Linköping Studies in Science and Technology
Licentiate Thesis No. 1964

Interaction and Uncertainty-Aware Motion Planning for Autonomous Vehicles Using Model Predictive Control

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Linköping 2023

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To my family

Populärvetenskaplig sammanfattning

Rörelseplanering spelar en betydande roll för att möjliggöra framsteg inom autonoma fordon med potential att rädda liv genom att undvika olyckor och förbättra trafikeffektiviteten. I en prediktiv rörelseplaneringsstrategi predikterar det egna fordonet rörelsen hos omgivande fordon och använder dessa prediktioner för att planera en säker trajektoria. I dynamiska trafiksituationer med multipla omgivande fordon är en central forskningsfråga hur man ska ta hänsyn till de omgivande fordonens interaktioner och rörelseosäkerheter för att åstadkomma en robust rörelseplanering.

Den här licentiatavhandlingen föreslår en modellprediktiv regleringsansats (MPC) för rörelseplanering i osäkra och dynamiska flerfordonsmiljöer. Robust och säker modellprediktiv regleringsbaserad rörelseplanering som tar hänsyn till interaktioner och osäkerheter hos rörelsen för omgivande fordon är ett öppet och utmanande problem, vilket är den primära motiveringen för den forskning som presenteras i denna avhandling.

Modellprediktiv reglering (MPC) är en vanlig ansats för rörelseplanering för autonoma fordon. Denna avhandling presenterar metoder som är steg mot att lösa rörelseplaneringsproblemet där interaktion mellan fordon och osäkerhet i rörelser för omgivande fordon beaktas. Det första bidraget fokuserar på interaktionen mellan omgivande fordon. En modellprediktiv regulator har utvecklats baserat på en modell för hur omgivande fordon interagerar och påverkar varandras beteende. Denna modell integreras sedan som tidsvarierande referensmål för det optimala styrningsproblemet vilket ger en förutseende och robust planering för det egna fordonet.

I det andra bidraget utökas den föreslagna MPC-metoden för att ta hänsyn till de multimodala rörelseosäkerheterna hos omgivande fordon; det finns en osäkerhet i vad de omgivande fordonen kommer göra härnäst och det är osäkert hur de kommer genomföra det. Baserat på en modellering av osäkerheterna, en delvis datadriven ansats, inkluderas en säkerhetsparameter i regulatorn som möjliggör en avvägning mellan prestanda och robusthet hos MPC-planeraren.

Den tredje bidraget i avhandlingen är nya metoder för att kvantifiera rörelseosäkerheten hos omgivande fordon och att använda denna för robust planering, utan att det egna fordonet blir för konservativ i sitt agerande. Den föreslagna ansatsen bygger på robust MPC där en riskmedvetenhet introduceras. Simuleringar av motorvägskörning med omgivande fordon med rörelseosäkerhet visar att metoden är mindre konservativ än en konventionell robust MPC och mer robust än en deterministisk MPC.

Abstract

Motion planning plays a significant role in enabling advances of autonomous vehicles in saving lives and improving traffic efficiency. In a predictive motion-planning strategy, the ego vehicle predicts the motion of surrounding vehicles and uses these predictions to plan collision-free reference trajectories. In dynamic multi-vehicle traffic environments, a key research question is how to consider vehicle-to-vehicle interactions and motion uncertainties of the surrounding vehicles in the motion planner to achieve resilient motion planning of the autonomous ego vehicle.

This Licentiate Thesis proposes a model predictive control (MPC)-based approach to achieve safe motion planning in uncertain and dynamic multi-vehicle driving environments. MPC has been widely applied for the motion planning of autonomous vehicles. However, designing resilient MPC-based motion planners that consider interactions and uncertainties of surrounding vehicles remains an open and challenging problem, which is the primary motivation for the research presented in this thesis.

This thesis makes several contributions toward solving the interaction and uncertainty-aware motion-planning problems. The first contribution is an MPC, which is called interaction-aware moving target MPC. It is designed based on the combination of an interaction-aware motion-prediction model and time-varying reference targets of the optimal control problem for proactive and non-local trajectory planning in multi-vehicle dynamic scenarios.

In the second contribution, the proposed MPC is extended to account for the multi-modal motion uncertainties of surrounding vehicles, including the maneuver and trajectory uncertainties, which are predicted by combining an interaction-aware motion-prediction model and a data-driven approach. Based on the modeling of uncertainties, a safety-awareness parameter is included in the design to compute the obstacle occupancy for achieving a trade-off between the performance and robustness of the MPC planner. The efficiency of the method is illustrated in challenging highway-driving simulation scenarios and a driving scenario from a recorded traffic dataset.

The third contribution of this thesis is quantifying the motion uncertainty of surrounding obstacles to reduce the conservativeness of the motion planner while pursuing robustness. To this end, a robust motion-planning method is designed for robotic systems based on uncertainty quantification of surrounding obstacles. The proposed MPC is called risk-aware robust MPC, as the risk of robustness reduction through uncertainty quantification is analyzed. Simulations in highway merging scenarios of an autonomous vehicle with uncertain surrounding vehicles show that the approach is less conservative than a conventional robust MPC and more robust than a deterministic MPC.

Acknowledgments

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Linköping, April 2023
Jian Zhou

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Contents

Chapter 1

Introduction

1.1 Research Background and Problem Description

This section provides an overview of autonomous driving systems and outlines the specific research questions addressed in this thesis.

1.1.1 Research Background

Autonomous driving technologies have attracted increasing attention from both academia and industry, as a result of the high potential that autonomous vehicles have shown in mitigating damages in safety-critical situations and improving traffic efficiency (Olofsson and Nielsen, 2021), (Guériau and Dusparic, 2020). Developing an autonomous vehicular system is comprehensive and requires a combination of several core technical components, which include sensing, perception, motion planning, and motion control (Murphey et al., 2022, p. 6–7). An overview of an architecture of an autonomous vehicle is shown in Fig. 1. In a physical driving environment, an autonomous vehicle first uses sensors, e.g., cameras, radar, lidar, etc., to acquire environmental information and vehicular state information. The sensory data are fused and analyzed in the perception system for situational awareness, which enables the vehicle to have knowledge about its state and the surrounding environment. Based on the perception module output, the motion-planning system predicts the motion of surrounding obstacles, e.g., surrounding vehicles, pedestrians, etc., and then computes a feasible reference path or trajectory to reach a user-specified

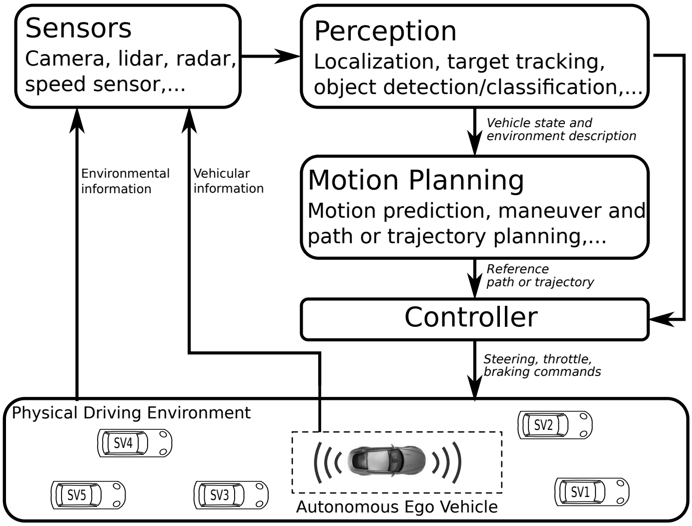


Figure 1: An overview of the autonomous driving system, SV is the abbreviation of surrounding vehicle (adapted from (Murphey et al., 2022, Fig. 1)).

goal. The difference between a path and a trajectory is that a path is a sequence of waypoints that represents a route, while a trajectory is a time-parameterized path (Paden et al., 2016). Finally, a lower-level path or trajectory-tracking controller adjusts actuation outputs, which include steering, throttle, and braking signals, to minimize errors in following the reference (Hua et al., 2020), (Paden et al., 2016).

It is clear that each of the four fundamental components in Fig. 1 plays an important role in enabling the functionality of the autonomous vehicle. However, each component also presents its own unique set of challenges, from designing sensors that are robust to different driving conditions, to developing algorithms that accurately understand the vehicle states and complex traffic situations, to constructing motion planners that are safe and resilient in uncertain traffic environments. While progress has been made in each of these areas, the problems are far from completely solved (Murphey et al., 2022, p. 7–14). Furthermore, these components are highly interdependent as they form a closed loop in an autonomous driving system. For example, inaccurate information from the perception system may lead to an unsafe reference trajectory computed by the motion planner and finally result in a collision with an obstacle. The challenges in designing each module and the interplay of the modules make the autonomous vehicle an extensive and complex system, and significant

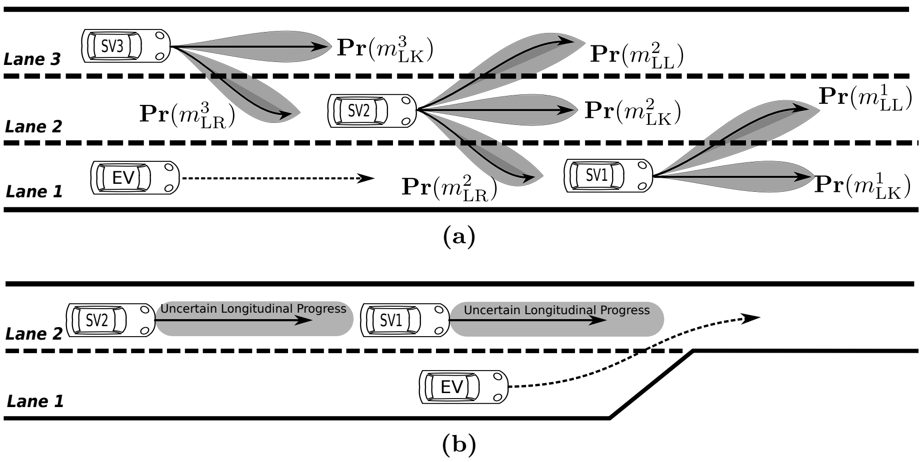


Figure 2: Examples of motion planning of an autonomous ego vehicle in multiple-vehicle dynamic environments. (a) A highway driving scenario where there are uncertainties in the intended maneuvers and planned trajectories of the surrounding vehicles. (b) A forced merging scenario where the surrounding vehicles have uncertain longitudinal accelerations.

research efforts are required in each component to release the full potential of autonomous vehicles.

1.1.2 Problem Description

Given the challenges mentioned in the previous section, the primary objective of this thesis is to develop motion-planning algorithms for autonomous vehicles. This is motivated by the crucial role of the reference path or trajectory, as depicted in Fig. 1, in ensuring the safety and efficiency of the autonomous ego vehicle in complex and dynamic driving environments that involve multiple surrounding vehicles (Dixit et al., 2018). The challenging motion-planning problem in such environments can be illustrated by the examples in Fig. 2, where EV and SV mean the ego vehicle and the surrounding vehicle, respectively. It is assumed that the EV and SV are not connected such that they can not communicate with each other.

The first observation from Fig. 2a is that the intended maneuvers of the SVs are uncertain to the EV, as each SV has the option to either change lanes or maintain its current lane. Furthermore, the planned trajectory of each SV to execute its intended maneuver is also unknown to the EV, as indicated by the shaded area surrounding each maneuver of the SVs. The unknown intended maneuvers and planned trajectories of the SVs are referred to as motion uncertainty of SVs (Brüdigam, 2022), which will inevitably impact the decision-making and motion-planning strategy of the EV, as the EV has to consider the collision avoidance with the SVs (Carvalho et al., 2015). Secondly, Fig. 2a implicitly indicates that the vehicles involved in the scenario are mutually influencing each other, and this is referred to as vehicle-to-vehicle interactions (Liu et al., 2023c). For instance, if SV2 suddenly brakes, it is reasonable to assume that SV3 would have a low probability of changing to lane 2 because of the risk of a rear-end collision related to SV2. As a result, the motion planner of the EV can potentially ignore some unnecessary constraints related to SV2 by understanding these vehicle-to-vehicle interactions. By doing so, the performance of the EV can be improved while still ensuring safe navigation in uncertain driving environments.

A similar mechanism applies for the forced merging scenario in Fig. 2b, where the EV has to either stop before the end of lane 1 or merge to lane 2 when the SVs approach. The scenario presents a challenge where the longitudinal accelerations of SVs are uncertain, making it difficult for the EV to accurately predict the longitudinal progress of SVs over the planning horizon. Overestimating the uncertain longitudinal progress of the SVs may lead to an overly conservative decision by the EV, resulting in the EV giving up the merging and instead decelerating to wait for the SVs to pass. On the other hand, inadequate awareness of uncertainties may lead to a collision with the SVs. Therefore, it is essential to quantify the uncertainty of SVs to achieve a balance between safety and performance, reducing the conservativeness of the planner while ensuring safety.

Inspired by the examples in Fig. 2, this thesis aims at finding solutions for complex motion-planning problems for an autonomous ego vehicle in multiple-vehicle dynamic and uncertain driving environments, based on the understanding of vehicle-to-vehicle interactions and quantification of motion uncertainties of surrounding vehicles. Therefore, the motion-planning method should be interaction and uncertainty-aware in complex environments and have good robustness against uncertainties. This is expected to be achieved by a predictive motion-planning strategy, as it has the ability to directly incorporate a proactive motion-prediction model with a resilient motion planner (Chen et al., 2022), (Villagra et al., 2023). The specific problems that should be addressed in designing the predictive motion planner are formulated in three sub-problems:

1. For motion prediction of surrounding vehicles, the prediction strategy should solve two sub-problems:

   (a) **Problem 1: Describing vehicle-to-vehicle interactions.** Modeling the vehicle-to-vehicle and vehicle-to-environment interactions between surrounding vehicles, such that the planner is considering the interactive behaviors of other vehicles.

   (b) **Problem 2: Quantifying obstacle uncertainties.** Quantifying motion uncertainties of surrounding vehicles to achieve a trade-off between performance and robustness of the motion-planning strategy of the autonomous ego vehicle.

2. **Problem 3: Designing a functional planning strategy.** For motion planning of the ego vehicle, designing a resilient and efficient motion-planning method based on the awareness of interactions and uncertainties of surrounding vehicles in multiple-vehicle dynamic environments.

1.2 Basic Concepts of Motion Planning

This section introduces the technical background and concepts of motion-planning problems of autonomous vehicles. To be specific, Section 1.2.1 presents general motion-planning methods of autonomous vehicles and addresses the methods that are suitable for solving the problems discussed in Section 1.1.2. Section 1.2.2 elaborates on the details of the modeling of vehicle-to-vehicle interactions and the quantification of obstacle uncertainties in dynamic driving environments. Section 1.2.3 describes how the interaction and uncertainty awareness of the surrounding traffic can be incorporated into an online predictive motion-planning framework to generate the optimal reference trajectory for the ego vehicle. In this chapter, the n-dimensional real number vector space is represented as $\mathbb{R}^n$, $\mathbb{R}^n_+$ means the n-dimensional non-negative real number vector space, $\mathbb{N}^n$ means the n-dimensional natural number vector space, and $\mathbb{N}^n_+$ means the n-dimensional positive integer vector space.

1.2.1 Motion Planning of Autonomous Vehicles

This subsection reviews common motion-planning methods for autonomous vehicles. At first, some general and basic definitions in motion-planning problems are introduced:

- **Dynamic model:** The dynamic model describes the evolution of the state of a system based on its current state and control input. A continuous-time dynamic model is often expressed as ordinary differential equations (ODEs)

$$\dot{x}(t) = f(x(t), u(t)),\ x(t_0) = x_0, \tag{1.1}$$

where t is the continuous-time instant, $x(t) \in \mathbb{R}^{n_x}$ is the system state at time t, $u(t) \in \mathbb{R}^{n_u}$ is the control input at time t, t_0 indicates the current time instant, and x_0 is the current state.

The model (1.1) can be discretized by different approaches with a suitable sampling interval T to get a discrete-time model (Ascher and Petzold, 1998, Chapter 2–5), which results in a relation

$$x_{k+1} = f(x_k, u_k). \tag{1.2}$$

Both the ego vehicle and the surrounding vehicles in a motion-planning problem can be described in the form of (1.1) or (1.2), while the specific formulations are adapted based on the problem.

- **State and control admissible sets:** The control admissible set of model (1.1) or model (1.2) is defined as $\mathcal{U}$, which means $u(t) \in \mathcal{U}$, $\forall t \in \mathbb{R}_+$, or $u_k \in \mathcal{U}$, $\forall k \in \mathbb{N}$. Similarly, the state admissible set is defined as $\mathcal{X}$. For model (1.1), it follows that $x(t) \in \mathcal{X}$, $\forall t \in \mathbb{R}_+$, and for model (1.2) it has $x_k \in \mathcal{X}$, $\forall k \in \mathbb{N}$.

- **Prediction horizon:** Denote the current time instant of model (1.1) by t_0. The prediction horizon, which is defined as $t_h = t_f - t_0$, refers to the length of time into the future that the system is being predicted or planned over. Here, t_f is the time at the end of the horizon. The prediction horizon for the discrete-time model (1.2) is defined as $N \in \mathbb{N}_+$. Denote by k the current time step. The time steps over the horizon are then expressed as $h = k+1, \ldots, k+N$ and $t_h = N \cdot T$. In motion-planning problems, the prediction horizon is an important design parameter that affects the performance of the motion planner. Generally, a longer prediction horizon allows for a more proactive forecast of future states and improves path or trajectory stability, while at the same time increasing the computational complexity. Conversely, a shorter horizon reduces computation time but also results in more reactive and less stable motion planning.

- **Collision-free state space:** Denote the collision-free state space by $\mathcal{X}_{\text{free}}$. It is defined as

$$\mathcal{X}_{\text{free}} = \{x \mid x \in \mathcal{X},\ Hx \cap \mathcal{O}_{\text{obs}} = \emptyset\}, \tag{1.3}$$

  where $\mathcal{O}_{\text{obs}}$ is the obstacle region and matrix H fetches the position-related elements from the state vector x.

In the context of autonomous systems design, motion planning refers to the computational problem of finding a sequence of feasible paths or trajectories, in the collision-free state space $\mathcal{X}_{\text{free}}$, to move the system from its initial state x_I to a goal state x_G (Bergman, 2019), (LaValle, 2006, Chapter 1.3). The motion-planning process usually involves satisfying movement constraints while potentially optimizing some aspects of the movement. The movement constraints typically contain the differential constraints of the dynamic system, collision-avoidance constraints with both static and dynamic obstacles, and constraints specifically defined by the user, e.g., constraints by traffic rules and user preferences. A motion-planning example of an autonomous vehicle's parking scenario is shown in Fig. 3, where a kinematically infeasible path that violates the differential constraints, an infeasible path that collides with the obstacle, a suboptimal path that is feasible but not efficient, and an optimal path in terms of the path length in the collision-free state space, are presented and compared.

As illustrated in Fig. 3, the main challenge in a motion-planning problem is to find a feasible reference path or trajectory for the autonomous vehicle and satisfy all pre-defined constraints, while the reference trajectory should be designed by optimizing some criterion, like the execution time, energy consumption, comfort, etc., which makes the problem even harder. In many cases, the feasible or optimal solution is not unique, and many approaches have been proposed in the literature to handle motion-planning problems in various situations. Classical algorithms include graph search-based approaches, sampling-based approaches, and optimal control-based approaches (González et al., 2015).

Graph search-based approach: A graph search-based approach formulates the motion-planning problem as finding a finite number of actions from the control admissible set $\mathcal{U}$ to move the controlled system, which is described by model (1.2), from an initial state x_I to the goal state x_G in a collision-free state space $\mathcal{X}_{\text{free}}$. If the dynamic system contains differential constraints, $\mathcal{X}_{\text{free}}$ is then usually described by state lattices, as shown in Fig. 4, where a set of motion primitives that are computed offline to satisfy the system constraints are applied in sequence to construct a

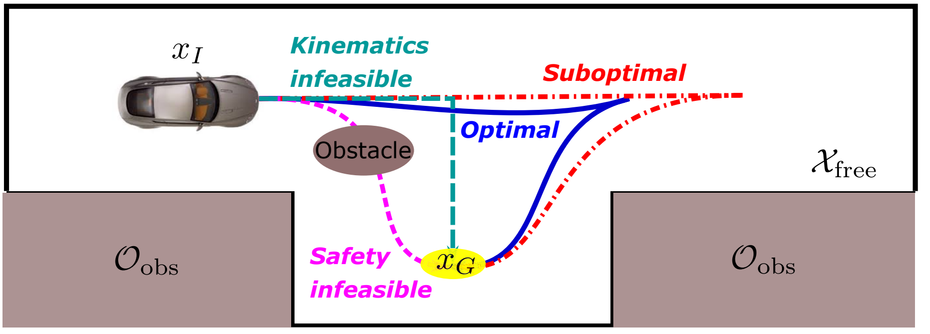


Figure 3: Motion planning of autonomous vehicle parking and different planned paths for the maneuver.

reference path or trajectory connecting x_I and x_G. An example of motion primitives is shown in Fig. 6a. Given $\mathcal{X}_{\text{free}}$, $\mathcal{U}$, and the dynamic system model (1.2), the most important aspect in the graph search approach is to determine the search strategy, e.g., depth-first, breadth-first, minimum cost first, etc. To this end, the graph search approaches further contain Dijkstra's algorithm that finds the path with the lowest cost, the A* algorithm that efficiently finds the lowest-cost path using heuristics, the anytime A* algorithm that achieves a trade-off between optimality and efficiency (Likhachev et al., 2003), etc. The graph search process using motion primitives in a state-lattice space is illustrated in Fig. 4. The details of search strategies are given in (LaValle, 2006, Chapter 2).

Sampling-based approach: The sampling-based approaches contain the rapidly-exploring random tree (RRT) (LaValle, 1998) and RRT* (Karaman and Frazzoli, 2011). The RRT, as indicated by the name, incrementally constructs a tree in the collision-free state space $\mathcal{X}_{\text{free}}$ using random or deterministic samples. The tree has a feature that any two vertices can be connected by exactly one path, such that a feasible path from x_I to x_G is found when x_G becomes a vertex of the tree. As an extension of RRT, RRT* aims at optimal sampling-based planning by using the path with the minimum cost to add a new state to the tree and also applying incremental rewiring of the edges. In addition, RRT and RRT* with differential motion constraints can be achieved by simulating the motion equations to compute the motion primitives with different inputs, then the motion primitives are used online to connect the vertex of the tree and a sampled state. Implementations and comparisons of RRT and RRT* with a particle model are shown in Fig. 5, where the

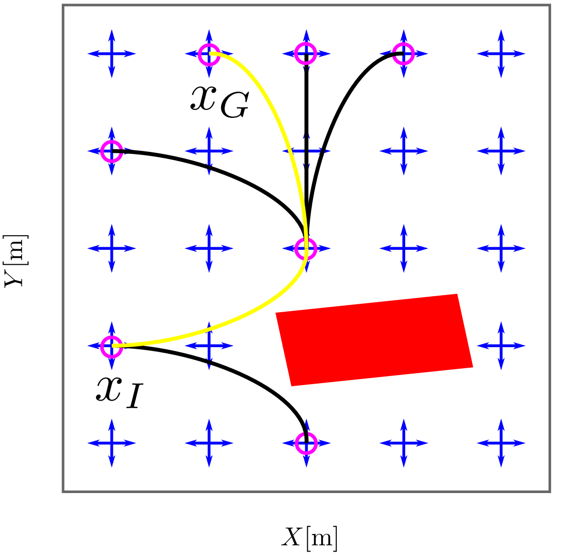


Figure 4: The graph-search lattice planning, the planned reference path is highlighted in yellow (adapted from (Likhachev and Ferguson, 2009, Fig. 1(a))).

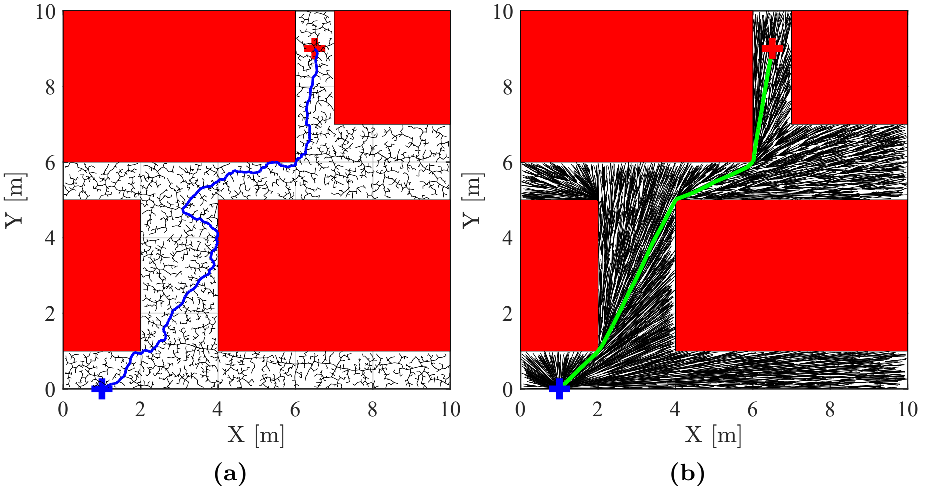
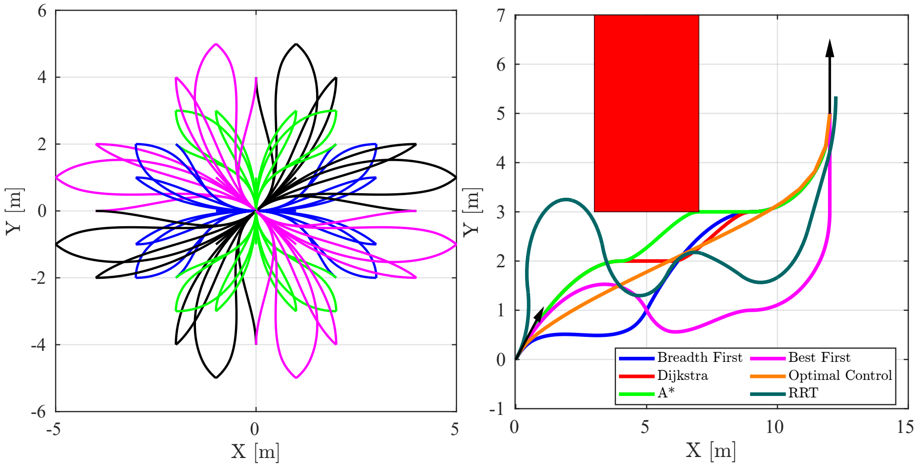


Figure 5: RRT and RRT* with a particle model. (a) RRT, the planning takes 0.22 s. (b) RRT*, the planning takes 3.22 s.

algorithms are implemented according to (Karaman and Frazzoli, 2011).

Optimal control-based approach: This approach essentially formulates the motion-planning problem as an optimal control problem



(a) **(b)**

Figure 6: Motion primitives and comparison of planned paths by different algorithms with a static obstacle. Note that the velocity profile along the path planned by the optimal control approach is time-varying, while the velocities by the other methods are constant. (a) Motion primitives that are generated by balancing the length of the path and the control effort of the vehicle. (b) Path-planning results by different approaches.

(OCP) that contains a cost function and generally multiple constraints. The OCP is solved to obtain the optimal control input to steer the dynamic model of the controlled system to generate the reference trajectory. The cost function commonly minimizes the deviation between the system state and the reference state, the control input, and the collision risk, over a prediction horizon (Mohseni, 2021), while the constraints may contain the differential constraints on the controlled system, collision-avoidance constraints with obstacles (Zhang et al., 2021), etc. Therefore, the formulation of an OCP for motion planning of autonomous vehicles can be highly tailored. Furthermore, if the OCP is solved with a receding horizon, this is referred to as model predictive control (MPC) (Rawlings et al., 2017, Chapter 1), which uses the first element of the sequence of the optimal control input to plan the reference trajectory for the system to reach the next time step. Aspects of OCP for motion planning of autonomous systems are discussed in (Bergman, 2021).

The planned paths of a vehicle system that is described by a single-track model (Mohseni, 2021, Eq. (2.1)) by different motion-planning methods are shown in Fig 6b. The motion primitives in Fig. 6a, which are generated according to a modified Dubins path (Dubins, 1957), are applied for the implementation of the graph search-based and sampling-based algorithms in Fig. 6b. The path planned by the optimal control approach, which is implemented according to (Zhang et al., 2021), is shorter than that by other methods. This means that the optimal control approach here provides a better solution than the other methods in terms of the length of the planned path.

Other methods, such as feedback motion planning, have also been extensively studied (LaValle, 2006, Chapter 8). It is noteworthy that these approaches are not mutually exclusive and can be combined in various ways to address different challenges and requirements in motion-planning applications. For example, in (Fors et al., 2022), a state-lattice planning algorithm with A* was applied to obtain an initial guess of the OCP in an MPC framework, which further computed the reference trajectory adapting to environmental disturbances and uncertainties. To conclude, the graph search-based approach and sampling-based approach have shown good performance in structured and unstructured environments with static obstacles. However, extending the methods for complex dynamic systems subjected to intricate constraints, particularly in the presence of uncertain obstacles in dynamic environments, can be a challenging task. The advantage of OCP is the straightforward inclusion of dynamic systems and the involved constraints to fulfill the motion-planning mission; if the motion planning is performed in an MPC framework, it is suitable for trajectory replanning in dynamic environments by updating the OCP at every time step (Eiras et al., 2022). The primary concerns in optimal control for motion planning are constructing feasible OCPs and solving the OCPs efficiently.

This thesis concerns motion-planning problems of autonomous vehicles in time-varying uncertain environments with multiple surrounding vehicles. To this end, the motion-planning methods in this thesis are developed using the optimal control method based on modeling and prediction of the motion of surrounding vehicles. The examples in Fig. 2 have reflected that two main complications for performing motion planning in such traffic environments are to address the interactions and uncertainties of surrounding vehicles, and they will be further discussed in Section 1.2.2.

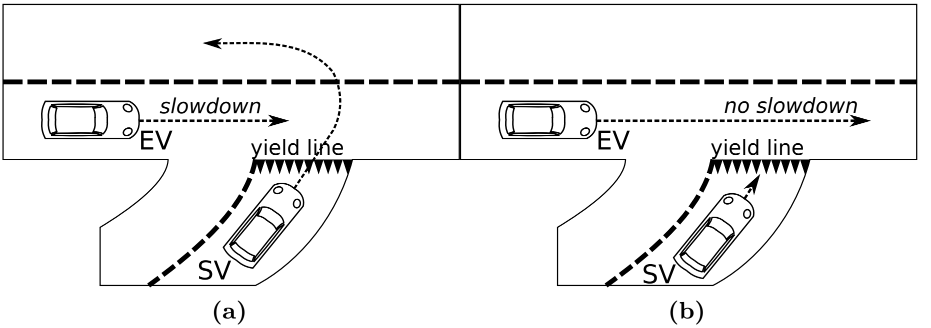


Figure 7: Motion planning of the EV under the impact of the interaction between the SV and the road (adapted from (Lefèvre et al., 2014, Figure 8)) (a) The EV does not consider that the SV will yield. (b) The EV considers that the SV will yield due to the yield line.

1.2.2 Interaction and Uncertainty in Motion Planning

This subsection starts by separately discussing the interaction awareness and uncertainty quantification in motion planning, and finally introduces how to connect them in the motion prediction of surrounding vehicles.

Interaction Awareness in Motion Planning

This part elaborates on **Problem 1** defined in Section 1.1.2, and presents general ideas to handle the problem. In real-world multi-vehicle driving scenarios, the interactions between vehicle to vehicle and vehicle to the environment occur frequently and dynamically, making it crucial and challenging for an effective motion-planning strategy to precisely and efficiently incorporate these interactions to ensure safe navigation. An example of motion planning of an ego vehicle (EV) under the impact of the interaction between the surrounding vehicle (SV) and the environment is illustrated in Fig. 7. In Fig. 7a, the EV does not consider the maneuver of the SV that can be affected by the yield line, such that the EV will decelerate to yield to the SV. In Fig. 7b, the EV considers the yield line and infers that the SV should give priority to the EV. As a result, the motion-planning strategy of the EV is optimized to avoid the unnecessary decelerating maneuver, thus improving driving efficiency while still ensuring collision avoidance. This example reflects that an awareness of the interactions in the driving environments contributes to a more proactive understanding of the situation and a more reliable evaluation of the associated risks, which will finally improve the quality of motion planning (Lefèvre et al., 2014). However, developing interaction-aware motion models to describe the mechanisms behind the complex interplay between vehicles is not trivial, as the decision made by the controller or the human driver of a surrounding vehicle is a hidden variable.

Methods to model the interactions in multiple-vehicle traffic environments include optimization-based approaches (Lefkopoulos et al., 2021), game-theoretic approaches (Li et al., 2018), (Liu et al., 2023b), and learning-based approaches (Westny et al., 2021), (Westny et al., 2023b), (Westny et al., 2023a). While it can be challenging to apply a type of method to uniformly describe the interactions between vehicles in every driving scenario, these methods can help enhance the planner's understanding of vehicle-to-vehicle and vehicle-to-environment interactions to a certain extent. Additionally, these methods have proven effective in modeling complex driving scenarios, demonstrating their usefulness in motion-planning strategies for autonomous vehicles (Zhou et al., 2022b).

Uncertainty Quantification in Motion Planning

This part specifies **Problem 2** defined in Section 1.1.2 and introduces the general strategies to approach the problem. Apart from the vehicle-to-vehicle interactions in multiple-vehicle dynamic driving situations, inherent uncertainties in predicting the motion of surrounding vehicles also play a significant role in affecting the robustness of the motion-planning algorithm (Gao, 2020). The key difficulty of incorporating the motion uncertainties of surrounding vehicles into a motion planner is determining an appropriate method for quantifying the motion uncertainties of a target vehicle (Gao et al., 2022). An overestimation of the uncertainty will shrink the collision-free driving space of the ego vehicle and may result in an infeasible motion-planning problem, while an underestimation of the uncertainty will reduce the robustness of the motion planner. The approaches for estimating the motion uncertainties of dynamic systems, not only limited to vehicular systems, can generally be divided into probabilistic approaches and deterministic approaches (Liu et al., 2023a).

The probabilistic approaches estimate the distribution of uncertainties, e.g., the mean and standard deviation of the position and velocity of a surrounding vehicle, over the prediction horizon. Considering that a surrounding vehicle usually has multiple candidate maneuvers to choose at one moment, and each maneuver could be executed by various candidate trajectories (Brüdigam et al., 2018), (Benciolini et al., 2023), the uncertain states over the prediction horizon are typically described as a multi-modal distribution

$$\mathbf{p}(x(t)) = \sum_{m\in\mathcal{M}} \mathbf{p}(x(t)|m)\mathbf{p}(m), \tag{1.4}$$

where $t \in [t_0, t_f]$, $\mathbf{p}(\cdot)$ means the probability density function (PDF), and $\mathcal{M}$ is the maneuver set. This requires estimating the maneuver probability distribution, i.e., $\mathbf{p}(m)$, and the state probability distribution given the maneuver, i.e., $\mathbf{p}(x(t)|m)$.

The deterministic approaches compute the forward reachable set (Seo et al., 2023) of a surrounding vehicle over a prediction horizon to include all possible states that can be reached by the vehicle with the control action from an admissible input set $\mathcal{U}$, or more generally, the input from an admissible disturbance set $\mathcal{W}$. Given the dynamic model of a surrounding vehicle described as in (1.1), the forward reachable set of the state of the model at time instant t is (Althoff and Magdici, 2016)

$$\mathcal{R}(t) = \left\{ x(t_0) + \int_{t_0}^{t} f(x(\tau), u(\tau))\mathrm{d}\tau \;\middle|\; \forall x(t_0) \in \mathcal{X}(0), \forall \tau : u(\tau) \in \mathcal{U} \right\}, \tag{1.5}$$

where $\mathcal{X}(0)$ is the set of states at time t_0. It follows that the reachable set over a time interval $t \in [t_0, t_f]$ is

$$\mathcal{R}([t_0, t_f]) = \bigcup_{t\in[t_0,t_f]} \mathcal{R}(t). \tag{1.6}$$

The formulas (1.4) or (1.5) are finally applied to construct the obstacle occupancy, which is defined as the space filled by uncertain positions of the obstacle at a time point or over a time interval. Denote by $\mathcal{O}(t)$ the occupancy of the obstacle at time t. Based on (1.4), the occupancy can be formulated as

$$\mathcal{O}(t) = \mathcal{F}(\mathbf{p}(x(t))), \tag{1.7}$$

where the function $\mathcal{F}(\cdot)$ formulates an occupancy region based on $\mathbf{p}(x(t))$. For example, $\mathcal{F}(\cdot)$ can truncate the support of $\mathbf{p}(x(t))$ according to some risk assignment (Zhou et al., 2022a). Note that the obstacle occupancy defined in (1.7) is a probabilistic region that indicates the space occupied by the obstacle with a certain level of confidence.

The obstacle occupancy based on $\mathcal{R}(t)$ can be directly expressed as

$$\mathcal{O}(t) = \{p \mid p = Hx,\ x \in \mathcal{R}(t)\}, \tag{1.8}$$

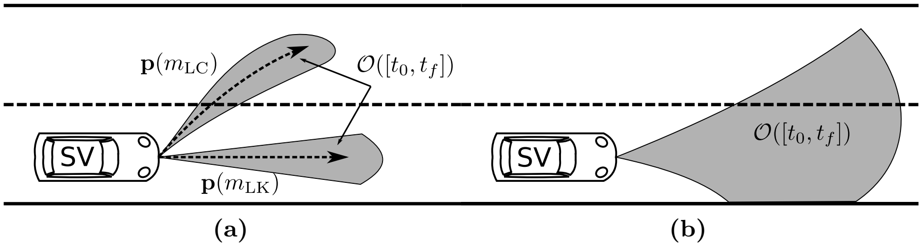


Figure 8: Uncertainty description of an SV over the prediction horizon (a) The probabilistic approach for uncertainty estimation, m_{LC} and m_{LK} mean the maneuver of lane-changing and lane-keeping, respectively. (b) The deterministic approach for uncertainty estimation. The occupancy is truncated according to the construction of the road structure.

where H is defined in the same way as in (1.3). The occupancy over a time interval $t \in [t_0, t_f]$ is defined as:

$$\mathcal{O}([t_0, t_f]) = \bigcup_{t \in [t_0, t_f]} \mathcal{O}(t). \tag{1.9}$$

In (1.5), if $t = +\infty$ then $\mathcal{R}(+\infty)$ is defined as the invariant set (Raković et al., 2005). A distinction between the reachable set and the invariant set is that the reachable set is time-varying, while the invariant set is immutable over time (Berntorp et al., 2020). The benefit of describing the uncertainties by the invariant set is that the invariant set can be computed offline, while the reachable set needs to be evaluated at every time instant. However, the invariant set, since it includes the state of the system at an infinite time instant, can be overly conservative when quantifying the uncertainty of the system. Motion-planning algorithms based on invariant sets can be found in (Berntorp et al., 2020), (Danielson et al., 2020), (Gao et al., 2014). To reduce the conservatism of adopting the worst-case uncertainties in the robust MPC framework, the minimum robust positive invariant (RPI) set of a discrete-time linear system based on quantifying a less conservative disturbance set of the system is proposed in (Gao et al., 2023).

The obstacle occupancy for the discrete-time model (1.2) is calculated similarly. The uncertainty prediction by the probabilistic approach and the deterministic approach is illustrated in Fig. 8, where a surrounding vehicle is driving on a two-lane road. The probabilistic approach predicts the probability of the lane-keeping maneuver ($\mathbf{p}(m_{\mathrm{LK}})$) and the lane-changing maneuver ($\mathbf{p}(m_{\mathrm{LC}})$), then propagates the uncertainty of each maneuver over the prediction horizon. Note that it is possible to incorporate the probabilistic occupancy of different maneuvers in a convex hull for convenience when formulating the collision-avoidance constraints (Cesari et al., 2017). The deterministic approach in Fig. 8b quantifies the forward occupancy over the prediction horizon to cover all reachable positions of the vehicle. The grey areas in Fig. 8, which indicate the occupancy of the vehicle over the prediction horizon, can be included in a planner for robust motion planning, i.e., solving **Problem 3** defined in Section 1.1.2. The details will be given in Section 1.2.3.

In this thesis, the focus is on unconnected driving scenarios where autonomous vehicles cannot exchange information with other traffic participants. In these scenarios, the ego vehicle must predict the motion of surrounding vehicles to ensure safe motion planning. Unlike unconnected scenarios, connected and automated vehicles (CAVs) can share information such as states, intended maneuvers, and planned trajectories (Zhou et al., 2020), eliminating the need for motion predictions. However, CAVs may face research issues such as cooperative planning problems (Mohseni et al., 2021). Methods and techniques regarding CAVs are explored in (Murphey et al., 2022).

Connections between Interaction Awareness and Uncertainty Quantification

The formulations (1.4) and (1.5) provide opportunities for connecting the interaction awareness and uncertainty quantification in the motion prediction of the surrounding vehicle. For example, the maneuver probability $\mathbf{p}(m)$ and the PDF $\mathbf{p}(x(t)|m)$ in (1.4) can be the output of an interaction-aware motion-prediction model (Lefkopoulos et al., 2021), (Zhou et al., 2022a). The control admissible set in (1.5), which is usually assumed to be known or overly estimated, can be quantified by considering the interactions between the surrounding vehicle and the driving situations.

1.2.3 Model Predictive Control for Interaction-Aware and Resilient Motion Planning

Formulation of the OCP

Model predictive control (MPC), also named receding horizon optimization, is regarded as an applicable approach for motion planning in time-varying scenarios because of the inherent planning and replanning capabilities (Nilsson, 2016), (Villagra et al., 2023). As discussed in Section 1.2.1, an MPC motion planner essentially solves an OCP at every time step and applies the first element of the optimal control sequence to refresh the reference trajectory of the ego vehicle, such that it can react to the changes in dynamic environments and avoid moving obstacles.

A continuous-time constrained OCP in an MPC framework for trajectory planning of the ego vehicle that is described by (1.1) can be generally formulated as:

$$\begin{aligned} \underset{u(t)}{\text{minimize}} \quad & \boldsymbol{V}\left(x(t), u(t), x_G\right) && (1.10a) \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)), && (1.10b) \\ & x(t) \in \mathcal{X}_{\text{free}}(t), && (1.10c) \\ & u(t) \in \mathcal{U}, && (1.10d) \end{aligned}$$

where $t \in [t_0, t_f]$ and $\boldsymbol{V}(\cdot)$ is the cost function. The optimization variable of the OCP is $u(t)$, such that the optimal solution to this OCP is denoted by $u^\star(t)$. Based on (1.7) or (1.8), the collision-free state space $\mathcal{X}_{\text{free}}$ in (1.10c) can be defined as:

$$\begin{aligned} \mathcal{X}_{\text{free}}(t) &= \left\{x \mid x \in \mathcal{X},\ \mathcal{O}(Hx) \cap \mathcal{O}_{\text{obs}}(t) = \emptyset\right\}, && (1.11a) \\ \mathcal{O}_{\text{obs}}(t) &= \mathcal{O}(t) \cup \mathcal{O}_{\text{s}}, && (1.11b) \end{aligned}$$

where $\mathcal{O}(Hx)$ means the occupancy of the ego vehicle associated with the vehicle size and state x in the admissible set $\mathcal{X}$, and $\mathcal{O}_{\text{s}}$ means the occupancy of static obstacles. Therefore, the constraint (1.10c) is equivalent to (Zhang et al., 2021):

$$\begin{aligned} \mathcal{O}(Hx(t)) \cap \mathcal{O}_{\text{obs}}(t) &= \emptyset, && (1.12a) \\ x(t) &\in \mathcal{X}. && (1.12b) \end{aligned}$$

The constraint (1.12a) can be further represented as (Schulman et al., 2014)

$$\min_{\sigma} \left\{||\sigma||_2 : (\mathcal{O}(Hx(t)) + \sigma) \cap \mathcal{O}_{\text{obs}}(t) \neq \emptyset\right\} > 0. \qquad (1.13)$$

If the occupancy $\mathcal{O}(Hx(t))$ and $\mathcal{O}_{\text{obs}}(t)$ are represented by polytopes, then the dual of the problem (1.13) of finding the minimum distance between two polytopes has a nice geometric interpretation in terms of separating hyperplanes between the polytopes (Boyd and Vandenberghe, 2004, p. 399–404). This allows to equivalently formulate (1.13) as smooth and tractable constraints in an optimization problem using Farkas' lemma or deriving its Lagrange dual problem (Zhang et al., 2021), (Helling and Meurer, 2023, Section II).

The safety constraint (1.10c) or (1.12) can be designed based on three MPC approaches, the nominal MPC, the robust MPC, and the stochastic MPC (Kouvaritakis and Cannon, 2016). Denote by $\tilde{p}(t)$ the uncertain position of the obstacle at time instant t with the distribution Δ, and $\hat{p}(t)$ the predicted nominal position of the obstacle at time instant t (e.g., the dashed line in Fig. 8a), then the safety constraint in the nominal MPC is constructed as

$$\mathcal{O}(Hx(t)) \cap (\mathcal{O}(\hat{p}(t)) \cup \mathcal{O}_{\mathrm{s}}) = \emptyset. \tag{1.14}$$

Here $\mathcal{O}(\hat{p}(t))$ means the occupancy of the surrounding vehicle associated with the predicted nominal state $\hat{p}(t)$ and the vehicle size. Therefore, the tube $\mathcal{O}(t)$ in (1.12a) is represented by the tube $\mathcal{O}(\hat{p}(t))$, which does not involve any uncertainty. One way to incorporate uncertainty into the safety constraint is to design a robust constraint, which is expressed as (Batkovic et al., 2021)

$$\mathcal{O}(Hx(t)) \cap (\mathcal{O}(\tilde{p}(t)) \cup \mathcal{O}_{\mathrm{s}}) = \emptyset, \ \forall \tilde{p}(t) \sim \Delta. \tag{1.15}$$

Here, the tube $\mathcal{O}(t)$ in (1.12a) is represented by the tube that contains all realizations of the random variable $\tilde{p}(t)$ in the distribution Δ. Another way to formulate safety constraints containing uncertainty is to design the stochastic constraint

$$\mathbf{Pr}[\mathcal{O}(Hx(t)) \cap (\mathcal{O}(\tilde{p}(t)) \cup \mathcal{O}_{\mathrm{s}}) = \emptyset] \geq \varepsilon, \ \forall \tilde{p}(t) \sim \Delta. \tag{1.16}$$

Here $\mathbf{Pr}[\cdot]$ means the probability of an event and $\varepsilon \in [0, 1]$. This is referred to as a chance constraint or probabilistic constraint, which means the probability that the ego vehicle's position does not intersect with the obstacle occupancy at time instant t is no less than ε.

Note that the robust safety constraint in (1.15) and the stochastic safety constraint in (1.16) concern motion uncertainty of the surrounding vehicle. This is different from the problems studied in (Dixit et al., 2019) and (Carvalho et al., 2014), where the modeling uncertainty of the ego vehicle is handled by robust MPC and stochastic MPC, respectively. However, the ideas for dealing with the problems are essentially similar.

Technical Challenges in Formulating the OCP

The formulation (1.10c) can be directly extended for consideration of multiple surrounding obstacles. If the occupancy $\mathcal{O}(t)$ is formulated by considering the motion uncertainty of the surrounding vehicles and the interaction between the surrounding vehicles and the environments, then the MPC planner is interaction and uncertainty-aware for the ego vehicle. However, in many motion-planning applications of autonomous vehicles, it can be challenging to construct the specific OCP, as a result of the following technical challenges:

- **Deciding the interaction and uncertainty-aware goal state in the OCP:** The reference state, or goal state x_G in (1.10a), is significant for the safety of the reference trajectory in uncertain environments. A fixed goal state can be provided by a higher-level decision maker, but it might be unsafe in the presence of abrupt obstacles, e.g., in a cut-in scenario. A time-varying goal state that is computed based on risk assessment can benefit the collision avoidance with obstacles, while updating the goal state at every time instant needs additional computational effort. Therefore, how to efficiently evaluate the goal state x_G of the OCP combined with the description of interactions and uncertainties of surrounding vehicles is difficult in multiple-vehicle uncertain dynamic scenarios.

- **Reformulating tractable safety constraints in the OCP:** The safety constraint (1.10c) forces the ego vehicle to plan the reference trajectory in a collision-free space when following the reference target x_G. Note that a constraint in the form of (1.10c) or (1.12) is not tractable for an optimization algorithm. As such, the safety constraint must be reformulated as a computationally tractable expression when solving the OCP. The complexity and precision of the reformulation will influence the efficiency of solving the OCP and the quality of the generated reference trajectory, such that finding an appropriate reformulation can be problematic in many cases. In particular, if the safety constraint is formulated in the form of a chance constraint as in (1.16), we need to find an analytical expression to represent or approximate the probabilistic constraint to be handled in optimization algorithms. This makes the problem of reformulating the safety constraint even harder.

The problem regarding computing the goal state of the ego vehicle in the OCP is essentially a decision-making process, which can be handled, e.g., by solving an optimization problem (Zhou et al., 2022a), (Dixit et al., 2019), a partially observable Markov decision process (POMDP) (Ulfsjöö and Axehill, 2022), or a discrete search problem (Danielson et al., 2016). The challenge regarding reformulating the safety constraint in the OCP requires constructing tractable constraints in the optimization problem that make the ego vehicle's state stay outside the obstacle occupancy over the prediction horizon. This can be tackled by either exact approaches or approximate approaches. Suppose that in (1.12a) the occupancy of the ego vehicle and the obstacle can be described by a polytope, then exact approaches for representing the constraint include mixed integer programming that introduces extra integer decision variables to make two polytopes to not intersect (Schouwenaars et al., 2001), and the duality approach that reformulates (1.13) as smooth but nonconvex expressions using duality theory (Zhang et al., 2021). The approximate approaches include linearization of the constraint (1.13) around the predicted ego vehicle state $x(t)$ (Carvalho et al., 2014), (Schulman et al., 2014), (Brüdigam et al., 2023), and the ellipsoidal reformation that describes the occupancy by an ellipse (Brüdigam et al., 2021), (Eiras et al., 2022).

The problem regarding the reformulation of the chance constraint (1.16) has also gained a lot of research attention (Farina et al., 2016). It is noteworthy that in very few cases an analytical expression that is equivalent to the probabilistic constraint (1.16) can be obtained. One typical case is that the chance constraint (1.16) with respect to the optimization variable $u(t)$ is linear and the uncertain parameter in the constraint satisfies a normal distribution (Kouvaritakis et al., 2010). However, this is not always realistic in real-world motion-planning problems of autonomous vehicles, as the uncertainty distribution of the surrounding vehicle can be complex and even unknown. This means that approximations are usually applied to reformulate the constraint, e.g., scenario optimization (Rawlings et al., 2017, Chapter 3.7). The techniques for handling the chance constraint with respect to linear systems are reviewed in (Farina et al., 2016), and (Brüdigam, 2022) introduced the application of chance constraints in autonomous vehicles motion-planning problems where the surrounding vehicles are modeled by linear time-invariant models.

Solving the OCP

This subsection finally discusses how to solve the continuous-time constrained OCP (1.10a)–(1.10d), where general solution approaches are summarized in Fig. 9. The first approach obtains $u^\star(t)$ by solving a par-

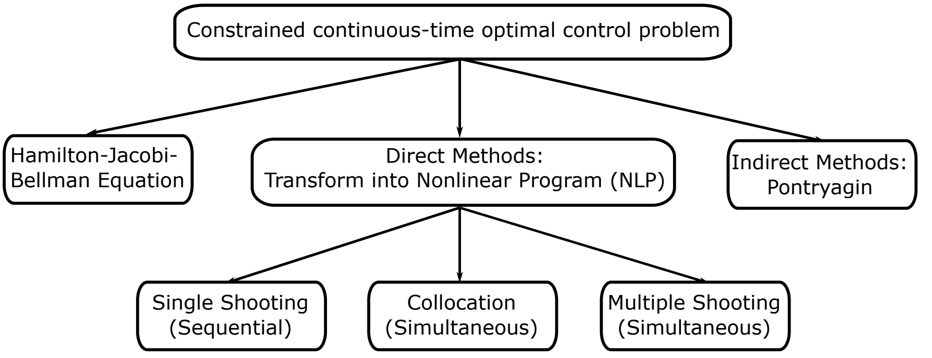


Figure 9: A summary of different methods to solve constrained continuous-time optimal control problem like (1.10a)–(1.10d) (adopted from (Diehl and Gros, 2011, Figure 9.2)).

tial differential equation, the Hamilton-Jacobi-Bellman (HJB) equation. The direct methods discretize the original OCP into a finite-dimensional NLP problem, which is solved to get the optimal control sequence in discrete time. Finally, indirect methods utilize the necessary conditions of optimality, described by Pontryagin's maximum principle (PMP), to transform the OCP into a boundary value problem (BVP) consisting of a set of ODEs (Bergman, 2021), (Diehl and Gros, 2011).

Due to the complex formulations of (1.10a) and (1.10c), it can be hard to formulate and solve the HJB and BVP from the original OCP (Bergman, 2021). Therefore, this thesis applies direct methods for solving the OCPs, as this formulation can fit different numerical optimization solution methods, like active-set methods, penalty and augmented Lagrangian methods, interior-point methods, etc. (Jorge and Stephen, 2006). In addition, the existence of several software packages, like `CasADi` (Andersson et al., 2019), `YALMIP` (Löfberg, 2004), etc., have shown strong performance in efficiently solving large-scale nonlinear optimization problems.

To conclude, this thesis constructs the collision-free space $\mathcal{X}_{\text{free}}(t)$ in (1.10c), or more specifically, the obstacle occupancy $\mathcal{O}_{\text{obs}}(t)$ in (1.11a), based on the awareness of interactions and motion uncertainties of surrounding vehicles. This contributes to solving **Problem 1**–**Problem 2** described in Section 1.1.2. To further realize interaction and uncertainty-aware motion planning, i.e., solving **Problem 3** described in Section 1.1.2, the interaction and uncertainty-aware obstacle occupancy $\mathcal{O}_{\text{obs}}(t)$ is substituted into the OCP as in (1.10) in an MPC framework to compute the optimal reference trajectory for the ego vehicle at every discrete time step.

The two technical challenges regarding formulating the OCP in MPC, as introduced in Section 1.2.3, are solved in each paper in the thesis using different approaches. The primary goal of this thesis is to realize proactive and resilient motion planning of an autonomous ego vehicle in multiple-vehicle driving scenarios, and the specific contributions of each article toward this objective are outlined in Section 1.3.

1.3 Thesis Focus and Contributions

The primary focus of this thesis is to address the research problem, as presented in Section 1.1.2, of motion planning for an autonomous ego vehicle in uncertain driving environments with multiple vehicles. The challenges associated with this problem arise from predicting interactions and quantifying motion uncertainties of surrounding vehicles. The ultimate goal is to develop a robust predictive motion planner that can efficiently compute an optimal reference trajectory based on the awareness of vehicle-to-vehicle interactions and motion uncertainties in dynamic traffic scenarios. The papers presented in this thesis are focused on this topic and contribute to solving the aforementioned problems. Each paper makes specific contributions, which are introduced separately.

Paper I: Interaction-Aware Moving Target Model Predictive Control for Autonomous Vehicles Motion Planning

by Jian Zhou, Björn Olofsson, and Erik Frisk. *2022 European Control Conference (ECC)*, London, United Kingdom, pp. 154-161, 2022.

This paper contributes to addressing **Problem 1** and **Problem 3**, while the uncertainties are not considered. The paper integrates traffic-environment modeling with an MPC to realize interaction-aware dynamic motion planning of an autonomous ego vehicle with multiple surrounding vehicles. The interaction-aware interacting multiple model Kalman filter (IAIMM-KF), which was proposed in (Lefkopoulos et al., 2021), is adopted to hierarchically predict maneuvers and trajectories of surrounding vehicles and to compute safe targets for the ego vehicle. The targets contain the terminal reference speed and the reference lane, which are moving targets as they are updated at each time step. Then, an MPC controller is designed for the ego vehicle to generate an optimal trajectory by following the moving targets and including the prediction results to formulate collision-free constraints. The proposed interaction-aware planning method has a proactive planning ability and can avoid collisions by non-local replanning. The performance is demonstrated by simulations in multiple-vehicle scenarios with comparisons to an MPC planner that tracks constant targets and an MPC planner without interaction awareness, respectively.

Paper II: Interaction-Aware Motion Planning for Autonomous Vehicles with Multi-Modal Obstacle Uncertainties Using Model Predictive Control

by Jian Zhou, Björn Olofsson, and Erik Frisk. *Submitted to journal.*

The paper builds upon the interaction-aware motion-planning method proposed in Paper I by considering the multi-modal motion uncertainties of surrounding vehicles in multiple-vehicle traffic environments. It contributes to solving **Problem 1**–**Problem 3**, i.e., both interaction awareness and probabilistic uncertainty estimation are involved in an MPC motion planner. In contrast to Paper I, which only considers the single prediction output of each surrounding vehicle, this paper takes into account the multi-modal prediction output of each vehicle by the interaction-aware motion-prediction model. The multi-modal prediction uncertainties include both maneuver and trajectory uncertainties of surrounding vehicles, which are integrated into the predictive motion-planning strategy of the ego vehicle. The optimal reference trajectory of the ego vehicle is computed using model predictive control (MPC) based on the prediction of the surrounding vehicles, following time-varying reference targets while avoiding collisions with surrounding vehicles. Additionally, a tunable safety-awareness parameter is incorporated into the MPC to balance performance and robustness when considering the multi-modal obstacle uncertainties. The proposed method is validated in both multi-vehicle simulation scenarios and a scenario from a recorded traffic dataset, which is called the highD dataset (Krajewski et al., 2018). It also includes comparisons to a scenario MPC, a deterministic MPC, and a trajectory of a human-driven vehicle.

Paper III: Robust Tube Model Predictive Control with Uncertainty Quantification for Discrete-Time Linear Systems

by Yulong Gao, Shuhao Yan, Jian Zhou, Mark Cannon, Alessandro Abate, and Karl H. Johansson. *Submitted to conference.*

This paper contributes to solving **Problem 2** from a theoretical perspective, providing a method to quantify the uncertainties of a controlled system in a robust model predictive control (MPC) framework. In particular, the controlled system is described by a discrete-time linear system subject to a bounded additive disturbance and hard constraints on the state and input, whereas the true disturbance set is unknown. Unlike most existing work on robust MPC, the proposed MPC algorithm incorporates online uncertainty quantification that builds on prior knowledge of the disturbance, i.e., a known but conservative disturbance set. The true disturbance set is approximated at each time step with a parameterized set, which is referred to as a quantified disturbance set, using the scenario approach with additional disturbance realizations collected online. A key novelty of this paper is that the parameterization of these quantified disturbance sets have desirable properties such that the quantified disturbance set and its corresponding rigid tube bounding disturbance propagation can be efficiently updated online. Statistical gaps between the true and quantified disturbance sets are provided, based on which, the probabilistic recursive feasibility of MPC optimization problems is discussed. Numerical simulations are provided to demonstrate the efficacy of the MPC algorithm compared with conventional robust MPC algorithms.

The author of this thesis primarily contributed to the formulation of the case study and the implementation of the MPC algorithm.

Paper IV: Risk-Aware Robust MPC for Motion Planning through Uncertainty Quantification

by Jian Zhou, Yulong Gao, Björn Olofsson, and Erik Frisk. *Manuscript.*

This paper develops a robust motion-planning strategy based on the uncertainty-quantification approach proposed in Paper III, and contributes to solving **Problem 2** and **Problem 3**. Specifically, a robust motion-planning strategy for an autonomous robotic system that addresses the uncertainty quantification of multiple uncertain surrounding obstacles is proposed in this paper. The motion planner incorporates obstacle uncertainties by computing forward reachable sets of obstacles with uncertain control actions over a prediction horizon. To reduce the conservatism of considering the worst-case uncertainties, the forward reachability analysis relies on quantifying the actual control-input set of the obstacle. Then a robust model predictive control (MPC) computes a safe reference trajectory for the robotic system that remains outside the reachable sets of the obstacle over the prediction horizon. The uncertainty quantification used in the robust MPC planner helps to balance the performance and robustness of the motion-planning method. Furthermore, disregarding the worst-case uncertainties may pose a risk to the safety guarantees, and this risk is explicitly examined, such that the designed robust MPC planner is risk-aware. The proposed method is applied to an autonomous vehicle in challenging forced merging simulations to demonstrate its effectiveness.

1.4 Future Research

The thesis has addressed motion-planning problems in multi-vehicle dynamic scenarios, with a focus on interaction awareness and uncertainty quantification of surrounding vehicles. Given the observed limitations of the proposed approaches in this thesis, three research directions have been identified for extensions of the methods.

Interaction-aware robust MPC with uncertainty quantification: Papers III–IV of this thesis propose a method to quantify motion uncertainties of a controlled linear system based on the observed disturbances or control signals. However, the method can only capture the control-input set or disturbance set of the obstacle using historical information, and it is unable to account for future uncertain control actions of the obstacle that might be affected by the interactions with other agents and the environment. To address this limitation, future research could explore combining the uncertainty-quantification approach with interaction-aware motion-prediction models for obstacles. By doing so, the forward reachability analysis could be more proactive and better able to account for the interactive behavior of obstacles.

Stochastic MPC with uncertainty quantification: Papers II–IV of this thesis have considered motion uncertainties of surrounding vehicles to predict the obstacle occupancy over the prediction horizon. However, these approaches do not fully utilize the statistical distribution information of the uncertainties. By incorporating distribution information, it is possible to build probabilistic collision-avoidance constraints like (1.16) that can increase the feasibility of motion-planning problems compared to robust constraints (1.15). Therefore, future research could focus on estimating the distribution of motion uncertainties of surrounding vehicles and using this information to design a stochastic MPC for more flexible and resilient motion planning.

Reachability analysis of nonlinear systems: Papers III–IV consider linear models for obstacles, which are limited in their ability to capture the full range of motion characteristics of the obstacle systems. To overcome this limitation, future research could focus on performing the forward reachability analysis of nonlinear obstacle models in an MPC framework. While established methods exist for doing so, a major challenge will be achieving an appropriate trade-off between computational efficiency and modeling accuracy, particularly in an online motion-planning framework.

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