis in closed-loop settings can be viewed as a JIO approach, considering the ratio of the cross-spectra estimates of reference to input and output, respectively. The JIO approach is also used in Pintelon and Schoukens [2013] in the context of FRF measurement of nonlinear systems operating in closed loop. Aspects of input design and properties of the resulting FRF estimate, called the best linear approximation (BLA), are discussed.

The indirect and joint input-output approaches convert the closed-loop problem to an open-loop problem, seeing the reference signal as input. Since the reference signal is uncorrelated with the output noise, these approaches can be used together with any open-loop method, including spectral analysis, instrumental variables and standard subspace methods. Disadvantageous are prerequisites that the controller is known (indirect approach) and that the reference signal must be measured.

3.2.3 Nonparametric averaging methods

In order to improve the quality of the FRF estimate (3.7), the effect of disturbances and nonlinear effects must be dealt with. Therefore, and for calculating the uncertainty of the FRF estimate, multiple periods are measured and additional experiments are performed in each robot configuration. Averaging over multiple periods will reduce random noise, whereas averaging over different experiments will also reduce the effect of nonlinearities, since different realizations of the random phase multi-sine input signal are used [Wernholt and Gunnarsson, 2006]. Here, multiple experiments are grouped in one block M, and an FRF is estimated for each block. By averaging and comparing these estimates, it is possible to both improve the final FRF estimate $\hat{G}^{(i)}$ for each configuration i as well as to calculate its uncertainty $\Lambda_{\hat{G}}^{(i)}$. Note that the configuration index i will be dropped in the remainder of this section. Assuming that M blocks of data are collected, i.e. $n_e = n_u\, M$ experiments in each configuration, the DFT matrices can be partitioned into M blocks such as

$$
\begin{aligned}
\mathbf{U}(\omega_k) &= \left[\mathbf{U}^{[1]}(\omega_k)\ldots\mathbf{U}^{[M]}(\omega_k)\right],\\
\mathbf{Y}(\omega_k) &= \left[\mathbf{Y}^{[1]}(\omega_k)\ldots\mathbf{Y}^{[M]}(\omega_k)\right].
\end{aligned}
\tag{3.9}
$$

Then, the FRF can be estimated using e.g. the H_1-estimator by

$$
\hat{G}^{H_1} = \left[\frac{1}{M}\sum_{m=1}^{M}\mathbf{Y}^{[m]}\mathbf{U}^{[m]^H}\right]\left[\frac{1}{M}\sum_{m=1}^{M}\mathbf{U}^{[m]}\mathbf{U}^{[m]^H}\right]^{-1}
\tag{3.10}
$$

where $(\,\cdot\,)^H$ denotes the complex conjugate transpose. Another estimation method is the arithmetic mean (ARI) estimator [Guillaume, 1998, Pintelon and Schoukens, 2012]:

$$
\hat{G}^{\text{ARI}} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{Y}^{[m]}\left[\mathbf{U}^{[m]}\right]^{-1} = \frac{1}{M}\sum_{m=1}^{M}\hat{G}^{[m]}.
\tag{3.11}
$$

The averaging can be generalized to nonlinear techniques, as in Guillaume et al. [1992] for SISO systems and in Guillaume [1998] for MIMO systems. The logarithmic averaging technique has shown to give best results for the robotic system [Wernholt and Moberg, 2008b]. The FRF is then estimated by

$$
\hat{G}^{\text{LOG}} = P^{-1}\exp\left(\frac{1}{M}\sum_{m=1}^{M}\log\left(P\,\hat{G}^{[m]}\right)\right)
\tag{3.12}
$$

where the matrix P is used to avoid phase wrapping problems when averaging the phase. It is chosen as

$$P = V^{[1]}\mathrm{diag}\left\{e^{-j\arg\lambda_l^{[1]}}\right\}_{l=1}^{n}[V^{[1]}]^{-1} \tag{3.13}$$

with the eigenvalue decomposition $\hat{G}^{[1]} = V^{[1]}\Lambda^{[1]}[V^{[1]}]^{-1}$, $\Lambda^{[1]} = \mathrm{diag}\left\{\lambda_l^{[1]}\right\}_{l=1}^{n}$.

Assuming the closed-loop system of Figure 3.3, and that the reference signal r is available, the following Joint-Input-Output approach can be applied (Wellstead [1981], Ljung [1999, p. 438]): First, the FRFs $\hat{G}_{ru}$ and $\hat{G}_{ry}$ from the reference r to the input u, and from the reference r to the output y are estimated. Second, the FRF from u to y is computed as the ratio $\hat{G}_{ry}/\hat{G}_{ru}$. Applying this approach to the multiple-experiment framework introduced above, gives

$$\hat{G}^{JIO} = \left[\frac{1}{M}\sum_{m=1}^{M}\mathbf{Y}^{[m]}\mathbf{R}^{[m]^H}\right]\left[\frac{1}{M}\sum_{m=1}^{M}\mathbf{U}^{[m]}\mathbf{R}^{[m]^H}\right]^{-1} \tag{3.14}$$

with $\mathbf{R}$ analogous to (3.9). The JIO estimator is consistent and asymptotically unbiased and is therefore expected to give the best performance when the number of measured data blocks M increases. This is also concluded in Wernholt and Gunnarsson [2007], where different averaging estimation techniques are compared in a simulation study with a linear robot model. However, the experimental results in Wernholt and Moberg [2008b] show that JIO gives "the worst overall performance" of all considered averaging techniques, mainly due to large errors at low frequencies.

3.2.4 Parametric local methods

The key idea of the method proposed by Pintelon et al. [2010] is that $G(\Omega_k)$ is a smooth function of the frequency so that it can be approximated in a narrow frequency band (window) around a central frequency f_k by a complex polynomial or rational function. Figure 3.4 visualizes this idea of estimating local approximations of $G(\Omega_k)$ from the measured input and output signals. The most appealing feature of such local methods compared to averaging techniques is their data efficiency: Only one experiment in each robot configuration is sufficient for estimating the $n_y \times n_u$ FRF matrix of the system. The following paragraphs introduce the local parametric modeling approach and some variants of the basic method. Targeting to reduce the experiments needed for identification, Paper D contributes with an experimental comparison of the FRF estimates obtained from local models with the FRFs gained from averaging techniques. The experimental study furthermore shows that the choice of parametrization in the local modeling approach has significant impact on the FRF quality, and that considering the reference signal (JIO-approach) improves the estimate.

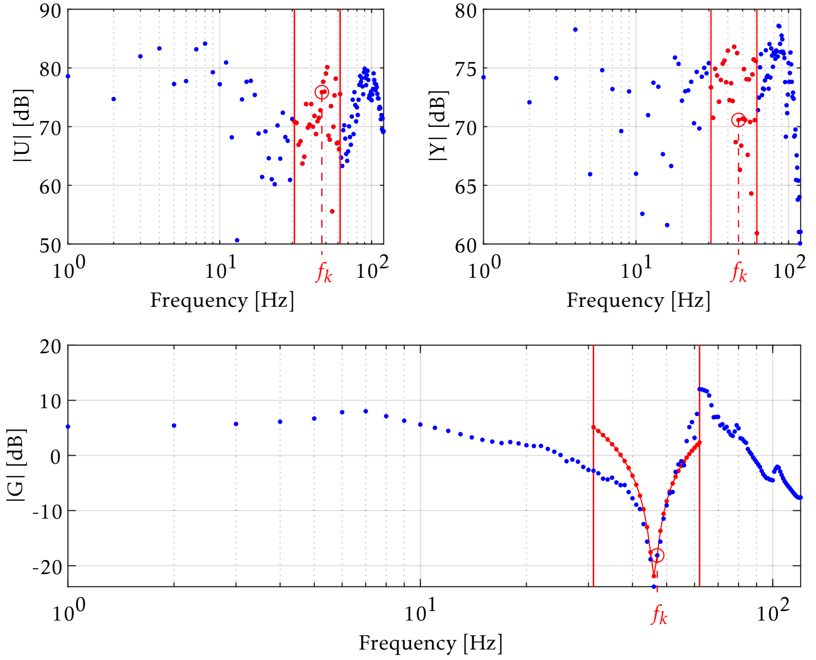


Figure 3.4: *Visualization of the local parametric modeling method for FRF estimation. The local frequency window is marked between the vertical red lines; the selected data points as well as the estimated polynomial are marked in red.*

Local polynomial method (LPM)

Assuming a good signal-to-noise ratio, the open loop system indicated in Figure 3.3 is considered. The goal is to obtain a non-parametric estimate of the system's transfer function G, as well as the noise covariance matrix $C_V = \mathrm{Cov}(V)$, by applying the local polynomial modeling (LPM) method. To begin with, every row $i = 1, \ldots, n_y$ of G is estimated separately by considering all n_u inputs $U = [U_1, \ldots, U_{n_u}]^T$ and one output Y_i at the time (MISO case). Then, (3.5) at DFT line Ω_{k+r} can be written as

$$Y_i(\omega_{k+r}) = G_i(\Omega_{k+r})U(\omega_{k+r}) + V_i(\omega_{k+r}) \tag{3.15}$$

where $G_i(\Omega_{k+r}) = \left[G_{i1}(\Omega_{k+r}), \ldots, G_{in_u}(\Omega_{k+r})\right]$ with

$$G_{ij}(\Omega_{k+r}) = G_{ij}(\Omega_k) + \sum_{s=1}^{R} g_{s,ij}(\omega_k) r^s \tag{3.16}$$

for $j = 1, \ldots, n_u$. $G(\Omega_k)$ is approximated with polynomials of order R in a sliding window of width $w = 2b$, which is centered around a central frequency k. Except near the frequency borders $r \in \{-b, \ldots, -1, 0, 1, \ldots, b-1\}$. Collecting all w samples within a window, (3.15) can be re-written as

$$Y_{w,i} = K_{w,i}\Theta_i(\omega_k) + V_{w,i} \tag{3.17}$$

where $Y_{w,i}$ and $V_{w,i}$ are $w \times 1$ vectors, and Θ_i is the $(R+1)n_u \times 1$ vector of unknown complex parameters, i.e.:

$$\begin{aligned}\Theta_i(\omega_k) = [&G_{i1}(\Omega_k), g_{1,i1}(\omega_k), \ldots, g_{R,i1}(\omega_k), \ldots, \\ &G_{i\,n_u}(\Omega_k), g_{1,i\,n_u}(\omega_k), \ldots, g_{R,i\,n_u}(\omega_k)]^T\end{aligned} \tag{3.18}$$

$K_{w,i}$ is the $w \times (R+1)n_u$ matrix containing the input data U_w within the window:

$$\begin{aligned}K_{w,i} &= \Big[[1\; r \ldots r^R] \otimes U_w\Big] \\ &= [U_{w,1}, rU_{w,1}, \ldots, r^R U_{w,1}, \ldots, U_{w,n_u}, \ldots, r^R U_{w,n_u}]\end{aligned} \tag{3.19}$$

Note that the window width w must fulfill $w \geq w_{min} = (R+1)n_u$ in order to estimate all parameters Θ_i.

If $w > (R+1)n_u$, (3.15) is an overdetermined set of equations that can be solved using least squares as

$$\hat{\Theta}_i(\omega_k) = (K_{w,i}^{\mathrm{H}} K_{w,i})^{-1} K_{w,i}^{\mathrm{H}} Y_{w,i} \tag{3.20}$$

where $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose. For each central frequency k, the estimate of the FRF related to output channel i is contained in $\hat{\Theta}_i$ as indicated in (3.18). The residual of the least-squares fit (3.20) is given by

$$\hat{V}_{w,i} = Y_{w,i} - K_{w,i}\hat{\Theta}_i(\omega_k) \tag{3.21}$$

and an estimate of the noise covariance is

$$\hat{C}_{V,i}(\omega_k) = \frac{1}{q}\hat{V}_{w,i}^{\mathrm{H}}\hat{V}_{w,i} \tag{3.22}$$

where $q = w - \mathrm{rank}(K_{w,i})$, see Pintelon et al. [2010].

Increasing w, i.e. taking a larger number of frequencies in the frequency window, reduces the variance of the parameter estimate since the noise is averaged over a larger number of data. On the other hand, the larger the window, the larger the interpolation error caused by the fact that the transfer function varies over the interval. In practice, the LPM is mostly used with polynomials of degree two, i.e. $R = 2$ [Gevers et al., 2011].

Local rational method (LRM), MISO

Considering rational functions as local approximations, (3.15) is modified to

$$Y_i(\omega_{k+r}) = \frac{G_i(\Omega_{k+r})}{D_i(\Omega_{k+r})}U(\omega_{k+r}) + V_i(\omega_{k+r}) \tag{3.23}$$

where G_i is defined as in (3.16) and D_i as

$$D_i(\Omega_{k+r}) = 1 + \sum_{s=1}^{R} d_{s,i}(\omega_k) r^s \,, \tag{3.24}$$

see Parametrization 2 in Voorhoeve et al. [2018]. Multiplying with D_i gives

$$\begin{aligned} Y_i(\omega_{k+r}) = &\left(G_i(\Omega_k) + \sum_{s=1}^{R} g_{s,i}(\omega_k) r^s \right) U(\omega_{k+r}) \\ &- \left(\sum_{s=1}^{R} d_{s,i}(\omega_k) r^s \right) Y_i(\omega_{k+r}) + V_i(\omega_{k+r}) D_i(\Omega_{k+r}) \end{aligned} \tag{3.25}$$

and using matrix notation yields

$$Y_{w,i} = K_{w,i}\Theta_i(\omega_k) + \tilde{V}_{w,i} \tag{3.26}$$

where $D_{w,i}$ contains the polynomial (3.24) at all frequencies of the window, $\tilde{V}_{w,i}$ is the noise term scaled with $D_{w,i}$ and Θ_i now is a $(R+1)n_u + R \times 1$ matrix of unknown complex parameters

$$\begin{aligned} \Theta_i(\omega_k) = [&G_{i1}(\Omega_k),\, g_{1,i1}(\omega_k), \ldots, g_{R,i1}(\omega_k), \ldots, \\ &G_{i\,n_u}(\Omega_k),\, g_{1,i\,n_u}(\omega_k), \ldots, g_{R,i\,n_u}(\omega_k), d_{1,i}(\omega_k), \ldots, d_{R,i}(\omega_k)]^T \end{aligned} \tag{3.27}$$

The matrix $K_{w,i}$ has the dimension $w \times (R+1)n_u + R$ and is defined as

$$K_{w,i} = \Big[[1\; r \ldots r^R] \otimes U_w \;,\; -[r \ldots r^R] \otimes Y_{w,i} \Big] \tag{3.28}$$

The window width must be $w \geq w_{min} = (R+1)n_u + R$ in order to estimate all parameters Θ_i. (3.26) is linear in the parameters and can be solved with (3.20), which is referred to as Local Levy method in Pintelon et al. [2021]. It is noted, but not further considered in the scope of this thesis, that the linearization of the output error yields a biased estimate [Pintelon and Schoukens, 2012, p. 301 f.]. Note also that no transient term is estimated here, but instead the data is truncated such that only steady-state samples are used.

Local rational method (LRM), MIMO

Since the robot operates in closed loop, it can be expected that all outputs affect the system through feedback. Therefore, all outputs i are considered simultaneously in the following such that $Y = [Y_1, \ldots, Y_{n_y}]$, and D becomes a $n_y \times n_y$-matrix of polynomials (full MFD parametrization, see Parametrization 3 in Voorhoeve et al. [2018]). With D_w being the block-matrix containing the matrix of polynomials for each frequency of the window, it can be written analogous to (3.26)

$$Y_w = K_w \Theta(\omega_k) + \tilde{V}_w \tag{3.29}$$

where Y_w and $\tilde{V}_w$ are $w \times n_y$ matrices and $\Theta(\omega_k) = [\Theta_1(\omega_k), \ldots, \Theta_{n_y}(\omega_k)]$ is the $(R+1)n_u + R\,n_y \times n_y$ matrix of unknown parameters. For $i = 1 \ldots n_y$,

$$\begin{aligned} \Theta_i(\omega_k) = [&G_{i1},\ g_{1,i1}, \ldots, g_{R,i1}, \ldots, G_{i\,n_u},\ g_{1,i\,n_u}, \ldots, g_{R,i\,n_u}, \\ &d_{1,i1}, \ldots, d_{R,i1}, \ldots, d_{1,i\,n_y}, \ldots, d_{R,i\,n_y}]^T \end{aligned} \tag{3.30}$$

Now, $w_{min} = (R+1)n_u + R\,n_y$, and the matrix K_w is $w \times (R+1)n_u + R\,n_y$:

$$K_w = \Big[[1\ r \ldots r^R] \otimes U_w\ ,\ -[r \ldots r^R] \otimes Y_w\Big] \tag{3.31}$$

This approach has the advantage that the cross-influence of the different outputs is taken into account through the parameters of the polynomials in the matrix D.

Analogous as in (3.21) and (3.22), the uncertainty of the FRF estimate is derived from the residuals as

$$\hat{C}_{\tilde{V}}(\omega_k) = \frac{1}{q}\hat{\tilde{V}}^{\mathrm{H}}(\omega_k)\hat{\tilde{V}}(\omega_k) \tag{3.32}$$

$$\hat{\tilde{V}}(\omega_k) = Y_w - K_w\hat{\Theta}(\omega_k)\,. \tag{3.33}$$

Local rational method (LRM), JIO

Assuming the closed-loop system of Figure 3.3, and assuming that the reference signal r is available, the following Joint-Input-Output approach can be applied (Wellstead [1981], [Ljung, 1999, p. 438]): First, the FRFs $\hat{G}_{ru}$ and $\hat{G}_{ry}$ from the reference r to the input u, and from the reference r to the output y are estimated using LRM as described in Sec. 3.2.4. Second, the FRF from u to y is computed as the ratio $\hat{G}_{ry}/\hat{G}_{ru}$.

3.2.5 Dealing with transients

When exciting a system, the response will show transient (leakage) effects before reaching the steady-state. For local parametric methods, transient effects may be handled by additional parameters, i.e. by approximating and estimating a transient term. Assuming periodic excitation and that an integer number of periods is measured in steady-state, "the spectra of the signals calculated using the DFT are free of leakage errors due to the plant dynamics" [Pintelon and Schoukens, 2012, p. 518].

Here, we avoid the estimation of transient terms and cut the data such that the estimation data only contains the steady-state response. The waiting time until the steady state is reached mainly depends on the damping of the system. In practice, the user needs to wait until the transient effects are less than other sources of errors, such as measurement noise.

3.3 Choices in identification

Many choices are to be made when performing system identification of robotic manipulators. Following the procedure outlined in the introduction of this chapter, some exemplary questions and considerations are outlined below.

The first step of constructing the model should be based on the intended usage of the model. The model structure should be as simple as possible, describing the relevant dynamic behavior of the robot. A reasonable level of detail should be chosen, e.g. related to the equations describing friction or transmission behavior of the robotic joints. Another task in this first step of the identification procedure is to decide which parameters need to be identified from data and which can be assumed to be known accurately enough from other sources such as specification data of components. Furthermore, a technique to linearize the constructed model must be chosen such that as little as possible information is lost.

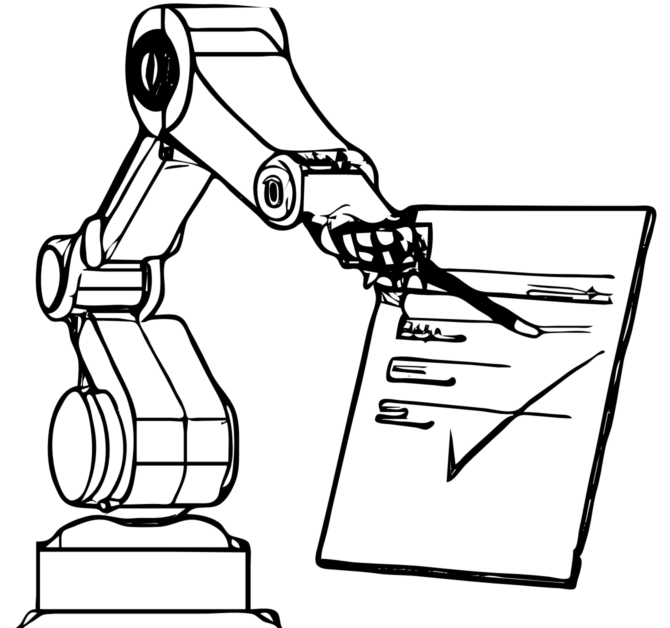
The step of experiment design involves many decisions that must be carefully made to ensure informative data and obtain convincing identification results. The type and design of the excitation signal plays an important role. Assuming periodic excitation, exemplary questions to be answered are: *What frequencies should be excited with what amplitude?*, or *How many periods are needed, and what period length is suitable?*. Physical constraints need to be considered, both related to excitation signal and the lab environment. When designing the experiments, the system's nonlinearities must be considered and it needs to be ensured that there is enough information about them in the data. Even the choice of payload that is attached to the end-effector is part of the experiment design problem.

Attempting to estimate the system's FRF from measurement data, a suitable method needs to be selected. Effects related to the present feedback must be considered, as well as the system's nonlinearities. Estimating the model parameters based on the FRFs requires also a careful selection of an optimization method and a suitable optimization criterion.

The above outlined complexity demonstrates the necessity to automate the identification procedure in order to make it applicable in practice. The ultimate goal of this work is therefore to reduce the choices that the user needs to make during the identification. The procedure should be simplified and shortened in order to enable on-site identification of robot individuals.

4

Experiment design



Estimating parametric robot models from experimental data is challenging since the system must operate in closed loop and since different types on nonlinearities act as disturbances. Phenomena such as friction, torque and resolver ripple, transmission backlash, and hysteresis can hardly be neglected, and the mechanical structure of the manipulator as well as the transmission behavior are elastic. Because of these challenges, identification from data requires well-designed experiments, especially the design of the input signal for data collection is very important. Choices related to the design of identification experiments are often called design variables and are in many cases crucial for an accurate and efficient identification. Since it is often costly and time-consuming to do new measurements, it is worthwhile to design the experiments such that the collected data is sufficiently informative.

After a brief introduction to the broad field of experiment design, this chapter focuses on two aspects that are relevant for frequency-domain identification of a parametric robot model. Firstly discussed are design variables related to the excitation signal that is used for data collection. Secondly, the problem of selecting the best manipulator configurations for estimating the system's FRFs is addressed. The chapter is concluded with some practical notes on how to optimally exploit the time that is available for doing experiments.

4.1 The idea of experiment design

Experiment design often relates to maximizing the information that is gained from the experiment while considering physical constraints. The classical approach is to minimize a scalar measure of the covariance matrix, which depends both on the experimental conditions χ and the true parameters θ_0. In the case of robot identification, the experimental conditions χ can be described in terms of excitation signal, the sampling period, amplitudes, frequency content, etc. Constraints are often related to the measurement time and the power or amplitude of the excitation signals. In system identification theory, an estimator is called maximally efficient if the parameter covariance matrix achieves the Cramér-Rao lower bound (inverse of the Fisher information matrix) with an unbiased estimator, see Ljung [1999] and Pintelon and Schoukens [2012] as basic references.

Many aspects of input design for general linear time-invariant systems are well-understood. See e.g. Goodwin and Payne [1977] for an early reference and Bombois et al. [2011] for a more recent overview. However, research on experiment design for nonlinear systems is limited to specific sub-classes of nonlinear systems, such as finite-impulse-response-type systems (see e.g. de Cock et al. [2016]) or nonlinear models that are composed of a known linear dynamic system interconnected with unknown static nonlinearities (see e.g. Vincent et al. [2010]).

Optimal experiment design in robotics has been studied for over two decades. Most methods parametrize the excitation as a Fourier series in order to find an optimal excitation trajectory. The method for optimizing the excitation trajectory that is presented in Swevers et al. [1997] aims directly at estimating the robot model parameters with minimal uncertainty. Compared to that, an information matrix is maximized in Vantilt et al. [2015] for guaranteeing an informative data set. A similar optimality criterion for finding a persistently exciting trajectory is chosen in Tika et al. [2020] and the optimal trajectory is computed by using an extended genetic algorithm.

The frequency-domain identification approach presented in Sec. 3.1 requires linearization of the dynamic equations around an operating point such that the system's FRFs can be estimated. Two aspects of experiment design are in this case of particular interest: First, optimal excitation for best possible quality of the FRF estimate (see Sec. 4.2), and second, optimal linearization points (configurations) for collecting informative data (see Sec. 4.3). Another design variable is, for example, the payload that is attached to the robot during data collection: What impact does the payload have, and is it beneficial if the estimation data contains experiments with different payloads?

Achieving an optimal experiment design related to all variables is often hard or impossible. From a practical point of view, it could instead be the goal to achieve an improvement compared to a reference. Referring to the research goals of this thesis (see Sec. 1.1), the experiment design is considered to be improved if an appropriate compromise can be found between the following criteria:

- The total experiment time is reduced, e.g. by fewer robot configurations for collecting data, or fewer experiments in each configuration,

- The model's FRFs are closer to the measured FRFs, i.e. the error (3.1) is reduced,

- The average and worst-case standard deviation of the parameter estimate $\hat{\theta}$ is reduced.

The goal of reducing the excitation amplitude is set aside for the moment, and the investigation of minimum amplitudes is left to future research. Instead, the maximal possible amplitude is realized for ensuring good excitation of the system's flexibilities and a high signal-to-noise ratio.

4.2 Design of the excitation signal

Choices related to the type of excitation, input amplitude, and excited frequencies are important to consider in order to obtain valid and good quality FRF estimates. The textbook Pintelon and Schoukens [2012] gives a very pedagogic and complete theoretical basis on the design of excitation signals for frequency domain identification, see especially Chapter 5. An extension to nonlinear systems operating in closed loop has been published in Pintelon and Schoukens [2013]. See also Saupe and Knoblach [2015], Boukhebouz et al. [2020] and Dirkx et al. [2022] for approaches on optimal input signals for FRF estimation. The purpose of the following section is to summarize and motivate the choices made for this work.

Type of excitation

For achieving a certain input spectrum, many different signals exist, e.g. (filtered) random noise, pseudo-random binary sequence, swept sine (chirp), and sum of sinusoids (multisine). In this thesis, a random phase multisine signal is used, since the advantages of a periodic excitation can be exploited, and this signal is very flexible in the sense that the user can specify exactly which frequencies to use from the available frequency grid. The multisine can be written as

$$u_{ms}(t) = \sum_{k=1}^{N_f} A_k \cos(\omega_k t + \phi_k) \tag{4.1}$$

where A_k are the amplitudes, ω_k the frequencies chosen from a grid

$$\left\{ \frac{2\pi l}{N_P T_s},\ l = 1, \dots, \frac{N_P}{2} - 1 \right\}$$

(N_P even) and ϕ_k the random phases uniformly distributed on the interval $[0, 2\pi)$. The signal (4.1) is periodic with N_P samples in each period, and given a desired power spectrum $\Phi_u(\omega)$, the amplitudes are $A_k = 2\sqrt{\frac{\Phi_u(\omega_k)}{N_P}}$.

Even though the multisine excitation might be optimal from a theoretical point of view, the nonlinear components of a robotic system require additional features of the excitation. For example, in order to reduce the effects of static friction around zero speed, a single sine wave with low frequency and high amplitude is superimposed with the multisine [Saupe and Knoblach, 2015, Wernholt and Gunnarsson, 2008], giving $u(t) = u_{ms}(t) + u_{ss}(t)$, see Figure 4.1.

For estimating multivariable nonparametric FRFs, the orthogonal random phase multisine signal is suggested in Dobrowiecki and Schoukens [2007]. The blocks of experiments in (3.9) are then given by

$$\mathbf{U}^{[m]}(\omega_k) = \operatorname{diag}\left\{ U_c^{[m]}(\omega_k) \right\}_{c=1}^{n_u} \mathbf{T}, \quad m = 1, \dots, M \tag{4.2}$$

where

$$\mathbf{T}_{jc} = e^{\frac{2\pi j}{n_u}(j-1)(c-1)} \tag{4.3}$$

is an orthogonal matrix for input channels $c = 1, \dots, n_u$ and experiments j. For each of the M blocks of measurement data, n_u random phase multisine signals are generated that are orthogonally shifted in the subsequent $n_u - 1$ experiments of each block.

As an example, Figure 4.1 shows the motor speed reference signal for Axis 2 of the robot. The data that is used for identification is marked in red, while motor torque limit and friction level are indicated as dashed horizontal lines. Figure 4.3 shows the measurement of the corresponding motor torque time series.

Excited frequencies

In order to avoid leakage effects in the DFT, the excitation signal is assumed to be periodic with N_P samples in each period. An integer number of periods P of the steady-state response are collected. A vector with logarithmically spaced frequencies is generated and all odd frequencies are excited, i.e. $\omega_k = \frac{2\pi(2k+1)}{N_P T_s}$ in (4.1). This is recommended in order to find the best linear approximation of the nonlinear system, see e.g. Schoukens et al. [2005]. The frequency range that is most relevant for robot motion control reaches up to 100 Hz, i.e. high frequent structural modes are not considered. Note that a periodic signal with period T_0 excites the plant only at frequencies kf_0 with $f_0 = 1/T_0$, and is insensitive to what happens between these frequencies. That is, very narrow resonance peaks (compared with f_0) can be missed. Assuming that several periods P are measured, the frequency resolution is increased to P/T_0.

Figure 4.2 shows the excited frequency lines of the designed reference signal shown in Figure 4.1, which excites 120 log-space frequencies between 4 and 80 Hz. Figure 4.4 shows the excited frequency lines for the measured torque of Figure 4.3.

Input amplitude

As mentioned above, the amplitude of the multisine delivers the desired power spectrum. Physical constraints such as the maximal available motor torque need to be considered, as well as possible damage and wear of components or the manipulator structure during identification experiments. However, large amplitudes are needed in order to achieve a high signal-to-noise ratio.

For the amplitude of the superimposed sine wave, the following trade-off needs to be considered: A small amplitude is favorable for ensuring that the manipulator does not move too far away from a quasi-static configuration and for ensuring the assumptions related to constant gravity torque, etc. On the other hand, the amplitude must be large enough such that static friction regime is overcome.

Figure 4.3 shows the measured input signal of an exemplary excitation of Axis 2 of the robot. The motor's torque limit as well as the friction level are indicated as horizontal dashed lines.

Optimal excitation

Periodic excitation signals based on Fourier series have become most popular for robot identification (see Swevers et al. [1997] as basic reference and Wu et al. [2012a] for a more recent application). For such signals, mostly two criteria are used to find optimal excitation: One is based on minimization of the condition number of the regressor matrix (see e.g. Presse and Gautier [1993], Armstrong [1987], Gautier and Khalil [1992], Wu et al. [2012a]). This criterion exploits that the sensitivity of a least squares solution to measurement noise depends on the condition number of the regressor matrix. The second criterion is based on the

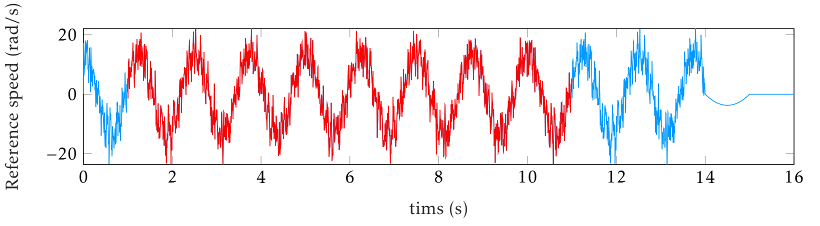


Figure 4.1: *Speed reference signal for Axis 2, Configuration 1. Data used for FRF estimation in red.*

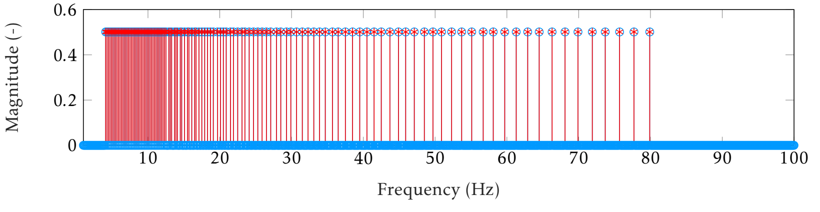


Figure 4.2: *FFT of the reference signal in Figure 4.1.*

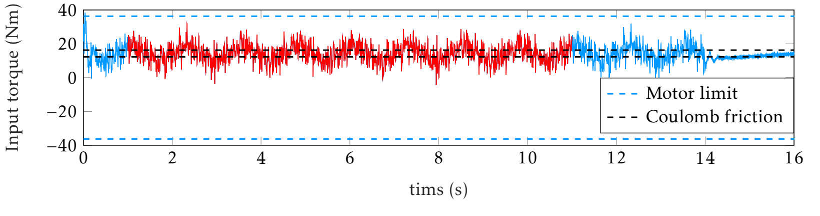


Figure 4.3: *Measured torque input signal for Axis 2, Configuration 1. Data used for FRF estimation in red. Torque limit and (gravity compensated) friction level as horizontal lines.*

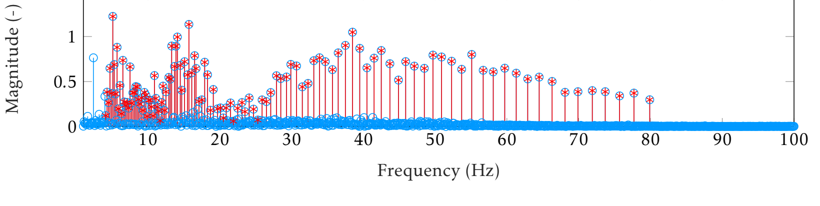


Figure 4.4: *FFT of the measured input signal in Figure 4.3.*

minimization of some measure (e.g. log det) of the Fisher information matrix (see e.g. Calafiore et al. [2001], Swevers et al. [1997]). Jin and Gans [2015] propose another criterion for optimizing an excitation trajectory using Hadamard's inequality, which states that the determinant of a positive definite matrix is less than or equal to the product of its diagonal entries.

It should be noted that it might be difficult to solve the resulting optimization problem, due to a large number of parameters describing the Fourier series and due to constraints on the trajectory, such as initial and final conditions and bounds on position, velocity and acceleration. Furthermore, most research related to optimal excitation trajectories targets only the estimation of geometric and inertial parameters (see references above), not the identification of dynamic parameters such as stiffness.

4.3 Choice of manipulator configurations

In the following, the primary goal of experiment design is to reduce the time and effort that are needed for collecting the identification data. This goal shall be achieved by reducing the number of robot configurations for experiments. Figure 4.5 shows six exemplary robot configurations as schematic 2-D drawings. In the scope of this section, it is assumed that a suitable excitation signal for FRF estimation is known and that it is not part of the experiment design problem to optimize the excitation signal according to any criteria.

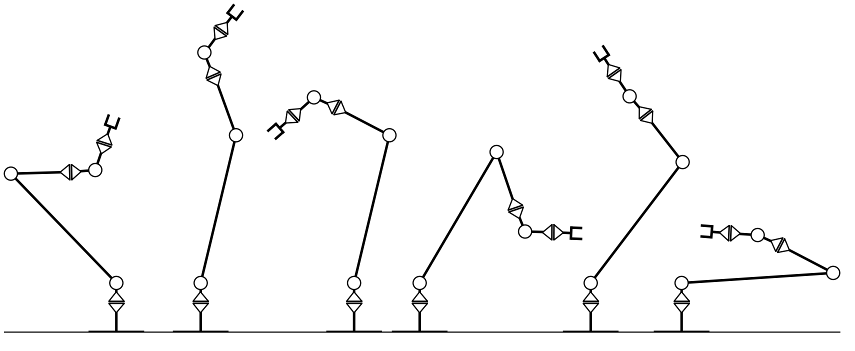


Figure 4.5: *Exemplary robot configurations.*

The idea of finding optimal manipulator configurations for parameter identification has mostly been treated related to robot calibration, which involves static experiments only. The goal in this context is to minimize the TCP position error by identifying the geometric parameters and by estimating a static stiffness compensation. In order to find optimal robot configurations w.r.t. kinematic performance methods based on Jacobians of the robot's generalized coordinates are most common, see e.g. Hu et al. [2020], Dumas et al. [2012]. An optimal compliance error compensation is derived in Wu et al. [2012b] by minimizing the covariance matrix, and a method for simultaneously identifying geometric and elasticity parameters is presented in Gadringer et al. [2022].

Compared to robot calibration, this work aims to improve experiment design for the identification of a dynamic robot model. Since the information content about the dynamic parameters θ differs between different configurations, experiments should be performed in the most informative configuration(s) w.r.t. θ. Wernholt and Löfberg [2007] propose a method which formulates the experiment design as a convex optimization problem: The optimal manipulator configurations are selected from a set of candidates based on the information matrix of each candidate configuration. The method, which is also used in Papers B and C, can be summarized as follows:

1. Choose a set of candidate configurations Q_c, that is sufficiently large to cover the workspace of the manipulator.
2. For each candidate configuration, derive an estimate of the FRF $\hat{G}^{(i)}$, together with the total co-variance matrix $\Lambda_0^{(i)}$.
3. Derive the information matrix for each candidate:
$$H_i = 2Re\left\{\overline{\Psi^{(i)}(\theta_0)}\left[\Lambda_0^{(i)}\right]^{-1}\left[\Psi^{(i)}(\theta_0)\right]^T\right\} \tag{4.4}$$
with the Jacobian $\left[\Psi^{(i)}(\theta_0)\right]^T = \frac{\partial G^{(i)}(\theta_0)}{\partial \theta}$, where $G^{(i)}(\theta_0)$ are the parametric model FRFs, and θ_0 the nominal parameters.
4. Solve an optimization problem for finding the best combination λ of candidate configurations:
$$\begin{aligned} &\text{minimize} \quad && \log\det\left[\textstyle\sum_{i\in Q_c} \lambda_i H_i\right]^{-1} \\ &\text{subject to} \quad && \lambda \geq 0, \quad 1^T\lambda = 1 \end{aligned} \tag{4.5}$$

The total co-variance matrix $\Lambda_0^{(i)}$ is a block-diagonal matrix with the uncertainty of the FRF estimate $\Lambda_{\hat{G}}^{(i)}$ on the diagonal:

$$\Lambda_0^{(i)} = \text{diag}\left[\Lambda_{\hat{G}}^{(i)}(\omega_1), \ldots, \Lambda_{\hat{G}}^{(i)}(\omega_f)\right] \tag{4.6}$$

Note that $\Lambda_0^{(i)}$ is usually unknown and needs to be estimated. A matrix $W^{(i)}(\omega)$ can optionally be introduced for weighting different frequencies. Paper B suggests to use simulations and estimate $\Lambda_0^{(i)}$ by averaging the signals over different periods and realizations of the excitation. Since the robot system is nonlinear, the experiment configurations, i.e. the linearization points, must contain information about all nonlinear effects for enabling the estimation of a globally valid model. Obtaining $\Lambda_{\hat{G}}^{(i)}$ from simulation implies multiple simulations (experiments) in each candidate configuration, which is a costly procedure. The effort can be motivated if the resulting experiment design can be used for the identification of many robots of the same type. Once the optimal configurations are found, only few experiments need to be conducted for identifying robot individuals of the same design, i.e. non-parametric FRFs $\hat{G}^{(i)}$ need to be estimated only for the optimal configurations Q_{opt}.

Another approach for obtaining $\Lambda_0^{(i)}$ is suggested in Wernholt and Löfberg [2007]. By assuming the FRF estimates to be the sum of the parametric FRF $G^{(i)}(l, \theta_0)$ and a zero mean measurement noise $\eta^{(i)}(l)$, i.e.

$$\hat{G}^{(i)}(l) = G^{(i)}(\theta_0) + \eta^{(i)}(l), \tag{4.7}$$

the variance matrix $\Lambda_0^{(i)}$ is determined by the noise contribution. If the noise is independent and identically distributed for different experiments with power spectrum $\Phi_v = C_v \omega^2 I$, then $\Lambda_0^{(i)}$ depends on the ratio of C_v and the power spectrum Φ_r of the multisine reference signal. Assuming the nominal model $G^{(i)}(l, \theta_0) = G_0^{(i)}$ linearized around configuration i together with an arbitrary controller F gives an approximation of the FRF uncertainty as

$$\Lambda_{\hat{G}}^{(i)} = \frac{1}{\Phi_r} \left[(G_0^{(i)} + F^{-1})^H (G_0^{(i)} + F^{-1}) \right]^T \otimes \Phi_v \,. \tag{4.8}$$

Besides the estimate of $\Lambda_0^{(i)}$, the Jacobian $\Psi^{(i)}(\theta_0)$ is needed in (4.4) to obtain the information matrices H_i. $\Psi^{(i)}(\theta_0)$ is estimated using the central difference approximation, assuming that the nominal parameters θ_0 are known. In most cases, a good guess of θ_0 can be provided, and if simulated experiments are used, the nominal parameters are known.

The results in Paper B show that a realistic estimate of the FRFs' uncertainty is crucial for successful experiment design. The potential of improving the experiment design by the proposed method has been demonstrated both with help of a simulation study and by an experimental validation with a medium size industrial robot (see Paper C). A significantly shorter time is needed for conducting data collection experiments and the average standard deviation of the parameter estimate is reduced, when using data from experiments with improved design.

4.4 Exploitation of experiment time

Until now, the goal of experiment design has been to reduce the experimental time as much as possible while keeping (or improving) the identification accuracy. Another formulation of experiment design builds on the assumption of a certain available time for doing experiments. This leads to the question of how to use the time optimally in the sense of collecting the most suitable data for identification. The optimal exploitation of the available experiment time can be seen as a tradeoff between

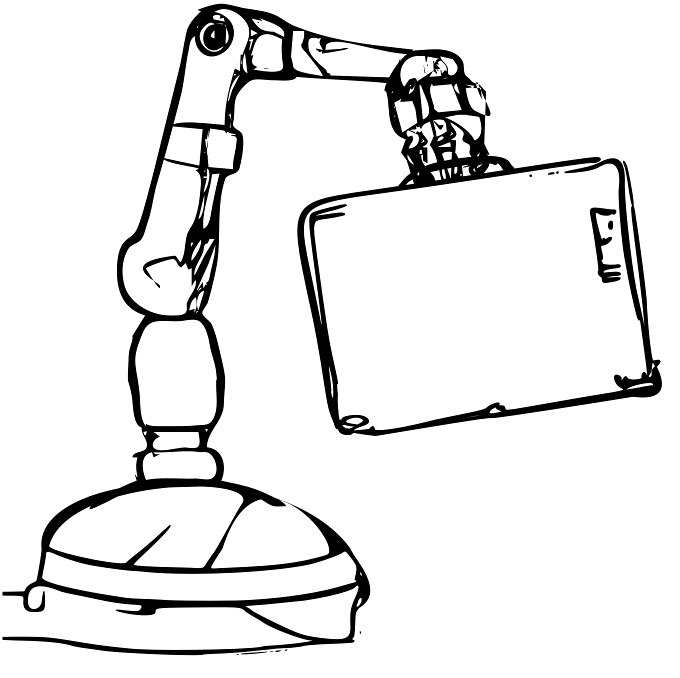
- The number of robot configurations Q_c,
- The number of experiments m in each configuration,
- The length of each experiment (period length and number of periods).

This is a difficult problem to solve, involving many constraints, design variables, and both time and frequency-domain. Furthermore, the method chosen for the FRF estimation plays an important role, since the estimated FRFs are treated as measurements.

By simulation is was shown in Paper B that additional configurations are more beneficial compared to longer experiments. However, a trade-off needs to be made since fewer experiments per configuration make it harder, or impossible, to estimate accurate FRFs using averaging techniques. Depending on the FRF estimation method, the number of experiments in each configuration has different impact: Using logarithmic averaging, for example, the choice of $M = 4$ blocks of each 6 experiments has turned out to be a good compromise between FRF quality and measurement time [Moberg et al., 2014]. However, an estimate derived with LRM does not significantly improve if more than 6 experiments are used, see Paper D. Compared to that the LRM method opens the possibility for more robot configurations in the same total measurement time assuming that only one experiment per configuration is sufficient for getting a high-quality FRF estimate.

5

Concluding remarks



This first part of the thesis has served as an introduction to modeling of robotic manipulators and to frequency-domain system identification methods. The aim has been to create a framework to relate the publications in the following second part to previous research results and to each other. Basic concepts for modeling of robotic manipulators were summarized in Chapter 2, as well as some aspects on modeling the transmission and friction occurring in the joints, and the structural dynamics of the robot's links. Data-driven estimation of a parametric robot model in frequency-domain was addressed in Chapter 3. The gray-box idea was introduced and the handling of nonlinearities, in particular friction, was addressed. Furthermore, techniques for FRF estimation were re-called. Since experiment design is a key factor for successful system identification, and essential for reaching the research goals of this thesis, Chapter 4 discussed some aspects of experiment design for frequency-domain identification of robotic manipulators. The design of the excitation signal, the problem of selecting the best robot configurations, and an optimal exploitation of available experiment time were addressed.

The contributions in Part II of this thesis deal with efficiency aspects of data-driven modeling of robotic manipulators. Considering the research goals that were stated in Sec. 1.1, the contributions mostly address goals 2 and 3, i.e. the reduction of experiment time while maintaining or improving the quality of the parametric robot model. Starting from the idea to completely avoid identification experiments, Paper A suggests a modeling approach restricted to component specifications and development data. An accurate robot model is derived by combining specific measurements of separated components with the reduced Finite Element description of the robot's links. If the robot to be modeled is available for experiments, measurement data is commonly used to further improve the model accuracy and particularly to estimate parameters of a lumped parameter gray-box model. In order to reduce the risk of wear or damage during data collection, the amount of identification experiments should be reduced. Paper B therefore presents an approach to limit the experiment configurations to a minimum number that contain most information and therefore allow the best parameter estimate. The effectiveness of the approach is demonstrated by a simulation study and completed by a validation with real measurement data in Paper C. Paper D addresses the aspect of reducing the experiment time by shorter experiments. A local parametric modeling method for estimating the FRFs from less data is analyzed w.r.t. its potential for robot identification. It is shown that the conventional averaging technique with appropriate excitation gives more accurate FRF estimate than the data-efficient LRM method and that a compromise needs to be found between estimation accuracy and experiment time.

Future work could be considered in terms of further improved experiment design, a modified or novel system identification method, or related applications such as modeling of a different robot type such as a Delta-type manipulator.

For further improving the experiment design method of Papers B and C, a nonlinear transmission model should be considered. The method could be adapted in order to account for the changed stiffness behavior of the robot in different areas of its workspace. In order to correctly describe the global dynamic behavior, the experimental data needs to contain information about all stiffness regimes, and the experiment design method needs to find the most valuable robot configurations in that sense. Combining the results of Paper D and the idea of data-efficient FRF estimation with the experiment design approach is another idea for future research. In particular, can the LRM method be used for estimating the FRF uncertainty that is needed for computing the information matrix of each configuration? A third direction for further research related to experiment design is the optimization of the excitation signal w.r.t. low excitation amplitudes. This is of particular interest in order to reach research goal 1.

Future research targeting a modified or novel system identification method can be thought of very broadly. Analyzing a time-domain approach could have scientific value within the field of identification of a nonlinear system operating in closed loop. A comparison of this approach with the frequency-domain method of this thesis would be interesting. Developing an identification approach that uses IMU measurements at the robot's end-effector could contribute to a better understanding of the system's nonlinear behavior and might result in more accurate parameter estimates. More general, the potential of additional sensors on the arm side of the gearboxes could be studied, both in terms of model accuracy and lowered excitation amplitudes. An idea that is close to the current identification method is to study the effect of different payloads during data collection on the identification accuracy.

All these suggestions aim to reach the original research goals of this thesis, i.e. the efficient data-driven modeling and identification of robotic manipulators for obtaining high quality models.

Bibliography

Nabtesco Reducers: Terminology and Understanding, 2013. URL https://www.nabtescomotioncontrol.com/pdfs/nmce13029-terminolog-and-understanding-nabtesco.pdf.

E. Abele, J. Bauer, T. Hemker, R. Laurischkat, H. Meier, S. Reese, and O. von Stryk. Comparison and validation of implementations of a flexible joint multibody dynamics system model for an industrial robot. *CIRP Journal of Manufacturing Science and Technology*, 4(1):38–43, 2011. ISSN 17555817. doi: 10.1016/j.cirpj.2011.01.006.

A. Albu-Schaffer and G. Hirzinger. Parameter identification and passivity based joint control for a 7 DOF torque controlled light weight robot. In *Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation (Cat. No.01CH37164)*, pages 2852–2858. IEEE, 2001. ISBN 0-7803-6576-3. doi: 10.1109/ROBOT.2001.933054.

B. Armstrong. On finding 'exciting' trajectories for identification experiments involving systems with non-linear dynamics. In *Proceedings. 1987 IEEE International Conference on Robotics and Automation*, pages 1131–1139. Institute of Electrical and Electronics Engineers, 1987. doi: 10.1109/ROBOT.1987.1087968.

B. Armstrong-Hélouvry, P. Dupont, and C. C. de Wit. A survey of models, analysis tools and compensation methods for the control of machines with friction. *Automatica*, 30(7):1083–1138, 1994. ISSN 00051098. doi: 10.1016/0005-1098(94)90209-7.

O. A. Bauchau. *Flexible Multibody Dynamics*, volume 176. Springer Netherlands, Dordrecht, 2011. ISBN 978-94-007-0334-6. doi: 10.1007/978-94-007-0335-3.

X. Bombois, M. Gevers, R. Hildebrand, and G. Solari. Optimal experiment design for open and closed-loop system identification. *Communications in Information and Systems*, 11(3):197–224, 2011. ISSN 15267555. doi: 10.4310/CIS.2011.v11.n3.a1.

B. Bona and M. Indri. Friction Compensation in Robotics: an Overview. In *Proceedings of the 44th IEEE Conference on Decision and Control*, pages 4360–4367. IEEE, 2005. ISBN 0-7803-9567-0. doi: 10.1109/CDC.2005.1582848.

B. Boukhebouz, G. Mercere, M. Grossard, X. Lamy, and E. Laroche. Identification of single flexible-joint robot dynamics: a nonparametric approach. In *2020 28th Mediterranean Conference on Control and Automation (MED)*, pages 357–362. IEEE, 2020. ISBN 978-1-7281-5742-9. doi: 10.1109/MED48518.2020.9182999.

G. Calafiore, M. Indri, and B. Bona. Robot dynamic calibration: Optimal excitation trajectories and experimental parameter estimation. *Journal of Robotic Systems*, 18(2):55–68, 2001. ISSN 0741-2223. doi: 10.1002/1097-4563(200102)18:2%3C55::AID-ROB1005%3E3.0.CO;2-O.

C. Canudas de Wit, H. Olsson, K. J. Astrom, and P. Lischinsky. A new model for control of systems with friction. *IEEE Transactions on Automatic Control*, 40(3):419–425, 1995. ISSN 00189286. doi: 10.1109/9.376053.

R. R. Craig and M. C. C. Bampton. Coupling of substructures for dynamic analyses. *AIAA Journal*, 6(7):1313–1319, 1968. ISSN 0001-1452. doi: 10.2514/3.4741.

S. H. Crandall. On using non-Gaussian distributions to perform statistical linearization. *International Journal of Non-Linear Mechanics*, 39(9):1395–1406, 2004. ISSN 00207462. doi: 10.1016/j.ijnonlinmec.2004.02.001.

P. R. Dahl. A Solid Friction Model, 1968. URL `https://apps.dtic.mil/sti/pdfs/ADA041920.pdf`.

A. de Cock, M. Gevers, and J. Schoukens. D-optimal input design for nonlinear FIR-type systems: A dispersion-based approach. *Automatica*, 73:88–100, 2016. ISSN 00051098. doi: 10.1016/j.automatica.2016.04.052.

A. de Luca. Feedforward/feedback laws for the control of flexible robots. In *Proceedings 2000 ICRA. Millennium Conference. IEEE International Conference on Robotics and Automation. Symposia Proceedings (Cat. No.00CH37065)*, pages 233–240. IEEE, 2000. ISBN 0-7803-5886-4. doi: 10.1109/ROBOT.2000.844064.

N. Dirkx, M. Bosselaar, and T. Oomen. Peak Amplitude-Constrained Experiment Design for FRF Identification of MIMO Motion Systems. In *2022 IEEE 17th International Conference on Advanced Motion Control*, pages 256–261. IEEE, 2022. ISBN 978-1-7281-7711-3. doi: 10.1109/AMC51637.2022.9729277.

T. Dobrowiecki and J. Schoukens. Measuring a linear approximation to weakly nonlinear MIMO systems. *Automatica*, 43(10):1737–1751, 2007. ISSN 00051098. doi: 10.1016/j.automatica.2007.03.007.

H. Du, M. K. Lim, and K. M. Liew. A nonlinear finite element model for dynamics of flexible manipulators. *Mechanism and Machine Theory*, 31(8):1109–1119, 1996. ISSN 0094114X. doi: 10.1016/0094-114X(96)84602-7.

C. Dumas, S. Caro, M. Cherif, S. Garnier, and B. Furet. Joint stiffness identification of industrial serial robots. *Robotica*, 30(4):649–659, 2012. ISSN 0263-5747. doi: 10.1017/S0263574711000932.

S. K. Dwivedy and P. Eberhard. Dynamic analysis of flexible manipulators, a literature review. *Mechanism and Machine Theory*, 41(7):749–777, 2006. ISSN 0094114X. doi: 10.1016/j.mechmachtheory.2006.01.014.

O. Egeland and J. T. Gravdahl. *Modeling and simulation for automatic control*. Marine Cybernetics, Trondheim, Norway and Great Britain, 2002. ISBN 8292356010.

U. Forssell. *Closed-loop identification: Methods, theory, and applications / Urban Forssell*, volume no.566 of *Linköping studies in science and technology. Dissertations, 0345-7524*. Department of Electrical Engineering, Linköping, 1999. ISBN 91-7219-432-4.

U. Forssell and L. Ljung. Closed-loop identification revisited. *Automatica*, 35(7): 1215–1241, 1999. ISSN 00051098. doi: 10.1016/S0005-1098(99)00022-9.

S. Gadringer, P. Klement, H. Gattringer, A. Mueller, and R. Naderer. Simultaneous Calibration and Stiffness Identification of Flexible Link Robots Using Lumped Parameter Model. In A. Müller and M. Brandstötter, editors, *Advances in Service and Industrial Robotics*, volume 120 of *Mechanisms and Machine Science*, pages 52–59. Springer International Publishing, Cham, 2022. ISBN 978-3-031-04869-2. doi: 10.1007/978-3-031-04870-8{\textunderscore}7.

M. Gautier and W. Khalil. Exciting Trajectories for the Identification of Base Inertial Parameters of Robots. *The International Journal of Robotics Research*, 11(4):362–375, 1992. ISSN 0278-3649. doi: 10.1177/027836499201100408.

M. Gautier and P. Poignet. Extended Kalman Filtering and Weighted Least Squares Dynamic Identification of Robot. *IFAC Proceedings Volumes*, 33(15): 935–940, 2000. ISSN 14746670. doi: 10.1016/S1474-6670(17)39873-7.

M. Gevers, R. Pintelon, and J. Schoukens. The Local Polynomial Method for nonparametric system identification: Improvements and experimentation. In *IEEE Conference on Decision and Control and European Control Conference*, pages 4302–4307. IEEE, 2011. ISBN 978-1-61284-801-3. doi: 10.1109/CDC.2011.6160311.

M. C. Good, L. M. Sweet, and K. L. Strobel. Dynamic Models for Control System Design of Integrated Robot and Drive Systems. *Journal of Dynamic Systems, Measurement, and Control*, 107(1):53–59, 1985. ISSN 0022-0434. doi: 10.1115/1.3140707.

G. C. Goodwin and R. L. Payne. *Dynamic system identification: Experiment design and data analysis*, volume 136 of *Mathematics in science and engineering*. Academic Press, New York and London, 1977. ISBN 0122897501.

M. Grotjahn, M. Daemi, and B. Heimann. Friction and rigid body identification of robot dynamics. *International Journal of Solids and Structures*, 38(10-13): 1889–1902, 2001. ISSN 00207683. doi: 10.1016/S0020-7683(00)00141-4.

P. Guillaume. Frequency response measurements of multivariable systems using nonlinear averaging techniques. *IEEE Transactions on Instrumentation and Measurement*, 47(3):796–800, 1998. ISSN 1557-9662. doi: 10.1109/19.744353.

P. Guillaume, R. Pintelon, and J. Schoukens. Nonparametric frequency response function estimators based on nonlinear averaging techniques. *IEEE Transactions on Instrumentation and Measurement*, 41(6):739–746, 1992. ISSN 1557-9662. doi: 10.1109/19.199393.

P. Guillaume, R. Pintelon, and J. Schoukens. Accurate Estimation of Multivariable Frequency Response Functions. *IFAC Proceedings Volumes*, 29(1):4351–4356, 1996. ISSN 14746670. doi: 10.1016/S1474-6670(17)58365-2.

I. Gustavsson, L. Ljung, and T. Söderström. Identification of processes in closed loop—identifiability and accuracy aspects. *Automatica*, 13(1):59–75, 1977. ISSN 00051098. doi: 10.1016/0005-1098(77)90009-7.

H.-J. Gutt, F. D. Scholl, and J. Blattner. High precision servo drives with DSP-based torque ripple reduction. In *AFRICON '96*, pages 632–637, New York, 1996. Institute of Electrical and Electronics Engineers. ISBN 0-7803-3019-6. doi: 10.1109/AFRCON.1996.562963.

R. J. Guyan. Reduction of stiffness and mass matrices. *AIAA Journal*, 3(2):380, 1965. ISSN 0001-1452. doi: 10.2514/3.2874.

D. C. Hanselman. Resolver signal requirements for high accuracy resolver-to-digital conversion. *IEEE Transactions on Industrial Electronics*, 37(6):556–561, 1990. ISSN 02780046. doi: 10.1109/41.103461.

T. Hardeman, R. G. K. M. Aarts, and J. B. Jonker. A finite element formulation for dynamic parameter identification of robot manipulators. *Multibody System Dynamics*, 16(1):21–35, 2006. ISSN 1384-5640. doi: 10.1007/s11044-006-9010-x.

R. Hensen, M. van de Molengraft, and M. Steinbuch. Frequency domain identification of dynamic friction model parameters. *IEEE Transactions on Control Systems Technology*, 10(2):191–196, 2002. ISSN 10636536. doi: 10.1109/87.987064.

M. Hu, H. Wang, and X. Pan. Optimal configuration selection for stiffness identification of 7-Dof collaborative robots. *Intelligent Service Robotics*, 13(3):379–391, 2020. ISSN 1861-2776. doi: 10.1007/s11370-020-00322-x.

J. T. Huang. A New Approach to Parametric Identification of a Single-Link Flexible-Joint Manipulator. *Journal of Intelligent and Robotic Systems*, 37(3): 273–284, 2003. ISSN 09210296. doi: 10.1023/A:1025435418131.

B. Irons. Structural eigenvalue problems - elimination of unwanted variables. *AIAA Journal*, 3(5):961–962, 1965. ISSN 0001-1452. doi: 10.2514/3.3027.

J. Jin and N. Gans. Parameter identification for industrial robots with a fast and robust trajectory design approach. *Robotics and Computer-Integrated Manufacturing*, 31(1):21–29, 2015. doi: 10.1016/j.rcim.2014.06.004.

B. Jonker. A Finite Element Dynamic Analysis of Flexible Manipulators. *The International Journal of Robotics Research*, 9(4):59–74, 1990. ISSN 0278-3649. doi: 10.1177/027836499000900404.

W. Khalil and E. Dombre. *Modeling, identification and control of robots*. HPS, London, 2002. ISBN 9781903996133.

W. Khalil and M. Gautier. Modeling of mechanical systems with lumped elasticity. In *Proceedings 2000 ICRA. Millennium Conference. IEEE International Conference on Robotics and Automation. Symposia Proceedings (Cat. No.00CH37065)*, pages 3964–3969. IEEE, 2000. ISBN 0-7803-5886-4. doi: 10.1109/ROBOT.2000.845349.

K. Kozlowski. *Modelling and Identification in Robotics*. Advances in Industrial Control. Springer London, London, 1998. ISBN 978-1-4471-1139-9.

Y. Li, M. Zhang, and Y. Zhu. A novel method for constructing a dynamics model with the flexible characteristics. *Journal of Physics: Conference Series*, 1939(1): 012017, 2021. ISSN 1742-6588. doi: 10.1088/1742-6596/1939/1/012017.

L. Ljung. *System identification: Theory for the user*. Prentice Hall, Upper Saddle River, N.J., 2nd ed. edition, 1999. ISBN 0136566952.

S. Moberg, E. Wernholt, S. Hanssen, and T. Brogårdh. Modeling and Parameter Estimation of Robot Manipulators Using Extended Flexible Joint Models. *Journal of Dynamic Systems, Measurement, and Control*, 136(3), 2014. ISSN 0022-0434. doi: 10.1115/1.4026300.

C. A. My, D. X. Bien, C. H. Le, and M. Packianather. An efficient finite element formulation of dynamics for a flexible robot with different type of joints. *Mechanism and Machine Theory*, 134:267–288, 2019. ISSN 0094114X. doi: 10.1016/j.mechmachtheory.2018.12.026.

S. Nicosia, P. Valigi, and L. Zaccarian. Dynamic modelling of a two link flexible robot and experimental validation. In *Proceedings of IEEE International Conference on Robotics and Automation*, pages 1953–1958. IEEE, 1996. ISBN 0-7803-2988-0. doi: 10.1109/ROBOT.1996.506158.

J. Öhr, S. Moberg, E. Wernholt, S. Hanssen, J. Pettersson, S. Persson, and S. Sander-Tavallaey. Identification of flexibility parameters of 6-axis industrial manipulator models. *Proceedings of ISMA2006: International Conference on Noise and Vibration Engineering*, 6, 2006.

H. Olsson, K. J. Åström, C. Canudas de Wit, M. Gäfvert, and P. Lischinsky. Friction Models and Friction Compensation. *European Journal of Control*, 4(3): 176–195, 1998. ISSN 09473580. doi: 10.1016/S0947-3580(98)70113-X.

M. T. Pham, M. Gautier, and P. Poignet. Accelerometer based identification of mechanical systems. In *Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No.02CH37292)*, pages 4293–4298. IEEE, 2002. ISBN 0-7803-7272-7. doi: 10.1109/ROBOT.2002.1014433.

R. Pintelon and J. Schoukens. *System identification: A frequency domain approach*. John Wiley & Sons Inc, Hoboken, N.J., 2nd ed. edition, 2012. ISBN 9780470640371.

R. Pintelon and J. Schoukens. FRF Measurement of Nonlinear Systems Operating in Closed Loop. *IEEE Transactions on Instrumentation and Measurement*, 62 (5):1334–1345, 2013. ISSN 1557-9662. doi: 10.1109/TIM.2012.2220033.

R. Pintelon, J. Schoukens, G. Vandersteen, and K. Barbé. Estimation of nonparametric noise and FRF models for multivariable systems—Part I: Theory. *Mechanical Systems and Signal Processing*, 24(3):573–595, 2010. ISSN 08883270. doi: 10.1016/j.ymssp.2009.08.009.

R. Pintelon, D. Peumans, G. Vandersteen, and J. Lataire. Frequency Response Function Measurements via Local Rational Modeling, Revisited. *IEEE Transactions on Instrumentation and Measurement*, 70:1–16, 2021. ISSN 1557-9662. doi: 10.1109/TIM.2020.3020601.

C. Presse and M. Gautier. New criteria of exciting trajectories for robot identification. In *[1993] Proceedings IEEE International Conference on Robotics and Automation*, pages 907–912. IEEE Comput. Soc. Press, 1993. ISBN 0-8186-3450-2. doi: 10.1109/ROBOT.1993.292259.

M. Ruderman and T. Bertram. FRF based identification of dynamic friction using two-state friction model with elasto-plasticity. In *2011 IEEE International Conference on Mechatronics*, pages 230–235. IEEE, 2011. ISBN 978-1-61284-982-9. doi: 10.1109/ICMECH.2011.5971286.

F. Saupe and A. Knoblach. Experimental determination of frequency response function estimates for flexible joint industrial manipulators with serial kinematics. *Mechanical Systems and Signal Processing*, 52-53:60–72, 2015. ISSN 08883270. doi: 10.1016/j.ymssp.2014.08.011.

H. Schempf. *Comparative design, modeling, and control analysis of robotic transmissions*. Massachusetts Institute of Technology and Woods Hole Oceanographic Institution, Woods Hole, MA, 1990. doi: 10.1575/1912/5431.

J. Schoukens, R. Pintelon, T. Dobrowiecki, and Y. Rolain. Identification of linear systems with nonlinear distortions. *Automatica*, 41(3):491–504, 2005. ISSN 00051098. doi: 10.1016/j.automatica.2004.10.004.

L. Sciavicco, L. Villani, B. Siciliano, and G. Oriolo. *Robotics: Modelling, planning and control*. Advanced textbooks in control and signal processing. Springer, London, 2010. ISBN 978-1-849-96634-4.

W. Seyfferth, A. J. Maghzal, and J. Angeles. Nonlinear modeling and parameter identification of harmonic drive robotic transmissions. In *Robotics and automation*, 1050-4729, pages 3027–3032. IEEE, 1995. ISBN 0-7803-1965-6. doi: 10.1109/ROBOT.1995.525714.

B. Siciliano and O. Khatib. *Springer handbook of robotics*. Springer Handbooks. Springer, Berlin, 2nd edition edition, 2016. ISBN 9783319325521.

M. W. Spong, S. Hutchinson, and M. a. Vidyasagar. *Robot Modeling and Control*. 2nd edition, 2020. ISBN 978-1-119-52404-5.

L. Sweet and M. Good. Redefinition of the robot motion-control problem. *IEEE Control Systems Magazine*, 5(3):18–25, 1985. ISSN 0272-1708. doi: 10.1109/MCS.1985.1104955.

J. Swevers, C. Ganseman, J. de Schutter, and H. van Brussel. EXPERIMENTAL ROBOT IDENTIFICATION USING OPTIMISED PERIODIC TRAJECTORIES. *Mechanical Systems and Signal Processing*, 10(5):561–577, 1996. ISSN 08883270. doi: 10.1006/mssp.1996.0039.

J. Swevers, C. Ganseman, D. B. Tukel, J. de Schutter, and H. van Brussel. Optimal robot excitation and identification. *IEEE Transactions on Robotics and Automation*, 13(5):730–740, 1997. ISSN 1042296X. doi: 10.1109/70.631234.

R. J. Theodore and A. Ghosal. Comparison of the Assumed Modes and Finite Element Models for Flexible Multilink Manipulators. *The International Journal of Robotics Research*, 14(2):91–111, 1995. ISSN 0278-3649. doi: 10.1177/027836499501400201.

A. Tika, J. Ulmen, and N. Bajcinca. Dynamic Parameter Estimation Utilizing Optimized Trajectories. In *2020 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 7300–7307. IEEE, 2020. ISBN 978-1-7281-6212-6. doi: 10.1109/IROS45743.2020.9341518.

P. Tomei. A simple PD controller for robots with elastic joints. *IEEE Transactions on Automatic Control*, 36(10):1208–1213, 1991. ISSN 00189286. doi: 10.1109/9.90238.

C. J. Tsaprounis and N. A. Aspragathos. Adaptive Tracking Controller for Rigid-Link Elastic-Joint Robots with Link Acceleration Estimation. *Journal of Intelligent and Robotic Systems*, 27(1):67–83, 2000. ISSN 09210296. doi: 10.1023/A:1008198029642.

J. Vantilt, E. Aertbeliën, F. de Groote, and J. de Schutter. Optimal excitation and identification of the dynamic model of robotic systems with compliant actuators. In *2015 IEEE International Conference on Robotics and Automation*, pages 2117–2124, 2015. doi: 10.1109/ICRA.2015.7139478.

T. L. Vincent, C. Novara, K. Hsu, and K. Poolla. Input design for structured nonlinear system identification. *Automatica*, 46(6):990–998, 2010. ISSN 00051098. doi: 10.1016/j.automatica.2010.02.029.

R. Voorhoeve, A. van der Maas, and T. Oomen. Non-parametric identification of multivariable systems: A local rational modeling approach with application to a vibration isolation benchmark. *Mechanical Systems and Signal Processing*, 105:129–152, 2018. ISSN 08883270. doi: 10.1016/j.ymssp.2017.11.044.

P. E. Wellstead. Non-parametric methods of system identification. *Automatica*, 17(1):55–69, 1981. ISSN 00051098. doi: 10.1016/0005-1098(81)90084-4.

E. Wernholt. Multivariable Frequency-Domain Identification of Industrial Robots. *0345-7524*, (1138), 2007. ISSN 0345-7524.

E. Wernholt and S. Gunnarsson. Detection and Estimation of Nonlinear Distortions in Industrial Robots. In *2006 IEEE Instrumentation and Measurement Technology Conference Proceedings*, pages 1913–1918. IEEE, 2006. ISBN 0-7803-9359-7. doi: 10.1109/IMTC.2006.328310.

E. Wernholt and S. Gunnarsson. Analysis of methods for multivariable frequency response function estimation in closed loop. In *46th IEEE Conference on Decision and Control*, pages 4881–4888, Piscataway, N.J., 2007. Institute of Electrical and Electronics Engineers. ISBN 978-1-4244-1497-0. doi: 10.1109/CDC.2007.4434780.

E. Wernholt and S. Gunnarsson. Estimation of Nonlinear Effects in Frequency Domain Identification of Industrial Robots. *IEEE Transactions on Instrumentation and Measurement*, 57(4):856–863, 2008. ISSN 1557-9662. doi: 10.1109/TIM.2007.911698.

E. Wernholt and J. Löfberg. Experiment design for identification of nonlinear gray-box models with application to industrial robots. In *46th IEEE Conference on Decision and Control*, pages 5110–5116, Piscataway, N.J., 2007. Institute of Electrical and Electronics Engineers. ISBN 978-1-4244-1497-0. doi: 10.1109/CDC.2007.4434059.

E. Wernholt and S. Moberg. Frequency-Domain Gray-Box Identification of Industrial Robots. *IFAC Proceedings Volumes*, 41(2):15372–15380, 2008a. ISSN 14746670. doi: 10.3182/20080706-5-KR-1001.02600.

E. Wernholt and S. Moberg. Experimental comparison of methods for multivariable frequency response function estimation. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 17(1 PART 1), 2008b. doi: 10.3182/20080706-5-KR-1001.3731.

W. Wu, S. Zhu, X. Wang, and H. Liu. Closed-Loop Dynamic Parameter Identification of Robot Manipulators Using Modified Fourier Series. *International Journal of Advanced Robotic Systems*, 9(1):29, 2012a. ISSN 1729-8814. doi: 10.5772/45818.

Y. Wu, A. Klimchik, A. Pashkevich, S. Caro, and B. Furet. Optimality Criteria for Measurement Poses Selection in Calibration of Robot Stiffness Parameters. In *Proceedings of the ASME 11th biennial conference on engineering systems design and analysis 2012*, pages 185–194, New York, 2012b. ASME. ISBN 978-0-7918-4486-1. doi: 10.1115/ESDA2012-82213.

T. Yoshikawa and K. Hosoda. Modeling of flexible manipulators using virtual rigid links and passive joints. In *Proceedings IROS '91:IEEE/RSJ International Workshop on Intelligent Robots and Systems '91*, pages 967–972. IEEE, 1991. ISBN 0-7803-0067-X. doi: 10.1109/IROS.1991.174614.

S. Zimmermann. *Dynamic modeling of robotic manipulators for accuracy evaluation*. Master's thesis, Technical University of Munich, München, 2018.

Part II

Publications

Papers

The papers associated with this thesis have been removed for copyright reasons. For more details about these see:

https://doi.org/10.3384/9789180751988

Licentiate Theses
Division of Automatic Control
Linköping University

P. Andersson: Adaptive Forgetting through Multiple Models and Adaptive Control of Car Dynamics. Thesis No. 15, 1983.

B. Wahlberg: On Model Simplification in System Identification. Thesis No. 47, 1985.

A. Isaksson: Identification of Time Varying Systems and Applications of System Identification to Signal Processing. Thesis No. 75, 1986.

G. Malmberg: A Study of Adaptive Control Missiles. Thesis No. 76, 1986.

S. Gunnarsson: On the Mean Square Error of Transfer Function Estimates with Applications to Control. Thesis No. 90, 1986.

M. Viberg: On the Adaptive Array Problem. Thesis No. 117, 1987.

K. Ståhl: On the Frequency Domain Analysis of Nonlinear Systems. Thesis No. 137, 1988.

A. Skeppstedt: Construction of Composite Models from Large Data-Sets. Thesis No. 149, 1988.

P. A. J. Nagy: MaMiS: A Programming Environment for Numeric/Symbolic Data Processing. Thesis No. 153, 1988.

K. Forsman: Applications of Constructive Algebra to Control Problems. Thesis No. 231, 1990.

I. Klein: Planning for a Class of Sequential Control Problems. Thesis No. 234, 1990.

F. Gustafsson: Optimal Segmentation of Linear Regression Parameters. Thesis No. 246, 1990.

H. Hjalmarsson: On Estimation of Model Quality in System Identification. Thesis No. 251, 1990.

S. Andersson: Sensor Array Processing; Application to Mobile Communication Systems and Dimension Reduction. Thesis No. 255, 1990.

K. Wang Chen: Observability and Invertibility of Nonlinear Systems: A Differential Algebraic Approach. Thesis No. 282, 1991.

J. Sjöberg: Regularization Issues in Neural Network Models of Dynamical Systems. Thesis No. 366, 1993.

P. Pucar: Segmentation of Laser Range Radar Images Using Hidden Markov Field Models. Thesis No. 403, 1993.

H. Fortell: Volterra and Algebraic Approaches to the Zero Dynamics. Thesis No. 438, 1994.

T. McKelvey: On State-Space Models in System Identification. Thesis No. 447, 1994.

T. Andersson: Concepts and Algorithms for Non-Linear System Identifiability. Thesis No. 448, 1994.

P. Lindskog: Algorithms and Tools for System Identification Using Prior Knowledge. Thesis No. 456, 1994.

J. Plantin: Algebraic Methods for Verification and Control of Discrete Event Dynamic Systems. Thesis No. 501, 1995.

J. Gunnarsson: On Modeling of Discrete Event Dynamic Systems, Using Symbolic Algebraic Methods. Thesis No. 502, 1995.

A. Ericsson: Fast Power Control to Counteract Rayleigh Fading in Cellular Radio Systems. Thesis No. 527, 1995.

M. Jirstrand: Algebraic Methods for Modeling and Design in Control. Thesis No. 540, 1996.

K. Edström: Simulation of Mode Switching Systems Using Switched Bond Graphs. Thesis No. 586, 1996.

J. Palmqvist: On Integrity Monitoring of Integrated Navigation Systems. Thesis No. 600, 1997.

A. Stenman: Just-in-Time Models with Applications to Dynamical Systems. Thesis No. 601, 1997.

M. Andersson: Experimental Design and Updating of Finite Element Models. Thesis No. 611, 1997.

U. Forssell: Properties and Usage of Closed-Loop Identification Methods. Thesis No. 641, 1997.

M. Larsson: On Modeling and Diagnosis of Discrete Event Dynamic systems. Thesis No. 648, 1997.

N. Bergman: Bayesian Inference in Terrain Navigation. Thesis No. 649, 1997.

V. Einarsson: On Verification of Switched Systems Using Abstractions. Thesis No. 705, 1998.

J. Blom, F. Gunnarsson: Power Control in Cellular Radio Systems. Thesis No. 706, 1998.

P. Spångéus: Hybrid Control using LP and LMI methods – Some Applications. Thesis No. 724, 1998.

M. Norrlöf: On Analysis and Implementation of Iterative Learning Control. Thesis No. 727, 1998.

A. Hagenblad: Aspects of the Identification of Wiener Models. Thesis No. 793, 1999.

F. Tjärnström: Quality Estimation of Approximate Models. Thesis No. 810, 2000.

C. Carlsson: Vehicle Size and Orientation Estimation Using Geometric Fitting. Thesis No. 840, 2000.

J. Löfberg: Linear Model Predictive Control: Stability and Robustness. Thesis No. 866, 2001.

O. Härkegård: Flight Control Design Using Backstepping. Thesis No. 875, 2001.

J. Elbornsson: Equalization of Distortion in A/D Converters. Thesis No. 883, 2001.

J. Roll: Robust Verification and Identification of Piecewise Affine Systems. Thesis No. 899, 2001.

I. Lind: Regressor Selection in System Identification using ANOVA. Thesis No. 921, 2001.

R. Karlsson: Simulation Based Methods for Target Tracking. Thesis No. 930, 2002.

P.-J. Nordlund: Sequential Monte Carlo Filters and Integrated Navigation. Thesis No. 945, 2002.

M. Östring: Identification, Diagnosis, and Control of a Flexible Robot Arm. Thesis No. 948, 2002.

C. Olsson: Active Engine Vibration Isolation using Feedback Control. Thesis No. 968, 2002.

J. Jansson: Tracking and Decision Making for Automotive Collision Avoidance. Thesis No. 965, 2002.

N. Persson: Event Based Sampling with Application to Spectral Estimation. Thesis No. 981, 2002.

D. Lindgren: Subspace Selection Techniques for Classification Problems. Thesis No. 995, 2002.

E. Geijer Lundin: Uplink Load in CDMA Cellular Systems. Thesis No. 1045, 2003.

M. Enqvist: Some Results on Linear Models of Nonlinear Systems. Thesis No. 1046, 2003.

T. Schön: On Computational Methods for Nonlinear Estimation. Thesis No. 1047, 2003.

F. Gunnarsson: On Modeling and Control of Network Queue Dynamics. Thesis No. 1048, 2003.

S. Björklund: A Survey and Comparison of Time-Delay Estimation Methods in Linear Systems. Thesis No. 1061, 2003.

M. Gerdin: Parameter Estimation in Linear Descriptor Systems. Thesis No. 1085, 2004.

A. Eidehall: An Automotive Lane Guidance System. Thesis No. 1122, 2004.

E. Wernholt: On Multivariable and Nonlinear Identification of Industrial Robots. Thesis No. 1131, 2004.

J. Gillberg: Methods for Frequency Domain Estimation of Continuous-Time Models. Thesis No. 1133, 2004.

G. Hendeby: Fundamental Estimation and Detection Limits in Linear Non-Gaussian Systems. Thesis No. 1199, 2005.

D. Axehill: Applications of Integer Quadratic Programming in Control and Communication. Thesis No. 1218, 2005.

J. Sjöberg: Some Results On Optimal Control for Nonlinear Descriptor Systems. Thesis No. 1227, 2006.

D. Törnqvist: Statistical Fault Detection with Applications to IMU Disturbances. Thesis No. 1258, 2006.

H. Tidefelt: Structural algorithms and perturbations in differential-algebraic equations. Thesis No. 1318, 2007.

S. Moberg: On Modeling and Control of Flexible Manipulators. Thesis No. 1336, 2007.

J. Wallén: On Kinematic Modelling and Iterative Learning Control of Industrial Robots. Thesis No. 1343, 2008.

J. Harju Johansson: A Structure Utilizing Inexact Primal-Dual Interior-Point Method for Analysis of Linear Differential Inclusions. Thesis No. 1367, 2008.

J. D. Hol: Pose Estimation and Calibration Algorithms for Vision and Inertial Sensors. Thesis No. 1370, 2008.

H. Ohlsson: Regression on Manifolds with Implications for System Identification. Thesis No. 1382, 2008.

D. Ankelhed: On low order controller synthesis using rational constraints. Thesis No. 1398, 2009.

P. Skoglar: Planning Methods for Aerial Exploration and Ground Target Tracking. Thesis No. 1420, 2009.

C. Lundquist: Automotive Sensor Fusion for Situation Awareness. Thesis No. 1422, 2009.

C. Lyzell: Initialization Methods for System Identification. Thesis No. 1426, 2009.

R. Falkeborn: Structure exploitation in semidefinite programming for control. Thesis No. 1430, 2010.

D. Petersson: Nonlinear Optimization Approaches to $\mathcal{H}_2$-Norm Based LPV Modelling and Control. Thesis No. 1453, 2010.

Z. Sjanic: Navigation and SAR Auto-focusing in a Sensor Fusion Framework. Thesis No. 1464, 2011.

K. Granström: Loop detection and extended target tracking using laser data. Thesis No. 1465, 2011.

J. Callmer: Topics in Localization and Mapping. Thesis No. 1489, 2011.

F. Lindsten: Rao-Blackwellised particle methods for inference and identification. Thesis No. 1480, 2011.

M. Skoglund: Visual Inertial Navigation and Calibration. Thesis No. 1500, 2011.

S. Khoshfetrat Pakazad: Topics in Robustness Analysis. Thesis No. 1512, 2011.

P. Axelsson: On Sensor Fusion Applied to Industrial Manipulators. Thesis No. 1511, 2011.

A. Carvalho Bittencourt: On Modeling and Diagnosis of Friction and Wear in Industrial Robots. Thesis No. 1516, 2012.

P. Rosander: Averaging level control in the presence of frequent inlet flow upsets. Thesis No. 1527, 2012.

N. Wahlström: Localization using Magnetometers and Light Sensors. Thesis No. 1581, 2013.

R. Larsson: System Identification of Flight Mechanical Characteristics. Thesis No. 1599, 2013.

Y. Jung: Estimation of Inverse Models Applied to Power Amplifier Predistortion. Thesis No. 1605, 2013.

M. Syldatk: On Calibration of Ground Sensor Networks. Thesis No. 1611, 2013.

M. Roth: Kalman Filters for Nonlinear Systems and Heavy-Tailed Noise. Thesis No. 1613, 2013.

D. Simon: Model Predictive Control in Flight Control Design — Stability and Reference Tracking. Thesis No. 1642, 2014.

J. Dahlin: Sequential Monte Carlo for inference in nonlinear state space models. Thesis No. 1652, 2014.

M. Kok: Probabilistic modeling for positioning applications using inertial sensors. Thesis No. 1656, 2014.

J. Linder: Graybox Modelling of Ships Using Indirect Input Measurements. Thesis No. 1681, 2014.

G. Mathai: Direction of Arrival Estimation of Wideband Acoustic Wavefields in a Passive Sensing Environment. Thesis No. 1721, 2015.

I. Nielsen: On Structure Exploiting Numerical Algorithms for Model Predictive Control. Thesis No. 1727, 2015.

C. Veibäck: Tracking of Animals Using Airborne Cameras. Thesis No. 1761, 2016.

N. Evestedt: Sampling Based Motion Planning for Heavy Duty Autonomous Vehicles. Thesis No. 1762, 2016.

H. Nyqvist: On Pose Estimation in Room-Scaled Environments. Thesis No. 1765, 2016.

Y. Zhao: Position Estimation in Uncertain Radio Environments and Trajectory Learning. Thesis No. 1772, 2017.

P. Kasebzadeh: Parameter Estimation for Mobile Positioning Applications. Thesis No. 1786, 2017.

K. Radnosrati: On Timing-Based Localization in Cellular Radio Networks. Thesis No. 1808, 2018.

G. Lindmark: Methods and Algorithms for Control Input Placement in Complex Networks. Thesis No. 1814, 2018.

M. Lindfors: Frequency Tracking for Speed Estimation. Thesis No. 1815, 2018.

D. Ho: Some results on closed-loop identification of quadcopters. Thesis No. 1826, 2018.

O. Ljungqvist: On motion planning and control for truck and trailer systems. Thesis No. 1832, 2019.

P. Boström-Rost: On Informative Path Planning for Tracking and Surveillance. Thesis No. 1838, 2019.

K. Bergman: On Motion Planning Using Numerical Optimal Control. Thesis No. 1843, 2019.

M. Klingspor: Low-rank optimization in system identification. Thesis No. 1855, 2019.

A. Bergström: Timing-Based Localization using Multipath Information. Thesis No. 1867, 2019.

F. Ljungberg: Estimation of Nonlinear Greybox Models for Marine Applications. Thesis No. 1880, 2020.

E. Hedberg: Control, Models and Industrial Manipulators. Thesis No. 1894, 2020.

R. Forsling: Decentralized Estimation Using Conservative Information Extraction. Thesis No. 1897, 2020.

D. Arnström: On Complexity Certification of Active-Set QP Methods with Applications to Linear MPC. Thesis No. 1901, 2021.

M. Malmström: Uncertainties in Neural Networks: A System Identification Approach. Thesis No. 1902, 2021.

K. Nielsen: Robust LIDAR-Based Localization in Underground Mines. Thesis No. 1906, 2021.

H. Haghshenas: Time-Optimal Cooperative Path Tracking for Multi-Robot Systems. Thesis No. 1915, 2021.

A. Kullberg: On Joint State Estimation and Model Learning using Gaussian Process Approximations. Thesis No. 1917, 2021.

J. Nordlöf: On Landmark Densities in Minimum-Uncertainty Motion Planning. Thesis No. 1927, 2022.

FACULTY OF SCIENCE AND ENGINEERING

Linköping Studies in Science and Technology, Licentiate No. 1963, 2023
Department of Electrical Engineering

Linköping University
SE-581 83 Linköping, Sweden

www.liu.se

LINKÖPING
UNIVERSITY