Phase Calibration of Distributed Antenna Arrays

Erik G Larsson and Joao Vieira

The self-archived postprint version of this journal article is available at Linköping University Institutional Repository (DiVA):
https://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-196867

N.B.: When citing this work, cite the original publication.

Original publication available at:
https://doi.org/10.1109/LCOMM.2023.3266836
Copyright: Institute of Electrical and Electronics Engineers
https://www.ieee.org/
©2023 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.
Phase Calibration of Distributed Antenna Arrays

Erik G. Larsson* and Joao Vieira†

Abstract—Antenna arrays can be either reciprocity calibrated (R-calibrated), which facilitates reciprocity-based beamforming, or fully calibrated (F-calibrated), which additionally facilitates transmission and reception in specific physical directions. We first expose, to provide context, the fundamental principles of over-the-air R- and F-calibration of distributed arrays. We then describe a new method for calibration of two arrays that are individually F-calibrated, such that the combined array becomes jointly F-calibrated.

I. INTRODUCTION

Coherent beamforming with antenna arrays is a cornerstone technology in co-located and distributed (massive) MIMO systems. In co-located MIMO, the antennas are located close by one another in an array; in distributed MIMO, they are spread out geographically, typically in the form of interconnected antenna panels that together constitute a large array.

For antenna arrays to work efficiently, they must be calibrated in amplitude and phase— the phase is the most important. There are two types of phase calibration: reciprocity (R) calibration, and full (F) calibration. R-calibration enables reciprocity-based operation, relying on uplink pilots for downlink beamforming. F-calibration is stronger, and additionally enables the use of geometrically parameterized array models.

We are concerned with calibration of antenna arrays via over-the-air (OtA) measurements. OtA calibration is attractive as it circumvents the use of synchronization cables. To provide context, and for completeness, we first give a unified treatment of the basic principles of R- and F-calibration OtA, using elementary mathematics and a phasor formalism (Sections II–III)– something missing from the literature. We then present our main technical contribution: a method for OtA F-calibration of two individually F-calibrated arrays (Section IV). Finally, we dispel some misconceptions that we have encountered when working on array calibration (Section V).

II. PRELIMINARIES

We consider a system with a mobile user and a set of service antennas, \{A_i\}. For now we make no assumptions on the locations of the antennas: they may be co-located, or distributed. In either case, we refer to \{A_i\} collectively as the array (Figure 1).

Each antenna \{A_i\} has a transmit and a receive branch; the same goes for the user. To each antenna we associate two local phasors that revolve \(f_c\) times per second: one for the transmit branch and one for the receive branch. Similarly, to the user we associate a local transmit phasor and a local receive phasor. Additionally, we consider a fictitious, global, unobservable phasor that represents an absolute reference. This global phasor does not exist physically, but serves as a useful abstraction. Note that the notion of a revolving (rotating) phasor is the standard one used in physics and engineering [1].

Define:

- \(t_u, r_u\): the user’s transmit and receive phasor values when the global phasor points to zero.
- \(t_i, r_i\): the transmit and receive phasors of \(A_i\) when the global phasor is zero.\(^1\)
- \(T_{ij}\): propagation (coupling) delay between \(A_i\) and \(A_j\); owing to reciprocity, \(T_{ij} = T_{ji}\).
- \(T_i\): propagation delay between the user and \(A_i\).

In general, \(t_u \neq r_u\), \(t_i \neq r_i\), and \(t_i \neq t_j\) because of differences in the circuitry, and because of oscillator drifts.

We consider passband signals with carrier frequency \(f_c\). Hence, we can equivalently refer to time as phase: phase equals \((2\pi f_c \times \text{time}) \mod 2\pi\). A “signal” at an antenna means the voltage across the antenna terminal. When saying that a signal is received (transmitted) “at local [or global] phase \(\phi\)”, we mean that the signal undergoes a positive zero-crossing when the corresponding local [or global] phasor points to \(\phi\). All quantities are in radians, and all phasor values and phase measurements are defined \(\mod 2\pi\). Our focus will be on identifiability questions, thus we neglect measurement noise.

Throughout, propagation delay between a transmit and receive antenna means the phase lag between the corresponding signals. This delay includes the effect of the antenna radiation patterns, which are completely reciprocal. If antennas are closely located, the delay is mainly determined by the mutual coupling. In this paper, calibration means determining relevant functions of \(\{t_i, r_i\}\) to facilitate phase-synchronous transmission or reception, or both. When calibrating multiple arrays together we will use the term “phase alignment”.

Rather than working with \(\{t_i, r_i\}\), some related literature works with “calibration coefficients” \(\{\gamma_t, \gamma_r\}\) defined in complex baseband [2], [3]. All results in this paper can be

---

\(^1\)The subscript \((\cdot)_u\) refers to the user; \((\cdot)_i\) refers to service antenna \(A_i\).
re-derived using that formalism instead, by setting \( \gamma_r = e^{j\pi} \) and \( \gamma_t = e^{-j\pi} \). (Note the minus sign: a positive shift of \( t \) has the opposite effect on the signal phase as a positive shift of \( r \), since the signals travel in opposite directions.) This means, for example, that the statement “\( r_1 = t \)” is equivalent to “\( r_1, t_1 = 1 \).” The advantage of the phasor formalism is that it connects more naturally to fundamental physics, and yields simple linear equations.

**A. Fully Calibrated Array**

If the differences \( \{r_i - r_j\} \) are known for all pairs \( (i, j) \), we say that the array is **fully receive-calibrated** (FR-calibrated). The circuitry can then post-compensate the received signals such that we can without loss of generality assume that \( r_i = c \) for all antennas \( A_i \) and some indeterminable constant \( c \). This ensures that an observed phase-shift between uplink signals received at two antennas can be mapped onto a specific physical direction, facilitating angle-of-arrival estimation and ensuring that an observed phase-shift between uplink signals is known; compensate the phase of \( A_i \) such that we can without loss of generality assume that \( r_i = c \); this residual phase shift can be eliminated by using a demodulation pilot or by blind decoding.

Note that FR-plus-FT-calibration (or F-calibration) implies R-calibration: if \( \{r_i - r_j\} \) and \( \{t_i - t_j\} \) are known for all \( (i, j) \), then \( \{t_j - t_i + r_j - r_i\} \) can be computed for all \( (i, j) \). The converse, however, does not hold.

**III. A Unified Description of OTA Calibration**

**A. F-calibration Methodology**

Methods for F-calibration OTA date back quite far [4], [5]. These methods require at least three antennas, and the propagation (coupling) delay between these antennas, \( \{T_{ij}\} \), must be known. Obtaining \( \{T_{ij}\} \) is feasible for a co-located array, although it requires electromagnetic simulations or measurements in an anechoic chamber.

The basic principle of OTA F-calibration is to perform bi-directional measurements between pairs of antennas. First, \( A_1 \) transmits at local phase 0, which is global phase \(-t_1\). \( A_2 \) receives at global phase \(-t_1 + T_{12}\), which is local phase \( r_2 - t_1 + T_{12} \); after subtracting the known \( T_{12} \), we obtain the measurement \( r_2 - t_1 \). Continuing for all three pairs, in both directions, we obtain six measurements:

\[
\begin{align*}
&d_{12} = r_2 - t_1, & d_{21} = r_1 - t_2, & d_{13} = r_3 - t_3 \\
&d_{31} = r_1 - t_3, & d_{32} = r_2 - t_3, & d_{23} = r_3 - t_2.
\end{align*}
\]

Equations (1)–(2) are easily rearranged to obtain

\[
\begin{align*}
& r_1 - t_1 = d_{21} + d_{13} - d_{23}, \\
& r_1 - r_2 = d_{31} - d_{12}, \\
& t_1 - t_2 = d_{23} - d_{13},
\end{align*}
\]

and so forth. This way, we can determine \( \{r_i - r_j\} \), \( \{t_i - t_j\} \), and \( \{r_i - t_i\} \), as required for the array to be F-calibrated.

The above equations can also be found in [6], although [6] failed to point out the crucial fact that \( \{T_{ij}\} \) must be known to solve for \( \{r_i - r_j\} \), \( \{t_i - t_j\} \), and \( \{r_i - t_i\} \) – something that seems infeasible with distributed antennas.

**B. R-calibration Methodology**

R-calibration can be performed by pairwise OTA measurements between the antennas, without knowing \( \{T_{ij}\} \), both for co-located and distributed arrays [2], [3], [7]–[11]. It is the fortunate fact that \( \{T_{ij}\} \) need not be known, that makes reciprocity-based operation possible in distributed MIMO.

To explain the principle it is sufficient to consider two antennas, say, \( A_1 \) and \( A_2 \). A bi-directional measurement is
performed in the same way as for F-calibration, but now, $T_{12}$ cannot be subtracted. This gives the two observations,
\begin{equation}
\begin{aligned}
d_{12} &= r_2 - t_1 + T_{12}, \\
d_{21} &= r_1 - t_2 + T_{12}.
\end{aligned}
\end{equation}
Subtracting $d_{12}$ from $d_{21}$ yields the required quantity
\begin{equation}
\begin{aligned}
d_{21} - d_{12} &= t_1 - t_2 + r_1 - r_2.
\end{aligned}
\end{equation}
For more than two antennas, one continues by bi-directional measurements between $A_i$ and $A_j$ for all $i$, or between other, appropriately selected, pairs. From these measurements, $\{t_j - t_i + r_j - r_i\}$ can be computed for all $(i,j)$.

IV. PHASE-ALIGNING TWO R- OR F-CALIBRATED ARRAYS

We now consider the following question: Given two arrays (that may themselves be co-located or distributed), of which each one is either F-calibrated or R-calibrated, what is required to calibrate the two arrays relative to one another? We call the two arrays $A$ and $B$, and their respective antennas $\{A_i\}$ and $\{B_j\}$; we write $AB$ for the array collectively comprised by $\{A_i\}$ and $\{B_j\}$ (Figure 2). This problem is of specific interest when $A$ and $B$ are antenna panels (“access points”) in distributed MIMO, in which case the calibration task is sometimes called phase alignment. Nothing precludes $A$ or $B$ from being a mobile user, although that may not be a typical scenario. In what follows, we assume that there is a central entity that can co-process measurements from both $A$ and $B$.

If $A$ and $B$ are separated, then $\{T_{A_iB_j}\}$ cannot reasonably be known a priori. Yet, as we will see in Section IV-B2, F-calibration of the joint array $AB$ is possible.

A. Aligning Two R-Calibrated Arrays

First consider the case when $A$ and $B$ are individually R-calibrated. Clearly, $AB$ cannot then be made F-calibrated. But $A$ and $B$ can be phase-aligned such that $AB$ becomes R-calibrated. This only takes a simple bi-directional measurement between any of $\{A_i\}$ and any of $\{B_j\}$:
\begin{equation}
\begin{aligned}
d_{A_kB_j} &= r_{B_j} - t_{A_k} + T_{A_kB_j}, \\
d_{B_jA_k} &= r_{A_k} - t_{B_j} + T_{A_kB_j}.
\end{aligned}
\end{equation}
Say we use $A_1$ and $B_1$: this gives the measurements $d_{A_1B_1}$ and $d_{B_1A_1}$. Subtraction of $d_{A_1B_1}$ from $d_{B_1A_1}$ gives $t_{A_1} - t_{B_1} + r_{A_1} - r_{B_1}$. Once $t_{A_1} - t_{B_1} + r_{A_1} - r_{B_1}$ is known, along with $\{t_{A_i} - t_{A_j} + r_{A_i} - r_{A_j}\}$ and $\{t_{B_j} - t_{B_i} + r_{B_j} - r_{B_i}\}$ (which are known by assumption), $\{t_{A_i} - t_{B_j} + r_{A_i} - r_{B_j}\}$ can be computed for any $(i,j)$ by pairwise addition and subtraction.

Alternatively, more combinations of antenna pairs $(A_iB_j)$ can be measured. Beamforming can be beneficially exploited to improve the SNR if the arrays are distant from one another, for example using a protocol similar to that in [12].

A useful fact, though unrelated to reciprocity-based beamforming, is the following. Once $AB$ is R-calibrated, then after appropriate compensation, the “channels” from $A_i$ to $B_j$ and from $B_j$ to $A_i$ are equal. Specifically, consider the two measurements between $A_i$ and $B_j$ in (8)–(9). They differ precisely by $t_{A_i} - t_{B_j} + r_{A_i} - r_{B_j}$, which is known, and hence can be compensated for.

B. Aligning Two F-Calibrated Arrays

Next, consider the case when $A$ and $B$ are (individually) F-calibrated. We can assume, after appropriate pre- and post-compensation of the transmitted and received signals, that $t_{A_i} = r_{A_i} = c_A$ and $t_{B_j} = r_{B_j} = c_B$ for all $i$ and some indeterminate constants $c_A$ and $c_B$.

To illustrate the methodology, we pick an arbitrary antenna from $A$ and an arbitrary antenna from $B$, enabling us to drop the index $i$. Anything said in what follows, if desired, can be repeated for any other pair $(A_iB_j)$ of antennas, and followed by appropriate averaging over antenna pairs to suppress measurement noise, and consequently obtaining a processing gain that improves the accuracy.

1) R-Calibration of the Joint Array $AB$: Since F-calibration is stronger than R-calibration, it comes as no surprise that one can phase-align $A$ and $B$, such that $AB$ becomes R-calibrated.

The goal is to obtain,
\begin{equation}
\begin{aligned}
t_A - t_B + r_A - r_B &= 2(c_A - c_B), \\
&= 0
\end{aligned}
\end{equation}
without knowledge of $\{T_{A_iB_j}\}$. This is easily achieved by a bi-directional measurement between $A$ and $B$:
\begin{equation}
\begin{aligned}
d_{AB} &= r_B - t_A + T_{AB} = c_B - c_A + T_{AB}, \\
&= 2(c_A - c_B) - T_{AB}, \\
&= 2(c_A - c_B) - d_{BA}.
\end{aligned}
\end{equation}
Subtracting (11) from (12) yields the sought-after quantity required for $AB$ to be R-calibrated (see (10)):
\begin{equation}
\begin{aligned}
d_{BA} - d_{AB} &= 2(c_A - c_B),
\end{aligned}
\end{equation}
Note that had $T_{AB}$ been known (which is unrealistic in practice), then $c_A - c_B$ could have been immediately obtained from a single measurement (11) or (12) alone.

2) F-Calibration of the Joint Array $AB$: Now consider the more intricate case when $A$ and $B$ are individually F-calibrated and we want to phase-align $A$ and $B$ such that $AB$ becomes F-calibrated. We must then, without knowing $T_{AB}$, obtain $c_A - c_B$, rather than $2(c_A - c_B)$. This is not possible from a bi-directional measurement as in (11)–(12), since dividing (13) by 2 yields an ambiguity of $\mod \pi$.

Our proposed resolution of this problem is to use measurements with two signals, say two sinusoids with frequencies $f$ and $f'$, where $f' < f$. (In an OFDM system, $f$ and $f'$ could correspond to different subcarriers.) The frequencies $f$ and $f'$
are associated with different propagation delays (phase lags), say $T_{AB}$ and $T'_{AB}$ (in radians). Specifically,

$$T'_{AB} = \frac{f'}{f} T_{AB}. \quad (14)$$

Since $f'/f < 1$, a higher frequency means a shorter wavelength, which means a larger propagation delay when expressed in radians. In the presence of multipath, $f - f'$ must be less than the channel coherence bandwidth (reciprocal delay spread).

Taking a bi-directional measurement at frequency $f$ (similar to in Section IV-B1), and a uni-directional measurement at frequency $f'$, yields three phase measurements (mod $2\pi$),

$$d_{AB} = c_B - c_A + T_{AB}, \quad (15)$$
$$d_{BA} = c_A - c_B + T_{AB}, \quad (16)$$
$$d'_{BA} = c_A - c_B + T'_{AB} \quad (17)$$

where $(d_{AB}, d_{BA})$ are obtained by measurements on the $f$-signal, and $d'_{BA}$ is obtained from the $f'$-signal.

There are three unknowns: $c_A - c_B, T_{AB}$ and $T'_{AB}$, where the first is of interest, and the latter two are nuisance parameters (which are proportionally related). By subtracting (15) from (16), $T_{AB}$ disappears and we conclude that $c_A - c_B$ is one of the following two values (mod $2\pi$),

$$(c_A - c_B)_i = (d_{BA} - d_{AB})/2, \quad (18)$$
$$(c_A - c_B)_{ii} = \pi + (d_{BA} - d_{AB})/2. \quad (19)$$

To determine which one of $(c_A - c_B)_i$ and $(c_A - c_B)_{ii}$ in (18)–(19) is the correct one, we proceed as follows. By adding (15) and (16), we infer that $T_{AB}$ is one of the two values (mod $2\pi$),

$$T_{AB}^i = (d_{AB} + d_{BA})/2, \quad (20)$$
$$T_{AB}^{ii} = \pi + (d_{AB} + d_{BA})/2. \quad (21)$$

But from (16) and (17), we concurrently deduce that

$$(1 - f'/f)T_{AB} = T_{AB} - T'_{AB} = d_{BA} - d'_{BA} \quad (22)$$

(mod $2\pi$), from which it follows that

$$T_{AB} = \hat{T}_{AB} \mod \frac{2\pi}{1 - f'/f}, \quad (23)$$

where we defined

$$\hat{T}_{AB} = \frac{d_{BA} - d'_{BA}}{1 - f'/f}. \quad (24)$$

We can then check which of $T_{AB}^i + m2\pi$ and $T_{AB}^{ii} + m2\pi$ in (20)–(21) matches most closely with $\hat{T}_{AB} + m2\pi/(1 - f'/f)$, for some combination of integers $m$ and $n$. (Absent measurement noise, one of them will match exactly.) If the match is best for $T_{AB}^i$ then we conclude that,

$$T_{AB} = T_{AB}^i \mod 2\pi \quad (25)$$

(hereafter referred to as case (i)); otherwise

$$T_{AB} = T_{AB}^{ii} \mod 2\pi \quad (26)$$

(referred to as case (ii)). In case (i), insertion of $T_{AB}$ into (15) gives that (mod $2\pi$)

$$c_A - c_B = T_{AB} - d_{AB} = (d_{BA} - d_{AB})/2, \quad (27)$$

i.e., $c_A - c_B = (c_A - c_B)_i$. In case (ii), (15) instead gives that

$$c_A - c_B = T_{AB} - d_{AB} = \pi + (d_{BA} - d_{AB})/2, \quad (28)$$

that is, $c_A - c_B = (c_A - c_B)_{ii}$. The physical interpretation of the algorithm is that we estimate, mod $2\pi/(1 - f'/f)$ radians, the propagation delay between A and B, using a probing signal with bandwidth $f - f'$. The estimated distance is then used as side information when resolving the $\pi$-ambiguity in the phase alignment.

The objective here has been to demonstrate how the $\pi$-ambiguity can be resolved, rather than to describe an algorithm that is optimal in the presence of measurement noise with some particular distribution. Variations and improvements are possible, and useful if noise is non-negligible. For example, one can obtain a fourth measurement, making the measurement at frequency $f'$ bi-directional as well:

$$d'_{AB} = c_B - c_A + T'_{AB}. \quad (29)$$

Then, estimates similar to (18)–(19), expressed as a function of $(d'_{AB}, d'_{BA})$, can be formed from (17) and (29), instead. Averages of estimates formed from (15)–(16), and from (17) and (29) can be formed. Also, instead of forming $T_{AB}^i$ and $T_{AB}^{ii}$, from (15)–(16), one can use (17) and (29) instead, or, use an average of the two estimates formed this way. Furthermore, rather than obtaining $\hat{T}_{AB}$ in (24) from (16) and (17), one can obtain it from (15) and (29), or, from the average of two such estimates. Formulation as a regression problem is also possible. We have to leave the precise formulation of statistically optimal estimators as future work.

To exemplify the principle, Figure 3 shows a simulation of the estimation performance for the proposed algorithm, compared to a genie baseline that knows $T_{AB}$. We took $f = 2$ GHz, $f' = f - 50$ MHz, and somewhat arbitrarily, a distance from A to B of 50 wavelengths, and selected $c_A$ and $c_B$ randomly. To obtain a processing gain, 100 samples were coherently averaged for each phase measurement. A closest-neighbor fit was used to find the integer pair $(m, n)$ that defines the best match between $\hat{T}_{AB} + m2\pi/(1 - f'/f)$
and one of the values \( \{T_{AB}^1 + m2\pi, T_{AB}^2 + m2\pi\} \). Due to the non-linearity of the ambiguity resolution operation, a threshold effect is observed. Once the SNR is high enough, the algorithm’s performance meets the genie bound, and improves \( \sqrt{10} \) times in RMSE per 10 dB increase in SNR – as expected, since once the \( \pi \)-ambiguity is resolved with high enough probability, the estimate is essentially a linear function of the noisy observations.

We end by pointing out that the propagation distance must be the same at \( f \) and \( f' \). In practice, one would have to operate in or near line-of-sight conditions, or, if A and B have multiple antennas, apply beamforming to ensure that no distant multipath is present. Similarly, corrections may have to be applied to compensate for phase variations in the antenna frequency response.

V. MISCONCEPTIONS

We discuss some misconceptions that we have encountered.

1. **Myth:** Two individually calibrated arrays A and B can be used together for joint reciprocity-based beamforming.

   **Reality:** A bi-directional, OtA measurement is required to phase-align the arrays, irrespective of whether the arrays are (individually) R- or F-calibrated.

2. **Myth:** R-calibration is sufficient to perform grid-of-beams beamforming with fully digital arrays.

   **Reality:** F-calibration is required for this.

3. **Myth:** For reciprocity-based beamforming, \( A_i \) should pre-compensate the phase with \( t_i - r_i \). (For example, [6] states that “The difference in time reference between the transmitter and receiver in a given AP represents the reciprocity calibration error.”)

   **Reality:** \( A_i \) should pre-compensate with \( t_i + r_i + c \) (for some arbitrary but common-for-all-\( A_i \) c).

4. **Myth:** A drift in the oscillator that drives the radio-frequency mixer of \( A_i \) does not require reciprocity re-calibration. (For example, [13] assumes this.)

   **Reality:** Such a shift affects \( t_i \) and \( r_i \) equally, so \( t_i + r_i - (t_j + r_j) \), which must be known for reciprocity, changes. Hence, an oscillator drift calls for re-calibration. This misconception, and its prevalence, were also pointed out in [14].

5. **Myth:** Channel aging (physically moving \( A_i \)) has the same effect on the signal phases as a shift in the sampling clock (oscillator phase), of say, \( \phi \).

   **Reality:** Channel aging has the same effect on measurements as shifting \( t_i \) by \( +\phi \) and \( r_i \) by \( -\phi \). But an oscillator drift is equivalent to shifting \( t_i \) by \( +\phi \) and \( r_i \) by \( +\phi \).

6. **Myth:** Beamforming to a user on downlink with an R-calibrated array requires demodulation pilots.

   **Reality:** Demodulation pilots allow for simpler implementation, but they are fundamentally unnecessary. Unless the user is jointly R-calibrated with the array, the signal received at the user will be randomly phase-rotated. This phase rotation (along with a partially unknown amplitude gain) can be estimated using a demodulation pilot, or with a joint equalization and decoding algorithm [15], or even blindly.

Note also that nothing, except implementation complexity, prevents joint R-calibration of the user and the array: take the array to be A and the user to be B in Section IV-A. In this case, the phase rotation at the user vanishes, and the remaining unknown amplitude gain can be estimated blindly through simple methods [16].

VI. CONCLUDING REMARKS

We considered antenna arrays, in their most general meaning: an array can be co-located or distributed, and have an arbitrary number of antennas. We showed that the concepts of F- and R-calibration OtA can be rigorously explained using a phasor formalism in which all relations are described through simple, linear equation systems. We then used this framework to develop a method for joint OtA F-calibration of two arrays A and B that have been previously individually F-calibrated. Finally, we discussed a number of misconceptions related to array calibration.

REFERENCES


